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VCE Mathematical Methods $\frac{3}{4}$
SAC 1 Revision IV [0.19]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:

Section A: SAC 1 Success

Welcome to the third SAC 1 workshop!



Context: SAC 1 Workshops

- SAC 1 - 50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC 1 ~ 8.

Successful SAC



Study Score = How much you know × How much you show

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key!**

Analogy: Skipping Questions



- Let's say if you were to fight them and win, you get assigned marks.



- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.



SAC Proficiency List

Before the SAC

- Prepare your stationery, including a ruler, eraser, and your mechanical pencil lead.
- Skim through the bound reference (if applicable).
- Do not speak to other people and lock in.
- TI & Mathematica Only: Check your Contour UDFS.
- TI Only: Check technology settings.

Document Settings

Display Digits:	Float 6	▶
Angle:	Radian	▶
Exponential Format:	Normal	▶
Real or Complex:	Real	▶
Calculation Mode:	Exact	▶
CAS Mode:	On	▶

OK Cancel

Reading Time

- **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and use what technology commands.
- Identify questions to **skip**.

For difficult SACs, it's not necessarily about getting 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.
- Identify questions to **start first** – You don't have to start from Q1!
- Look for potential pitfalls – **Units, domain restriction of the unknown, variable and function meaning.**

Writing Time

- **Circle what the question is asking** for in the question.
- Spend the first **50%** of the time on all the **easy questions** you identified.
- Spend the next **25%** of doing the **difficult questions** you left blank.

- Spend the last 25% of the time on **checking your answers**.
- Check your answer by reading the question again and see if you answered the question.
- Check in the order of:
Domino effect (check a, b, c first) > *Questions with high marks* (3+) > *Hard Questions*.
- **TI ONLY:** Use new document - **doc 4, 1**.

After the SAC

- Think about how each mark loss can be prevented using this proficiency list.
- Think about the big picture and improve the marks -
Instead of spending 10 minutes on 10c) (1 mark), I should have checked 5a) (3 marks).

Space for Personal Notes

Section B: SAC Questions - Tech Active (52 Marks)

INSTRUCTION:

➤ 52 Marks. 15 Minutes Reading, 75 Minutes Writing.



Question 1 (16 marks)

A function $y = \frac{x^3}{(x+2)^2}$ is to undergo the following sequence of transformations:

1. Dilation factor $\frac{1}{2}$ from the x -axis.
 2. Reflection in the x -axis.
 3. Reflection in the line $y = x$.
 4. Dilation factor 8 from the y -axis.
 5. Reflection in the y -axis.
- a. Show that this sequence of transformations can be represented by the transformation: (2 marks)

$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (4y, x)$$

- b.** Describe a sequence of 2 transformations that would result in the same transformation as T . (2 marks)

- c.**
- i.** Determine the equation of image of y after applying the transformations represented by T . Leave your answer in the form $x = f(y)$. (2 marks)

- ii.** By carefully considering the graph of the preimage and the linear transformations used to transform it, explain why you would not expect the image to be a function. (2 marks)

d.

- i. Find a sequence of transformations which take the graph of $y = 5e^{\frac{x}{2}-3} + 4$ to the graph of $y = e^x$. (2 marks)

- ii. The transformation $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2, S(x, y) = (ax + b, cy + d)$ takes the graph of $y = e^x$ to the graph of $y = -3e^{-4(x+6)} + 2$.

Find the values of $a, b, c, d \in \mathbb{R}$. (3 marks)

- iii. Hence, or otherwise, determine a sequence of transformations that would transform the graph of $y = 5e^{\frac{x}{2}-3} + 4$ to the graph of $y = -3e^{-4(x+6)} + 2$.

Give your answer in an order where translations are applied last. (3 marks)

Space for Personal Notes

Question 2 (9 marks)

Let f be the function that is given by $f(x) = \frac{ax^2 + b}{x^2 - c}$, and that has the following properties:

1. $\lim_{x \rightarrow \infty} f(x) = 1$
2. $\lim_{x \rightarrow 3^-} f(x) = -\infty$
3. $f'(1) = -1$

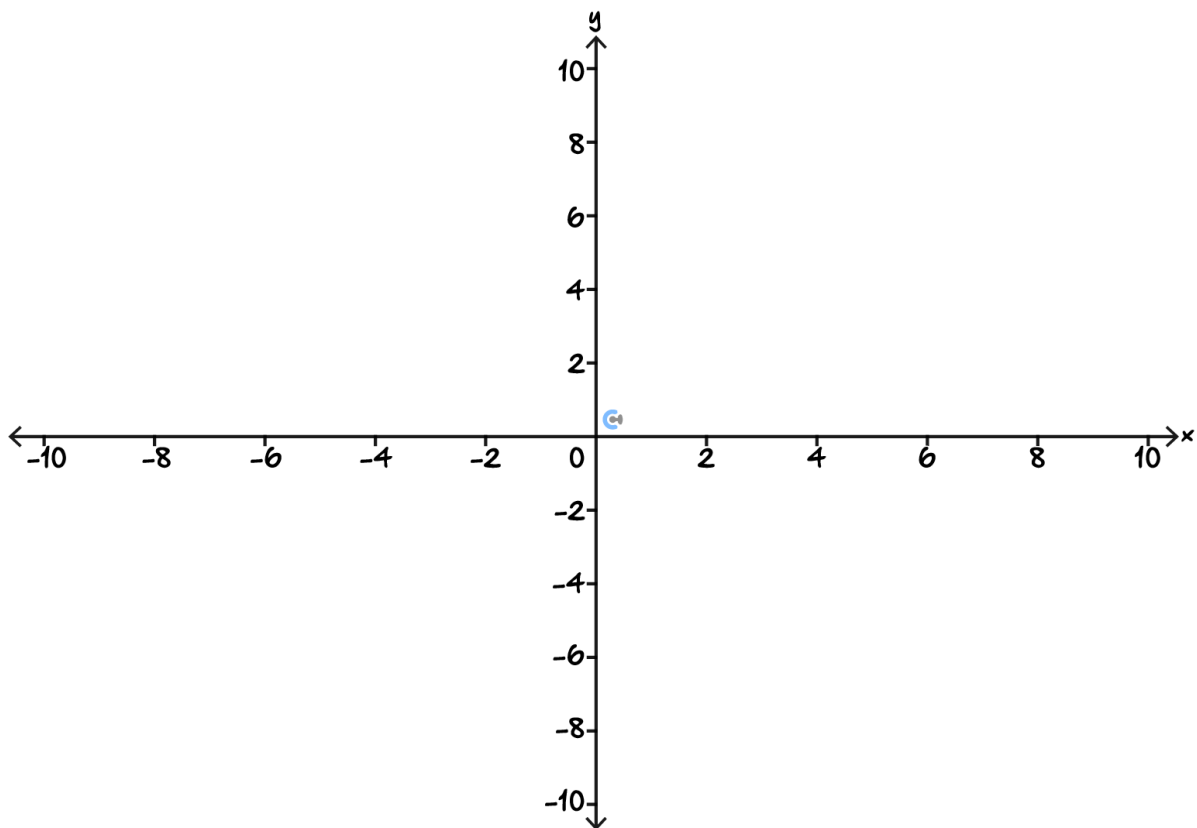
Where $a, b, c \in \mathbb{R}$.

