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VCE Mathematical Methods ¾
SAC 1 Revision III [0.18]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
Pg / Q #: ____________ Notes:	Pg / Q #: ____________ Notes:

Section A: SAC 1 Success

Welcome to the third SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1 - 50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that, the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

Study Score = How much do you know × How much do you show

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key**!

Tutor's Comment: Explain how even if you know everything, if you cannot show in the SAC, you will get 0.

Analogy: Skipping Questions

- Let's say if you were to fight them and win, you get assigned marks.

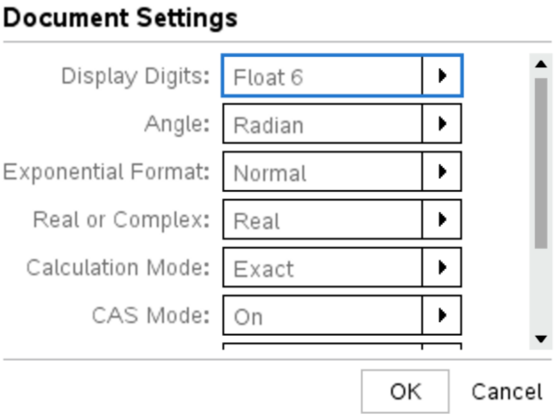
- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.

Definition

SAC Proficiency List

Before the SAC:

- Prepare your stationery, including a ruler, eraser and your mechanical pencil lead.
- Skim through the bound reference (if applicable).
- Do not speak to other people and lock in.
- TI & Mathematica Only: Check your Contour UDFS.
- TI Only: Check technology settings.



Reading Time:

- **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and use what technology commands.
- Identify questions to **skip.**

  For difficult SACs, it's not necessarily about getting the 100%. *It's about getting better than everyone else. While others are stuck on a hard* 1*-marker question, you can do an easy* 3*-marker first.*

- Identify questions to **start first** – You don't have to start from Q1!
- Look for potential pitfalls – **Units, Domain restriction of the unknown, variable and function meaning.**

Writing Time:

- **Circle what the question is asking** for in the question.
- Spend the first **50**% of the time on all the **easy questions** you identified.
- Spend the next **25**% of doing the **difficult questions** you left blank.

- Spend the last 25% of the time on **checking your answers**.
- Check your answer by reading the question again and see if you answered the question.
- Check in the order of:

$$\boldsymbol{Domino\ effect}\ (check\ a, b, c\ first) > \boldsymbol{Questions\ with\ high\ marks}\ (\mathbf{3+}) > Hard\ Questions$$

- $\mathbf{TI}$ ONLY: Use new document - **doc** $\mathbf{4, 1.}$

After the SAC:

- Think about how each mark loss can be prevented using this proficiency list.
- Think about the big picture and improve the marks -

Instead of spending 10 *minutes on* 10*c) (*1 *mark), I should have checked* 5*a) (*3 *marks)*

Space for Personal Notes

Section B: SAC Questions - Tech-Active (51 Marks)

INSTRUCTION: 51 Marks. 15 Minutes Reading. 75 Minutes Writing.

Question 1 (12 marks)

A mountain can be modelled with the function:

$$h(x) = 10xe^{-x} - 1$$

where h and x are in hundreds of metres, and h measures the height of the mountain above ground level at a given x.

Suppose b is where the right side of the mountain touches the ground.

a. It is given that $b > 2$. Estimate b as x_2, the second iteration of Newton's method, using $x_0 = 2$. Give your answer correct to two decimal places. (2 marks)

$x_{n+1} = x_n - \dfrac{h(x_n)}{h'(x_n)}$. (1M, some Newton formula).
$x_1 = 3.26109$ and $x_2 = 3.55$. (1A)

b. Find the actual value of b. Give your answer correct to two decimal places. (1 mark)

$b = 3.58$. (1A)

c. State the rule for $h'(x)$. (1 mark)

$h'(x) = 10e^{-x}(1 - x)$. (1A)

d. Hence, find the x-value(s) where the **magnitude** of the mountain's slope is $45°$. Give your answer to two decimal places. (2 marks)

We solve $h'(x) = 1 \implies x = 0.78$. (1M)
But the mountain could also be sloping downward so we solve $h'(x) = -1 \implies x = 1.41, 2.99$.
Thus all values are $0.78, 1.41, 2.99$. (1A)

e. A bridge is to be built tangentially to the mountain, starting at the point $(-2, 0)$. Find the equation of the bridge, giving your answer to two decimal places. (3 marks)

Tangent at $x = a$ has equation $t(x) = 10e^{-a}a^2 + (10e^{-a} - 10ae^{-a})\,x - 1$. (1M, or compare gradients).
Tangent passes through $(-2, 0) \implies a = 0.794828$. (1M)
Thus tangent line is $y = 1.85 + 0.93x$. (1A)

f. The council allows bridges to have an incline of at most $30°$, to the positive direction of the horizontal axis, and the bridge must meet the mountain tangentially. Determine the equation for the steepest permitted bridge, giving your answer to two decimal places. (2 marks)

$\tan(30°) = \dfrac{1}{\sqrt{3}}$. So we solve $h'(x) = \dfrac{1}{\sqrt{3}} \implies x = 0.863$. (1M)
The bridge is given by the tangent at $x = 0.863$ which is $y = 0.58x + 2.14$. (1A)

g. What is the difference in the angle of incline between the two bridges found in **parts e.** and **f.**? Give your answer to the nearest degree. (1 mark)

$$\boxed{\arctan(0.93) - 30 = 13^{\circ}. \ (1\text{A})}$$

Space for Personal Notes

Question 2 (11 marks)

The old mountain is too steep, causing hundreds of hiker casualties per year. The council tries to fix this problem by dumping a huge pile of dirt onto the side of the mountain, creating a second peak. This new mountain can be modelled with the hybrid function:

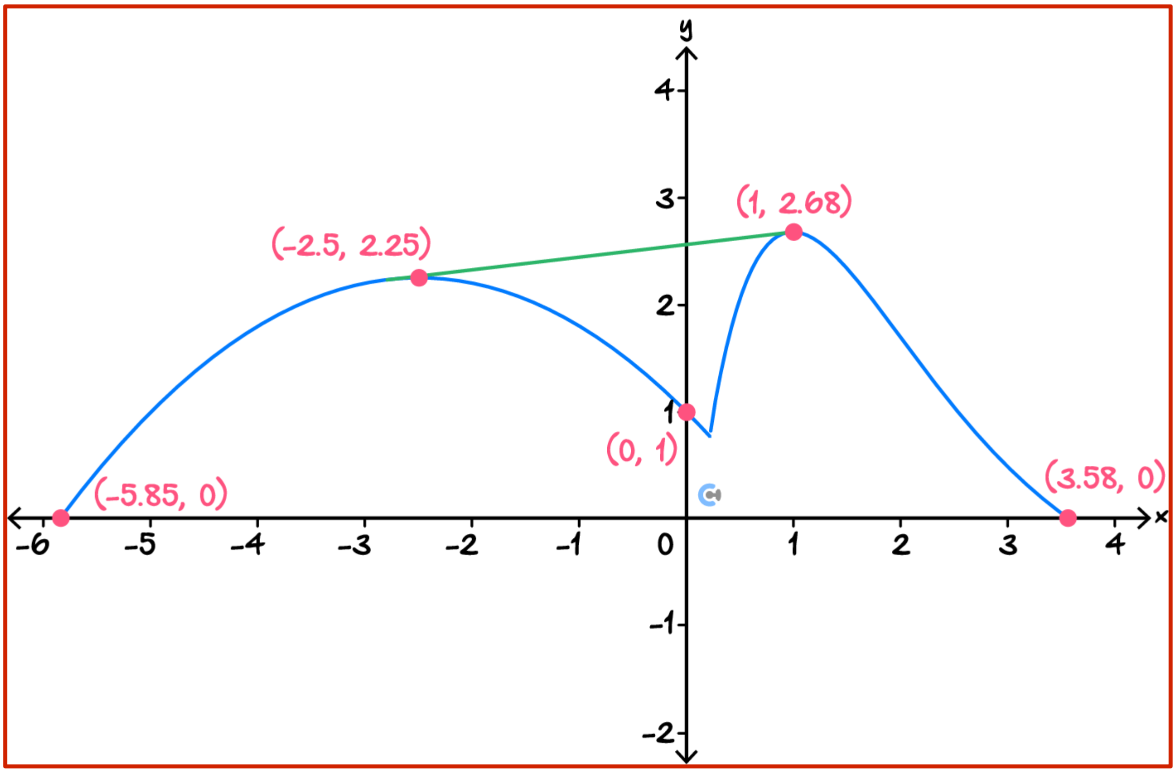
$$h(x) = \begin{cases} -\frac{1}{5}x^2 - x + 1, \text{if } 5.85 \leq x < a \\ 10xe^{-x} - 1, \text{if } a \leq x \leq b \end{cases}$$

where b is the largest x-intercept of $y = 10xe^{-x} - 1$.

