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VCE Mathematical Methods ¾
SAC 1 Revision II [0.17]
Workshop Solutions

Error Logbook:

New Ideas / Concepts	Didn't Read Question
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:
Algebraic / Arithmetic / Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ____________________ Notes:	Pg / Q #: ____________________ Notes:

Section A: SAC 1 Success

Welcome to the first SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1 - 50% of Sacs, 20% of the study score.
- Will be running all the way till mid-June.
- After that the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

$$\text{Study Score} = \text{How much you know} \times \text{How much you show}$$

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key!**

Tutor's Comment: Explain how even if you know everything, if you cannot show in the sac, you will get 0.

Analogy: Skipping Questions

- Let's say if you were to fight them and win, you get assigned marks.

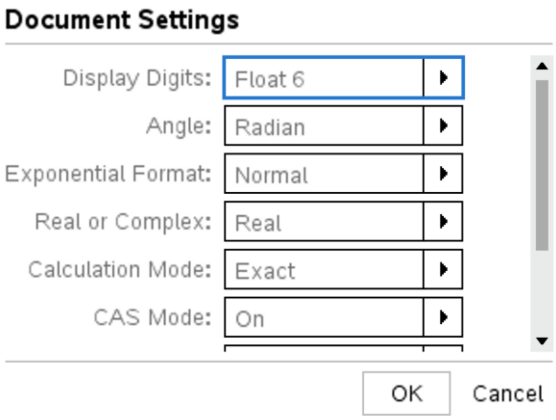
- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.

Definition

SAC Proficiency List

Before the SAC

- ☐ Prepare your stationery including a ruler, eraser, and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your ContourUDFS.
- ☐ TI Only: Check technology settings.



Reading Time

- ☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and using what technology commands.
- ☐ Identify questions to **skip.**

  For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** – You don't have to start from Q1!
- ☐ Look for potential pitfalls – **Units, Domain restriction of the unknown, variable and function meaning.**

Writing Time

- ☐ **Circle what the question is asking** for in the question.
- ☐ Spend the first **50%** of the time on all the **easy questions** you identified.
- ☐ Spend the next **25%** of doing the **difficult questions** you left blank.

- [ ] Spend the last 25% of the time on **checking your answers**.
- [ ] Check your answer by reading the question again and see if you answered the question.
- [ ] Check in the order of:

  Domino effect (check a, b, c first) > ***Questions with high marks*** $(3+)$ > *Hard Questions*

- [ ] $\mathbf{TI}$ ONLY: Use new document - **doc** $\mathbf{4, 1.}$

After the SAC

- [ ] Think about how each mark loss can be prevented using this proficiency list.
- [ ] Think about the big picture and improve the marks -

  Instead of spending 10 minutes on 10c) (1 mark), I should have checked 5a) (3 marks).

Space for Personal Notes

Section B: SAC Questions - Tech Active (56 Marks)

INSTRUCTION:

- Regular: 56 Marks. 10 Minutes Reading. 70 Minutes Writing.

Question 1 (11 marks)

a. Consider the function $f: [0, 2] \to \mathbb{R}, f(x) = x(x + 4)$. The graph of $y = g(x)$ is the graph of $y = f(x)$ reflected in the line $y = x$.

i. Find the rule for $y = g(x)$. (2 marks)

Reflecting in $y = x$ gives us the inverse function. So solve $f(y) = x \implies y = -2 \pm \sqrt{x+4}$. (1M).
Consider the range of g which is the domain of f in order to choose the right option.
$g(x) = -2 + \sqrt{x+4}$. (1A)

ii. State the domain for $g(x)$. (1 mark)

$[0, 12]$. (1A)

b. List a sequence of transformations, that does **not** include a reflection in the x-axis, which maps the graph of $y = \frac{1}{x}$ to the graph of: (4 marks)

$$y = -\frac{A}{x-b} - k \text{ where } A, b \text{ and } k \in \mathbb{Z}^+.$$

- A dilation by factor A from the x-axis.
- A reflection in the y-axis.
- A translation b units to the right
- A translation k units down.

(1A for each correct transformation).

c. The graph of $y = h(x)$ joins smoothly with the graph of $y = g(x)$ at the point where $x = 12$.

Given $h(15) = 8$ and $h(x) = -\frac{A}{x-b} - k$, find the rule for $h(x)$. (4 marks)

$h'(x) = \frac{A}{(x-b)^2}$ and $g'(x) = \frac{1}{2\sqrt{x+4}}$. (1M)

Must solve the system of equations

$$g'(12) = h'(12) \quad \text{and} \quad g(12) = h(12) \quad \text{and} \quad h(15) = 8$$

(1M for the system of equations)

$A = \frac{32}{25}, b = \frac{76}{5}$ and $k = -\frac{8}{5}$. (1A for these values). So

$$h(x) = \frac{8(x-16)}{5x-76} \quad \text{(1A)}$$

Question 2 (17 marks)

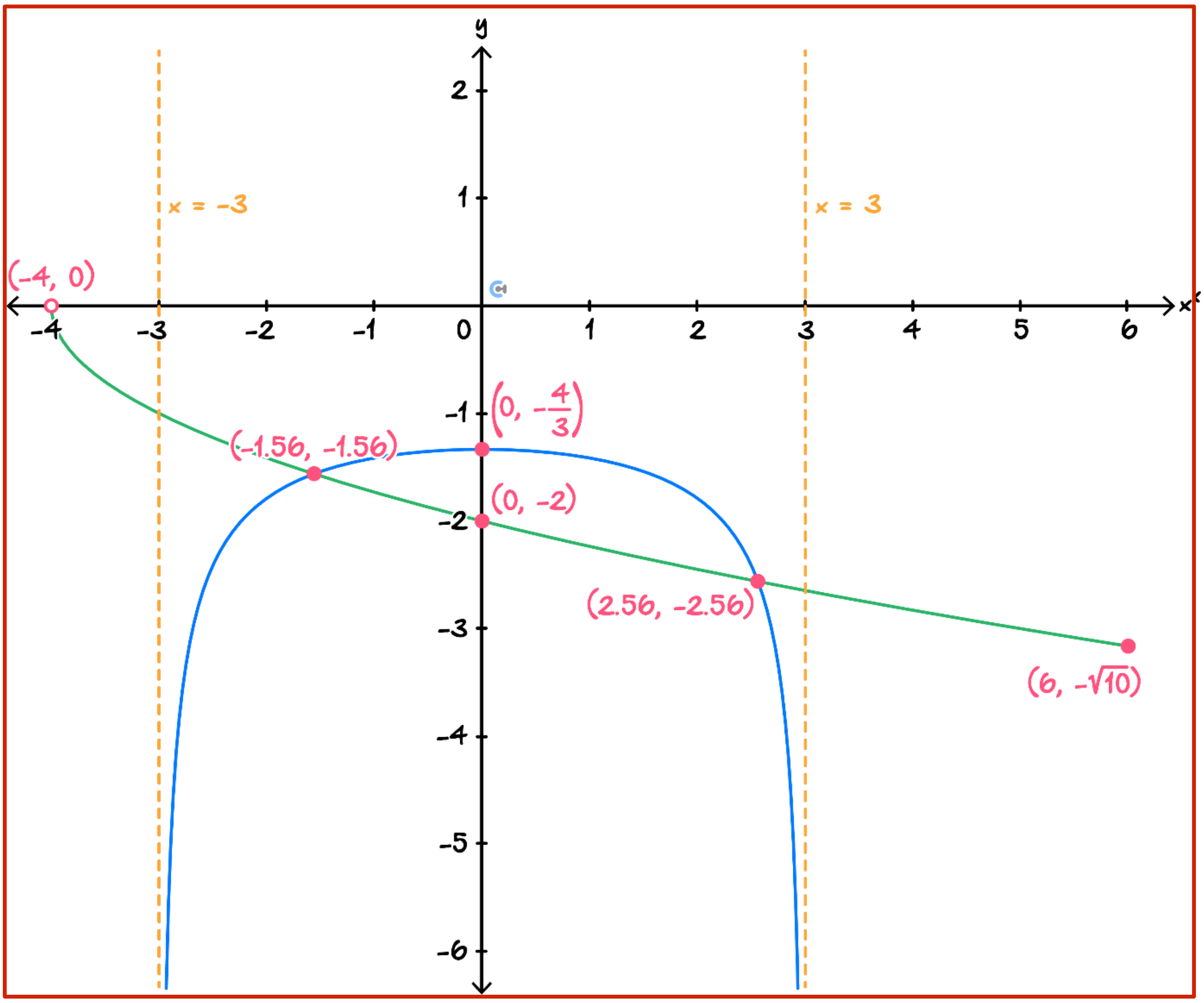
Consider the function f such that:

$$f: D \to \mathbb{R}, \qquad f(x) = -\frac{4}{\sqrt{9 - x^2}}$$

a. Find D, the maximal domain of f. (1 mark)

$\boxed{x \in (-3, 3).\ (1\text{A})}$

b. Sketch the graph of $y = f(x)$ over its maximal domain. Label all axis intercepts and stationary points with coordinates. Label any asymptotes with their equation. (3 marks)



$\boxed{(1\text{M asymptotes labelled, 1M intercepts labelled, 1M shape})}$

c. Hence, or otherwise, find the absolute maximum value of f. (1 mark)