- a.** Show that f is an even function. (1 mark)

- b.** Show that $a = 1$, $b = 23$ and $c = 9$. (3 marks)

- c. Write an equation for each vertical and horizontal asymptote of the graph of f . (2 marks)

- d. Sketch the graph of $y = f(x)$ on the axes below. Label any asymptotes with equations and axial intercepts with coordinates. (3 marks)



Space for Personal Notes

Question 3 (14 marks)

An amusement park wants to open a new ride. Sarah, an architect working for the company, pitched a roller coaster using a model. The design of her ride is aimed towards thrill seekers who are chasing an adrenaline rush.

- a.** The ascent has two distinct sections. The first section is modelled by the following:

$$a_s : [-4, c], \quad a_s(x) = 2^{3x+d}$$

Where x is an arbitrary position, whilst $a_s(x)$ is the height above the ground, and c and d are real constants. The units (or exact scale) can be ignored.

- i.** Write $a_s(x)$ in the form of $e^{m(nx+p)}$, where $m, n, p \in \mathbb{R}$. (2 marks)

- ii.** Find the rule for the inverse of $a(x) = 2^{3x+d}$. Provide your answer in the form of $\frac{\log_e(x)}{n \log_e(p)} + q$, where $n, p, q \in \mathbb{R}$. (2 marks)

- iii. The latter half of the roller coaster ascent is modelled by the following:

$$a_l : \left[c, \frac{2}{\log_e(2)} \right] \rightarrow \mathbb{R}, \quad a_l(x) = a^{-1}(x)$$

Given that the two sections join **smoothly**, find the value of c and d . (3 marks)

- b. The descent of the roller coaster is a steep drop, modelled by **translations** of the following function:

$$d : [m, n] \rightarrow \mathbb{R}, d(x) = -x^3$$

Where $m, n \in \mathbb{R}$.

- i. Given that the descent joins continuously with the ascent at the stationary point of the image of $d(x)$, state the sequence of translations that are applied to d . (2 marks)

- ii. Can the two sections ever join smoothly? Explain your answer. (2 marks)

- iii. Given that the descent ends when it hits the ground (x -axis intercept), find the coordinates of the **endpoints** of $d(x)$ correct to 2 decimal places. (3 marks)

Space for Personal Notes

Question 4 (13 marks)

Consider the piecewise function:

$$g(x) = \begin{cases} x^2 & \text{for } -\infty < x < 0 \\ \sqrt{x} & \text{for } 0 \leq x < \infty \end{cases}$$

Where $g_1(x) = x^2$ for $-\infty < x < 0$, and $g_2(x) = \sqrt{x}$ for $0 \leq x < \infty$.

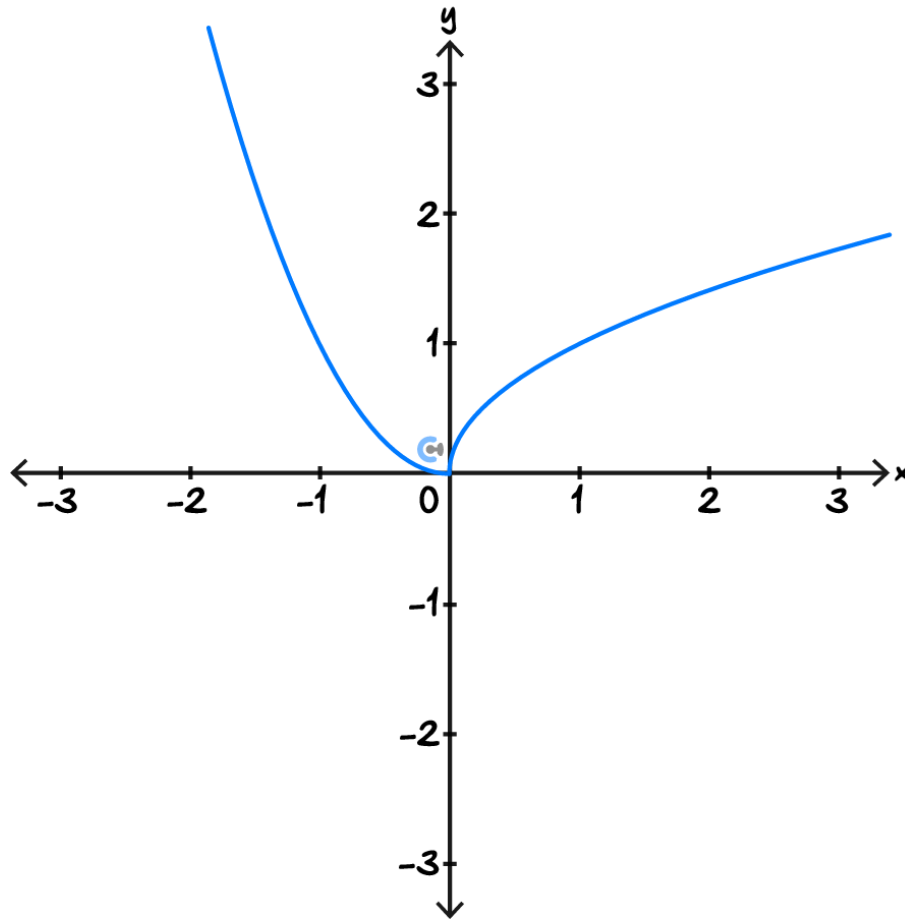
a.

- i.** Show that $g(x)$ is continuous at $x = 0$. (2 marks)

- ii.** Show that $g(x)$ is not smooth at $x = 0$. (2 marks)

b.