a. Assume $h(x)$ is continuous. Find the value of a. Give your answer correct to 2 decimal places. (2 marks)

Let $f(x) = -\frac{1}{5}x^2 - x + 1$ and $g(x) = 10xe^{-x} - 1$.
For continuous we require that $f(x) = g(x)$. (1M).
Thus $x = 0.22065$ so $a = 0.22$. (1A)

b. Plot the graph of $h(x)$, labelling the x-intercepts, y-intercepts and turning points. Give all values to two decimal places where appropriate. (4 marks)



The council wants to build a bridge spanning over the valley between the two peaks. This bridge will lie tangential to the mountain on both ends.
Consider the two points c and d, $c < a < d$. Let c and d be the x-coordinates of where the bridge meets the mountain.

c. Write an expression for the gradient of the line connecting c and d, in terms of the function h, c, and d. (1 mark)

$$\frac{h(d) - h(c)}{d - c}. \text{ (1A)}$$

d. Hence, find c and d. Give your answers correct to 2 decimal places. (2 marks)

The following system must be satisfied

$$\frac{h(d) - h(c)}{d - c} = h'(c)$$
$$\frac{h(d) - h(c)}{d - c} = h'(d)$$

(1M for system).
Solving we get $c = -2.80, d = 0.97$. (1A)

e. Give the equation of the bridge. Give your answer to 2 decimal places. (1 mark)

$$y = 0.12x + 2.56. \text{ (1A)}$$

f. Sketch the bridge on the graph in **part b.**, assuming it ends where it meets the mountain. (1 mark)

1M straight line as shown, ending at intersections with mountain.

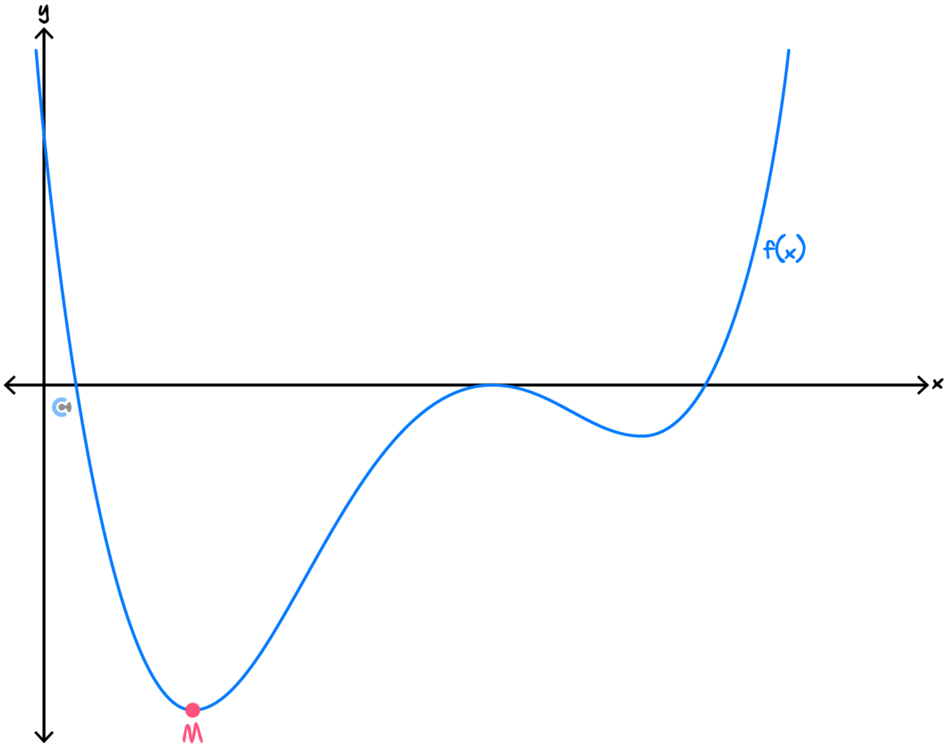
Question 3 (17 marks)

Dr. Evil is fleeing in his submarine The *Leviathan*, whose path follows a quartic function defined by:

$$f : R \to R, f(x) = (x-3)^2\,(3x^2 - 14x + 3)$$

where x and $y = f(x)$ represent coordinates in nautical units.

Dr. Evil is being pursued by the elite Poseidon Squadron, who are in their own submarine named *Justice*. Part of the graph of f is shown below.



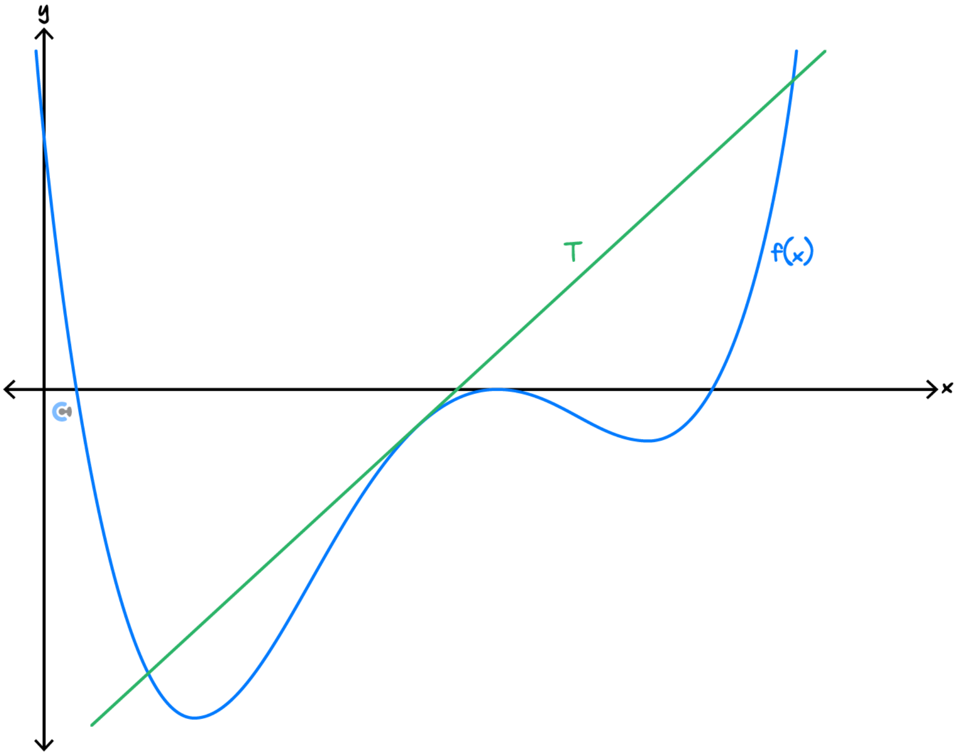
a. Find the coordinates of the point M, which is the absolute minimum of the function f. (1 mark)

$\boxed{(1, -32).\ (1\text{A})}$

b. State the values of b for which the graph of $y = f(x) + b$ has no x-intercepts. (1 mark)

$\boxed{b > 32 \text{ or equivalently } b \in (32, \infty).\ (1\text{A})}$

The *Leviathan* comes into view of the Poseidon Squadron, who immediately fires a torpedo at it. The diagram below shows the path of the torpedo and *The Leviathan*.