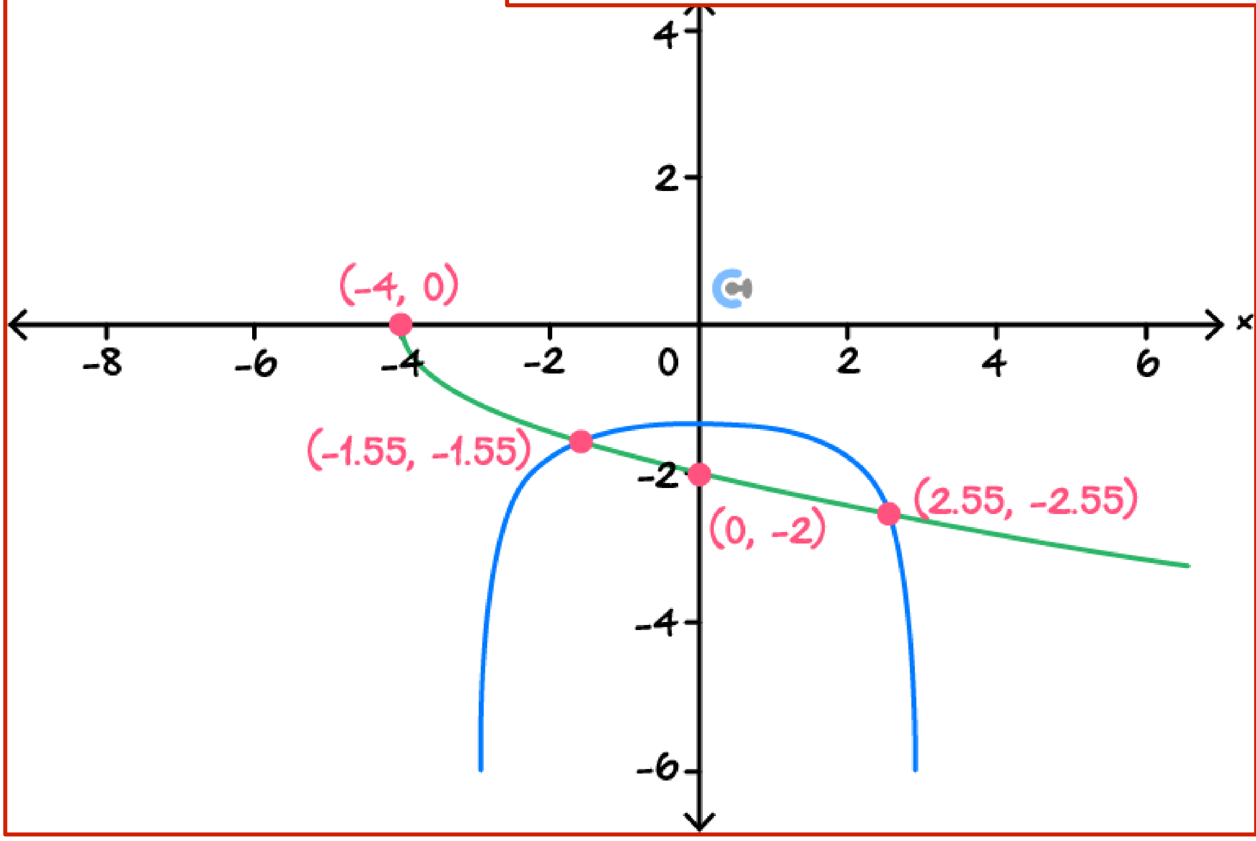
$-\frac{4}{3}$. (1A)

d. Consider the function g such that:

$$g: (-4, 6] \to \mathbb{R}, \qquad g(x) = -\sqrt{x+4}$$

On the axes, sketch $y = g(x)$, including the exact coordinates of all endpoints and intercepts, and the coordinates of the point(s) of intersection with $y = f(x)$ correct to two decimal places. (3 marks)

(1M endpoints labelled and correct circles, 1M intersections and axis intercepts, 1M shape)



e. Show that the function $f(g(x))$ does not exist. (1 mark)

ran $g = [-\sqrt{10}, 0) \not\subseteq (-3, 3)$. Thus $f(g(x))$ does not exist. (1A)

f. Let $h: X \to \mathbb{R}, h(x) = -\sqrt{x+4}$ where X is the domain of h.

i. Find the maximal domain X such that $f(h(x))$ exists. (1 mark)

$X = [-4, 5)$. (1A)

ii. Find the rule $f(h(x))$. (1 mark)

$f(h(x)) = -\dfrac{4}{\sqrt{5-x}}$. (1A)

g. Let $p: (0, 6) \to \mathbb{R}, p(x) = \frac{2}{\sqrt{x(6-x)}} - 1$. The graph of p can be obtained by applying the transformation T to the graph of f, where:

$$T: \mathbb{R}^2 \to \mathbb{R}^2, \qquad T(x, y) = (x + q, my + n)$$

Find the values of m, n, and q. (3 marks)

p is obtained from f by

- A dilation by factor $\frac{1}{2}$ from the x-axis.
- A reflection in the x-axis.
- A translation 3 units to the right.
- A dilation 1 unit down.

(1M for any reasonable working out attempt)

Thus, $q = 3$, $m = -\frac{1}{2}$ and $n = -1$. (1M for one correct value, 1A all correct)

h. Find the coordinates of the point P on the curve with equation $y = f(x)$ at which the tangent to $f(x)$ is normal to the line $3x + 7y = 18$, giving your answer correct to two decimal places. (3 marks)

The line has gradient $-\frac{3}{7}$. So our tangent line must have gradient $\frac{7}{3}$. (1M).

We then solve $f'(x) = \frac{7}{3} \implies x = -2.519$. (1M)

Thus the coordinates are $P(-2.52, -2.46)$. (1A)

Space for Personal Notes

Question 3 (12 marks)

Let $f(x) = 3e^{-\frac{1}{2}x}$ for $x \geq 0$ and let $g(x) = 3e^{\frac{1}{2}x}$ for $x < 0$. Consider the piecewise-defined function:

$$h(x) = \begin{cases} f(x), & x \geq 0 \\ g(x), & x < 0 \end{cases}$$

a. Find the equation of the tangent to the graph of h at the point $\left(1, 3e^{-\frac{1}{2}}\right)$. (2 marks)

$f'(x) = -\frac{3}{2}e^{-x/2}$, so $f'(1) = -\frac{3}{2\sqrt{e}}$. (1M)

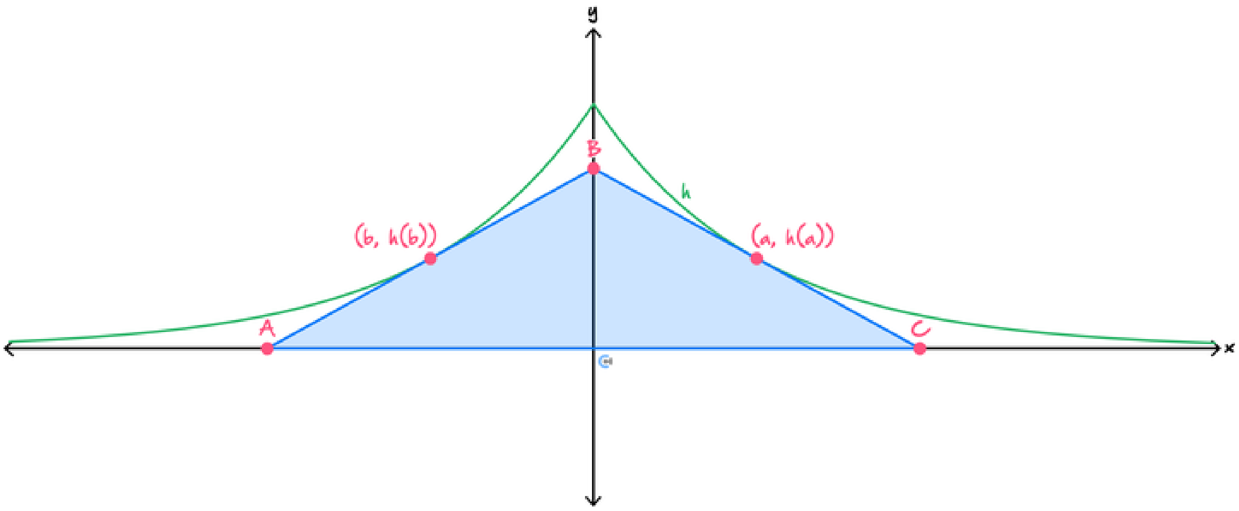
The tangent line is then $y = -\frac{3(x-3)}{2\sqrt{e}}$. (1A)

b. Find the area of the triangular region bounded by the tangent found in **part a.**, the positive x-axis and the positive y-axis. (1 mark)

$\frac{27}{4\sqrt{e}}$. (1A)

A tangent to the graph of h is drawn at the point $(a, h(a))$, where $a > 0$, and a second tangent is drawn to the graph of h at the point $(b, h(b))$, where $b < 0$.

These tangents are drawn such that they always intersect on the y-axis, and a triangle ABC is formed by the two tangents and the x-axis as shown below. The vertex B lies on the y-axis and the vertices A and C lie on the x-axis.



c. State the value of b in terms of a. (1 mark)

$b = -a$. (1A)

d. Given that the tangents intersect at the point $(0, 2)$, find the corresponding value of a, correct to three decimal places. (2 marks)

The tangent at $x = a$ has y-intercept $\frac{3}{2}(2 + a)e^{-a/2}$. (1M)

Thus we solve $\frac{3}{2}(2 + a)e^{-a/2} = 2$.