- i. Sketch the **inverse** of $g(x)$ on the axes provided. $g(x)$ has been drawn for you. (2 marks)

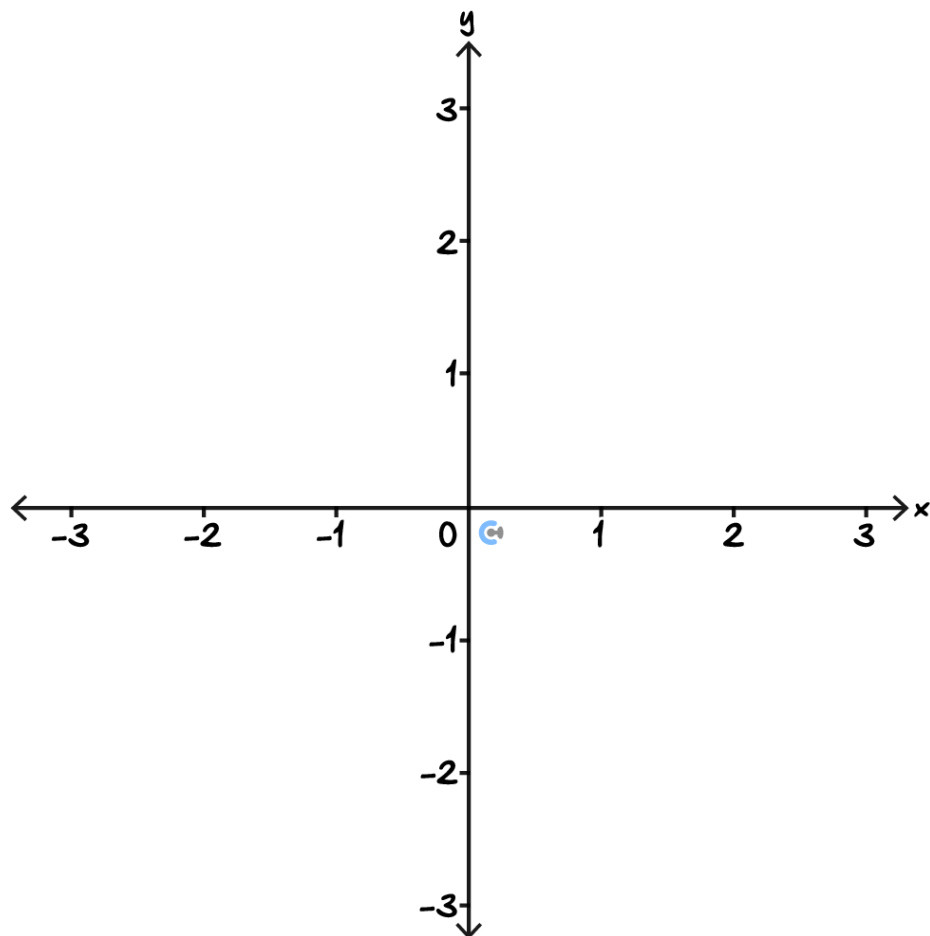


- ii. State why g^{-1} is not a function. (1 mark)

c.

- i. By investigating compositions of $g_1(x)$ and $g_2(x)$, find the **rule** for $g(g(x))$. (4 marks)

- ii. Hence, sketch $g(g(x))$ on the axes provided. (2 marks)



Section C: Marking Scheme

Question Number	Solutions
1a	We apply the transformations to the point (x, y) in order: $(x, y) \rightarrow \left(x, \frac{1}{2}y\right) \rightarrow \left(x, -\frac{1}{2}y\right) \rightarrow \left(-\frac{1}{2}y, x\right) \rightarrow (-4y, x) \rightarrow (4y, x)$ [1M for swapping x and y when reflecting in $y=x$, 1M for clear logical sequence]
1b	A reflection in the line $y = x$ [1M] followed by a dilation by factor 4 from the y-axis [1M]
1ci	We have that $x' = 4y \implies y = \frac{1}{4}x'$ and $y' = x \implies x = y'$. [1M] Substitute into the equation and remove the dashes to get: $\frac{1}{4}x = \frac{y^3}{(y+2)^2}$ $x = \frac{4y^3}{(y+2)^2} \quad [1A]$
1cii	The pre-image is not a 1:1 function. [1M] The transformation involves reflecting in the line $y = x$ which produces the inverse of the pre-image, but the pre-image is not 1:1 so its image will not be a function. [1M].
1di	• A translation 4 units down (or a dilation by factor $\frac{1}{5}$ from the x-axis) • A dilation by factor $\frac{1}{5}$ from the x-axis (a translation $\frac{4}{5}$ units down) [1M] Then we have $x' = \frac{x}{2} - 3 = \frac{1}{2}(x - 6)$, so • A translation 6 units to the left (or a dilation by factor $\frac{1}{2}$ from the y-axis) • A dilation by factor $\frac{1}{2}$ from the y-axis (a translation 3 units left) [1M] There could be many different combinations.
1dii	Immediately see that $y' = -3y + 2$. [1M] Then $-4(x' + 6) = x \implies x' = -\frac{1}{4}x - 6$. [1M] So $a = -\frac{1}{4}, b = 6, c = -3$ and $d = 2$. [1A]

1diii

Can do all the transformations in **part d.i** followed by the transformations in **part d.ii** and simplify.

It is probably simpler to just do it from scratch.

$$\text{Solve } \frac{y-4}{5} = \frac{y'-2}{-3} \implies y' = \frac{22}{5} - \frac{3y}{5} \quad [1M]$$

$$\text{Solve } \frac{x}{2} - 3 = -4(x' + 6) \implies x' = -\frac{21}{8} - \frac{x}{8} \quad [1M]$$

Therefore a sequence of transformations is:

- A reflection in the x axis
- A dilation by factor $\frac{3}{5}$ from the x -axis
- A reflection in the y -axis.
- A dilation by factor $\frac{1}{8}$ from the y -axis
- A translation $\frac{22}{5}$ units up
- A translation $\frac{21}{4}$ units to the left.

[1A] (order of reflections and dilations can be mixed around.)

2a

We require that $f(x) = f(-x)$ for all valid x for an even function.

$$f(-x) = \frac{a(-x)^2 + b}{(-x)^2 + c} = \frac{ax^2 + b}{x^2 + c} = f(x).$$

So is an even function. [1A]

2b

We can write $f(x) = \frac{ac+b}{x^2-c} + a$. So $\lim_{x \rightarrow \infty} f(x) = a = 1$. [1M]

Vertical asymptote when denominator is zero. Thus $x^2 - c = 0$ when $x = 3$. So $c = 9$. [1M]

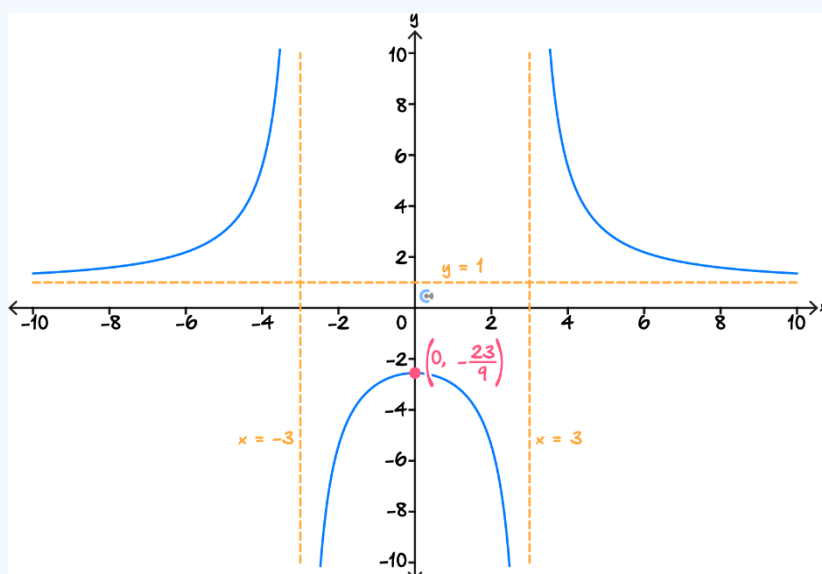
Then we use $f'(1) = -1$ to find $b = 23$. [1M]

2c

Vertical asymptotes: $x = \pm 3$ [1M]

Horizontal asymptote: $y = 1$ [1M]

2d



[1M] for shape
[1M] asymptotes
[1M] y -intercept.