c. A torpedo is launched and travels along a straight path T, which is tangent to *The Leviathan's* path at $x = \frac{5}{2}$. Find the equation of T in the form $T = mx + c$. (2 marks)

$$f'\left(\frac{5}{2}\right) = \frac{27}{2}. \text{ (1M)}$$
$$T = \frac{27}{2}x - \frac{593}{16}. \text{ (1A)}$$

d. The torpedo path intersects *The Leviathan's* path at two other points. State the x-coordinates of these points in the form $\frac{a \pm \sqrt{b}}{c}$, where $a, b, c \in Z$. (2 marks)

Solve $f(x) = T(x)$. (1M)

$$x = \frac{17 \pm \sqrt{166}}{6}. \text{ (1A answer in correct form.)}$$

The *Leviathan* changes direction and follows a new path given by:

$$l(x) = 3x^2 + (k - 3)x + 5,$$

where $k \in \mathbb{R}$. The *Justice* changes course in order to intercept, following the path:

$$j(x) = k - 4x.$$

e.

i. Find the value(s) of k for which the graphs of $l(x)$ and $j(x)$ intersect exactly once. (3 marks)

$l(x) = j(x) \implies 3x^2 + (k+1)x + 5 - k$. (1M)
Consider the discriminant. $\Delta = 0$ for one solution.
$\Delta = (k+1)^2 - 12(5-k) = 0 \implies k = -7 \pm 6\sqrt{3}$. (1M, find Δ and set to 0, 1A)

ii. Hence, find the value(s) of k for which the graphs of $l(x)$ and $j(x)$ never intersect. (1 mark)

$k \in (-7 - 6\sqrt{3}, -7 + 6\sqrt{3})$. (1A)

Suppose for the following that $k = -2$.

f. Show that the gradient m of the tangent to $l(x)$ at $x = a$ is given by $m = 6a - 5$. (2 marks)

When $k = -2$ we have $l(x) = 3x^2 - 5x + 5$. (1M)
Thus $l'(x) = 6x - 5$ and so $l'(a) = 6a - 5$. (1M)

g. Find the value of a for which the gradients of both $l(x)$ and $j(x)$ are equal. (1 mark)

Solve $6a - 5 = -4 \implies a = \frac{1}{6}$. (1A)

h. At the moment, *The Leviathan* is at the point $\left(\frac{1}{6}, l\left(\frac{1}{6}\right)\right)$, and the *Justice* travels along the line:

$$p(x) = -2 - 4x.$$

Show that the distance from the *Justice* to *The Leviathan* can be expressed as: (2 marks)

$$D = \sqrt{17x^2 + \frac{149x}{3} + \frac{5629}{144}}.$$

$$\begin{aligned} D &= \sqrt{\left(\frac{1}{6} - x\right)^2 + \left(l\left(\frac{1}{6}\right) - p(x)\right)^2} \quad \text{(1M)} \\ &= \sqrt{\left(\frac{1}{6} - x\right)^2 + \left(4x + \frac{25}{4}\right)^2} \\ &= \sqrt{x^2 - \frac{x}{3} + \frac{1}{36} + 16x^2 + 50x + \frac{625}{16}} \\ &= \sqrt{17x^2 + \frac{149x}{3} + \frac{5629}{144}} \quad \text{(1M clear working leading to conclusion)} \end{aligned}$$

i. Hence, find the minimum distance between the *Justice* and the point $\left(\frac{1}{6}, l\left(\frac{1}{6}\right)\right)$. (2 marks)

Solve $D'(x) = 0 \implies x = -\frac{149}{102}$. (1M)

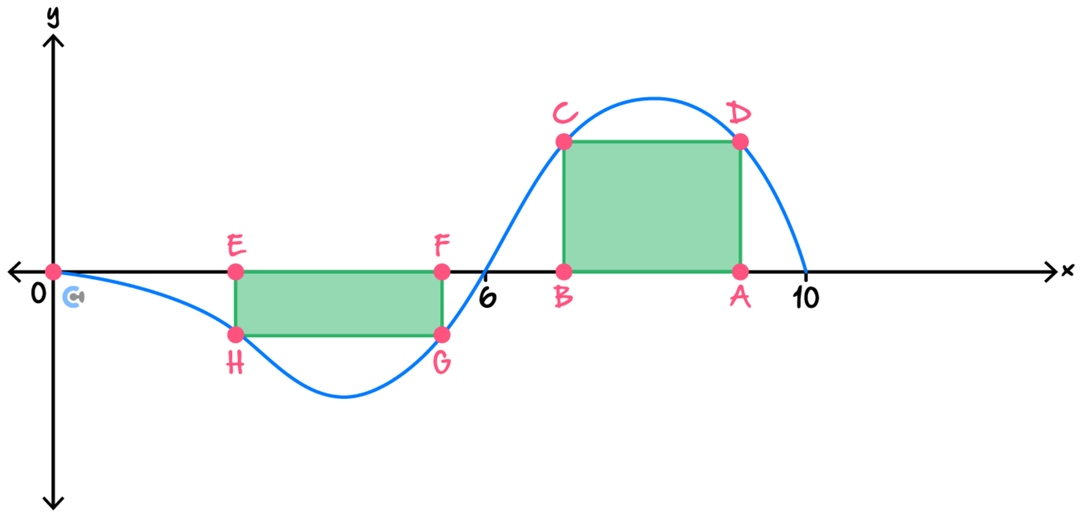
Then min distance is $D\left(-\frac{149}{102}\right) = \frac{83}{12\sqrt{17}} = \frac{83\sqrt{17}}{204}$. (1A)

Space for Personal Notes

Question 4 (11 marks)

Let

$$f(x) = \begin{cases} \frac{1}{3}x^3 - 2x^2, & 0 \le x \le 6 \\ -3x^2 + 48x - 180, & 6 < x \le 10 \end{cases}$$



A rectangle $ABCD$ is to be drawn with A and B on the x-axis and C and D on the parabola. Let $B = (x, 0)$.

a.

i. Show that the area of a rectangle $ABCD$ may be given by $A_1(x) = 6(x-10)(x-8)(x-6)$. (2 marks)

Let $h(x) = -3x^2 + 48x - 180$
Parabola has turning point when $x = 8$. (1M)
Base width $= 2(8-x)$, and height is $h(x)$.
Therefore $A_1(x) = 2(8-x)h(x) = 6(x-10)(x-8)(x-6)$. (1M)

ii. Use calculus to determine the dimensions of the rectangle when its area is a maximum and find the area. (4 marks)

$A_1'(x) = 18x^2 - 288x + 1128$. (1M)
Solve $A_1'(x) = 0 \implies x = \frac{2}{3}(12 \pm \sqrt{3})$. (1M).
Let $b = \frac{2}{3}(12 - \sqrt{3})$
Rectangle width is $2(8-b) = \frac{4}{\sqrt{3}}$.
Then height is $h(b) = 8$. (1A for height and width)
The max area is $A_1(b) = \frac{32}{\sqrt{3}}$. (1A)

b. A rectangle $EFGH$ is to be drawn with E and F on the x-axis and G and H on the cubic. Let $E = (x, 0)$ and $F = (x + h, 0)$, where $h > 0$.

i. Find an expression $A_2(x)$, in terms of x, for the area of a rectangle $EFGH$. (3 marks)

