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VCE Mathematical Methods ¾
Functions & Relations Exam Skills [0.2]
Workshop

Section A: Recap

Sub-Section: Maximal Domains

Starting with a domain!

Maximal Domain

- **Definition**: The largest possible set of input values (elements of the domain) for which the function is well-defined.

- **Three Important Rules:**

Functions	Maximal Domain
$\sqrt{z}$	
$\log(z)$	
$\frac{1}{z}$	

- **Steps:**

1. Find the restriction of the inside.
2. Sketch the graph if needed.
3. Solve for domain.

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Sub-Section: Domain of Sum, Difference and Product of Functions

What about a domain of the sum of two functions?

Sums, Differences and Products of Functions

- Rules:

$$(f+g)(x) = \underline{\qquad\qquad\qquad}$$

$$(f-g)(x) = \underline{\qquad\qquad\qquad}$$

$$(f\times g)(x) = \underline{\qquad\qquad\qquad}$$

- Idea:

$$\textbf{Domain of sum or product of two functions} = \underline{\qquad\qquad\qquad\qquad} \textbf{ of the two domains}$$

- Steps:

1. Find the domain of each function.
2. Find the intersection (draw a number line if needed).

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Sub-Section: Basics of Composition

What was the "composition" of functions?

Composite Functions

- **Definition**: A ______________ of functions.
- **Representation of the above:**

$$y = \underline{\hspace{6cm}}$$

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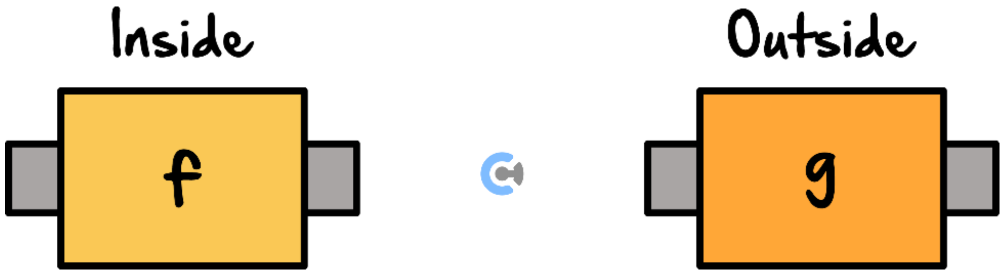
Sub-Section: Validity of Composite Functions

Did composite functions work all the time?

Validity of Composite Functions



- Output of $f(x)$: ___________________________________. *(Label Above)*
- Input of $g(x)$: ___________________________________. *(Label Above)*
- Composite Function is only valid if:

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \subseteq \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- Acronym:

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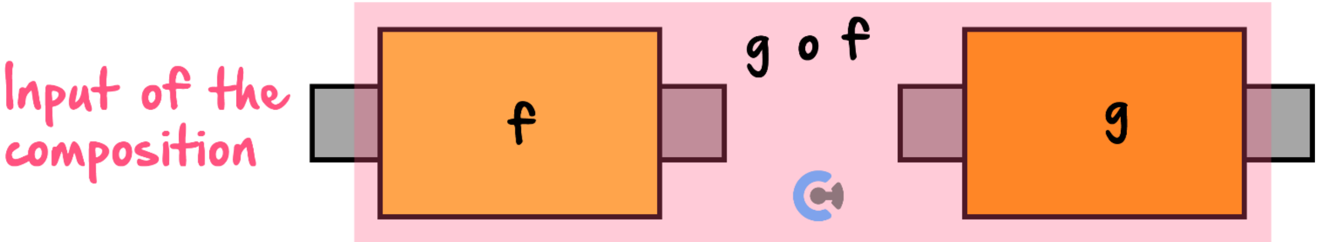
Sub-Section: Domain of Composite Functions

How did we find the domain of a composite function?

Domain of Composite Functions



$$\textit{Domain of Composite} = \textit{Domain of Inside}$$

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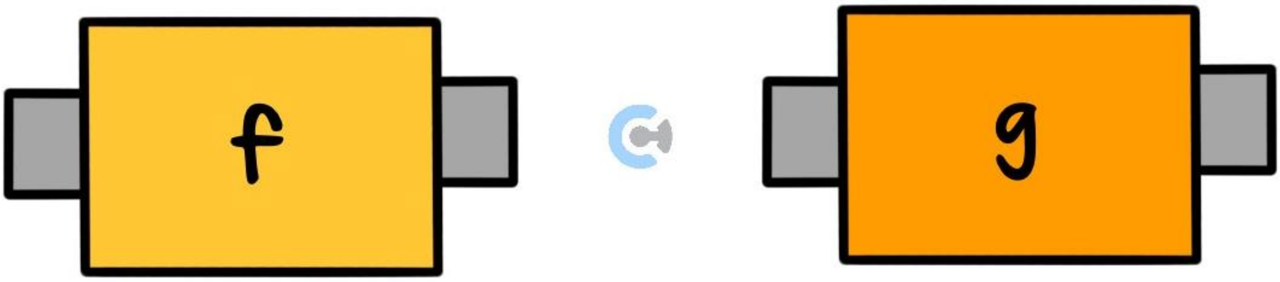
Sub-Section: Range of Composite Functions

Range of the Composite Functions



Range of Composite* $\subseteq$ *Range of the Outside

- **Finding the range of composition function**: Use the domain and the rule, just like another function.

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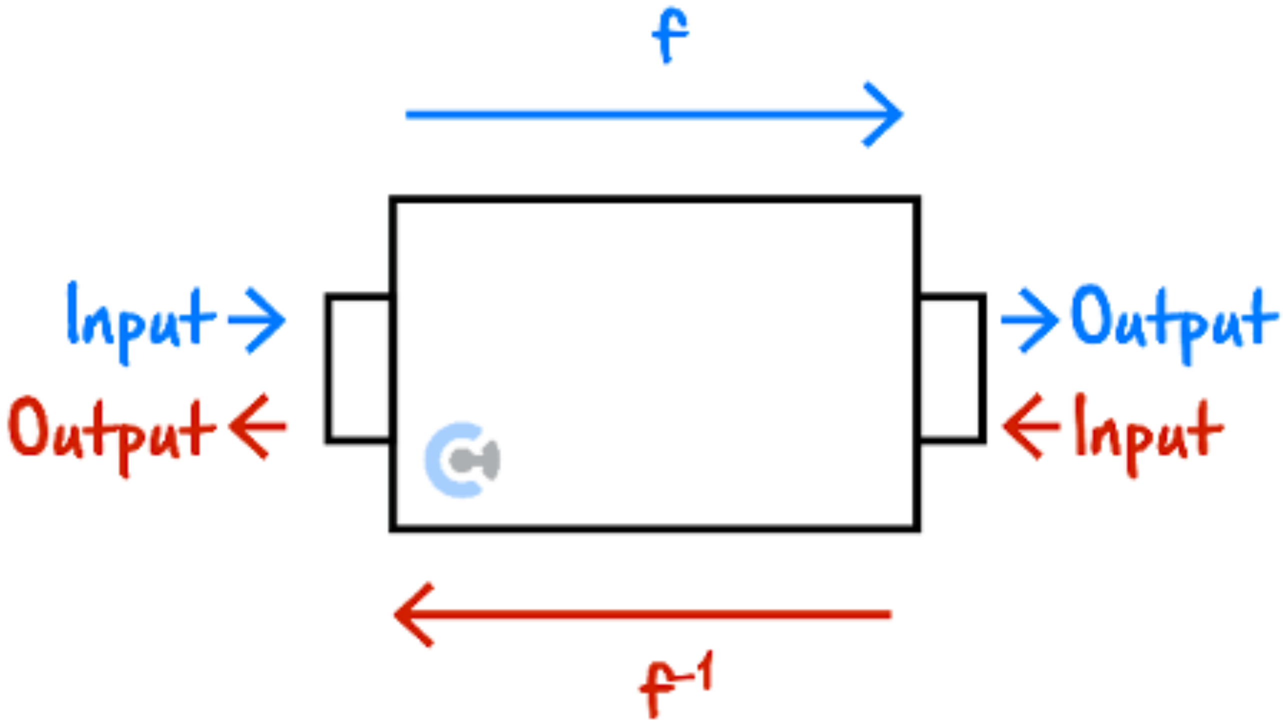
Sub-Section: Basics of Inverses