3ai	Note that $2 = e^{\ln(2)}$. [1M] Thus, $a_s(x) = e^{(3x+d)\ln(2)} \quad [1A]$
3aii	To find the inverse we solve $a(y) = x$ for y. [1M] $a^{-1}(x) = \frac{\log_e(x)}{3\log_e(2)} - \frac{d}{3} \quad [1A]$
3aiii	To join smoothly we require that $a_s(c) = a_l(c)$ [1M] and that $a'_s(c) = a'_l(c)$ Since they are inverses we can solve $a_s(c) = c$ and $a'_s(c) = 1$ simultaneously [1M] We get $c = \frac{1}{\ln(8)}$ and $d = -\frac{1 + \ln(\ln(8))}{\ln(2)}$ [1A] (Check against approx values of $c \approx 0.48$ and $d \approx -2.5$ to verify your answer)
3bi	The descent must join the ascent section at $\left(\frac{2}{\ln(2)}, \frac{\ln(6)+1}{\ln(8)}\right)$. [1M] d has stationary point at $(0, 0)$ So translations are $\frac{2}{\ln(2)}$ units right and $\frac{\ln(6)+1}{\ln(8)}$ units up. [1A]
3bii	No. The ascent section always has a gradient > 0 and the descent section has gradient ≤ 0, so they cannot joint smoothly. [1A for no, 1M for explanation]
3biii	$d(x)$ must start at its stationary point, so one endpoint is $(0, 0)$. [1A] $d(x)$ ends when it has gone $\frac{\ln(6)+1}{\ln(8)} \approx 1.34$ units down. [1M] Solve $d(x) = -1.34 \implies x = 1.10$. Therefore other endpoint is $(1.10, -1.34)$. [1A]
4ai	To show continuity at $x = 0$, we need to verify the following: $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^+} g(x)$ $g(0)$ is defined and equals the above limit. We evaluate: $\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} x^2 = 0 \quad \text{and} \quad \lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} \sqrt{x} = 0 \quad [1M]$ Since $g(0) = \sqrt{0} = 0$, the function is continuous at $x = 0$. [1A]

4aii

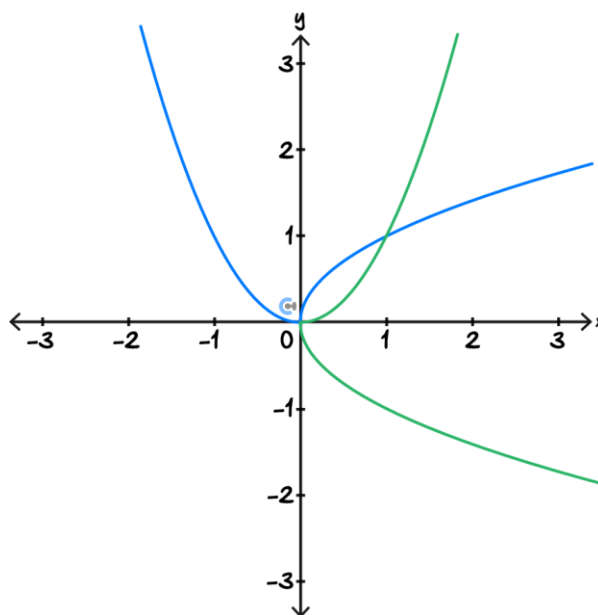
To show that $g(x)$ is not smooth at $x = 0$, we evaluate the left and right derivatives. We have that

$$g'(x) = \begin{cases} 2x, & -\infty < x < 0 \\ \frac{1}{2\sqrt{x}}, & 0 < x < \infty \end{cases} \quad [1M]$$

Then $\lim_{x \rightarrow 0^-} = \lim_{x \rightarrow 0} 2x = 0$
and $\lim_{x \rightarrow 0^+} g'(x) = \lim_{x \rightarrow 0} \frac{1}{2\sqrt{x}}$ which is undefined.

Thus not smooth at $x = 0$ because the left and right limits differ. [1A]

4bi



4bii

g^{-1} is not a function because it fails the vertical line test (or not 1:1); for some values of x , there are two possible y -values in the inverse relation. [1M]

4ci

We have that $\text{dom } g_1 = \mathbb{R}^-$ and $\text{ran } g_1 = \mathbb{R}^+$.
and that $\text{dom } g_2 = \mathbb{R}^+ \cup \{0\}$ and $\text{ran } g_2 = \mathbb{R}^+ \cup \{0\}$

Then we see that $g_1 \circ g_1$ does not exist and $g_1 \circ g_2$ does not exist.
 $g_2 \circ g_1$ does exist and $g_2 \circ g_2$ does exist.

[1M determine which functions exist.]

$$(g_2 \circ g_1)(x) = \sqrt{x^2} \text{ with domain } (-\infty, 0) \quad [1M]$$

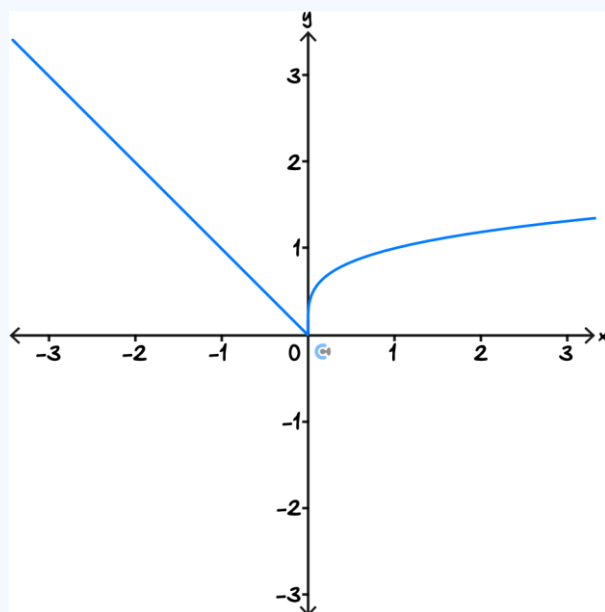
$$(g_2 \circ g_2)(x) = \sqrt{x^{1/2}} = x^{1/4} \text{ with domain } [0, \infty] \quad [1M]$$