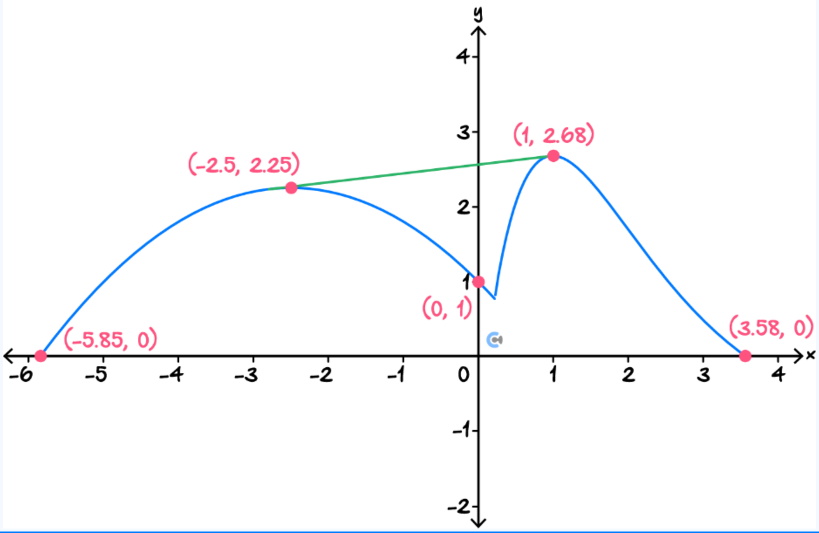
Let $g(x) = \frac{1}{3}x^3 - 2x^2$
Must have that $g(x) = g(x + h)$ and $h > 0$ (1M)
$\implies h = \frac{1}{2}\left(\sqrt{-3x^2 + 12x + 36} - 3x + 6\right)$. (1M choose correct h)
Then the height is $-g(x)$ and the base is h.
So $A_2(x) = -hg(x) = -\frac{1}{6}(x - 6)x^2\left(\sqrt{-3x^2 + 12x + 36} - 3x + 6\right)$. (1A)

ii. Determine the dimensions, correct to two decimal places, of the rectangle when its area is a maximum. (2 marks)

We solve $A_2'(x) = 0$ and sketch a graph to find a local max when $x = 2.397$. (1M)
Then sub this value into the equation for h to get $h = 2.85$.
Also $g(2.397) = -6.90$.
The dimensions are approximately 2.85 by 6.90 (1A)

Space for Personal Notes

Section C: Marking Scheme

Question Number	Solutions
1a	$x_{n+1} = x_n - \dfrac{h(x_n)}{h'(x_n)}$. (1M, some Newton formula). $x_1 = 3.26109$ and $x_2 = 3.55$. (1A)
1b	$b = 3.58$. (1A)
1c	$h'(x) = 10e^{-x}(1-x)$. (1A)
1d	We solve $h'(x) = 1 \implies x = 0.78$. (1M) But the mountain could also be sloping downward so we solve $h'(x) = -1 \implies x = 1.41, 2.99$. Thus all values are $0.78, 1.41, 2.99$. (1A)
1e	Tangent at $x = a$ has equation $t(x) = 10e^{-a}a^2 + (10e^{-a} - 10ae^{-a})x - 1$. (1M, or compare gradients). Tangent passes through $(-2, 0) \implies a = 0.794828$. (1M) Thus tangent line is $y = 1.85 + 0.93x$. (1A)
1f	$\tan(30°) = \dfrac{1}{\sqrt{3}}$. So we solve $h'(x) = \dfrac{1}{\sqrt{3}} \implies x = 0.863$. (1M) The bridge is given by the tangent at $x = 0.863$ which is $y = 0.58x + 2.14$. (1A)
1g	$\arctan(0.93) - 30 = 13°$. (1A)
2a	Let $f(x) = -\dfrac{1}{5}x^2 - x + 1$ and $g(x) = 10xe^{-x} - 1$. For continuous we require that $f(x) = g(x)$. (1M). Thus $x = 0.22065$ so $a = 0.22$. (1A)
2b	

2c	$\frac{h(d)-h(c)}{d-c}$. (1A)
2d	The following system must be satisfied $$\frac{h(d)-h(c)}{d-c}=h'(c)$$ $$\frac{h(d)-h(c)}{d-c}=h'(d)$$ (1M for system). Solving we get $c=-2.80, d=0.97$. (1A)
2e	$y=0.12x+2.56$. (1A)
2f	1M straight line as shown, ending at intersections with mountain.
3a	$(1,-32)$. (1A)
3b	$b>32$ or equivalently $b\in(32,\infty)$. (1A)
3c	$f'(\frac{5}{2})=\frac{27}{2}$. (1M) $T=\frac{27}{2}x-\frac{593}{16}$. (1A)
3d	Solve $f(x)=T(x)$. (1M) $x=\frac{17\pm\sqrt{166}}{6}$. (1A answer in correct form.)
3ei	$l(x)=j(x) \implies 3x^2+(k+1)x+5-k$. (1M) Consider the discriminant. $\Delta=0$ for one solution. $\Delta=(k+1)^2-12(5-k)=0 \implies k=-7\pm6\sqrt{3}$. (1M, find Δ and set to 0, 1A)
3eii	$k\in(-7-6\sqrt{3},-7+6\sqrt{3})$. (1A)
3f	When $k=-2$ we have $l(x)=3x^2-5x+5$. (1M) Thus $l'(x)=6x-5$ and so $l'(a)=6a-5$. (1M)
3g	Solve $6a-5=-4 \implies a=\frac{1}{6}$. (1A)
3h	$$\begin{aligned} D&=\sqrt{\left(\frac{1}{6}-x\right)^2+\left(l\left(\frac{1}{6}\right)-p(x)\right)^2} \quad \text{(1M)}\\ &=\sqrt{\left(\frac{1}{6}-x\right)^2+\left(4x+\frac{25}{4}\right)^2}\\ &=\sqrt{x^2-\frac{x}{3}+\frac{1}{36}+16x^2+50x+\frac{625}{16}}\\ &=\sqrt{17x^2+\frac{149x}{3}+\frac{5629}{144}} \quad \text{(1M clear working leading to conclusion)}\end{aligned}$$
3i	Solve $D'(x)=0 \implies x=-\frac{149}{102}$. (1M) Then min distance is $D\left(-\frac{149}{102}\right)=\frac{83}{12\sqrt{17}}=\frac{83\sqrt{17}}{204}$. (1A)

4ai	Let $h(x) = -3x^2 + 48x - 180$ Parabola has turning point when $x = 8$. (1M) Base width $= 2(8 - x)$, and height is $h(x)$. Therefore $A_1(x) = 2(8 - x)h(x) = 6(x - 10)(x - 8)(x - 6)$. (1M)
4aii	$A_1'(x) = 18x^2 - 288x + 1128$. (1M) Solve $A_1'(x) = 0 \implies x = \frac{2}{3}\left(12 \pm \sqrt{3}\right)$. (1M). Let $b = \frac{2}{3}\left(12 - \sqrt{3}\right)$ Rectangle width is $2(8 - b) = \frac{4}{\sqrt{3}}$. Then height is $h(b) = 8$. (1A for height and width) The max area is $A_1(b) = \frac{32}{\sqrt{3}}$. (1A)
4bi	Let $g(x) = \frac{1}{3}x^3 - 2x^2$ Must have that $g(x) = g(x + h)$ and $h > 0$ (1M) $\implies h = \frac{1}{2}\left(\sqrt{-3x^2 + 12x + 36} - 3x + 6\right)$. (1M choose correct h) Then the height is $-g(x)$ and the base is h. So $A_2(x) = -hg(x) = -\frac{1}{6}(x - 6)x^2\left(\sqrt{-3x^2 + 12x + 36} - 3x + 6\right)$. (1A)
4bii	We solve $A_2'(x) = 0$ and sketch a graph to find a local max when $x = 2.397$. (1M) Then sub this value into the equation for h to get $h = 2.85$. Also $g(2.397) = -6.90$. The dimensions are approximately 2.85 by 6.90 (1A)

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Section D: Mathematica Solutions