What does "Inverse" mean?

Inverse Relation

- **Definition**: Inverse is a relation which does the ________________.



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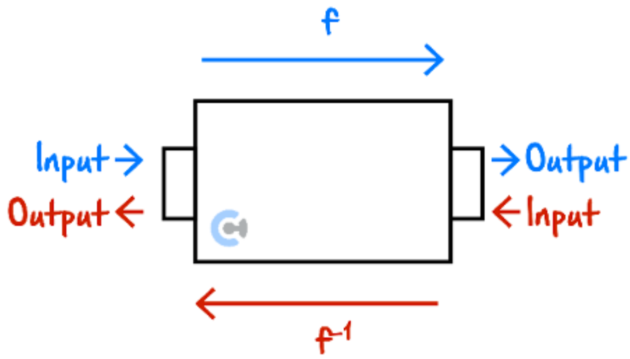
Sub-Section: Swapping x and y

Is there a better way of solving for an inverse relation?

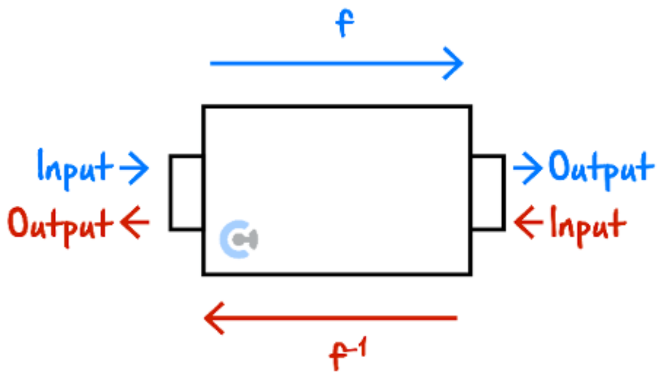
Solving for an Inverse Relation

- Swap x and y.



NOTE: $f(x) = y$.

Domain and Range of Inverse Functions



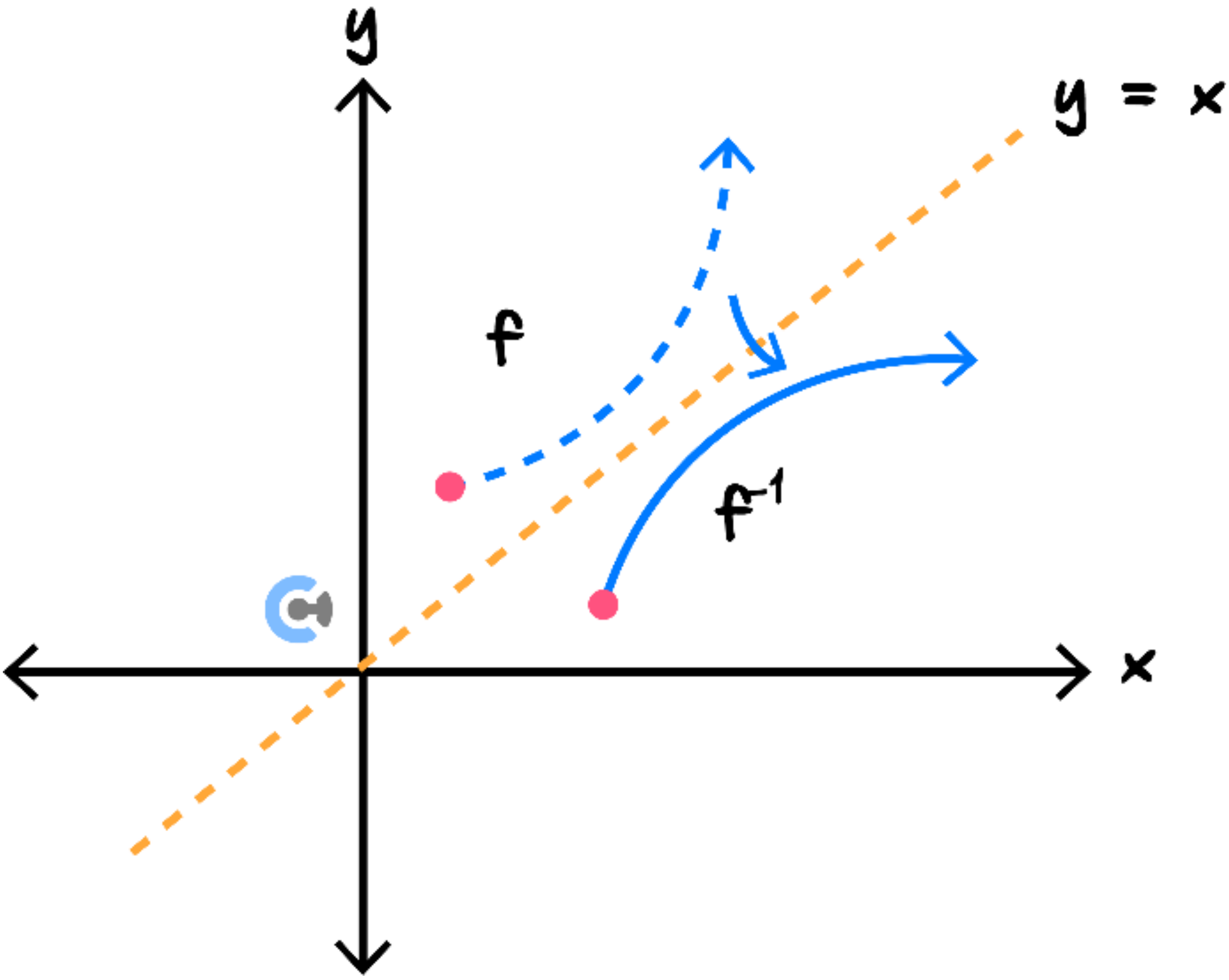
$$Dom\ f^{-1} = \underline{\hspace{4cm}}$$

$$Ran\ f^{-1} = \underline{\hspace{4cm}}$$

Sub-Section: Symmetry Around $y = x$

Why does this happen?

Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

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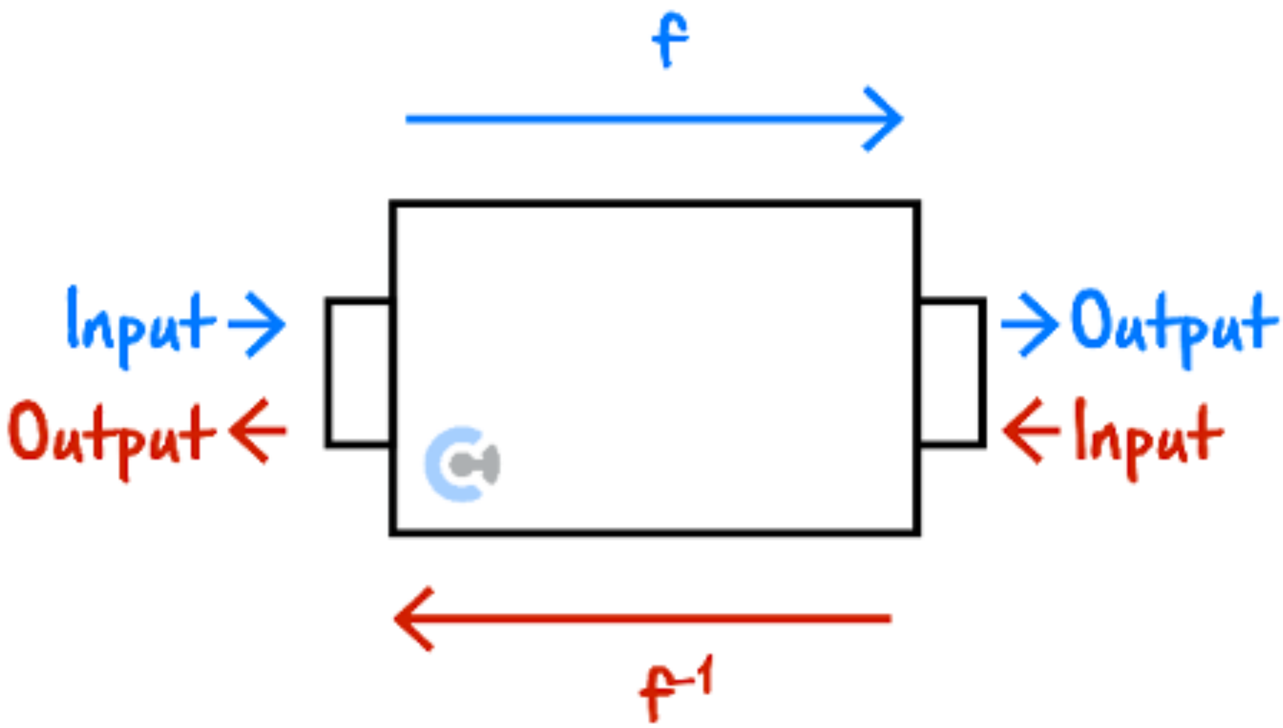
Sub-Section: Validity of Inverse Function

Does an inverse function always exist?

Validity of Inverse Functions



- Requirement for Inverse Function:

$$f \text{ needs to be } \_\_\_\_\_\_\_\_\_\_.$$

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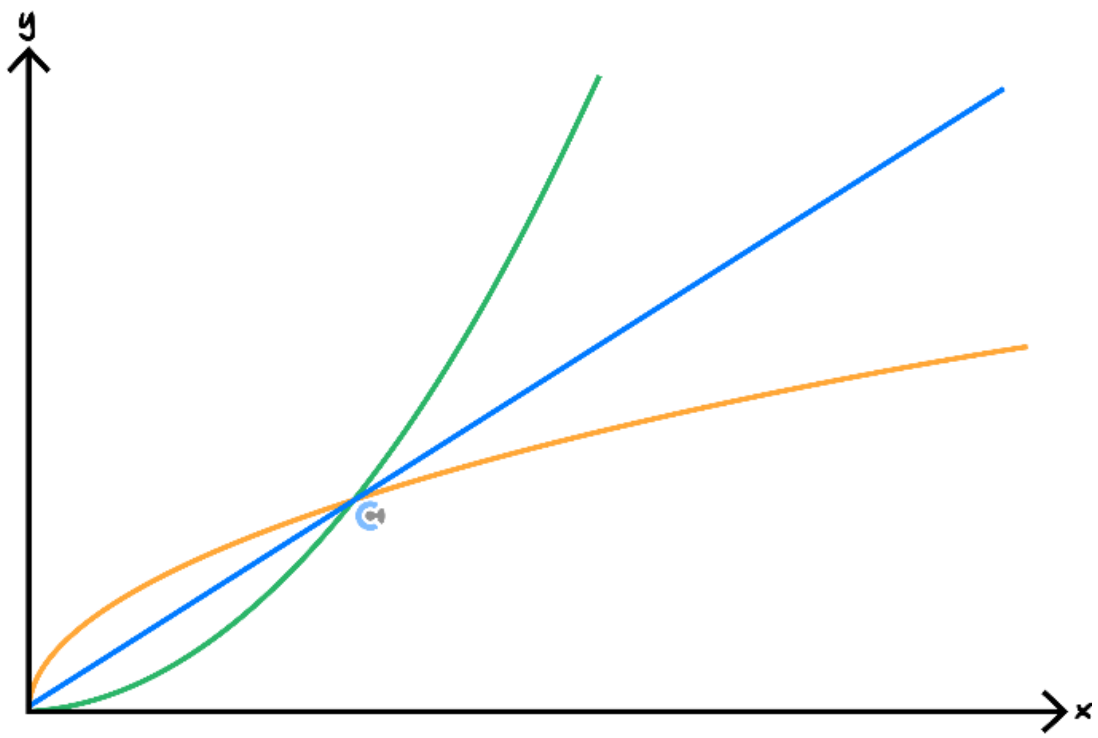
Sub-Section: Intersection Between Inverses

Where do inverses meet?

Intersection Between a Function and its Inverse



- Equate with ________ instead.

$$f(x) = x \text{ OR } f^{-1}(x) = x$$

- We cannot do this when the function is ____________________ function.

NOTE: This only works for an increasing function.