$a = 2.378$. (1A)

e. Find the value of a such that the angle $\angle ABC$ is $120°$. Give an exact answer in the form $\log_e\left(\frac{a}{b}\right)$, where a and b are positive integers. (2 marks)

The tangent at $x = a$ must make an angle of $\frac{\pi}{6}$ with the **negative** direction of the x-axis. (1M).
Thus we solve $f'(x) = \tan\left(-\frac{\pi}{6}\right)$.
$x = \log_e\left(\frac{27}{4}\right)$. (1A)

f. Use calculus to find the value of a that maximises the area of the triangle ABC. Also, state this maximum area. (4 marks)

The tangent at $x = a$ has x-intercept $x = 2 + a$. Thus the triangle has base $2(2 + a)$. (1M)
The triangle has area given by $A = \frac{1}{2} \times 2(2 + a) \times \frac{3}{2}(2 + a)e^{-a/2}$. (1M)
We now maximise A and get max area of $\frac{24}{e}$ when $a = 2$. (1A each).

Space for Personal Notes

Question 4 (16 marks)

A function $C(t)$ models the temperature (in degrees Celsius) of a drink inside a smart cup t minutes after being placed in a room. Initially, the drink is 5°C, so $C(0) = 5$.

If the drink's temperature reaches 15°C, (i.e. $C(t) = 15$), the cup's smart cooling feature activates, this feature cools the drink at a constant rate over the next 5 minutes, until the drink returns to its original temperature, 5°C, and the cooling stops.

The room has a highly unpredictable and faulty air conditioning system, so the temperature of the drink is modelled by:

$$C(t) = \frac{1}{8000}(at - 10)^3(b - t) + 6, \text{ where } t \geq 0, 5 \leq C(t) \leq 15, \text{ and } a, b \text{ are real constants.}$$

a.

i. Show that $b = 8$. (2 marks)

$C(0) = 6 - \frac{b}{8} = 5 \implies b = 8$. (1M for finding $C(0)$, 1A)

ii. When $a = 0$, calculate the time taken, in minutes, for the temperature to reach 10°C. (2 marks)

When $a = 0, C(t) = 6 + \frac{5}{4}(t - 8)$

Solve $C(t) = 10$ to get $t = 11.2$

iii. When $a = 2$, calculate the total time after the drink is placed in the room until the temperature returns to its original value. Give your answer correct to one decimal place. (2 marks)

When $a = 2$ we get $C(t) = 6 - \frac{(t-8)(t-5)^3}{1000}$

Solve $C(t) = 5 \implies t = 14.3\ minutes$

b.

i. State, in terms of a, the possible value(s) of t for which $(t, C(t))$ is a stationary point of the function $C(t)$. (2 marks)

We solve $C'(t) = 0$.
$t = \frac{10}{a}$ and $t = \frac{5 + 12a}{2a}$. (1A each)

ii. For what value(s) of a, $a \in [0, 7]$, does $C(t)$ have no stationary points? Give your answers correct to three decimal places where appropriate. (1 mark)

$a = 0$. (1A)

iii. For what value(s) of a, $a \in [0, 7]$, does $C(t)$ have one stationary point? Give your answers correct to three decimal places. (2 marks)

The stationary point at $t = \frac{10}{a}$ is always valid.
One stationary point if $\frac{10}{a} = \frac{5 + 12a}{2a} \implies a = \frac{5}{4}$. (1M)
The second stationary point is only valid if $5 \leq C\left(\frac{5 + 12a}{2a}\right) \leq 15$.
$0 < a \leq 0.0140$ or $a = 1.25$. (1A)

iv. What is the maximum number of stationary points $C(t)$ can have? (1 mark)

2 stationary points. (1A).

c.

i. What is the maximum temperature that the drink achieves when $a = 6$? Give your answer correct to one decimal place. (1 mark)

$\boxed{10.6. \ (1A)}$

ii. When $a = 6$, at what time is this maximum temperature reached? Give your answer in minutes, correct to one decimal place. (1 mark)

$\boxed{6.4. \ (1A)}$

d. a is restricted to being a positive integer.

i. What is the least value of a that will cause the smart cup to activate the cooling feature? (1 mark)

$\boxed{a = 8. \ (1A)}$

ii. The cup's smart cooling feature malfunctions, and instead only starts working once the drinks temperature starts to decrease. Thus, the function $C(t)$ is now valid for $t \geq 0$ and while it is non-decreasing. The drink is still cooled down to 5°C in 5 minutes.

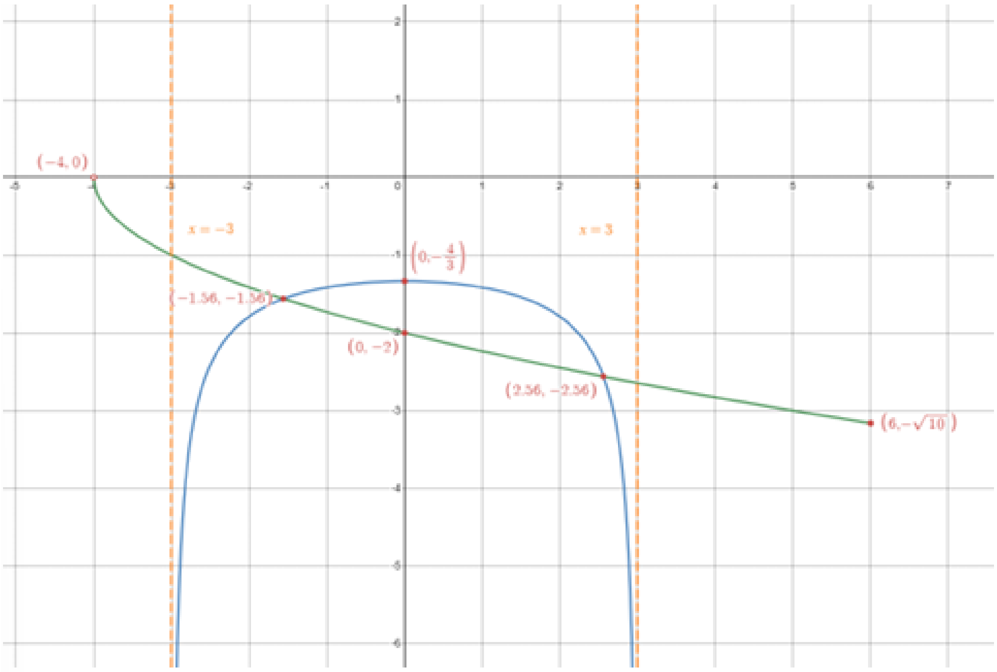
Using your answer to **part d.i.**, at what rate does the smart cup cooling feature cool the drink? Give your answer in °C / minute correct to one decimal place. (1 mark)

Using $a = 8$, we get a maximum value of 20.01.
Thus cools at $\frac{20.01 - 15}{5} = 3.0°\text{C/minute}$. (1A)

Section C: Marking Scheme

Question Number	Solutions
1	a) i) Reflecting in $y = x$ gives us the inverse function. So solve $f(y) = x \implies y = -2 \pm \sqrt{x+4}$. (1M). Consider the range of g which is the domain of f in order to choose the right option. $g(x) = -2 + \sqrt{x+4}$. (1A) ii) $[0, 12]$. (1A) b) • A dilation by factor A from the x-axis. • A reflection in the y-axis. • A translation b units to the right • A translation k units down. (1A for each correct transformation). c) $h'(x) = \dfrac{A}{(x-b)^2}$ and $g'(x) = \dfrac{1}{2\sqrt{x+4}}$. (1M) Must solve the system of equations $$g'(12) = h'(12) \quad \text{and} \quad g(12) = h(12) \quad \text{and} \quad h(15) = 8$$ (1M for the system of equations) $A = \dfrac{32}{25}, b = \dfrac{76}{5}$ and $k = -\dfrac{8}{5}$. (1A for these values). So $$h(x) = \frac{8(x-16)}{5x-76} \quad (1A)$$
2	a) $x \in (-3, 3)$. (1A)

b)



(1M asymptotes labelled, 1M intercepts labelled, 1M shape)

c) $-\frac{4}{3}$. (1A)

d) (1M endpoints labelled and correct circles, 1M intersections and axis intercepts, 1M shape)

e) ran $g = [-\sqrt{10}, 0) \not\subseteq (-3, 3)$. Thus $f(g(x))$ does not exist. (1A)

f)

i) $X = [-4, 5)$. (1A)

ii) $f(h(x)) = -\frac{4}{\sqrt{5-x}}$. (1A)

g) p is obtained from f by

- A dilation by factor $\frac{1}{2}$ from the x-axis.
- A reflection in the x-axis.
- A translation 3 units to the right.
- A dilation 1 unit down.