Question Number	Solutions

1	**Q1.**
	In[1]:= `h[x_] := 10 x Exp[-x] - 1`
	
	a.
	In[2]:= $x - \frac{h[x]}{h'[x]}$ // FullSimplify
	Out[2]= $\frac{e^x - 10x^2}{10 - 10x}$
	In[3]:= $n[x\_] := \frac{e^x - 10x^2}{10 - 10x}$
	In[4]:= `n[2.0]`
	Out[4]= 3.26109
	In[5]:= `n[n[2.0]]`
	Out[5]= 3.55002
	
	b.
	In[6]:= `Solve[h[x] == 0, x] // N`
	Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.
	Out[6]= {{x → 0.111833}, {x → 3.57715}}
	
	c.
	In[7]:= `h'[x] // Factor`
	Out[7]= $-10 e^{-x} (-1 + x)$
	
	d.
	In[8]:= `NSolve[h'[x] == 1, x, Reals]`
	Out[8]= {{x → 0.781521}}
	In[9]:= `NSolve[h'[x] == -1, x, Reals]`
	Out[9]= {{x → 1.40932}, {x → 2.99145}}
	
	e.
	In[10]:= `TangentLine[h[x], x, a]`
	Out[10]= $-1 + 10 a^2 e^{-a} + (10 e^{-a} - 10 a e^{-a}) x$
	In[11]:= $-1 + 10 a^2 e^{-a} + (10 e^{-a} - 10 a e^{-a}) x$ /. x → -2
	Out[11]= $-1 + 10 a^2 e^{-a} - 2 (10 e^{-a} - 10 a e^{-a})$
	In[12]:= NSolve[$-1 + 10 a^2 e^{-a} - 2 (10 e^{-a} - 10 a e^{-a}) == 0$, a, Reals]
	Out[12]= {{a → -2.73393}, {a → 0.794828}, {a → 6.18742}}
	In[13]:= `TangentLine[h[x], x, 0.79482754426972]`
	Out[13]= 1.85336 + 0.92668 x

f.

```
In[14]:= NSolve[h'[x] == 1/√3, x, Reals]
Out[14]= {{x → 0.863135}}

In[15]:= TangentLine[h[x], x, 0.8631345180721505`]
Out[15]= 2.1427 + 0.57735 x
```

g.

```
In[16]:= ArcTan[0.9266801068229706] / Degree // N
Out[16]= 42.8207

In[17]:= 42.820658305600176 - 30
Out[17]= 12.8207
```

2

Q2.

```
In[18]:= f[x_] := -1 / 5 x^2 - x + 1
In[19]:= g[x_] := 10 x Exp[-x] - 1
```

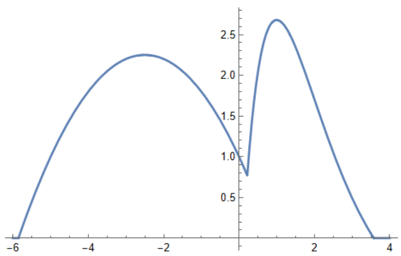
a.

```
In[20]:= Solve[f[x] == g[x], x, Reals] // N
Out[20]= {{x → 0.220651}}

In[21]:= Solve[g[x] == 0, x, Reals] // N
Out[21]= {{x → 0.111833}, {x → 3.57715}}
```

b.

```
In[22]:= h[x_] := Piecewise[{{f[x], -5.85 ≤ x < 0.22065076394834193}, {g[x], 0.220651 ≤ x ≤ 3.577152063957297}}]
In[23]:= Plot[h[x], {x, -6, 4}]
Out[23]=
```




```
In[24]:= Reduce[h'[x] == 0, x] // N
Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.
Out[24]= x == 1. || x == -2.5 || 0.220651 < x < 0.220651 || x < -5.85 || x > 3.57715

In[25]:= h[1.0]
Out[25]= 2.67879

In[26]:= h[-2.5]
Out[26]= 2.25
```

d.

In[27]:= `NSolve[(g[d] - f[c])/(d - c) == f'[c] && (g[d] - f[c])/(d - c) == g'[d], Reals]`

Out[27]= $\{\{c \to -2.79516,\ d \to 0.96889\}\}$

e.

In[28]:= `TangentLine[f[x], x, -2.795161271047774]`

Out[28]= $2.56259 + 0.118065\,x$

f.

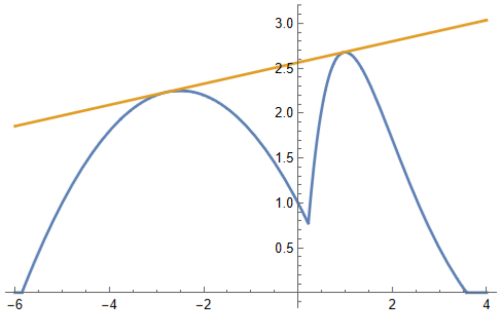
In[29]:= `b[x_] := 2.562585306233082` + 0.11806450841910965` x`

In[30]:= `b[1]`

Out[30]= 2.68065

In[31]:= `Plot[{h[x], b[x]}, {x, -6, 4}]`

Out[31]=



3

Q3.

a

In[32]:= `f[x_] := (x - 3)^2 (3 x^2 - 14 x + 3)`

In[33]:= `Minimize[f[x], x]`

Out[33]= $\{-32,\ \{x \to 1\}\}$

b

In[34]:= `Reduce[-32 + b > 0, b]`

Out[34]= $b > 32$

c

In[35]:= `f'[5/2]`

Out[35]= $\frac{27}{2}$

In[36]:= `TangentLine[f[x], x, 5/2]`

Out[36]= $-\frac{593}{16} + \frac{27\,x}{2}$

d

In[37]:= `Solve[-593/16 + 27 x/2 == f[x], x]`

Out[37]= $\left\{\left\{x \to \frac{5}{2}\right\},\ \left\{x \to \frac{5}{2}\right\},\ \left\{x \to \frac{1}{6}\left(17 - \sqrt{166}\right)\right\},\ \left\{x \to \frac{1}{6}\left(17 + \sqrt{166}\right)\right\}\right\}$

e.

```
In[38]:= l[x_] := 3 x^2 + (k - 3) x + 5

In[39]:= j[x_] := k - 4 x

In[40]:= l[x] - j[x]
```

Out[40]= $5 - k + 4x + (-3 + k)\,x + 3x^2$

```
In[41]:= Collect[l[x] - j[x], x]
```

Out[41]= $5 - k + (1 + k)\,x + 3x^2$

```
In[42]:= Discriminant[l[x] - j[x], x]
```

Out[42]= $-59 + 14k + k^2$

```
In[43]:= Solve[Discriminant[l[x] - j[x], x] == 0, k]
```

Out[43]= $\{\{k \to -7 - 6\sqrt{3}\}, \{k \to -7 + 6\sqrt{3}\}\}$

```
In[44]:= Reduce[Discriminant[l[x] - j[x], x] < 0, k]
```

Out[44]= $-7 - 6\sqrt{3} < k < -7 + 6\sqrt{3}$

f.

```
In[45]:= l[x] /. k → -2
```

Out[45]= $5 - 5x + 3x^2$

```
In[46]:= TangentLine[l[x] /. k → -2, x, a]
```

Out[46]= $5 - 3a^2 + (-5 + 6a)\,x$

g.

```
In[47]:= Solve[6 a - 5 == -4]
```

Out[47]= $\left\{\left\{a \to \frac{1}{6}\right\}\right\}$

h.

In[49]:= `p[x_] := -2 - 4 x`

In[50]:= `l[x] /. k → -2`

Out[50]= $5 - 5x + 3x^2$

In[51]:= `l1[x_] := 5 - 5 x + 3 x^2`

In[52]:= `EuclideanDistance[{1/6, l1[1/6]}, {x, p[x]}]`

Out[52]= $\sqrt{\text{Abs}\left[\frac{1}{6} - x\right]^2 + \text{Abs}\left[\frac{25}{4} + 4x\right]^2}$

In[53]:= `√((1/6 - x)^2 + (l1[1/6] - p[x])^2)`

Out[53]= $\sqrt{\left(\frac{1}{6} - x\right)^2 + \left(\frac{25}{4} + 4x\right)^2}$

In[54]:= $\left(\frac{1}{6} - x\right)^2$ // Expand

Out[54]= $\frac{1}{36} - \frac{x}{3} + x^2$

In[55]:= $\left(\frac{25}{4} + 4x\right)^2$ // Expand

Out[55]= $\frac{625}{16} + 50x + 16x^2$

In[56]:= Expand$\left[\left(\frac{1}{6} - x\right)^2 + \left(\frac{25}{4} + 4x\right)^2\right]$

Out[56]= $\frac{5629}{144} + \frac{149x}{3} + 17x^2$

i.

