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VCE Mathematical Methods ¾
SAC 1 Revision I [0.16]
Workshop Solutions

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ______________ Notes:	Pg / Q #: ______________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ______________ Notes:	Pg / Q #: ______________ Notes:

Section A: SAC 1 Success

Welcome to the first SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1 - 50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC $1 \sim 8$.

Successful SAC

Study Score = How much you know × How much you show

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key!**

Tutor's Comment: Explain how even if you know everything, if you cannot show in the SAC, you will get 0.

Analogy: Skipping Questions

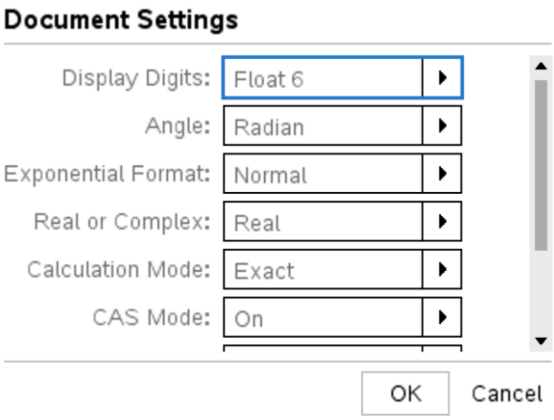
- Let's say if you were to fight them and win, you get assigned marks.

- Who would you fight first?
- Skip the hard question with little marks if it doesn't make sense during the reading time.

SAC Proficiency List

Before the SAC

- ☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.



Reading Time

- ☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and using what technology commands.
- ☐ Identify questions to **skip.**

  For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** – You don't have to start from Q1!
- ☐ Look for potential pitfalls – **Units, Domain restriction of the unknown, variable and function meaning.**

Writing Time

- ☐ **Circle what the question is asking** for in the question.
- ☐ Spend the first **50**% of the time on all the **easy questions** you identified.
- ☐ Spend the next **25**% of doing the **difficult questions** you left blank.

- [ ] Spend the last $\mathbf{25}\%$ of the time on **checking your answers**.
- [ ] Check your answer by reading the question again and see if you answered the question.
- [ ] Check in the order of

  Domino effect (check a, b, c first) $>$ ***Questions with high marks*** $(\mathbf{3}+)$ $>$ *Hard Questions*
- [ ] $\mathbf{TI}$ ONLY: Use new document - **doc** $\mathbf{4, 1.}$

After the SAC

- [ ] Think about how each mark loss can be prevented using this proficiency list.
- [ ] Think about the big picture and improve the marks -

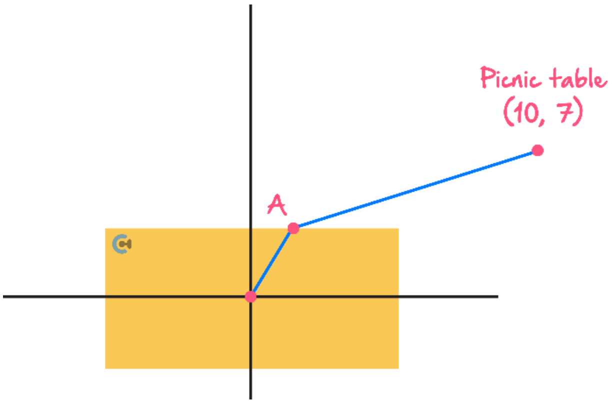
  Instead of spending 10 *minutes on* 10*c) (*1 *mark), I should have checked* 5*a) (*3 *marks)*

Space for Personal Notes

Section B: SAC Questions – Tech Active (53 Marks)

Question 1 (11 marks)

James is in the middle of a rectangular pool with dimensions of $4\ m$ (length) by $8\ m$ (width). Food has just been placed down on the picnic table, so James needs to get out of the pool and arrive at the picnic table as fast as possible. He can swim at a speed of $1\ m/s$ and run at a speed of $3\ m/s$. James is initially at the origin $(0, 0)$.



a. The point, $A(x, y)$, can be on any point along the horizontal section of the pool in the first quadrant. Find the distance that James needs to swim and state any domain restrictions. (2 marks)

Travels up 2 metres and across x metres. Cannot go left or more than 4 more metres right and $x > 0$ to be in the first quadrant.
$d_s = \sqrt{x^2 + 4}, x \in (0, 4]$. (1A function, 1A domain)

b. Find the distance that James needs to run and state any domain restrictions. (2 marks)

Gets out of the pool at $(x, 2)$, where $0 < x \leq 4$
$d_r = \sqrt{25 + (x - 10)^2}, x \in (0, 4]$. (1A function, 1A domain)

```
In[25]:= EuclideanDistance[{x, 2}, {10, 7}]
```

Out[25]= $\sqrt{25 + \text{Abs}[-10 + x]^2}$

```
In[26]:= Assuming[x ≥ 0, Refine[EuclideanDistance[{x, 2}, {10, 7}]]]
```

Out[26]= $\sqrt{25 + (-10 + x)^2}$

c. Hence, find the total time taken by James to reach the picnic table. (1 mark)

Swims d_s metres at 1m/s. Thus $T_s = d_s$.