(1M for any reasonable working out attempt)

Thus, $q = 3$, $m = -\frac{1}{2}$ and $n = -1$. (1M for one correct value, 1A all correct)

	h) The line has gradient $-\frac{3}{7}$. So our tangent line must have gradient $\frac{7}{3}$. (1M) We then solve $f'(x) = \frac{7}{3} \implies x = -2.519$. (1M) Thus the coordinates are $P(-2.52, -2.46)$. (1A)
3	a) $f'(x) = -\frac{3}{2}e^{-x/2}$, so $f'(1) = -\frac{3}{2\sqrt{e}}$. (1M) The tangent line is then $y = -\frac{3(x-3)}{2\sqrt{e}}$. (1A) b) $\frac{27}{4\sqrt{e}}$. (1A) c) $b = -a$. (1A) d) The tangent at $x = a$ has y-intercept $\frac{3}{2}(2+a)e^{-a/2}$. (1M) Thus we solve $\frac{3}{2}(2+a)e^{-a/2} = 2$. $a = 2.378$. (1A) e) The tangent at $x = a$ must make an angle of $\frac{\pi}{6}$ with the **negative** direction of the x-axis. (1M). Thus we solve $f'(x) = \tan\left(-\frac{\pi}{6}\right)$. $x = \log_e\left(\frac{27}{4}\right)$. (1A) f) The tangent at $x = a$ has x-intercept $x = 2 + a$. Thus the triangle has base $2(2+a)$. (1M) The triangle has area given by $A = \frac{1}{2} \times 2(2+a) \times \frac{3}{2}(2+a)e^{-a/2}$. (1M) We now maximise A and get max area of $\frac{24}{e}$ when $a = 2$. (1A each).
4	a) i) $C(0) = 6 - \frac{b}{8} = 5 \implies b = 8$. (1M for finding $C(0)$, 1A) ii) When $a = 0, C(t) = 6 + \frac{5}{4}(t-8)$ Solve $C(t) = 10$ to get $t = 11.2$ iii) When $a = 2$ we get $C(t) = 6 - \frac{(t-8)(t-5)^3}{1000}$ Solve $C(t) = 5 \implies t = 14.3\ minutes$

b)

i) We solve $C'(t) = 0$.
$t = \dfrac{10}{a}$ and $t = \dfrac{5+12a}{2a}$. (1A each)

ii) $a = 0$. (1A)

iii) The stationary point at $t = \dfrac{10}{a}$ is always valid.
One stationary point if $\dfrac{10}{a} = \dfrac{5+12a}{2a} \implies a = \dfrac{5}{4}$. (1M)
The second stationary point is only valid if $5 \leq C\left(\dfrac{5+12a}{2a}\right) \leq 15$.
$0 < a \leq 0.0140$ or $a = 1.25$. (1A)

iv) 2 stationary points. (1A).

c)

i) 10.6. (1A)

ii) 6.4. (1A)

d)

i) $a = 8$. (1A)

ii) Using $a = 8$, we get a maximum value of 20.01.
Thus cools at $\dfrac{20.01 - 15}{5} = 3.0°\text{C/minute}$. (1A)

Space for Personal Notes

Section D: Tech-Active Solutions – Mathematica

Question Number	Solutions

1	**Q1**
	
	a.
	In[126]:= `f[x_] := x (x + 4)`
	In[127]:= `Solve[f[y] == x, y]`
	Out[127]= $\{\{y \to -2 - \sqrt{4 + x}\}, \{y \to -2 + \sqrt{4 + x}\}\}$
	In[128]:= `FunctionRange[{f[x], 0 ≤ x ≤ 2}, x, y]`
	Out[128]= $0 \le y \le 12$
	In[129]:= `g[x_] := -2 +` $\sqrt{4 + x}$
	In[130]:= `(*dom g = [0,12]*)`
	
	b.
	In[131]:= `q[x_] :=` $\frac{1}{x}$
	In[132]:= `t[x_] :=` $-\frac{A}{x - b} - k$
	In[133]:= `A * q[-(x - b)] - k == t[x] // FullSimplify`
	Out[133]= `True`
	
	c.
	In[134]:= `h[x_] :=` $-\frac{A}{x - b} - k$
	In[135]:= `g'[x]`
	Out[135]= $\frac{1}{2\sqrt{4 + x}}$

In[136]:=

```
h'[x]
```

Out[136]=

$$\frac{A}{(-b+x)^2}$$

In[137]:=

```
g'[12]
```

Out[137]=

$$\frac{1}{8}$$

In[138]:=

$$\frac{A}{(12-b)^2}$$

Out[138]=

$$\frac{A}{(12-b)^2}$$

In[139]:=

```
h'[12]
```

Out[139]=

$$\frac{A}{(12-b)^2}$$

In[140]:=

```
h[15]
```

Out[140]=

$$-\frac{A}{15-b}-k$$

In[141]:=

```
Solve[g'[12] == h'[12] && g[12] == h[12] && h[15] == 8]
```

Out[141]=

$$\left\{\left\{A\to\frac{32}{25},\ b\to\frac{76}{5},\ k\to-\frac{8}{5}\right\}\right\}$$

In[142]:=

$$-\frac{A}{x-b}-k\ /.\ \left\{A\to\frac{32}{25},\ b\to\frac{76}{5},\ k\to-\frac{8}{5}\right\}\ //\ \text{FullSimplify}$$

Out[142]=

$$\frac{8\,(-16+x)}{-76+5\,x}$$

2

Q2.

a

n[143]:=

$$f[x\_]\ :=\ -\frac{4}{\sqrt{9-x^{\wedge}2}}$$

In[144]:=

```
FunctionDomain[f[x], x]
```

Out[144]=

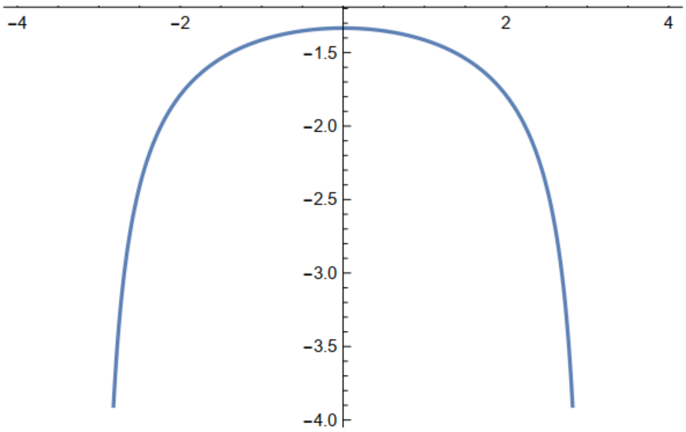
$-3 < x < 3$

b

In[145]:=

```
Plot[f[x], {x, -4, 4}]
```

Out[145]=



In[146]:=

```
Solve[f'[x] == 0]
```

Out[146]=

$\{\{x \to 0\}\}$

In[147]:=

```
f[0]
```

Out[147]=

$-\frac{4}{3}$

c.

In[148]:=

```
f[0]
```

Out[148]=

$-\frac{4}{3}$

d.

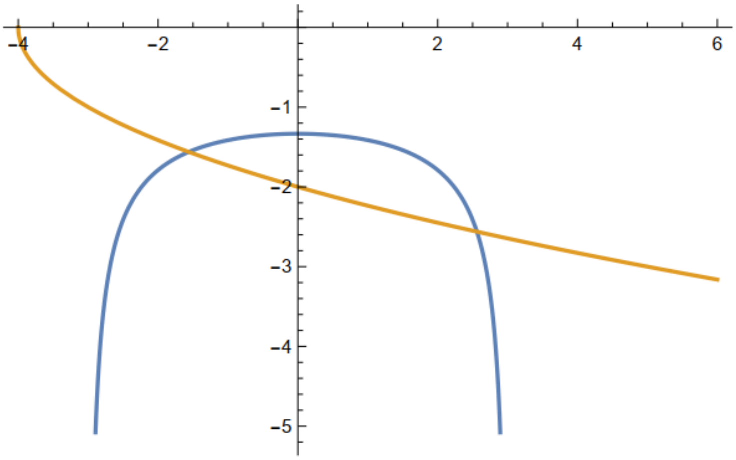
In[149]:=

$g[x\_] := -\sqrt{x+4}$

In[150]:=

```
Plot[{f[x], g[x]}, {x, -4, 6}]
```

Out[150]=



In[151]:=

```
Solve[f[x] == g[x] && y == f[x]] // N
```

Out[151]=

$\{\{x \to -1.56155,\ y \to -1.56155\},\ \{x \to 2.56155,\ y \to -2.56155\}\}$

e.

In[152]:=

```
FunctionRange[{g[x], -4 < x ≤ 4}, x, y]
```

Out[152]=

$-2\sqrt{2} \le y < 0$

f.

In[153]:=

```
Reduce[-3 < g[x] < 3, x]
```

Out[153]=

$-4 \le x < 5$

In[154]:=

```
f[g[x]]
```

Out[154]=

$$-\frac{4}{\sqrt{5-x}}$$

g.

In[155]:=

$$p[x\_] := \frac{2}{\sqrt{x\,(6-x)}} - 1$$

In[156]:=

```
-1 / 2 * f[x - 3] - 1 == p[x] // FullSimplify
```

Out[156]=

```
True
```

---

h.