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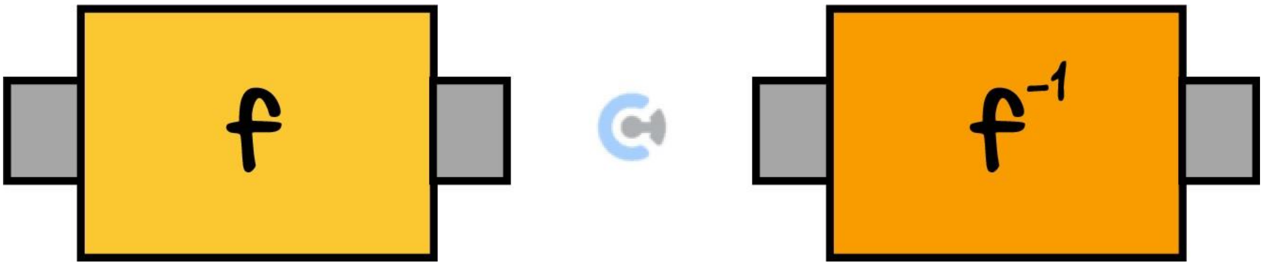
Sub-Section: Composition of Inverses

Composition of Inverse Functions



$$f \circ f^{-1}(x) = \_\_\_, \qquad for\ all\ x \in \_\_\_\_\_\_\_\_$$

$$f^{-1} \circ f(x) = \_\_\_, \qquad for\ all\ x \in \_\_\_\_\_\_\_\_$$

NOTE: Domain = Domain of Inside.

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Sub-Section: Find a New Domain to Fix Composite Functions

Fixing Broken Composite Functions

- **Aim:** Restrict the domain of the inside function so that the range of the inside function fits inside the domain of the outside.
- **Steps:**
  1. **Write down the RIDO statement** with the domain of the outside (as it is fixed).
  2. **Sketch the inside function** to see what domain is needed.

Space for Personal Notes

Sub-Section: Find the Range of Complex Composite Functions

Finding Range of Complex Composite Functions

- **Aim:** Find the range of complicated functions.
- **Steps:**

1. Break the function into ________________ of two simple functions.
2. Follow the ____________________ to find the range.

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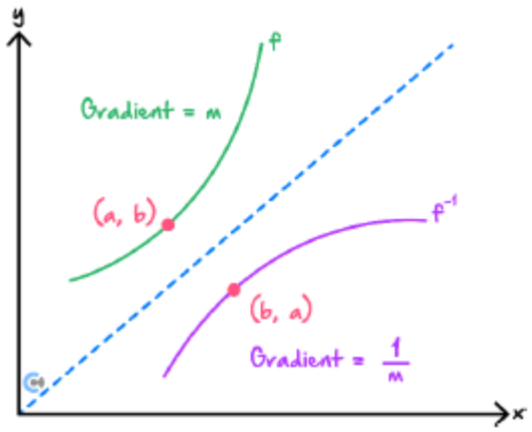
Sub-Section: Find the Gradient of Inverse Functions

This is a fun application of inverse with calculus!

REMINDER: Gradient of a Point

$$\text{Gradient at a point } = \frac{dy}{dx}$$

Gradient of an Inverse



$$\text{If Gradient of } f \text{ at } (a, f(a)) = m$$

$$\text{Gradient of } f^{-1} \text{ at } \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

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Section B: Warm Up

INSTRUCTION: 5 Minutes Writing.

Question 1

Consider $f(x) = \sqrt{2x}$ and $g(x) = 2x - 1$, both defined over their maximal domains.

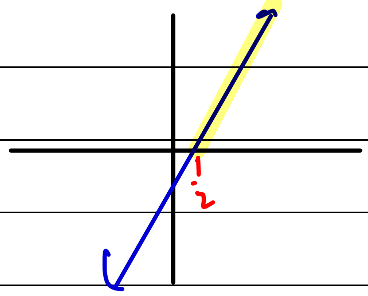
a. Is $f(g(x))$ defined?

ran g: R

dom f: $[0, \infty)$

$\therefore$ $f(g(x))$ is undef.

b. Find the largest domain of g such that $f(g(x))$ is defined.



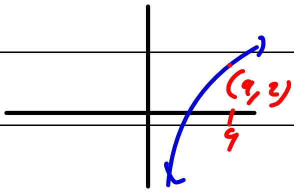
$\therefore$ dom g: $[\frac{1}{2}, \infty)$

c. Find the range of $f(x) = \log_3(x^2 + 9)$.

let $m(x) = x^2 + 9$

let $n(x) = \log_3(x)$

$\mathbb{R} \rightarrow \boxed{m}\ [9, \infty) \rightarrow \boxed{n}\ [2, \infty)$



d. Consider the one-to-one function h with the following properties:

$$h(2) = 3 \text{ and } h'(3) = 5.$$

Find the gradient of h^{-1} at $x = 2$.

$(2, 3)$

$(3, 2)$

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Section C: Exam 1 (20 Marks)

INSTRUCTION: 20 Marks. 26 Minutes Writing.

Question 2 (7 marks)

Consider the two functions:

$$f : D \to \mathbb{R}, f(x) = \frac{1}{x+2} + 1$$

$$g : [0, \infty) \to \mathbb{R}, g(x) = \sqrt{x+4}$$

a. Define g^{-1}, the inverse function of g. (2 marks)

Swap x and y

$x = \sqrt{y+4}$

$x^2 = y+4$

$\therefore g^{-1}(x) = x^2 - 4, \ x \in [2, \infty)$

b. The gradient of g^{-1} when $x = 3$ is 6. Find the gradient of g when $x = 5$. (2 marks)

$g^{-1}(3) = 9 - 4$

$= 5$

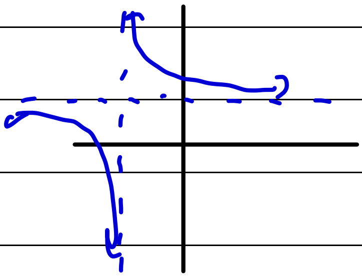
$\therefore m = \frac{1}{6}$

c. Find the maximal domain, D, of the function f such that the composite function $g \circ f$ is defined. (3 marks)

$R \setminus \{0\}$

ran f: $R \setminus \{1\}$

dom g: $[0, \infty)$



$g(x) = 0$

$\frac{1}{x+2} = -1$

$x + 2 = -1$

$x = -3$

$\therefore D: (-\infty, -3] \cup (-2, \infty)$

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Question 3 (6 marks)

Consider the functions, $f(x) = \frac{1}{x-2}$ and $g(x) = 2x + 3$ defined on their maximal domains.