Runs d_r metres at 3m/s. Thus $T_r = \dfrac{d_r}{3}$.

Total time $= T_s + T_r = \sqrt{x^2+4} + \dfrac{\sqrt{25+(x-10)^2}}{3}$. (1A)

d. Hence, or otherwise, find the coordinates of A, such that James minimises the total time taken and states this minimal time taken. Give your answers correct to two decimal places. (3 marks)

$T(x) = \sqrt{x^2+4} + \dfrac{\sqrt{25+(x-10)^2}}{3}$ we want to minimise this function.

Solve $T'(x) = \dfrac{x}{\sqrt{x^2+4}} + \dfrac{x-10}{3\sqrt{(x-10)^2+25}} = 0 \implies x = 0.615607$ (1M).

$T(0.615607) = 5.64$.

Thus $A(0.62, 2)$ and minimum time is 5.64 seconds (1A for coordinate, 1A for time).

```
In[37]:= T[x_] := Sqrt[x^2 + 4] + Sqrt[25 + (-10 + x)^2]/3
In[38]:= T'[x]
Out[38]= (-10 + x)/(3 Sqrt[25 + (-10 + x)^2]) + x/Sqrt[4 + x^2]
In[39]:= Solve[T'[x] == 0, x] // N
Out[39]= {{x -> 0.615607}}
In[40]:= T[0.6156067766766208`]
Out[40]= 5.63703
In[43]:= Minimize[{T[x], 0 ≤ x ≤ 4}, x] // N
Out[43]= {5.63703, {x -> 0.615607}}
```

e. James now swims at a speed of $q\ m/s$. Find the value(s) of q such that the time it takes James to reach the picnic table is minimised at an endpoint. Give your answer correct to two decimal places. (3 marks)

$T = \dfrac{\sqrt{x^2+4}}{q} + \dfrac{\sqrt{25+(x-10)^2}}{3}$. (1M).

We want the minimum of this function for $0 \le x \le 4$ to occur when $x = 0$ or $x = 4$.

The function will never have a minimum when $x = 0$, so we solve $T'(4) = 0$ for q

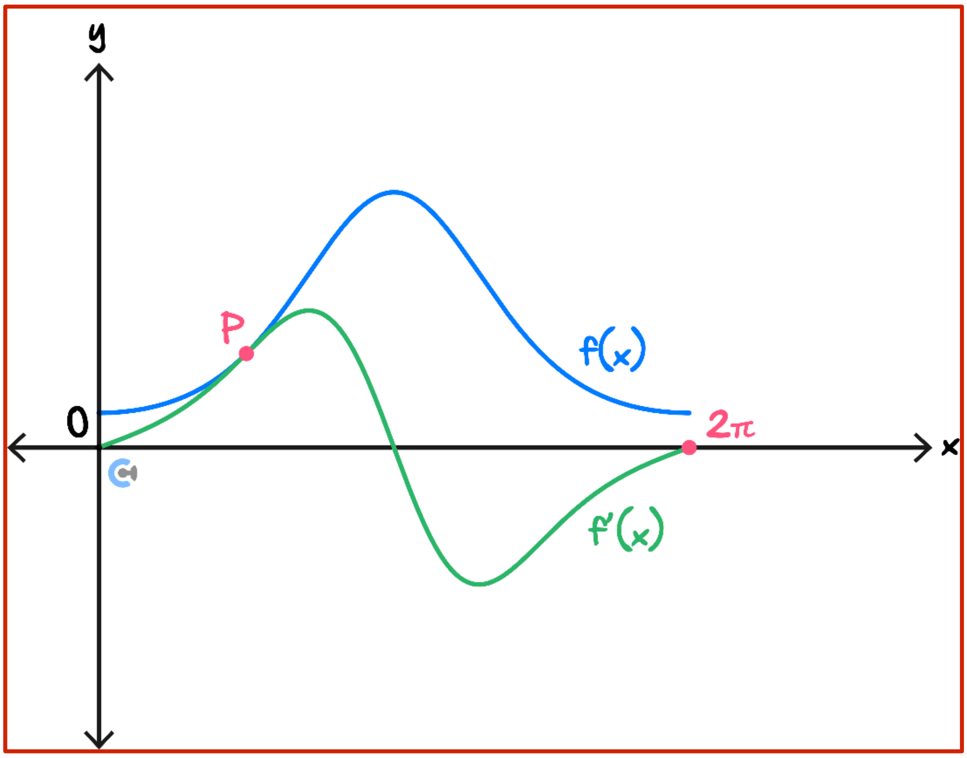
$\implies q = \sqrt{\dfrac{61}{5}}$. (1M).