In[157]:=

```
m = -3 / 7
```

Out[157]=

$-\frac{3}{7}$

In[158]:=

```
mtan = -1 / m
```

Out[158]=

$\frac{7}{3}$

In[159]:=

```
Solve[f'[x] == mtan && y == f[x], Reals] // N
```

Out[159]=

```
{{x → -2.51949, y → -2.45619}}
```

In[160]:=

```
TangentLine[f[x], x, -2.5194944158496226`]
```

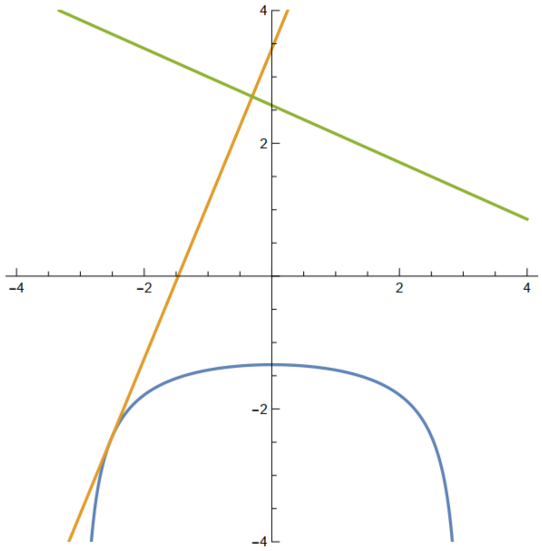
Out[160]=

```
3.42264 + 2.33333 x
```

In[161]:=

```
Plot[{f[x], 3.422635033206351` + 2.3333333333333357` x, (18 - 3 x)/7},
 {x, -4, 4}, PlotRange → {-4, 4}, AspectRatio → 1]
```

Out[161]=



3	**Q3.** **a.** In[162]:= `f[x_] := 3 Exp[-1/2 x]` In[163]:= `g[x_] := 3 Exp[1/2 x]` In[164]:= `f'[x]` Out[164]= $-\frac{3}{2} e^{-x/2}$ In[165]:= `f'[1]` Out[165]= $-\frac{3}{2\sqrt{e}}$

In[166]:=
```
f[1]
```
Out[166]=
$$\frac{3}{\sqrt{e}}$$

In[167]:=
```
TangentLine[f[x], x, 1] // FullSimplify
```
Out[167]=
$$-\frac{3\,(-3+x)}{2\sqrt{e}}$$

---

b.

In[168]:=
$$t[x\_] := -\frac{3\,(-3+x)}{2\sqrt{e}}$$

In[169]:=
```
t[0]
```
Out[169]=
$$\frac{9}{2\sqrt{e}}$$

In[170]:=
```
Solve[t[x] == 0, x]
```
Out[170]=
$$\{\{x \to 3\}\}$$

In[171]:=
```
3 * 1 / 2 * t[0]
```
Out[171]=
$$\frac{27}{4\sqrt{e}}$$

---

c.

In[172]:=
```
TangentLine[f[x], x, a] /. x → 0
```
Out[172]=
$$\frac{3}{2}\,(2+a)\,e^{-a/2}$$

In[173]:=
```
TangentLine[g[x], x, b] /. x → 0
```
Out[173]=
$$-\frac{3}{2}\,(-2+b)\,e^{b/2}$$

In[174]:=
```
-3/2 (-2 + b) e^(b/2) /. b → -a // FullSimplify
```
Out[174]=
$$\frac{3}{2}(2+a)\,e^{-a/2}$$

d.

In[175]:=
```
Solve[3/2 (2 + a) e^(-a/2) == 2 && a > 0, a] // N
```
Out[175]=
```
{{a → 2.37767}}
```

e.

In[219]:=
```
Tan[5 Pi / 6] == Tan[-Pi / 6]
```
Out[219]=
```
True
```
In[176]:=
```
Solve[f'[x] == Tan[-Pi / 6], x, Reals]
```
Out[176]=
```
{{x → -2 Log[2] + 3 Log[3]}}
```
In[177]:=
```
-2 Log[2] + 3 Log[3] == Log[27 / 4]
```
Out[177]=
```
True
```

f.

In[178]:=
```
(* y coord of B*)
```
In[179]:=
```
TangentLine[f[x], x, a] /. x → 0
```
Out[179]=
$$\frac{3}{2}(2+a)\,e^{-a/2}$$

In[180]:=
```
Solve[TangentLine[f[x], x, a] == 0, x]
```
Out[180]=
```
{{x → 2 + a}}
```
In[181]:=
```
(* Triangle base*)
```

In[182]:=
```
2 (2 + a)
```
Out[182]=

$2\,(2+a)$

In[183]:=

$$\texttt{r[a\_] := 1 / 2 * 2 (2 + a) * }\frac{3}{2}\texttt{ (2 + a) }e^{-a/2}$$

In[184]:=
```
r[a]
```
Out[184]=

$$\frac{3}{2}(2+a)^2\,e^{-a/2}$$

In[185]:=
```
r'[a] // FullSimplify
```
Out[185]=

$$-\frac{3}{4}\left(-4+a^2\right)e^{-a/2}$$

In[186]:=
```
Solve[r'[a] == 0, a]
```
Out[186]=

$\{\{a \to -2\}, \{a \to 2\}\}$

In[187]:=
```
r[2]
```
Out[187]=

$$\frac{24}{e}$$

In[188]:=

$$N\left[\frac{24}{e}\right]$$

Out[188]=

8.82911

In[189]:=
```
Maximize[{r[a], a ≥ 0}, a]
```
Out[189]=

$$\left\{\frac{24}{e}, \{a \to 2\}\right\}$$

4

Q4.

a

In[190]:=
```
c[t_] := 1 / 8000 (a * t - 10) ^3 (b - t) + 6
```

In[191]:=
```
c[0]
```
Out[191]=
$$6 - \frac{b}{8}$$

In[192]:=
```
Solve[c[0] == 5, b]
```
Out[192]=
```
{{b → 8}}
```

In[193]:=
```
c[t_] := 1 / 8000 (a * t - 10) ^3 (8 - t) + 6
```

In[194]:=
```
c[t] /. a → 0
```
Out[194]=
$$6 + \frac{1}{8}(-8 + t)$$

When $a = 0, C(t) = 6 + \frac{5}{4}(t - 8)$

Solve $C(t) = 10$ to get $t = 11.2$

When $a = 2$ we get $C(t) = 6 - \frac{(t-8)(t-5)^3}{1000}$

Solve $C(t) = 5 \Longrightarrow t = 14.3\ minutes$

---

b.

In[198]:=
```
c'[t] // Factor
```
Out[198]=
$$-\frac{(-10 + a t)^2 (-5 - 12 a + 2 a t)}{4000}$$

In[199]:=
```
Solve[c'[t] == 0, t]
```
Out[199]=
$$\left\{\left\{t \to \frac{10}{a}\right\}, \left\{t \to \frac{10}{a}\right\}, \left\{t \to \frac{5 + 12 a}{2 a}\right\}\right\}$$

In[200]:=
```
Reduce[5 ≤ c[10 / a] ≤ 15, a]
```
Out[200]=
```
True
```

In[201]:=
```
c[10 / a]
```
Out[201]=
```
6
```
In[202]:=
```
c''[10 / a]
```
Out[202]=
```
0
```
In[203]:=

$$\text{Reduce}\left[5 \le c\left[\frac{5+12a}{2a}\right] \le 15,\ a\right]$$

Out[203]=

$$a == -\frac{5}{12}\ ||\ 0.0140\ldots \le a \le 7.12\ldots$$

In[204]:=

$$\text{Solve}\left[c\left[\frac{5+12a}{2a}\right] == 15,\ a,\ \text{Reals}\right]$$

Out[204]=

$$\{\{a \to 0.0140\ldots\},\ \{a \to 7.12\ldots\}\}$$

In[205]:=

$$\text{Solve}\left[c\left[\frac{5+12a}{2a}\right] == 0,\ a,\ \text{Reals}\right]$$

Out[205]=

$$\{\{a \to -3.04\ldots\},\ \{a \to -0.0237\ldots\}\}$$

In[206]:=

$$\text{Solve}\left[\frac{10}{a} == \frac{5+12a}{2a},\ a\right]$$

Out[206]=

$$\left\{\left\{a \to \frac{5}{4}\right\}\right\}$$

In[207]:=
```
(* No stationary points if a=0,
one stationary point if 0<a≤0.0140 OR a=1.250,
two stationary points if 0.014<a≤7 and a≠1.250*)
```

c.

In[208]:=

$$c\left[\frac{5+12a}{2a}\right]\ /.\ a \to 6\ //\ N$$

Out[208]=
```
10.5816
```
In[209]:=
```
Maximize[c[t] /. a → 6, t] // N
```
Out[209]=
```
{10.5816, {t → 6.41667}}
```

In[210]:=

$$\frac{5+12a}{2a} \text{ /. } a \to 6 \text{ // N}$$

Out[210]=

6.41667

d.