So

$$g(g(x)) = \begin{cases} -x, & -\infty < x < 0 \\ x^{1/4}, & 0 \leq x < \infty \end{cases}$$

[1A]

4cii



Space for Personal Notes

Section D: Mathematica Solutions

Question Number	Solutions
1	Q1. d. <pre data-bbox="517 640 1217 1993"> In[2]:= f[x_] := 5 Exp[x / 2 - 3] + 4 In[3]:= 1 / 5 (f[x] - 4) Out[3]= e^{-3 + $\frac{x}{2}$} In[5]:= 1 / 5 f[x] - 4 / 5 // FullSimplify Out[5]= e^{-3 + $\frac{x}{2}$} In[6]:= f1[x_] := e^{-3 + $\frac{x}{2}$} In[12]:= f1[2 x + 6] // FullSimplify Out[12]= e^x In[14]:= f1[2 (x + 3)] Out[14]= e^x In[16]:= q[x_] := Exp[x] In[18]:= -3 q[-4 (x + 6)] + 2 Out[18]= 2 - 3 e^{-4 (6 + x)} In[15]:= g[x_] := -3 Exp[-4 (x + 6)] + 2 In[24]:= Solve[$\frac{y - 4}{5} == \frac{y1 - 2}{-3}, y1]$ // Expand Out[24]= $\left\{ \left\{ y1 \rightarrow \frac{22}{5} - \frac{3 y}{5} \right\} \right\}$ In[22]:= Solve[$\frac{x}{2} - 3 == -4 (x1 + 6), x1]$ // Expand Out[22]= $\left\{ \left\{ x1 \rightarrow -\frac{21}{4} - \frac{x}{8} \right\} \right\}$ In[25]:= -3 / 5 f[-8 (x + 21 / 4)] + 22 / 5 // FullSimplify Out[25]= 2 - 3 e^{-4 (6 + x)} </pre>

2

Q2.

$$\text{In[39]:= } f[x_]:= \frac{a x^2 + b}{x^2 - c}$$

$$\text{In[40]:= } f[x] == f[-x]$$

Out[40]= True

$$\text{In[48]:= } \text{Apart}[f[x]]$$

$$\text{Out[48]= } a + \frac{b + a c}{-c + x^2}$$

$$\text{In[50]:= } f'[1]$$

$$\text{Out[50]= } -\frac{2(a+b)}{(1-c)^2} + \frac{2a}{1-c}$$

$$\text{In[51]:= } \text{Solve}\left[-\frac{2(a+b)}{(1-c)^2} + \frac{2a}{1-c} == -1 /. \{a \rightarrow 1, c \rightarrow 9\}\right]$$

Out[51]= {{b -> 23}}

(* Confirm that the properties are satisfied*)

$$\text{In[52]:= } f1[x_]:= \frac{x^2 + 23}{x^2 - 9}$$

$$\text{In[54]:= } \text{Limit}[f1[x], x \rightarrow \text{Infinity}]$$

Out[54]= 1

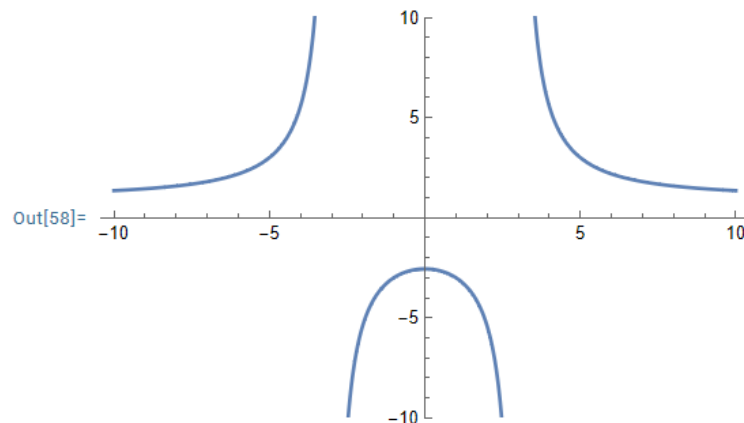
$$\text{In[56]:= } \text{Limit}[f1[x], x \rightarrow 3, \text{Direction} \rightarrow \text{"FromBelow"}]$$

Out[56]= -∞

$$\text{In[57]:= } f1'[1]$$

Out[57]= -1

$$\text{In[58]:= } \text{Plot}[f1[x], \{x, -10, 10\}, \text{PlotRange} \rightarrow \{-10, 10\}]$$



$$\text{In[59]:= } f1[0]$$

$$\text{Out[59]= } -\frac{23}{9}$$

3

Q3.

a

In[60]:= $a[x_] := 2^{3x+d}$

In[63]:= $\text{Exp}[(3x + d) \text{Log}[2]]$

Out[63]= 2^{d+3x}

In[65]:= $\text{Solve}[a[y] == x, y, \text{Reals}]$

Out[65]= $\left\{ \left\{ y \rightarrow -\frac{d}{3} + \frac{\text{Log}[x]}{3 \text{Log}[2]} \text{ if } x > 0 \right\} \right\}$

In[68]:= $\text{Solve}[a[c] == c \&\& a'[c] == 1] // \text{FullSimplify}$

*** Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution

Out[68]= $\left\{ \left\{ c \rightarrow \frac{1}{\text{Log}[8]}, d \rightarrow -\frac{1 + \text{Log}[\text{Log}[8]]}{\text{Log}[2]} \right\} \right\}$

(* Note that we dont get exact solution if we try to brute force by using the inverse*)

In[69]:= $a1[x_] := -\frac{d}{3} + \frac{\text{Log}[x]}{3 \text{Log}[2]}$

In[73]:= $\text{Solve}[a[c] == a1[c] \&\& a'[c] == a1'[c], \text{Reals}] // \text{FullSimplify}$

Out[73]= $\left\{ \left\{ c \rightarrow 0.481..., d \rightarrow -\frac{1 + \text{Log}[9 \text{Log}[2]^2 \cdot 0.481...]}{\text{Log}[2]} \right\} \right\}$

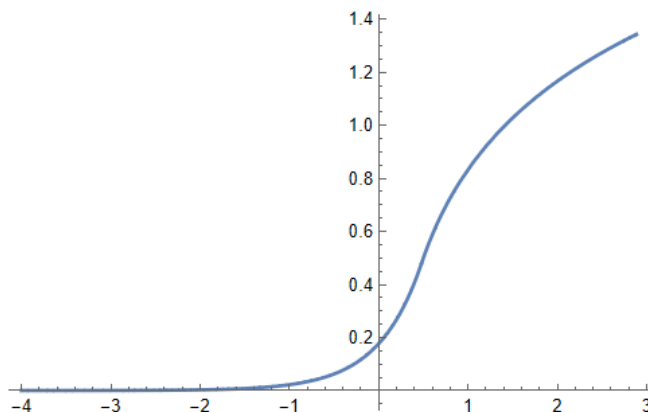
In[75]:= $b[x_] := 2^{3x - \frac{1 + \text{Log}[\text{Log}[8]]}{\text{Log}[2]}}$