Let $h(x) = g(f(x))$.

a. Write down the rule and domain of $h(x)$. (2 marks)

$h(x) = \frac{2}{x-2} + 3$

dom: $R \setminus \{2\}$

b. Define the inverse function, h^{-1}, of h. (2 marks)

Swap x and y

$x = \frac{2}{y-2} + 3$

$x - 3 = \frac{2}{y-2}$

$y - 2 = \frac{2}{x-3}$

$\therefore h^{-1}(x) = \frac{2}{x-3} + 2, \ x \in R \setminus \{3\}$

c. Find the point(s) of intersection between h and h^{-1}. (2 marks)

$h(x) = x$

$\frac{2}{x-2} + 3 = x$

$\frac{2}{x-2} = x - 3$

$2 = (x-3)(x-2)$

$x^2 - 5x + 6 = 2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\therefore x = 1, 4$

$\therefore (1,1), (4,4)$

Question 4 (7 marks)

Consider the functions:

$$f: [a, \infty) \to \mathbb{R}, f(x) = \frac{1}{2}x^2 - 2x + 5$$

a. Find the largest value of a such that the inverse function f^{-1} exists. (1 mark)

$f(x) = \frac{1}{2}(x^2 - 4x + 10)$

$= \frac{1}{2}((x-2)^2 - 4 + 10)$

$= \frac{1}{2}(x-2)^2 + 3$

$\therefore a = 2$

b. Define f^{-1}. (2 marks)

Swap x and y

$x = \frac{1}{2}(y-2)^2 + 3$

$2(x-3) = (y-2)^2$

$y - 2 = \pm\sqrt{2(x-3)}$

$\therefore f^{-1}(x) = 2 + \sqrt{2x-6}, \; x \in [3, \infty)$

c. Show that f and f^{-1} do not have any points of intersection. (2 marks)

$f(x)$ and f^{-1} should intersect at $y = x$

$f(x) = x$	$x = \frac{6 \pm \sqrt{36 - 4 \cdot 10}}{2}$
$\frac{1}{2}x^2 - 2x + 5 = x$	$= \frac{6 \pm \sqrt{-4}}{2}$
$\frac{1}{2}x^2 - 3x + 5 = 0$	Inside of root $= \ominus$
$x^2 - 6x + 10 = 0$	$\therefore \Delta < 0 \quad \therefore$ No solution

Let $g: \mathbb{R} \to \mathbb{R}, g(x) = \frac{1}{2}x^2 - 2x + 5$ and $h: \mathbb{R} \to \mathbb{R}, h(x) = 2^x$.

d. Find the range of $h(g(x))$. (1 mark)

$R \ \boxed{g} \ [3, \infty) \to \boxed{h}$

$\therefore [8, \infty)$

e. Find the value of c if the maximal domain that $h(g(x+c))$ is one-to-one on is $[1, \infty)$. (1 mark)

$c = 1$

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Section D: Technology Warmup

INSTRUCTION: 5 Minutes Writing.

Calculator Commands: Finding the domain and range

TI

- domain $(f(x), x)$, $f\ Min$ and $Fmax$.

Define $f(x)=\sqrt{9-x^2}$	Done
domain$(f(x),x)$	$-3\le x\le 3$
fMin$(f(x),x)$	$x=-3$ or $x=3$
fMax$(f(x),x)$	$x=0$
$f(3)$	0
$f(0)$	3

TI-UDF

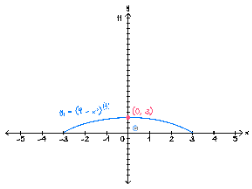
Analyse a Function: Find intercepts, critical points and their nature, maximal domain, asymptote.	analyse(<function>, <variable>) analysed(<function>, <variable>, <lower bound>, <upper bound>)

$$analysed\left(\frac{x^4-2\cdot x^3-3\cdot x^2+3\cdot x+1}{-3\cdot x^3-6\cdot x^2-x+1},x,-5,5\right)$$

- Start Point: $\begin{bmatrix}-5 & \frac{262}{77}\end{bmatrix}$
- End Point: $\begin{bmatrix}5 & \frac{-316}{529}\end{bmatrix}$
- Maximal Domain:
  $x\ne-1.68469$ and
  $x\ne-0.629579$ and
  $x\ne0.314273$ and
  $-5\le x\le 5$

Casio Classpad

- Graph the function and use G-Solve to find min and max values for the range.



Mathematica

In[127]:= `f[x_] := √9 - x^2` — $f[x\_] := \sqrt{9-x^2}$

In[128]:= `FunctionDomain[f[x], x]`

Out[128]= $-3 \le x \le 3$

In[129]:= `FunctionRange[f[x], x, y]`

Out[129]= $0 \le y \le 3$

Mathematica UDF :

- FInfo [f [x], $\{x, x\ min, x\ max\}, y$]

Returns useful information about a function, including derivative, domain, range, period, horizontal intercepts, vertical intercepts, stationary points, inflexion points, left and sided asymptotes, oblique asymptotes and vertical asymptotes.

$$\text{FInfo}\left[\frac{x^2-1}{x\,(x^2-3)}, \{x, -\text{Infinity}, \text{Infinity}\}, y\right]$$

The function is $\frac{x^2-1}{x\,(x^2-3)}$

The derivative is $-\frac{x^4+3}{x^2\,(x^2-3)^2}$

Domain: $x < -\sqrt{3} \lor -\sqrt{3} < x < 0 \lor 0 < x < \sqrt{3} \lor x > \sqrt{3}$

Range: $y\in\mathbb{R}$

Period: 0

Horizontal Intercepts: {-1, 1}

Vertical Intercepts: None

Stationary points: {}

Inflexion points: {{-0.871..., -0.123...}, {0.871..., 0.123...}}

Left sided asymtote: y=0

Right sided asymtote: y=0

Oblique asymtote: {0}

Vertical asymtote: $\{x=0, x=-\sqrt{3}, x=\sqrt{3}\}$

Calculator Commands: Finding the composite function

TI	CASIO	Mathematica
Define $f(x)=\ln(x)$ — *Done*	define f(x) = ln(x) — done	In[141]:= f[x_] := Log[x]
Define $g(x)=x^2+3$ — *Done*	define g(x) = x^2+3 — done	In[142]:= g[x_] := x^2 + 3
$f(g(x))$ — $\ln(x^2+3)$	f(g(x)) — $\ln(x^2+3)$	In[143]:= f[g[x]] Out[143]= Log[3 + x^2]

Calculator Commands: Finding the inverse function

TI	CASIO	Mathematica
Define $f(x)=x^2+4\cdot x+9$ — *Done*	define f(x) = x^2+4x+9 — done	In[154]:= f[x_] := x^2 + 4 x + 9
solve$(f(y)=x,y)$ — $y=-(\sqrt{x-5}+2)$ or $y=\sqrt{x-5}-2$	solve(f(y)=x, y) — $\{y=-\sqrt{x-5}-2,\ y=\sqrt{x-5}-2\}$	In[155]:= Solve[f[y] == x, y] Out[155]= {{y → -2 - √(-5 + x)}, {y → -2 + √(-5 + x)}}

NOTE: It doesn't tell us which branch is correct.