In[211]:=

$$\text{Solve}\left[c\left[\frac{5+12a}{2a}\right] == 15, a, \text{Reals}\right]$$

Out[211]=

{{a → 0.0140...}, {a → 7.12...}}

In[212]:=

```
(*a=8*)
```

In[213]:=

```
c[t] /. a → 8
```

Out[213]=

$$6 + \frac{(8-t)(-10+8t)^3}{8000}$$

In[214]:=

```
ClearAll[m]
```

In[215]:=

$$m[t\_] := 6 + \frac{(8-t)(-10+8t)^3}{8000}$$

In[221]:=

```
Solve[m[t] == 15, t, Reals]
```

Out[221]=

{{t → 4.77...}, {t → 7.39...}}

In[216]:=

```
Solve[m'[t] == 0 && y == m[t]] // N
```

Out[216]=

{{t → 1.25, y → 6.}, {t → 6.3125, y → 20.0126}}

In[217]:=

In[218]:=

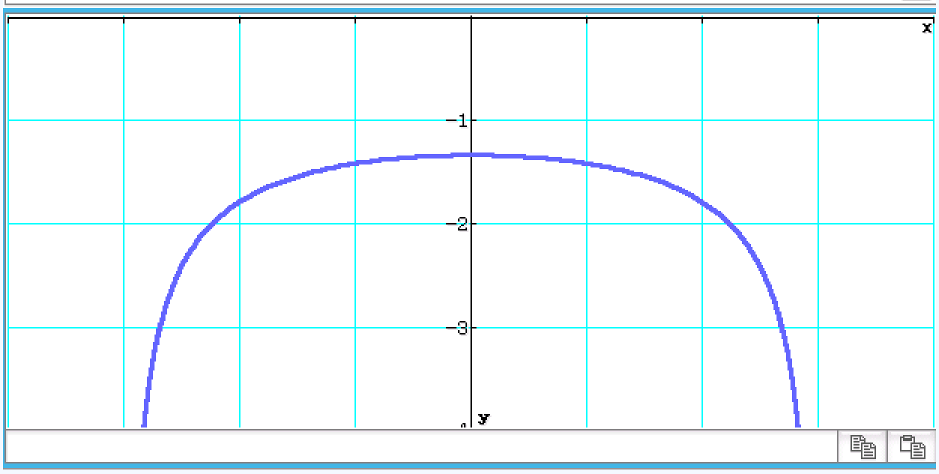
$$\frac{20.0126044921875 - 5}{5}$$

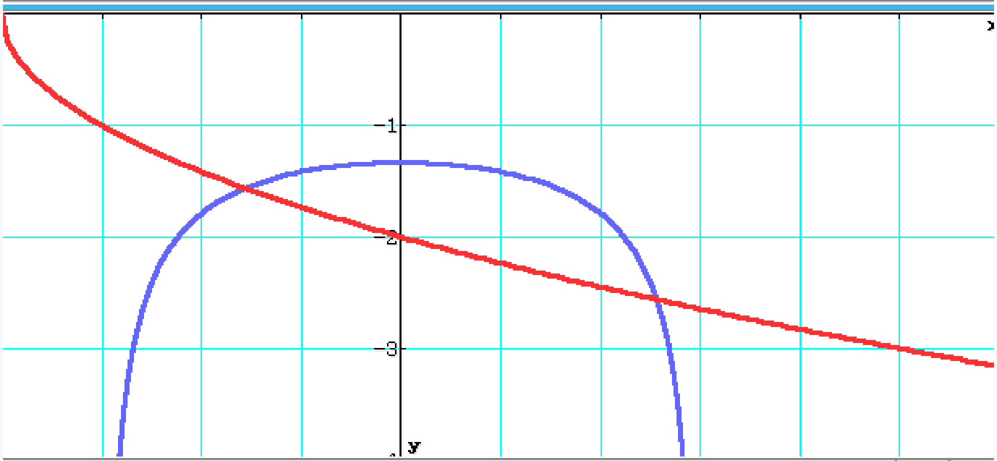
Out[218]=

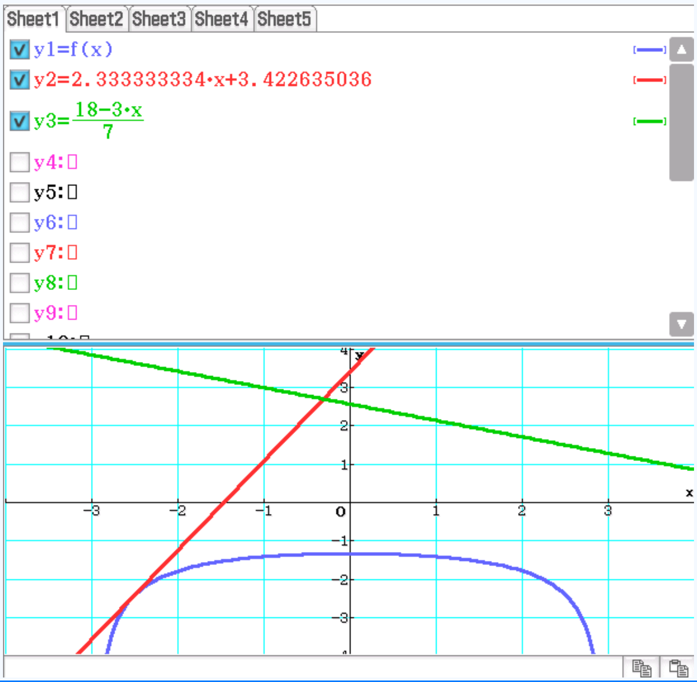
3.00252

Section E: Tech-Active Solutions – Casio

Question Number	Solutions
1a	Define $f(x)=x*(x+4)$ done solve$(f(y)=x, y$ $\{y=-\sqrt{x+4}-2, y=\sqrt{x+4}-2\}$ fmin$(f(x), x, 0, 2)$ $\{\text{MinValue}=0, x=0\}$ fmax$(f(x), x, 0, 2)$ $\{\text{MaxValue}=12, x=2\}$ Define $g(x)=-2+\sqrt{4+x}$ done
1b	Define $q(x)=1/x$ done Define $t(x)=-\frac{A}{x-b}-k$ done judge$(t(x)=A*q(-(x-b))-k$ TRUE
1c	Define $h(x)=-\frac{A}{x-b}-k$ done $\frac{d}{dx}(g(x))$ $\frac{1}{2\cdot\sqrt{x+4}}$ $\frac{d}{dx}(h(x))$ $\frac{A}{(x-b)^2}$ $\frac{d}{dx}(g(x))\|x=12$ $\frac{1}{8}$ $\frac{A}{(12-b)^2}$ $\frac{A}{(b-12)^2}$ $\frac{d}{dx}(h(x))\|x=12$ $\frac{A}{(b-12)^2}$ $h(15)$ $-k+\frac{A}{b-15}$

		$\begin{cases} \frac{d}{dx}(g(x))=\frac{d}{dx}(h(x)) \mid x=12 \\ g(12)=h(12) \\ h(15)=8 \end{cases} \Bigg\vert A,b,k$ $\left\{A=\frac{32}{25}, b=\frac{76}{5}, k=-\frac{8}{5}\right\}$ $h(x) \mid \left\{A=\frac{32}{25}, b=\frac{76}{5}, k=-\frac{8}{5}\right\}$ $\frac{-32}{25\cdot\left(x-\frac{76}{5}\right)}+\frac{8}{5}$ simplify(ans) $\frac{8\cdot(x-16)}{5\cdot x-76}$
2a		define $f(x)=\frac{-4}{\sqrt{9-x^{\wedge}2}}$ done solve($\sqrt{9-x^{\wedge}2}>0$, x {−3<x<3}
2b		Sheet1 Sheet2 Sheet3 Sheet4 Sheet5 ☑ y1=f(x) ☐ y2:□ ☐ y3:□ ☐ y4:□ ☐ y5:□ ☐ y6:□ ☐ y7:□ ☐ y8:□ ☐ y9:□ ☐ y10:□ 

2c		$\text{solve}\left(\frac{d}{dx}(f(x))=0, x\right)$ $\{x=0\}$ $f(0)$ $-\frac{4}{3}$
2d		Sheet1 Sheet2 Sheet3 Sheet4 Sheet5 y1=f(x) y2=g(x) y3:□ y4:□ y5:□ y6:□ y7:□ y8:□ y9:□ y10:□ 
2e		$\text{fmin}(g(x), x, -4, 4)$ $\{\text{MinValue}=-2\cdot\sqrt{2}, x=4\}$ $\text{fmax}(g(x), x, -4, 4)$ $\{\text{MaxValue}=0, x=-4\}$
2f		$\text{solve}(-3<g(x)<3, x)$ $\{-4\le x<5\}$ $f(g(x))$ $\frac{-4}{\sqrt{-x+5}}$

2g	Define $p(x)=\frac{2}{\sqrt{x*(6-x)}}-1$ done judge$\left(\frac{-1}{2}*f(x-3)-1=p(x)\right)$ TRUE
2h	$-3/7 \Rightarrow m$ $-\frac{3}{7}$ $-1/m \Rightarrow mtan$ $\frac{7}{3}$ $\left\{\begin{array}{l}\frac{d}{dx}(f(x))=mtan\\ y=f(x)\end{array}\right\|_{x,\,y}$ $\{x=-2.519494416, y=-2.45618527\}$ tanline$(f(x),x,-2.519494416)$ $2.333333334\cdot x+3.422635036$ 

3a	Define f(x)=3*e^-1/2*x done Define g(x)=3*e^1/2*x done $\frac{d}{dx}(f(x))$ $\frac{-3\cdot e^{\frac{-x}{2}}}{2}$ $\frac{d}{dx}(f(x))\|x=1$ $\frac{-3\cdot e^{-\frac{1}{2}}}{2}$ f(1) $3\cdot e^{-\frac{1}{2}}$ tanline(f(x), x, 1) $\frac{-3\cdot x\cdot e^{-\frac{1}{2}}}{2}+\frac{9\cdot e^{-\frac{1}{2}}}{2}$ simplify(ans) $\frac{-3\cdot(x-3)\cdot e^{-\frac{1}{2}}}{2}$
3b	Define $t(x)=\frac{-3\cdot(x-3)\cdot e^{-\frac{1}{2}}}{2}$ done t(0) $\frac{9\cdot e^{-\frac{1}{2}}}{2}$ solve(t(x)=0, x {x=3} 3*1/2*t(0) $\frac{27\cdot e^{-\frac{1}{2}}}{4}$