In[76]:= $b1[x_] := \frac{\text{Log}[x]}{3 \text{Log}[2]} + 1/3 * \frac{1 + \text{Log}[\text{Log}[8]]}{\text{Log}[2]}$

In[94]:= $af[x_] := \text{Piecewise}[\{ \{b[x], -4 \leq x \leq 1/\text{Log}[8]\}, \{b1[x], 1/\text{Log}[8] < x < 2/\text{Log}[2]\} \}]$

In[95]:= $\text{Plot}[af[x], \{x, -4, 2/\text{Log}[2]\}]$

Out[95]=



b.

```
In[97]:= b1[ $\frac{2}{\text{Log}[2]}$ ] // FullSimplify
```

```
Out[97]=  $\frac{1 + \text{Log}\left[\frac{\text{Log}[64]}{\text{Log}[2]}\right]}{\text{Log}[8]}$ 
```

```
In[101]:= FullSimplify[ $\frac{\text{Log}\left[\frac{\text{Log}[64]}{\text{Log}[2]}\right]}{\text{Log}[8]}$ ]
```

```
Out[101]=  $\frac{\text{Log}[6]}{\text{Log}[8]}$ 
```

```
In[104]:= d[x_] := -x^3
```

```
In[110]:= NSolve[d[x] == - $\frac{1 + \text{Log}\left[\frac{\text{Log}[64]}{\text{Log}[2]}\right]}{\text{Log}[8]}$ , Reals]
```

```
Out[110]= {{x -> 1.10317}}
```

```
In[112]:= 2 / Log[2] // N
```

```
Out[112]= 2.88539
```

```
In[111]:= d[1.1031733472385987]
```

```
Out[111]= -1.34255
```

Q4.

In[143]:= `g1[x_] := x^2`

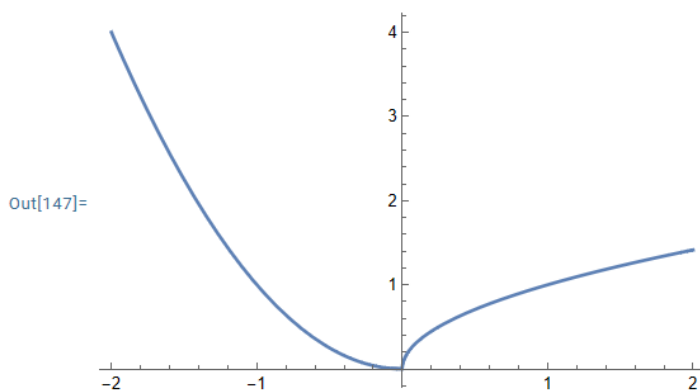
In[144]:= `g2[x_] := Sqrt[x]`

In[145]:= `g[x_] := Piecewise[{{g1[x], -Infinity < x < 0}, {g2[x], 0 ≤ x < Infinity}}]`

In[146]:= `g[x]`

Out[146]=
$$\begin{cases} x^2 & -\infty < x < 0 \\ \sqrt{x} & 0 \leq x < \infty \\ 0 & \text{True} \end{cases}$$

In[147]:= `Plot[g[x], {x, -2, 2}]`



In[148]:= `FunctionRange[{g1[x], x > 0}, x, y]`

Out[148]= `y > 0`

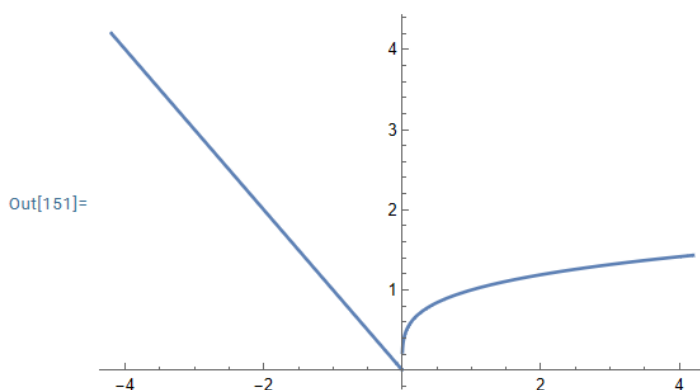
In[149]:= `FunctionRange[g2[x], x, y]`

Out[149]= `y ≥ 0`

In[150]:= `g[g[x]] // Simplify`

Out[150]=
$$\begin{cases} x & \sqrt{x} \in \mathbb{R} \ \&\& \ \sqrt{x} < 0 \ \&\& \ x \geq 0 \\ x^{1/4} & \sqrt{x} \in \mathbb{R} \ \&\& \ x \geq 0 \ \&\& \ \sqrt{x} \geq 0 \\ \sqrt{x^2} & (\sqrt{x} \in \mathbb{R} \ \&\& \ \sqrt{x} \geq 0) \mid \mid x < 0 \\ 0 & \text{True} \end{cases}$$

In[151]:= `Plot[Piecewise[{{x^(1/4), x ≥ 0}, {Sqrt[x^2], x < 0}}, 0], {x, -4.2, 4.2}]`



Section E: Casio Solutions

Question Number	Solutions
1	Part d. Define $f(x)=5e^{x/2-3}+4$ done $\frac{1}{5}(f(x)-4)$ $e^{\frac{x}{2}-3}$ Define $f1(x)=e^{\frac{x}{2}-3}$ done $f1(2x+6)$ $e^{\frac{2 \cdot x+6}{2}-3}$ simplify(ans) e^x $f1(2(x+3))$ e^x Define $q(x)=e^x$ done $-3q(-4(x+6))+2$ $-3 \cdot e^{-4 \cdot (x+6)}+2$ simplify(ans) $-3 \cdot e^{-4 \cdot x-24}+2$ solve($\frac{y-4}{5}=\frac{yone-2}{-3}$, yone $\left\{ yone=\frac{-3 \cdot y}{5}+\frac{22}{5} \right\}$ solve($\frac{x}{2}-3=-4(xone+6)$, xone $\left\{ xone=\frac{-x}{8}-\frac{21}{4} \right\}$ $-3/5f(-8(x+21/4))+22/5$ $\frac{-3 \cdot \left(5 \cdot e^{-4 \cdot \left(x+\frac{21}{4} \right)}-3 \right)+22}{5}$ simplify(ans) $-3 \cdot e^{-4 \cdot x-24}+2$