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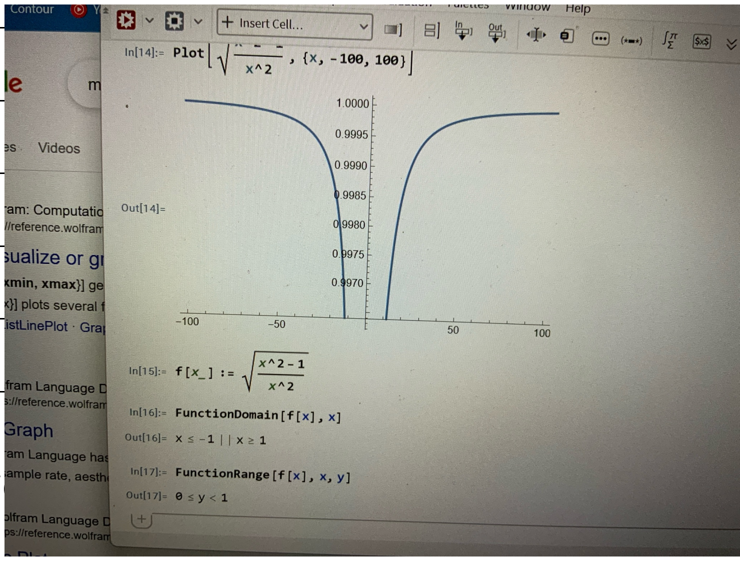
Question 5 Tech-Active.

a. Find the domain and range of $f(x) = \sqrt{\frac{x^2-1}{x^2}}$.



b. Let $f: [0, \infty) \to R, f$ has a gradient of 4 when $x = 1$.

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Section E: Exam 2 (23 Marks)

INSTRUCTION: 23 Marks. 30 Minutes Writing.

Question 6 (1 mark)

The function f defined by $f\ \ A \to \mathbb{R}, f(x) = (x-2)^2 + 3$ will have an inverse function if its domain A is:

A. $\mathbb{R}$

B. $(-\infty, 3]$

C. $[3, 6]$

D. $[0, \infty)$

Question 7 (1 mark)

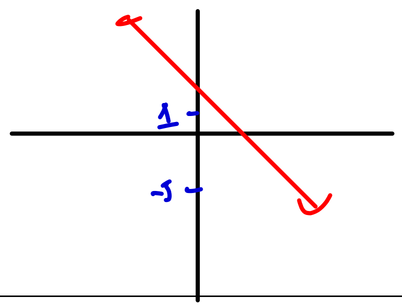
The linear function $f : D \to \mathbb{R}, f(x) = 2 - x$ has a range of $[-5,1)$. The domain of D is:

A. $(-5,1]$

B. $(-1,5]$

C. $(1,7]$

D. $[7,1)$



$2 - x = 1$

$x = 1$

$2 - x = -5$

$x = 7$

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Question 8 (1 mark)

Let f be a one-to-one differentiable function, and the following values are known:

$$f(2) = 3, f(3) = 4, f'(2) = 5, \text{ and } f'(3) = 6$$

Let $g(x) = f^{-1}(x)$. The value of $g'(3)$ is:

A. $\frac{1}{5}$

B. $\frac{1}{7}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$

Question 9 (1 mark)

Let $f : [-1,3] \to \mathbb{R}, f(x) = 2x - 1$ and $g: D \to \mathbb{R}, g(x) = x^2 - 1$.

The largest interval D such that $f \circ g$ exists is:

A. $(0,2)$

B. $[0,2]$

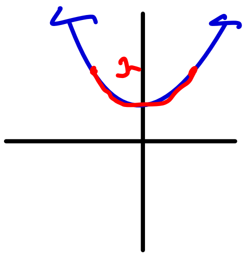
C. $[-2,2]$

D. $[0,3]$

rang: $[-1, \infty)$

dom f: $[-1, 3]$

$g(x) = 3$



$x^2 - 1 = 3$

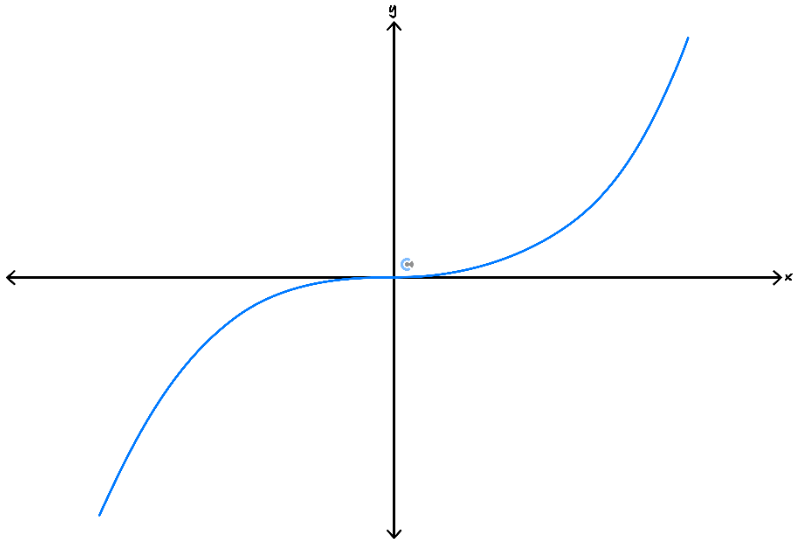
$x^2 = 4$

$x = \pm 2$

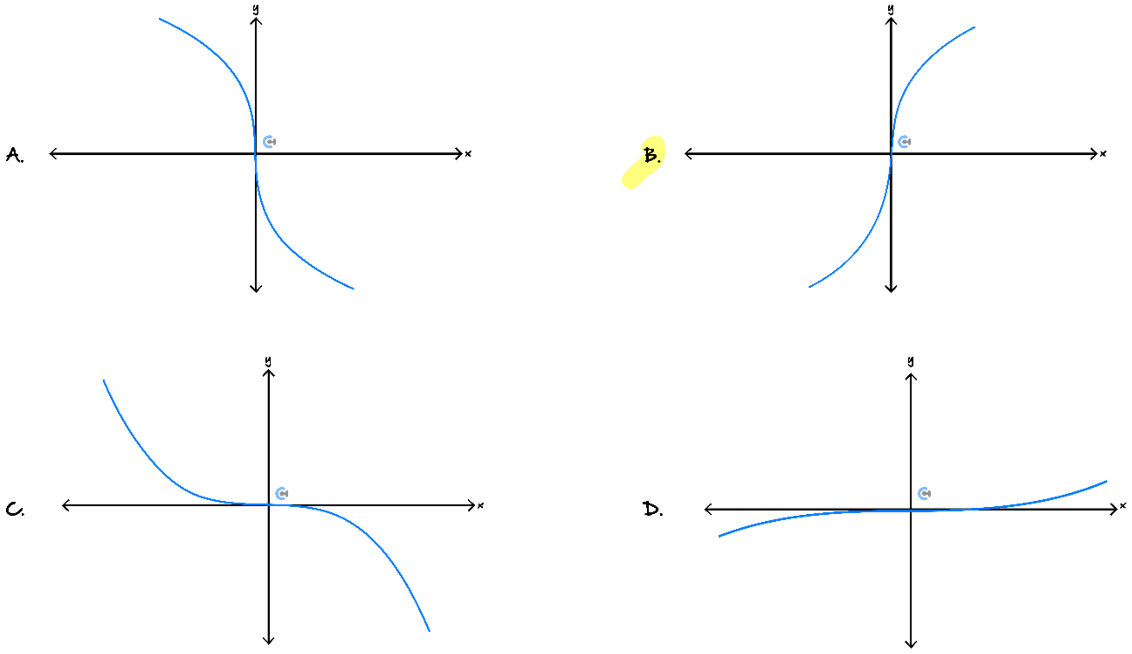
Space for Personal Notes

Question 10 (1 mark)

Part of the graph of $y = f(x)$ is shown below.