3c	tanline(f(x),x,a)|x=0 $\frac{3\cdot a\cdot e^{\frac{-a}{2}}}{2}+3\cdot e^{\frac{-a}{2}}$ tanline(g(x),x,b)|x=0 $\frac{-3\cdot b\cdot e^{\frac{b}{2}}}{2}+3\cdot e^{\frac{b}{2}}$ ans|b=−a $\frac{3\cdot a\cdot e^{\frac{-a}{2}}}{2}+3\cdot e^{\frac{-a}{2}}$ simplify(ans) $\frac{3\cdot(a+2)\cdot e^{\frac{-a}{2}}}{2}$
3d	solve$\left(\frac{3\cdot(a+2)\cdot e^{\frac{-a}{2}}}{2}=2\right.$|a>0,a {a=2.377668332}
3e	solve$\left(\frac{d}{dx}(f(x))=\tan(-\pi/6),x\right.$ {x=3·ln(3)−2·ln(2)} judge(3·ln(3)−2·ln(2)=ln(27/4) TRUE
3f	tanline(f(x),x,a)|x=0 $\frac{3\cdot a\cdot e^{\frac{-a}{2}}}{2}+3\cdot e^{\frac{-a}{2}}$ solve(tanline(f(x),x,a)=0,x {x=a+2} 2*(a+2) 2·(a+2) Define r(a)=1/2*2·(a+2)*$\left(\frac{3\cdot a\cdot e^{\frac{-a}{2}}}{2}+3\cdot e^{\frac{-a}{2}}\right)$ done r(a) $(a+2)\cdot\left(\frac{3\cdot a\cdot e^{\frac{-a}{2}}}{2}+3\cdot e^{\frac{-a}{2}}\right)$ simplify(ans) $\frac{3\cdot(a+2)^2\cdot e^{\frac{-a}{2}}}{2}$

	$\frac{d}{da}(r(a))$ $\frac{-(3\cdot a^2-12)\cdot e^{\frac{-a}{2}}}{4}$ simplify(ans) $\frac{-3\cdot(a^2-4)\cdot e^{\frac{-a}{2}}}{4}$ solve($\frac{d}{da}$(r(a))=0, a {a=−2, a=2} r(2) $24\cdot e^{-1}$ r(2) 8.829106588 fmax(r(a), a, 0, ∞) $\{MaxValue=24\cdot e^{-1}, a=2\}$
4a	Define c(t)=1/8000*(a*t−10)^3*(b−t)+6 done c(0) $\frac{-b}{8}+6$ solve(c(0)=5, b {b=8} Define c(t)=1/8000*(a*t−10)^3*(8−t)+6 done When $a = 0, C(t) = 6 + \frac{5}{4}(t-8)$ Solve $C(t) = 10$ to get $t = 11.2$ When $a = 2$ we get $C(t) = 6 - \frac{(t-8)(t-5)^3}{1000}$ Solve $C(t) = 5 \Longrightarrow t = 14.3\ minutes$

4b	$\text{factor}\left(\frac{d}{dt}(c(t))\right)$ $\frac{-(a\cdot t-10)^2\cdot(2\cdot a\cdot t-12\cdot a-5)}{4000}$ $\text{solve}\left(\frac{d}{dt}(c(t))=0,t\right)$ $\left\{t=\frac{5}{2\cdot a}+6,\ t=\frac{10}{a}\right\}$ $\text{solve}\left(5\le c\left(\frac{10}{a}\right)\le 15,a\right)$ TRUE $c\left(\frac{10}{a}\right)$ 6 $\frac{d^2}{dt^2}(c(t))\mid t=\frac{10}{a}$ $\frac{-\left(120\cdot a^2-\frac{10\cdot(12\cdot a^3+45\cdot a^2)}{a}+450\cdot a\right)}{2000}$ simplify(ans) 0
4c	$\text{solve}\left(5\le c\left(\frac{5}{2\cdot a}+6\right)\le 15,a\right)$ $\{a=-0.4166666667,\ 0.0140029947\le a\le 7.119048107\}$ $\text{solve}\left(c\left(\frac{5}{2\cdot a}+6\right)=15,a\right)$ $\{a=0.0140029947,\ a=7.119048107\}$ $\text{solve}\left(c\left(\frac{5}{2\cdot a}+6\right)=0,a\right)$ $\{a=-3.035430345,\ a=-0.02368601681\}$ $\text{solve}\left(\frac{10}{a}=\frac{5}{2\cdot a}+6,a\right)$ $\left\{a=\frac{5}{4}\right\}$ $c\left(\frac{5}{2\cdot a}+6\right)\mid a=6$ 10.58159766 $\text{fmax}(c(t)\mid a=6,t)$ $\{\text{MaxValue}=10.58159766,\ t=6.416666667\}$ $\frac{5}{2\cdot a}+6\mid a=6$ 6.416666667

4d	$\text{solve}\left(c\left(\frac{5}{2\cdot a}+6\right)=15, a\right)$ $\{a=0.0140029947, a=7.119048107\}$ $c(t)\mid a=8$ $\frac{-(t-8)\cdot(8\cdot t-10)^3}{8000}+6$ $\text{define } m(t)=\frac{-(t-8)\cdot(8\cdot t-10)^3}{8000}+6$ done $\left.\begin{cases}\frac{d}{dt}(m(t))=0\\ y=m(t)\end{cases}\right\vert_{t, y}$ $\{\{y=6, t=1.25\}, \{y=20.01260449, t=6.3125\}\}$ $\frac{20.01260449-5}{5}$ 3.002520898

Space for Personal Notes

Section F: Tech-Active Solutions - TI

Question Number	Solutions
1	a) i) Note: The inverse program requires a point to be specified so an appropriate inverse can be chosen. Here, we can choose 2 as a point in our domain. Define $f(x)=x\cdot(x+4)$ *Done* $methods\_func\backslash inverse(f(x),x,2)$ $\sqrt{x+4}-2$ *Done* ii) The domain for g is the range of f. The function f is increasing so the range can be found by evaluating the endpoints. $f(0)$ 0 $f(2)$ 12 c) Method 1: (Non-UDFs) Define $g(x)=\sqrt{x+4}-2$ *Done* Define $h(x)=\dfrac{-a}{x-b}-k$ *Done* Define $dg(x)=\dfrac{d}{dx}(g(x))$ *Done* Define $dh(x)=\dfrac{d}{dx}(h(x))$ *Done* solve$(g(12)=h(12)$ and $dg(12)=dh(12)$ and $h(15)=8,a,b,k)$ $a=\dfrac{32}{25}$ and $b=\dfrac{76}{5}$ and $k=\dfrac{-8}{5}$ $h(x)\vert a=\dfrac{32}{25}$ and $b=\dfrac{76}{5}$ and $k=\dfrac{-8}{5}$ $\quad \dfrac{8}{5}-\dfrac{32}{5\cdot(5\cdot x-76)}$ factor$\left(\dfrac{8}{5}-\dfrac{32}{5\cdot(5\cdot x-76)}\right)$ $\quad \dfrac{8\cdot(x-16)}{5\cdot x-76}$

Method 2: (UDFs) Not recommended for questions with more than 2 parameters.

Define $g(x)=\sqrt{x+4}-2$	*Done*
Define $h(x)=\frac{-a}{x-b}-k$	*Done*

Reduce to two parameters by solving for k and redefine the function.

$\text{solve}(h(15)=8,k)$	$k=\frac{a-8\cdot(b-15)}{b-15}$
Define $h(x)=\frac{-a}{x-b}-\frac{a-8\cdot(b-15)}{b-15}$	*Done*

Use the solve_smooth UDF

	$methods\_diffcalc\backslash solve\_smooth(g(x),h(x),x,\{a,b\},12)$ ▶ Left Derivative: $\frac{1}{2\cdot\sqrt{x+4}}$ ▶ Right Derivative: $\frac{a}{(x-b)^2}$ $\begin{bmatrix} \text{"At x=12:"} & \text{"Left Func."} & \text{"Right Func."} \\ \text{"Value:"} & 2 & \frac{-(3\cdot a-8\cdot(b-15)\cdot(b-12))}{(b-15)\cdot(b-12)} \\ \text{"Gradient:"} & \frac{1}{8} & \frac{a}{(b-12)^2} \end{bmatrix}$ ▶ Solutions: $a=\frac{32}{25}$ and $b=\frac{76}{5}$ *Done* Define $a=\frac{32}{25}$ *Done* Define $b=\frac{76}{5}$ *Done* factor$(h(x))$ $\frac{8\cdot(x-16)}{5\cdot x-76}$
2	a) Define $f(x)=\frac{-4}{\sqrt{9-x^2}}$ *Done* domain$(f(x),x)$ $-3<x<3$

b)

$methods\_func\backslash analyse(f(x),x)$

- Start Point: $\begin{bmatrix} -\infty & 0 \end{bmatrix}$
- End Point: $\begin{bmatrix} \infty & 0 \end{bmatrix}$
- Maximal Domain: $-3<x<3$
- No x –Intercepts Found
- Vertical Intercept: $\begin{bmatrix} 0 & \frac{-4}{3} \end{bmatrix}$

- Derivative: $\dfrac{-4\cdot x}{(9-x^2)^{\frac{3}{2}}}$
- Asymptotes: (3)

  $x=-3$ (Vertical)

  $x=3$ (Vertical)

  $y=0$ (Horizontal)

- No Inflection Points Found
- Stationary Point:

  $\begin{bmatrix} 0 & \frac{-4}{3} \end{bmatrix}$ (Local max.)

d)

$methods\_func\backslash analysed(g(x), x, -4, 6)$

- Start Point: $\begin{bmatrix} -4 & 0 \end{bmatrix}$
- End Point: $\begin{bmatrix} 6 & -\sqrt{10} \end{bmatrix}$
- Maximal Domain: $-4 \le x \le 6$
- x-Intercept: $\begin{bmatrix} -4 & 0 \end{bmatrix}$
- Vertical Intercept: $\begin{bmatrix} 0 & -2 \end{bmatrix}$
- Derivative: $\dfrac{-1}{2\cdot\sqrt{x+4}}$
- No Inflection Points Found
- No Stationary Points Found

$methods\_func\backslash intersect(f(x), g(x), x)$

- Intersection Points: (2.)