In[57]:= $d[x\_] := \sqrt{\frac{5629}{144} + \frac{149x}{3} + 17x^2}$

In[58]:= `Solve[d'[x] == 0, x]`

Out[58]= $\left\{\left\{x \to -\frac{149}{102}\right\}\right\}$

In[59]:= `d[-149/102]`

Out[59]= $\frac{83}{12\sqrt{17}}$

In[60]:= `Minimize[d[x], x]`

Out[60]= $\left\{\frac{83}{12\sqrt{17}}, \left\{x \to -\frac{149}{102}\right\}\right\}$

In[61]:= `12 * 17`

Out[61]= 204

In[62]:= $\frac{83\sqrt{17}}{204}$

Out[62]= $\frac{83}{12\sqrt{17}}$

4

Q4.

In[63]:= `g[x_] := 1/3 x^3 - 2 x^2`

In[64]:= `h[x_] := -3 x^2 + 48 x - 180`

---

a.

In[65]:= `Solve[h'[x] == 0, x]`

Out[65]= $\{\{x \to 8\}\}$

In[66]:= `2 (8 - x) * h[x] // FullSimplify`

Out[66]= $6(-10 + x)(-8 + x)(-6 + x)$

In[67]:= `a1[x_] := 6 (-10 + x) (-8 + x) (-6 + x)`

In[68]:= `a1'[x] // Expand`

Out[68]= $1128 - 288x + 18x^2$

In[69]:= `Solve[1128 - 288 x + 18 x^2 == 0, x]`

Out[69]= $\left\{\left\{x \to \frac{2}{3}\left(12 - \sqrt{3}\right)\right\}, \left\{x \to \frac{2}{3}\left(12 + \sqrt{3}\right)\right\}\right\}$

In[70]:= `Solve[1128 - 288 x + 18 x^2 == 0, x] // N`

Out[70]= $\{\{x \to 6.8453\}, \{x \to 9.1547\}\}$

In[71]:= $2 * \left(8 - \frac{2}{3}\left(12 - \sqrt{3}\right)\right)$ `// FullSimplify`

Out[71]= $\frac{4}{\sqrt{3}}$

In[72]:= $h\left[\frac{2}{3}\left(12 - \sqrt{3}\right)\right]$ `// FullSimplify`

Out[72]= 8

In[73]:= $a1\left[\frac{2}{3}\left(12 - \sqrt{3}\right)\right]$ `// FullSimplify`

Out[73]= $\frac{32}{\sqrt{3}}$

In[74]:= `Maximize[{a1[x], 6 ≤ x ≤ 8}, x] // FullSimplify`

Out[74]= $\left\{\frac{32}{\sqrt{3}}, \left\{x \to 8 - \frac{2}{\sqrt{3}}\right\}\right\}$

In[75]:= $8 - \frac{2}{\sqrt{3}}$ `// N`

Out[75]= 6.8453

b.

In[76]:= g[x]

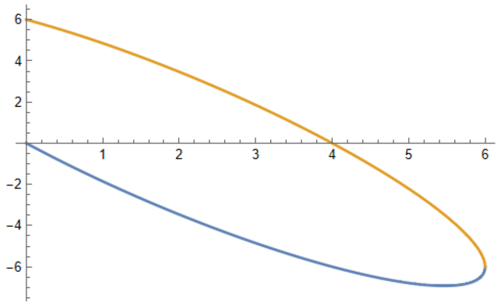
Out[76]= $-2x^2 + \frac{x^3}{3}$

In[77]:= Solve[g[x] == g[x + h], h] // FullSimplify

Out[77]= $\left\{\{h \to 0\}, \left\{h \to \frac{1}{2}\left(6 - 3x - \sqrt{36 + 12x - 3x^2}\right)\right\}, \left\{h \to \frac{1}{2}\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)\right\}\right\}$

In[78]:= $\text{Plot}\left[\left\{\frac{1}{2}\left(6 - 3x - \sqrt{36 + 12x - 3x^2}\right), \frac{1}{2}\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)\right\}, \{x, 0, 6\}\right]$

Out[78]=



In[79]:= $-\frac{1}{2}\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right) * g[x]$ // FullSimplify

Out[79]= $-\frac{1}{6}(-6 + x)\,x^2\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)$

In[80]:= $\text{hh}[x\_] := \frac{1}{2}\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)$

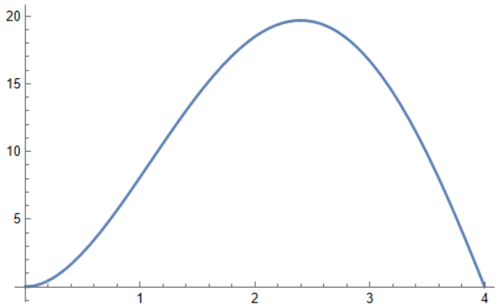
In[81]:= $\text{a2}[x\_] := -\frac{1}{6}(-6 + x)\,x^2\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)$

In[82]:= a2[x] // FullSimplify

Out[82]= $-\frac{1}{6}(-6 + x)\,x^2\left(6 - 3x + \sqrt{36 + 12x - 3x^2}\right)$

In[83]:= Plot[a2[x], {x, 0, 4}]

Out[83]=



In[84]:= Solve[a2'[x] == 0, x] // N

Out[84]= {{x → 0.}, {x → 5.24814}, {x → -1.98019}, {x → 2.39761}}

In[85]:= hh[2.397613211838814]

Out[85]= 2.85052

In[86]:= a2[2.397613211838814`]

Out[86]= 19.6767

In[87]:= g[2.397613211838814`]

Out[87]= -6.90283

In[88]:= 2.85 * 6.90

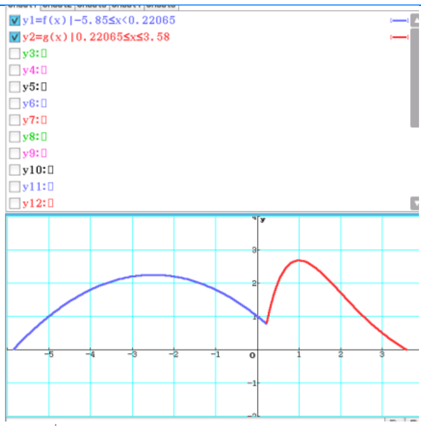
Out[88]= 19.665

Section E: Casio Solutions

Question Number	Solutions
1	a. Define $h(x)=10x*e^{-x}-1$ → done Define $n(x)=x-\frac{h(x)}{\frac{d}{dx}(h(x))}$ → done n(2) → 3.26109439 n(3.26109439) → 3.55002009 b. solve(h(x)=0, x → {x=0.1118325592, x=3.577152064} c. $\frac{d}{dx}(h(x))$ → $-(10\cdot x-10)\cdot e^{-x}$ d. solve($\frac{d}{dx}$(h(x))=tan(45°), x → {x=0.7815207694} solve($\frac{d}{dx}$(h(x))=tan(−45°), x → {x=1.409315108, x=2.991446203} e. tanline(h(x), x, a) → $10\cdot a\cdot e^{-a}-x\cdot(10\cdot a\cdot e^{-a}-10\cdot e^{-a})+a\cdot(10\cdot a\cdot e^{-a}-10\cdot e^{-a})-1$ solve(ans=0 | x=−2, a) → {a=−2.733925137, a=0.7948275443, a=6.18742285} $10\cdot a\cdot e^{-a}-x\cdot(10\cdot a\cdot e^{-a}-10\cdot e^{-a})+a\cdot(10\cdot a\cdot e^{-a}-10\cdot e^{-a})-1$ | a=0.7948275 → 0.9266801067·x+1.853360214 f. solve($\frac{d}{dx}$(h(x))=tan(30°), x → {x=0.8631345181} tanline(h(x), x, 0.8631345181) → 0.5773502691·x+2.142696282 g. $\tan^{-1}$(0.9266801067)−30 → 12.8206583
2	a. Define $f(x)=\frac{-1}{5}x\text{^}2-x+1$ → done Define $g(x)=10xe^{-x}-1$ → done solve(f(x)=g(x), x → {x=0.2206507639}

<table>
<tr><td></td><td>

b.



d.

$$\left\{\begin{array}{l}\frac{g(d)-f(c)}{d-c}=\frac{d}{dx}(f(x))\,|x=c\\ \frac{g(d)-f(c)}{d-c}=\frac{d}{dx}(g(x))\,|x=d\end{array}\right|_{c,\,d}$$

{{c=−2.795161271, d=0.9688897963}, {c=−1.436133888, d=4.37267▶

$\frac{d}{dx}(f(x))\,|x=-2.795161271$

0.1180645084

e.

solve(y−f(−2.795161271)=0.1180645084(x−−2.795161271), y

{y=0.1180645084·x+2.562585306}

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<tr><td>3</td><td>

a.