2

Define $f(x) = \frac{a \cdot x^2 + b}{x^2 - c}$

done

$f(-x)$

$$\frac{a \cdot x^2 + b}{x^2 - c}$$

$\left[\begin{array}{l} \lim_{x \rightarrow \infty} (f(x)) = 1 \\ x^2 - c = 0 \mid x = 3 \\ \frac{d}{dx} (f(x)) = -1 \mid x = 1 \end{array} \right| a, b, c$

$\{a=1, b=23, c=9\}$

$f(x) \mid \{a=1, b=23, c=9\}$

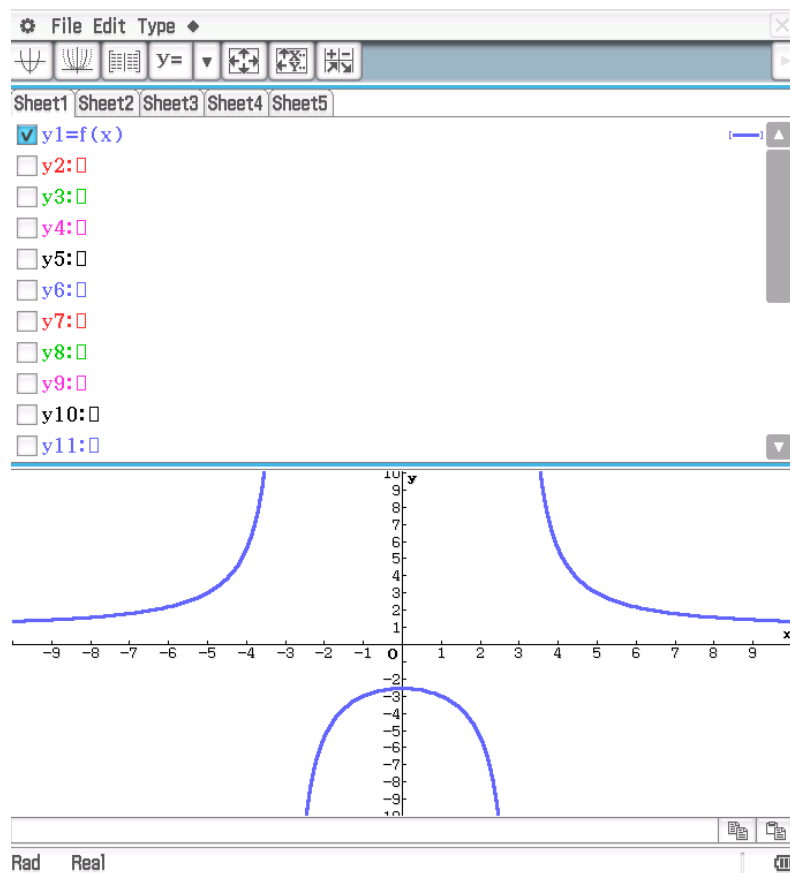
$$\frac{x^2 + 23}{x^2 - 9}$$

propFrac(ans)

$$\frac{32}{x^2 - 9} + 1$$

Define $f(x) = \frac{x^2 + 23}{x^2 - 9}$

done



3

```

Define a1(x)=23x+d
done

simplify(a1(x))
23·x+d

solve(a1(y)=x, y)
{y= $\frac{\ln(x)}{3 \cdot \ln(2)} - \frac{d}{3}$ }

Define a2(x)= $\frac{\ln(x)}{3 \cdot \ln(2)} - \frac{d}{3}$ 
done

{a1(x)=x
  $\left. \frac{d}{dx}(a1(x)) = 1 \right|_{x, d}$ 
}
{ $x=2^{\frac{-\ln(\ln(2))}{\ln(2)} - \frac{\ln(3)}{\ln(2)}}$ ,  $d=\frac{-\ln(\ln(2))}{\ln(2)} - \frac{\ln(3)}{\ln(2)} - 3 \cdot 2^{\frac{-\ln(\ln(2))}{\ln(2)} - \frac{\ln(3)}{\ln(2)}}$ }

simplify(ans)
{x= $\frac{1}{3 \cdot \ln(2)}$ ,  $d=\frac{-(\ln(\ln(2)) + \ln(3) + 1)}{\ln(2)}$ }

a2(2/ln(2)) | d= $\frac{-(\ln(\ln(2)) + \ln(3) + 1)}{\ln(2)}$ 
 $\frac{\ln(\ln(2)) + \ln(3) + 1}{3 \cdot \ln(2)} + \frac{-\ln(\ln(2)) + \ln(2)}{3 \cdot \ln(2)}$ 

simplify(ans)
 $\frac{\ln(6) + 1}{3 \cdot \ln(2)}$ 

solve(-x^3=- $\frac{\ln(6) + 1}{3 \cdot \ln(2)}$ , x)
{x=1.103173347}

- $\frac{\ln(6) + 1}{3 \cdot \ln(2)}$ 
-1.342552514

□