$\begin{bmatrix} -1.56155 & -1.56155 \end{bmatrix}$

$\begin{bmatrix} 2.56155 & -2.56155 \end{bmatrix}$

f) The lower bound of the range of h is already contained in the domain of f. It remains to bound the upper bound of the range of h to be no more than 3.

Define $h(x)=g(x)$	*Done*
$\text{solve}(h(x)=-3, x)$	$x=5$
⚠ $f(h(x))$	$\dfrac{-4}{\sqrt{5-x}}$

g) Use the transform program to check transformations.

$$\text{thods\_func\textbackslash transform}\left(f(x),x,\left\{x+3,\frac{-1}{2}\cdot y-1\right\}\right)$$

- Translation 3 units along the pos. x−dir.

$$\frac{-4}{\sqrt{-x\cdot(x-6)}}$$

- Dilation by factor of $\frac{1}{2}$ from the x−axis

$$\frac{-2}{\sqrt{-x\cdot(x-6)}}$$

- Reflection in the x−axis

$$\frac{2}{\sqrt{-x\cdot(x-6)}}$$

- Translation −1 unit along the neg. y−dir.

$$\frac{2}{\sqrt{-x\cdot(x-6)}}-1$$

$$methods\_diffcalc\backslash solve\_grad\left(f(x),x,\frac{7}{3}\right)$$

▶ Maximal Domain: $-3<x<3$

▶ Derivative: $\dfrac{-4\cdot x}{\left(9-x^2\right)^{\frac{3}{2}}}$

▶ Found point with gradient $\frac{7}{3}$:

$$\begin{bmatrix} -2.51949 & -2.45619 \end{bmatrix}$$

3	a) $methods\_diffcalc\backslash tangent\_line(f(x),x,1)$ ▶ Derivative: $\dfrac{-3\cdot e^{\frac{-x}{2}}}{2}$ ▶ Gradient: $\dfrac{-3\cdot e^{\frac{-1}{2}}}{2}$ ▶ Passes Through: $\begin{bmatrix} 1 & 3\cdot e^{\frac{-1}{2}} \end{bmatrix}$ ▶ x −Intercept: $\begin{bmatrix} 3 & 0 \end{bmatrix}$ ▶ Vertical Intercept: $\begin{bmatrix} 0 & \dfrac{9\cdot e^{\frac{-1}{2}}}{2} \end{bmatrix}$ ▶ Tangent Line: $\dfrac{9\cdot e^{\frac{-1}{2}}}{2}-\dfrac{3\cdot e^{\frac{-1}{2}}\cdot x}{2}$

b)

$$\frac{1}{2}\cdot 3\cdot \frac{9\cdot e^{\frac{-1}{2}}}{2} \qquad \frac{27\cdot e^{\frac{-1}{2}}}{4}$$

d)

$$\text{tangentLine}(f(x),x,a) \qquad \frac{3\cdot (a+2)\cdot e^{\frac{-a}{2}}}{2}-\frac{3\cdot e^{\frac{-a}{2}}\cdot x}{2}$$

$$\text{solve}\left(\frac{3\cdot (a+2)\cdot e^{\frac{-a}{2}}}{2}=2,a\right)|a>0 \qquad a=2.37767$$

e)

$$\text{Define } df(x)=\frac{d}{dx}(f(x)) \qquad \textit{Done}$$

$$\text{solve}\left(df(x)=\tan\left(\frac{-\pi}{6}\right),x\right) \qquad x=-\ln\left(\frac{4}{27}\right)$$

f)

$methods\_diffcalc\backslash tangent\_line(f(x),x,a)$

- Derivative: $\dfrac{-3\cdot e^{\frac{-x}{2}}}{2}$
- Gradient: $\dfrac{-3\cdot e^{\frac{-a}{2}}}{2}$
- Passes Through: $\begin{bmatrix} a & 3\cdot e^{\frac{-a}{2}} \end{bmatrix}$
- x −Intercept: $\begin{bmatrix} a+2 & 0 \end{bmatrix}$
- Vertical Intercept: $\begin{bmatrix} 0 & \dfrac{3\cdot (a+2)\cdot e^{\frac{-a}{2}}}{2} \end{bmatrix}$
- Tangent Line:

$$\frac{3\cdot (a+2)\cdot e^{\frac{-a}{2}}}{2} - \frac{3\cdot e^{\frac{-a}{2}}\cdot x}{2}$$

Define $t(a)=(a+2)\cdot \dfrac{3\cdot (a+2)\cdot e^{\frac{-a}{2}}}{2}$ *Done*

$methods\_func\backslash analyse(t(a),a)$

- Start Point: $\begin{bmatrix} -\infty & \infty \end{bmatrix}$
- End Point: $\begin{bmatrix} \infty & 0 \end{bmatrix}$
- Maximal Domain: $-\infty < a < \infty$
- a −Intercept: $\begin{bmatrix} -2 & 0 \end{bmatrix}$
- Vertical Intercept: $\begin{bmatrix} 0 & 6 \end{bmatrix}$
- Derivative:

$$\frac{-3\cdot(a-2)\cdot(a+2)\cdot \mathbf{e}^{\frac{-a}{2}}}{4}$$

- Stationary Points: (2)

$\begin{bmatrix} -2 & 0 \end{bmatrix}$ (Local min.)

$\begin{bmatrix} 2 & 24\cdot \mathbf{e}^{-1} \end{bmatrix}$ (Local max.)

4

a)
i)

$$\text{Define } c(t)=\frac{1}{8000}\cdot(a\cdot t-10)^3\cdot(b-t)+6 \qquad \textit{Done}$$

$$c(0) \qquad 6-\frac{b}{8}$$

$$\text{solve}(c(0)=5,b) \qquad b=8$$

ii)

When $a=0, C(t)=6+\frac{5}{4}(t-8)$ Solve $C(t)=10$ to get $t=11.2$

iii)

When $a=2$ we get $C(t)=6-\frac{(t-8)(t-5)^3}{1000}$
Solve $C(t)=5 \Rightarrow t=14.3\ minutes$

b)
i)

$$\text{Define } dc(t)=\frac{d}{dt}(c(t)) \qquad \textit{Done}$$

$$\text{solve}(dc(t)=0,t) \qquad t=\frac{12\cdot a+5}{2\cdot a} \text{ or } t=\frac{10}{a}$$

ii)

$$\text{solve}\left(\frac{12\cdot a+5}{2\cdot a}<0,a\right) \qquad \frac{-5}{12}<a<0$$

$$\text{solve}\left(\frac{10}{a}=\frac{12\cdot a+5}{2\cdot a},a\right) \qquad a=\frac{5}{4}$$

c)
i)

$methods\_func\backslash analyse(c(t),t)$

- Start Point: $\begin{bmatrix} -\infty & -\infty \end{bmatrix}$
- End Point: $\begin{bmatrix} \infty & -\infty \end{bmatrix}$
- Maximal Domain: $-\infty < t < \infty$
- t –Intercepts: (2.)
  $\begin{bmatrix} -1.22174 & 0. \end{bmatrix}, \begin{bmatrix} 8.65197 & 0. \end{bmatrix}$
- Vertical Intercept: $\begin{bmatrix} 0. & 5. \end{bmatrix}$
- Derivative:
  $-0.108 \cdot t^3 + 1.053 \cdot t^2 - 2.61 \cdot t + 1.925$
- Stationary Points: (2.)
  $\begin{bmatrix} 1.66667 & 6. \end{bmatrix}$ (Inflection)
  $\begin{bmatrix} 6.41667 & 10.5816 \end{bmatrix}$ (Local max.)

d)
i)

DelVar a — *Done*

$$\text{solve}\left(c\left(\frac{12 \cdot a + 5}{2 \cdot a}\right) = 15, a\right)$$

$a = 0.014003$ or $a = 7.11905$

ii)

$methods\_func\backslash analyse(c(t),t)$

- Start Point: $\begin{bmatrix} -\infty & -\infty \end{bmatrix}$
- End Point: $\begin{bmatrix} \infty & -\infty \end{bmatrix}$
- Maximal Domain: $-\infty < t < \infty$
- t –Intercepts: (2.)
  $\begin{bmatrix} -0.938915 & 0. \end{bmatrix}, \begin{bmatrix} 8.27089 & 0. \end{bmatrix}$
- Vertical Intercept: $\begin{bmatrix} 0. & 5. \end{bmatrix}$
- Derivative:
  $-0.256 \cdot t^3 + 2.256 \cdot t^2 - 4.44 \cdot t + 2.525$
- Stationary Points: (2.)
  $\begin{bmatrix} 1.25 & 6. \end{bmatrix}$ (Inflection)
  $\begin{bmatrix} 6.3125 & 20.0126 \end{bmatrix}$ (Local max.)

$$\frac{20.0126-5}{5} \qquad 3.00252$$

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