Define f(x)=(x−3)^2(3x^2−14x+3)

done

fmin(f(x), x)

{MinValue=−32, x=1}

c.

$\frac{d}{dx}(f(x))\,|x=5/2$

$\frac{27}{2}$

tanline(f(x), x, 5/2)

$\frac{27\cdot x}{2}-\frac{593}{16}$

d.

solve$\left(f(x)=\frac{27\cdot x}{2}-\frac{593}{16}, x\right.$

$\left\{x=\frac{5}{2}, x=\frac{-\sqrt{166}}{6}+\frac{17}{6}, x=\frac{\sqrt{166}}{6}+\frac{17}{6}\right\}$

e. i.

Define l(x)=3x^2+(k−3)*x+5

done

Define j(x)=k−4x

done

solve(l(x)=j(x), x

$\left\{x=\frac{-\left(k-\sqrt{k^2+14\cdot k-59}+1\right)}{6}, x=\frac{-\left(k+\sqrt{k^2+14\cdot k-59}+1\right)}{6}\right\}$

solve($k^2+14\cdot k-59=0$, k

$\{k=-6\cdot\sqrt{3}-7, k=6\cdot\sqrt{3}-7\}$

e. ii.

solve($k^2+14\cdot k-59<0$, k

$\{-6\cdot\sqrt{3}-7<k<6\cdot\sqrt{3}-7\}$

f.

l(x)|k=−2

$3\cdot x^2-5\cdot x+5$

$\frac{d}{dx}(l(x))\,|x=a\,|k=-2$

$6\cdot a-5$

g.

solve$\left(\frac{d}{dx}(l(x))=\frac{d}{dx}(j(x))\,|k=-2, x\right)$

$\left\{x=\frac{1}{6}\right\}$

</td></tr>
</table>

	h. Define p(x)=−2−4x done Define D(x)=$\sqrt{(1/6-x)^2+(l(1/6)-p(x))^2}$ | k=−2 done solve($\frac{d}{dx}$(D(x))=0, x $\left\{x=-\frac{149}{102}\right\}$ D(x) $\frac{\sqrt{2448\cdot x^2+7152\cdot x+5629}}{12}$ fmin(D(x), x) $\left\{\text{MinValue}=\frac{83\cdot\sqrt{17}}{204}, x=-\frac{149}{102}\right\}$
4	a. i. Define h(x)=−3x^2+48x−180 done fmax(h(x), x {MaxValue=12, x=8} Define A1(x)=2(8−x)*h(x) done A1(x) $2\cdot(x-8)\cdot(3\cdot x^2-48\cdot x+180)$ simplify(ans) $6\cdot(x-6)\cdot(x-8)\cdot(x-10)$ a. ii. $\frac{d}{dx}$(A1(x)) $18\cdot x^2-288\cdot x+1128$ solve($\frac{d}{dx}$(A1(x))=0, x $\left\{x=\frac{-2\cdot\sqrt{3}}{3}+8, x=\frac{2\cdot\sqrt{3}}{3}+8\right\}$ $\frac{-2\cdot\sqrt{3}}{3}+8 \Rightarrow b$ $\frac{-2\cdot\sqrt{3}}{3}+8$ 2(8−b) $\frac{4\cdot\sqrt{3}}{3}$ h(b) $-3\cdot\left(\frac{-2\cdot\sqrt{3}}{3}+8\right)^2+48\cdot\left(\frac{-2\cdot\sqrt{3}}{3}+8\right)-180$ simplify(ans) 8 A1(b) $\frac{4\cdot\sqrt{3}\cdot\left(-3\cdot\left(\frac{-2\cdot\sqrt{3}}{3}+8\right)^2+48\cdot\left(\frac{-2\cdot\sqrt{3}}{3}+8\right)-180\right)}{3}$ simplify(ans) $\frac{32\cdot\sqrt{3}}{3}$

Define g(x)=1/3x^3-2x^2

done

solve(g(x)=g(x+h)|h>0,h

$$\left\{h=\frac{-\left(3\cdot x-\sqrt{3}\cdot\sqrt{-x^2+4\cdot x+12}-6\right)}{2}, h=\frac{-\left(3\cdot x+\sqrt{3}\cdot\sqrt{-x^2+4\cdot x+12}-6\right)}{2}\right\}$$

Define $h(x)=\frac{-\left(3\cdot x-\sqrt{3}\cdot\sqrt{-x^2+4\cdot x+12}-6\right)}{2}$

done

Define A2(x)=-h(x)*g(x)

done

A2(x)

$$\frac{\left(\frac{x^3}{3}-2\cdot x^2\right)\cdot\left(3\cdot x-\sqrt{-3\cdot\left(x^2-4\cdot x-12\right)}-6\right)}{2}$$

simplify(ans)

$$\frac{x^2\cdot(x-6)\cdot\left(3\cdot x-\sqrt{-3\cdot x^2+12\cdot x+36}-6\right)}{6}$$

b. i.

solve($\frac{d}{dx}$(A2(x))=0,x

{x=-1.980188907,x=0,x=2.397613212,x=5.248138099}

h(2.397613212)

2.850524887

g(2.397613212)

-6.90283246

b. ii.

Space for Personal Notes

Section F: TI Solutions

Question Number	Solutions
1	a) Define $h(x)=10\cdot x\cdot e^{-x}-1$ *Done* $methods\_diffcalc\backslash newtons\_method(h(x),x,2)$ ▸ Derivative: $(10-10\cdot x)\cdot e^{-x}$ ▸ Iterative Formula: $\dfrac{10\cdot x^2-e^x}{10\cdot (x-1)}$ ▸ Number of Iterations: 2 $\begin{bmatrix} \text{"n"} & \text{"xn"} & \text{"\|xn-xn-1\|"} & \text{"f(xn)"} & \text{"f'(xn)"} \\ 0. & 2. & \_ & 1.70671 & -1.35335 \\ 1. & 3.26109 & 1.26109 & 0.250513 & -0.867049 \\ 2. & 3.55002 & 0.288926 & 0.01971 & -0.732469 \end{bmatrix}$ b) $\text{zeros}(h(x),x)\|x>2$ $\{3.57715\}$ c) Define $dh(x)=\frac{d}{dx}(h(x))$ *Done* $dh(x)$ $(10-10\cdot x)\cdot e^{-x}$ d) $\text{solve}(dh(x)=1,x)$ $x=0.781521$ $\text{solve}(dh(x)=-1,x)$ $x=1.40932$ or $x=2.99145$

e)

$\text{Define } t(x)=\text{tangentLine}(h(x),x,a)$ Done

$t(x)$ $\quad -10\cdot (a-1)\cdot e^{-a}\cdot x-(e^{a}-10\cdot a^{2})\cdot e^{-a}$

$\text{solve}(t(-2)=0,a)$

$a=-2.73393 \text{ or } a=0.794828 \text{ or } a=6.18742$

$t(x)|a=0.794828$ $\quad 0.926678\cdot x+1.85336$

f)

Compute Tangent Lines? 1

OK Cancel

$methods\_diffcalc\backslash solve\_grad\left(h(x),x,\tan\left(\frac{\pi}{6}\right)\right)$

- Maximal Domain: $-\infty<x<\infty$
- Derivative: $(10-10\cdot x)\cdot e^{-x}$
- Found point with gradient $\frac{\sqrt{3}}{3}$:

$$\begin{bmatrix} \text{"x"} & \text{"y"} & \text{"Eqn."} & \text{"x−Int."} & \text{"y−Int."} \\ 0.863135 & 2.64103 & 0.57735\cdot x+2.1427 & -3.71126 & 2.1427 \end{bmatrix}$$

g)