The inverse function f^{-1} is best represented by:



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Question 11 (8 marks)

Health insurance providers MediBear and Bopa offer cost-based health insurance plans represented by the functions:

$$C_m = \sqrt{4x-3}, 2 \leq x \leq 10 \text{ and } C_b = C_m^{-1}(x)$$

where C represents the amount paid for the plan in tens of dollars and x represents the plan's benefits rating.

a. Define the function C_b. (2 marks)

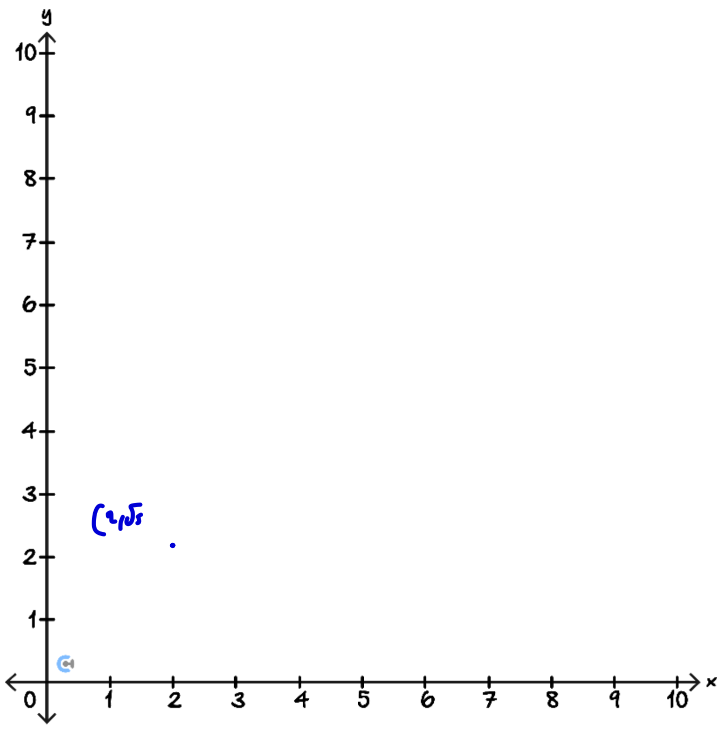
$C_b = C_m^{-1}(x)$

$C_m^{-1}(x) = \frac{1}{4}(3+x^2)$

range C_m: $[\sqrt{5}, \sqrt{37}]$

$\therefore C_b(x) = \frac{1}{4}(3+x^2), x \in [\sqrt{5}, \sqrt{37}]$

b. Graph the two functions on the axes below. Label all endpoints with coordinates. (2 marks)



c. What is the maximum plan benefit rating for a plan offered by Bopa? Give your answer correct to one decimal place. (1 mark)

$\sqrt{37} \approx 6.1$

d. What plan benefit rating results in both plans having the same cost? (1 mark)

3

e. Find for what benefit ratings, customers are better off with a MediBear plan. (2 marks)

$[2, \sqrt{5}) \cup (3, 10)$

Space for Personal Notes

Question 12 (10 marks)

Consider the function $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 2x - 2$.

a. State the smallest value of a for which f has an inverse function. (1 mark)

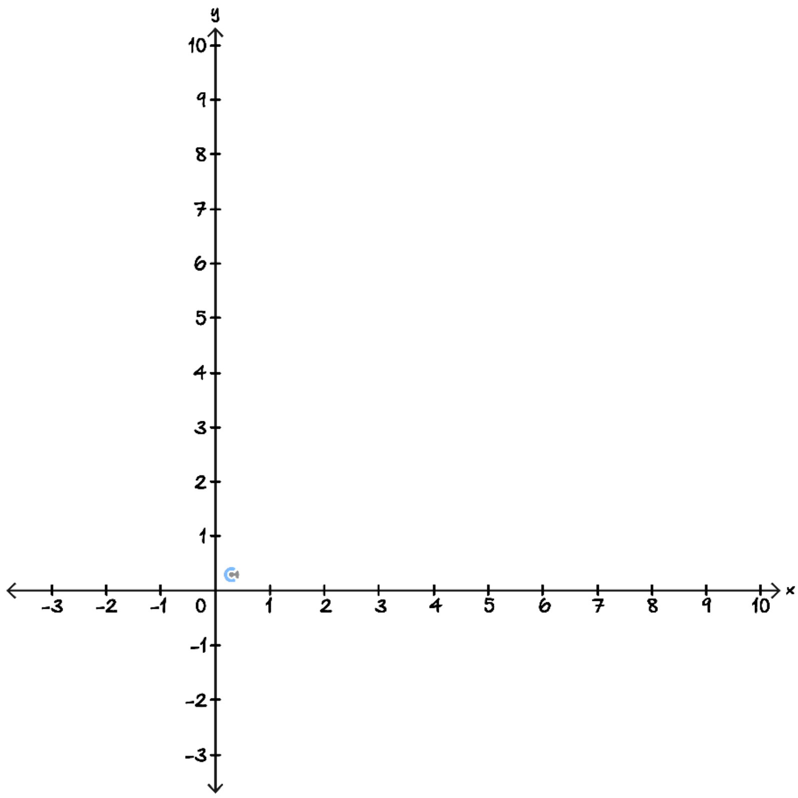
$(x-1)^2 - 3$

$a = 1$

b. Define the inverse function, f^{-1}. (2 marks)

$f^{-1}(x) = 1 + \sqrt{x+3}, x \in [-3, \infty)$

c. Sketch the graphs of f and its inverse on the axes below. Label all axes intercepts, endpoints and points of intersection with coordinates correct to two decimal places. (3 marks)



d. Let f and f^{-1} intersect at the point P. It is known that f has a gradient of $1 + \sqrt{17}$ at P. Find the angle made by the tangents to f and f^{-1} at P. Give your answer in degrees correct to two decimal places. (2 marks)

$m = 1+\sqrt{17}$ | $\theta = \tan^{-1}(1+\sqrt{17}) = 78.96°$

$m_i = \frac{1}{1+\sqrt{17}}$ | $\theta = \tan^{-1}\left(\frac{1}{1+\sqrt{17}}\right) = 11.04°$

$\therefore \theta = 78.96 - 11.04$

$= 67.91°$

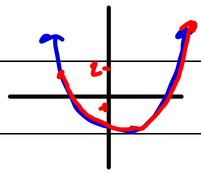
Consider the function $g : [2, \infty) \to \mathbb{R}, g(x) = \log_2(x + 1)$.

e. Find all possible values for a such that $g(f(x))$ does not exist. (1 mark)

Klns

ran f: $[-1, \infty)$

dom g: $[2, \infty)$



$f(x) = 2$

$x = 1 \pm \sqrt{5}$

$\therefore$ at $(1-\sqrt{5}, 1+\sqrt{5})$

f. If $a = 4$, find the range of $g(f(x))$. (1 mark)

$[\log_2(7), \infty)$

$[4, \infty) \boxed{f} [6, \infty) \to \boxed{g} (\log_2(7), \infty)$

Let's take a BREAK (Extension Stream)!