```

4

```

Define g1(x)=x^2
done

Define g2(x)= $\sqrt{x}$ 
done

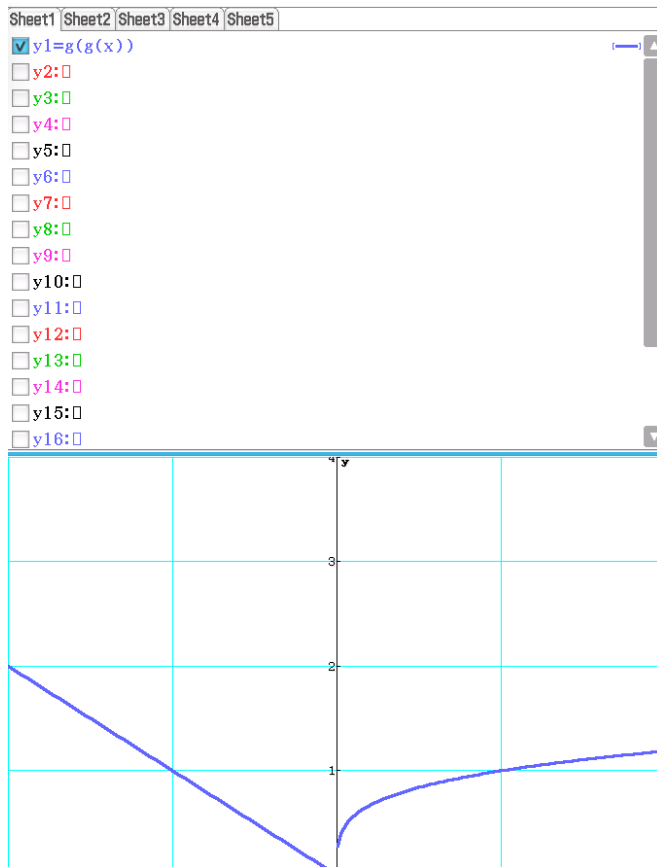
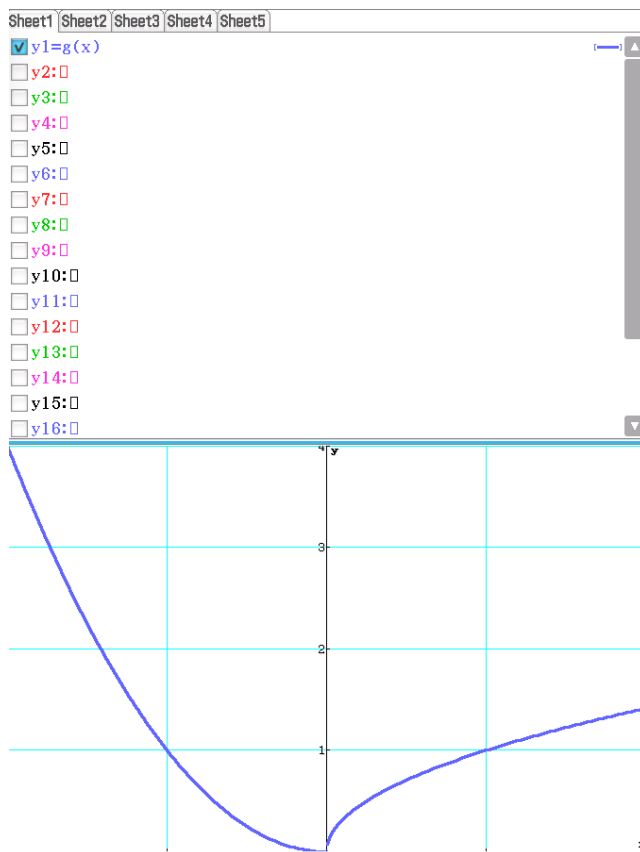
Define g(x)= $\begin{cases} x^2, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$ 
done

g(x)
Undefined

g(g(x))
Undefined

CASIO struggles – do by hand!

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Section F: TI solutions

Question Number	Solutions
1	d) The transform program can be used to check your transformations are correct. In the current version, it is not yet able to find the transformations. i) Done Define $f(x) = 5 \cdot e^{\frac{x}{2} - 3} + 4$ <code>MethodsFuncTransform(f(x), x, {y-4, y/5, x-6, x/2})</code> ► Translation -4 units along the neg. y-dir. $5 \cdot e^{\frac{x}{2} - 3}$ ► Dilation by factor of $\frac{1}{5}$ from the x-axis $e^{\frac{x}{2} - 3}$ ► Translation 6 units along the neg. x-dir. $e^{\frac{x}{2}}$ ► Dilation by factor of $\frac{1}{2}$ from the y-axis e^x ii) <code>MethodsFuncTransform(e^x, x, {-x/4 + 6, -3 * y + 2})</code> ► Dilation by factor of $\frac{1}{4}$ from the y-axis $e^{4 \cdot x}$ ► Reflection in the y-axis $e^{-4 \cdot x}$

		► Translation 6 units along the pos. x-dir. $e^{24-4 \cdot x}$ ► Dilation by factor of 3 from the x-axis $3 \cdot e^{24-4 \cdot x}$ ► Reflection in the x-axis $-3 \cdot e^{24-4 \cdot x}$ ► Translation 2 units along the pos. y-dir. $2-3 \cdot e^{24-4 \cdot x}$
2	b)	Define $f(x) = \frac{a \cdot x^2 + b}{x^2 - c}$ Done Define $df(x) = \frac{d}{dx}(f(x))$ Done expand($f(x)$) $\frac{a \cdot c}{x^2 - c} + \frac{b}{x^2 - c} + a$ Define $a=1$ Done Define $c=9$ Done $df(1)$ $\frac{-(b+9)}{32}$ solve($df(1) = -1, b$) $b=23$ Define $b=23$ Done

c,d)

```
methods_func\analyse(f(x),x)
```

▶ Start Point: $[-\infty \quad 1]$

▶ End Point: $[\infty \quad 1]$

▶ Maximal Domain:

 $x \neq -3$ and

 $x \neq 3$ and

 $-\infty < x < \infty$

▶ No x -Intercepts Found

▶ Vertical Intercept: $\left[0 \quad \frac{-23}{9} \right]$

▶ Derivative: $\frac{-64 \cdot x}{(x^2 - 9)^2}$

▶ Asymptotes: (3)

 $x = -3$ (Vertical)

 $x = 3$ (Vertical)

 $y = 1$ (Horizontal)

a)

ii) Note that the third argument in the inverse program specifies any point in the domain of the function. Since the exponential is defined and one-to-one for all real numbers, we can choose 0. In general, one must take care when choosing this point.

Define $as(x) = 2^{3 \cdot x + d}$ Done

```
methods_func\inverse(as(x),x,0)
```

$$\left\{ \frac{\ln(x) - d \cdot \ln(2)}{3 \cdot \ln(2)}, x > 0 \right.$$

Done

$$\text{expand}\left(\frac{\ln(x) - d \cdot \ln(2)}{3 \cdot \ln(2)}\right) \quad \frac{\ln(x)}{3 \cdot \ln(2)} - \frac{d}{3}$$

3

iii) We find the value of d where the function a_s touches the line $y = x$ smoothly. The point where it touches is exactly the point c .

methods_diffcalc\solve_touch(as(x),x,x,d)

- ▶ Derivative 1: $3 \cdot \ln(2) \cdot 2^d \cdot 8^x$
- ▶ Derivative 2: 1
- ▶ Equating functions and derivatives.
- ▶ Solutions:

$$x = \frac{1}{3 \cdot \ln(2)} \text{ and } d = \frac{-(\ln(3 \cdot \ln(2))) + 1}{\ln(2)}$$

b)
i)

$$\text{Define } a1(x) = \frac{\ln(x)}{3 \cdot \ln(2)} - \frac{d}{3} \mid d = \frac{-(\ln(3 \cdot \ln(2))) + 1}{\ln(2)}$$

Done

$$a1\left(\frac{2}{\ln(2)}\right) \qquad \frac{\ln(6) + 1}{3 \cdot \ln(2)}$$

ii)

$$\text{solve}\left(-x^3 = \frac{-(\ln(6) + 1)}{3 \cdot \ln(2)} \cdot x\right) \qquad x = 1.10317$$

$$-(1.10317)^3 \qquad -1.34254$$

c) The range of g is $[0, \infty)$ so only the g_2 branch of the outer g is required when computing the composition.

$$\text{Define } g1(x) = x^2 \qquad \text{Done}$$

$$\text{Define } g2(x) = \sqrt{x} \qquad \text{Done}$$

$$g2(g1(x)) \mid x < 0 \qquad -x$$

$$g2(g2(x)) \mid x \geq 0 \qquad \frac{1}{x^4}$$

4



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VCE Mathematical Methods $\frac{3}{4}$

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