$\tan^{-1}(0.93)-\frac{\pi}{6}$ $\quad 0.225546$

$\frac{0.22554584900771\cdot 180}{\pi}$ $\quad 12.9228$

2	a) $\text{Define } h2(x)=\frac{-x^2}{5}-x+1$ *Done* $\text{solve}(h(a)=h2(a),a)$ $a=0.220651$ d) $\text{Define } g(c,d)=\frac{h(d)-h2(c)}{d-c}$ *Done* $\text{Define } dh2(x)=\frac{d}{dx}(h2(x))$ *Done* $\text{solve}(g(c,d)=dh2(c) \text{ and } g(c,d)=dh(d),c,d)$ $c=-2.79516 \text{ and } d=0.96889 \text{ or } c=-1.4361$ e) $\text{tangentLine}(h(x),x,0.96889)$ $0.118064\cdot x+2.56259$

3	a) Define $f(x)=(x-3)^2\cdot(3\cdot x^2-14\cdot x+3)$ *Done* $methods\_func\backslash analyse(f(x),x)$ ▶ Stationary Points: (3) $\begin{bmatrix}1 & -32\end{bmatrix}$ (Local min.) $\begin{bmatrix}3 & 0\end{bmatrix}$ (Local max.) $\begin{bmatrix}4 & -5\end{bmatrix}$ (Local min.) Note: Some output has been omitted. c) $methods\_diffcalc\backslash tangent\_line\left(f(x),x,\frac{5}{2}\right)$ ▶ Derivative: $12\cdot(x-3)\cdot(x^2-5\cdot x+4)$ ▶ Gradient: $\frac{27}{2}$ ▶ Passes Through: $\begin{bmatrix}\frac{5}{2} & \frac{-53}{16}\end{bmatrix}$ ▶ x −Intercept: $\begin{bmatrix}\frac{593}{216} & 0\end{bmatrix}$ ▶ Vertical Intercept: $\begin{bmatrix}0 & \frac{-593}{16}\end{bmatrix}$ ▶ Tangent Line: $\frac{27\cdot x}{2}-\frac{593}{16}$

d)

Define $t(x)=\frac{27\cdot x}{2}-\frac{593}{16}$ *Done*

$methods\_func\backslash intersect(f(x),t(x),x)$

▶ Intersection Points: (3)

$$\begin{bmatrix} \frac{-(\sqrt{166}-17)}{6} & \frac{19}{16}-\frac{9\cdot\sqrt{166}}{4} \end{bmatrix}$$
$$\begin{bmatrix} \frac{5}{2} & \frac{-53}{16} \end{bmatrix}$$
$$\begin{bmatrix} \frac{\sqrt{166}+17}{6} & \frac{9\cdot\sqrt{166}}{4}+\frac{19}{16} \end{bmatrix}$$

e)
i)

Define $l(x)=3\cdot x^2+(k-3)\cdot x+5$ *Done*

Define $j(x)=k-4\cdot x$ *Done*

$\text{solve}(l(x)=j(x),x)$

$$x=\frac{\sqrt{k^2+14\cdot k-59}-k-1}{6} \text{ or } x=\frac{-(\sqrt{k^2+14\cdot k-5}}{6}$$

$\text{solve}(k^2+14\cdot k-59=0,k)$

$$k=-(6\cdot\sqrt{3}+7) \text{ or } k=6\cdot\sqrt{3}-7$$

ii)

$\text{solve}(k^2+14\cdot k-59<0,k)$

$$-(6\cdot\sqrt{3}+7)<k<6\cdot\sqrt{3}-7$$

f)

Define $k=-2$	*Done*
Define $dl(x)=\frac{d}{dx}(l(x))$	*Done*
$dl(a)$	$6\cdot a-5$

g)

$j(x)$	$-4\cdot x-2$
$\text{solve}(6\cdot a-5=-4,a)$	$a=\frac{1}{6}$

h)

Define $p(x)=-2-4\cdot x$	*Done*

$$methods\_misc\backslash linear\_info\left(\begin{bmatrix}\frac{1}{6} & l\left(\frac{1}{6}\right)\end{bmatrix},\begin{bmatrix}x & p(x)\end{bmatrix}\right)$$

▸ Midpoint: $\begin{bmatrix}\frac{6\cdot x+1}{12} & \frac{-(16\cdot x-9)}{8}\end{bmatrix}$

▸ Gradient: $\frac{-3\cdot(16\cdot x+25)}{2\cdot(6\cdot x-1)}$

▸ Distance: $\frac{\sqrt{2448\cdot x^2+7152\cdot x+5629}}{12}$

i)

$$\text{Define } d(x)=\frac{\sqrt{2448\cdot x^2+7152\cdot x+5629}}{12}$$

Done

$$methods\_func\backslash analyse(d(x),x)$$

- Start Point: $[-\infty \quad \infty]$
- End Point: $[\infty \quad \infty]$
- Maximal Domain: $-\infty<x<\infty$
- Vertical Intercept: $\left[0 \quad \frac{\sqrt{5629}}{12}\right]$
- Derivative:

$$\frac{2\cdot(102\cdot x+149)}{\sqrt{2448\cdot x^2+7152\cdot x+5629}}$$

- Stationary Point:

$$\left[\frac{-149}{102} \quad \frac{83\cdot\sqrt{17}}{204}\right] \text{(Local min.)}$$

<table>
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a)
i)

Define $f1(x)=-3\cdot x^2+48\cdot x-180$ Done

Define $f2(x)=\dfrac{x^3}{3}-2\cdot x^2$ Done

$\text{zeros}\left(\dfrac{d}{dx}(f1(x)),x\right)$ $\{8\}$

Define $a1(x)=2\cdot(8-x)\cdot f1(x)$ Done

$\text{factor}(a1(x))$ $6\cdot(x-10)\cdot(x-8)\cdot(x-6)$

ii)

$methods\_func\backslash analysed(a1(x),x,6,8)$

- Start Point: $[6 \quad 0]$
- End Point: $[8 \quad 0]$
- Maximal Domain: $6\leq x\leq 8$
- x –Intercepts: (2)
  $[6 \quad 0]$, $[8 \quad 0]$
- Vertical Intercept: $[0 \quad -2880]$
- Derivative: $18\cdot x^2-288\cdot x+1128$
- Stationary Point:
  $\left[\dfrac{-2\cdot(\sqrt{3}-12)}{3} \quad \dfrac{32\cdot\sqrt{3}}{3}\right]$ (Local max.)

$8-\dfrac{-2\cdot(\sqrt{3}-12)}{3}$ $\dfrac{2\cdot\sqrt{3}}{3}$

$f1\left(\dfrac{-2\cdot(\sqrt{3}-12)}{3}\right)$ 8

</td>
</tr>
</table>

b)
Find the local minimum of the cubic first.

$methods\_func\backslash analysed(f2(x), x, 0, 6)$

▶ Stationary Points: (2)

$\begin{bmatrix} 0 & 0 \end{bmatrix}$ (Local max.)

$\begin{bmatrix} 4 & \frac{-32}{3} \end{bmatrix}$ (Local min.)

$\text{solve}(f2(x)=f2(x+h),h)$

$h=\frac{-\left(\sqrt{-3\cdot(x^2-4\cdot x-12)}+3\cdot(x-2)\right)}{2}$ or $h=\frac{\sqrt{-3\cdot(x^2-4\cdot x-12)}-3\cdot(x-2)}{2}$

Take the positive branch to be the base h.

$\text{Define } h(x)=\frac{\sqrt{-3\cdot(x^2-4\cdot x-12)}-3\cdot(x-2)}{2}$ *Done*

$\text{Define } a2(x)=-f2(x)\cdot h(x)$ *Done*

ii)

$methods\_func\backslash analysed(a2(x), x, 0, 4)$

▶ Stationary Points: (2)

$\begin{bmatrix} 0 & 0 \end{bmatrix}$ (Local min.)

$\begin{bmatrix} 2.39761 & 19.6767 \end{bmatrix}$ (Local max.)

$h(2.39761)$ 2.85053

$f2(2.39761)$ -6.90282

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