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Section F: Extension Exam 1 (13 Marks)

INSTRUCTION: 13 Marks. 20 Minutes Writing.

Question 13 (5 marks)

Consider the two functions $f(x) = \frac{1}{x-2}$ and $g(x) = 1 + \cos(x)$ defined on their maximal domains.

a. Determine whether or not the functions $f \circ g(x)$ or $g \circ f(x)$ exist, and justify your answer. If the composite function exists, state its rule and domain. (2 marks)

b. Find the values of x for which $f^{-1}(x) > f(x)$. (3 marks)

Question 14 (4 marks)

Let $f: (-\infty, k) \to \mathbb{R},\ f(x) = \frac{1}{(x-k)^2}$, where k is a real constant.

a. Find the rule for f^{-1} in terms of k. (1 mark)

b. Find the exact value of k so that $f = f^{-1}$ has one unique solution.
Express your answer in the form $\frac{a}{b^c}$, for positive integers a, b and rational number c.

HINT: There will be one unique solution if $y = x$ is tangent to f and f^{-1} when they intersect. (3 marks)

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Question 15 (4 marks)

Let $f: \left[-\frac{a}{4}, a\right] \to \mathbb{R}, f(x) = \sqrt{4x + a} - 2$, where a is a positive real number.

a. The graphs of $y = f(x)$ and its inverse $y = f^{-1}(x)$ may have up to two points of intersection. Find the x-coordinates of any possible points of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$ in terms of a. (2 marks)

b. Determine the set of values of a for which the graphs of f and f^{-1} have two points of intersection. (2 marks)

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Section G: Extension Exam 2 (16 Marks)

INSTRUCTION: 16 Marks. 20 Minutes Writing.

Question 16 (1 mark)

Let $f(x) = \sqrt{ax+b}$ and let g be the inverse function of f.
Given that $f(0) = 1$ and $g'(x) > 0$, then all possible values of a are:

A. $a \in \mathbb{R}^-$

B. $a \in \mathbb{R}^+$

C. $a \in \mathbb{R}\backslash\{0\}$

D. $a \in [0,1)$

Question 17 (1 mark)

The range of the function given by $f: (0,3] \to \mathbb{R}, f(x) = x^2 - 2x + b$ is:

A. $(b-1, b+3)$

B. $[b-1, b+3]$

C. $(b, 3]$

D. $(b-1, b+3]$

Question 18 (1 mark)

The functions $f(x) = \log_2(a-x)$ and $g(x) = -\sqrt{x+a}$ are defined on their maximal domains and $a \in \mathbb{R}^+$. The domain of $\frac{f}{g}$ is:

A. $[-a, a)$

B. $[-a, a]$

C. $(-a, a)$

D. $\mathbb{R}\backslash\{a\}$

Question 19 (1 mark)

Let f be a one-to-one differentiable function, and the following values are known:

$$f(a) = b, f(b) = c, f'(c) = d, \text{ and } f'(b) = k.$$

Let $g(x) = f^{-1}(x)$. The value of $g'(c)$ is:

A. $\frac{1}{a}$

B. $\frac{1}{k}$

C. $\frac{1}{d}$

D. $\frac{1}{b}$

Question 20 (1 mark)

Consider the functions $f(x) = x^2 + b$, where $b \in \mathbb{R}$ and $g(x) = \sqrt{(x-2)(x-3)}$ defined on their maximal domains. The composite functions $g(f(x))$ will have domain equal to $\mathbb{R}$ if:

A. $b < -2$

B. $b < -1$

C. $b > 1$

D. $b > 2$

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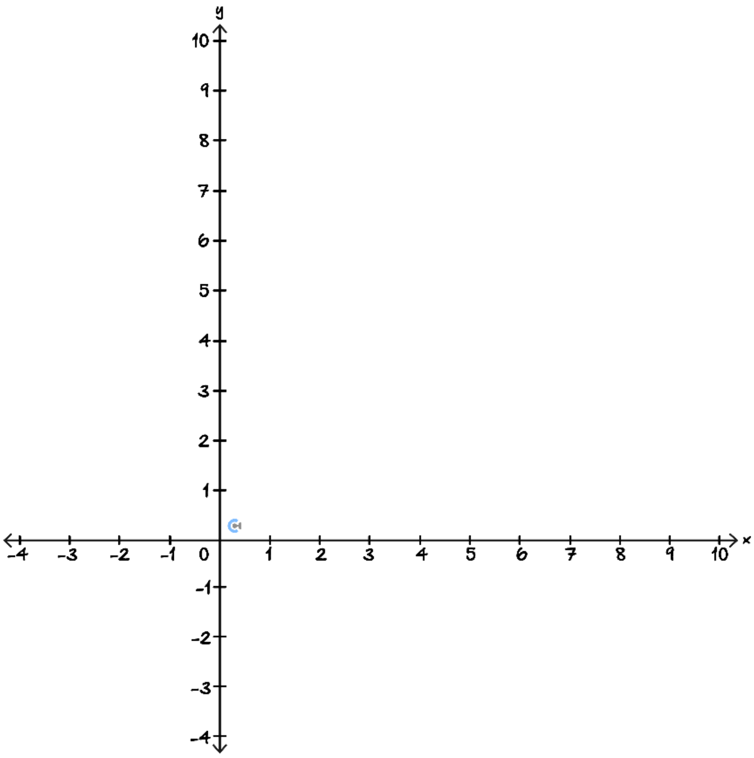
Question 21 (11 marks)

Consider the function $f : [\sqrt{2}, \infty) \to \mathbb{R}$, where $f(x) = \sqrt{2x^2 - 4}$.

a. Define f^{-1}, the inverse function of f. (2 marks)

b. Find the coordinates of the point P, which is the intersection between the graphs of $y = f(x)$ and $y = f^{-1}(x)$. (1 mark)

c. Sketch the graphs of $y = f(x), y = f^{-1}(x)$, on the axes below. Label all axes intercepts. (2 marks)



d. Find the angle between the tangents to the curves $y = f(x)$ and $y = f^{-1}(x)$ at the point P. Give your answer in degrees correct to two decimal places. (2 marks)

Now, consider the one-to-one increasing function $g : D \to \mathbb{R}$, where $g(x) = \sqrt{kx^2 - 4}$ and $k \in \mathbb{R}^+$.

e.

i. Find the domain D in terms of k. (1 mark)

ii. Find the value(s) of k such that g and g^{-1} do not intersect. (1 mark)

f. The two curves $y = g(x)$ and $y = g^{-1}(x)$ intersect at $x = c$, where $c > 1$. The angle between the tangents at $x = c$ is θ.

It is known that $\tan(\theta) = \frac{9}{40}$. Determine the value of c and k. (2 marks)

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