Checking with sliders we conclude that minimum occurs when $x = 4$ for $q \ge \sqrt{\dfrac{61}{5}} \approx$ 3.49. (1A)

```
In[19]:= Tq[x_] := Sqrt[x^2 + 4]/q + Sqrt[25 + (x - 10)^2]/3
In[20]:= Solve[Tq'[4] == 0, q]
Out[20]= {{q -> Sqrt[61/5]}}
In[21]:= Solve[Tq'[4] == 0, q] // N
Out[21]= {{q -> 3.49285}}
```

Question 2 (15 marks)

Consider the function $f: [0, 2\pi] \to R, f(x) = e^{-\cos(x)}$. The graph below shows $y = f(x)$ and $y = f'(x)$. The two functions touch at the point P.



a. State $f'(x)$. (1 mark)

$f'(x) = e^{-\cos(x)}\sin(x)$

```
In[56]:= f[x_] := Exp[-Cos[x]]
In[57]:= f'[x]
Out[57]= e^-Cos[x] Sin[x]
```

b. Find the exact coordinates of P. (2 marks)

Solve $f'(x) = f(x) \implies x = \frac{\pi}{2}$. $f(\frac{\pi}{2}) = 1$.

Thus $P\left(\frac{\pi}{2}, 1\right)$. (1A for each coordinate).

```
In[58]:= Solve[f[x] == f'[x] && 0 ≤ x ≤ 2 Pi]
Out[58]= {{x → π/2}, {x → π/2}}
In[59]:= f[Pi / 2]
Out[59]= 1
```

c. A normal line is drawn to the function f when $x = a$, and $a \neq \pi$.

i. Find the two possible values of a such that the normal line passes through the point $(\pi, 0)$. Give your answers correct to two decimal places. (2 marks)

We can find the normal line of $f(x)$ at $x = a$ in terms of a.
$n(x) = -xe^{\cos(a)}\csc(a) + e^{-\cos(a)} + ae^{\cos(a)}\csc(a)$ (1M or 1M for equating $f'(x)$ and a gradient of line).
$n(x)$ passes through $(\pi, 0)$, so we sub in $x = \pi$ equate to zero and solve for a.
$a = 1.75, 4.54$.

```
In[23]:= NormalLine[f[x], x, a]
Out[23]= e^-Cos[a] + a e^Cos[a] Csc[a] - e^Cos[a] x Csc[a]

In[27]:= e^-Cos[a] + a e^Cos[a] Csc[a] - e^Cos[a] x Csc[a] /. x → Pi
Out[27]= e^-Cos[a] + a e^Cos[a] Csc[a] - e^Cos[a] π Csc[a]

In[29]:= Solve[e^-Cos[a] + a e^Cos[a] Csc[a] - e^Cos[a] π Csc[a] == 0 && 0 ≤ a ≤ 2 Pi, a] // N
Out[29]= {{a → 1.74605}, {a → 4.53713}}
```

ii. Find the acute angle made by the two normal lines when they intersect. Give your answer correct to the nearest degree. (2 marks)

Gradient of normal lines are ± 0.853. (1M).
Thus acute angle between them is $|\arctan(0.835) - \arctan(-0.853)| = 81^\circ$. (1A)

```
In[30]:= NormalLine[f[x], x, 1.7460514603409005`]
Out[30]= 2.67997 - 0.853062 x

In[31]:= NormalLine[f[x], x, 4.537133846838686`]
Out[31]= -2.67997 + 0.853062 x

In[35]:= Abs[ArcTan[0.8530621074843592`] - ArcTan[-0.8530621074843592`]] /
           Degree // N
Out[35]= 80.9325
```

d. Find the maximum vertical distance between the functions $f(x)$ and $f'(x)$ and state the x value for which this maximum distance occurs. Give your answers correct to two decimal places. (2 marks).

We maximise $f(x) - f'(x)$.
Max distance is 3.57 which occurs when $x = 3.72$ (1A each).

```
In[42]:= Maximize[f[x] - f'[x], x] // N
Out[42]= {3.5732, {x → 3.71642}}
```

e.

i. Write down an equation that can be solved using Newton's method to approximate the x-value for a turning point of $f'(x)$. (2 marks)

Turning points of $f'(x)$ occur when $f''(x) = 0$. (1M)
Equation to solve is $e^{-\cos(x)}(\cos(x) + \sin^2(x)) = 0$. (1A or equivalent form, must have equals zero).

```
In[60]:= f''[x] // FullSimplify
Out[60]= e^-Cos[x] (Cos[x] + Sin[x]^2)
```

ii. Find an approximate x-value for a turning point of $f'(x)$ by completing 3 iterations of Newton's method with $x_0 = 2$, in the table below. Write your answers correctly to three decimal places. (2 marks).

$x_0 = 2, x_1 = 2.318, x_2 = 2.241, x_3 = 2.237$. (2A all correct, 1M one correct).

```
In[65]:= n[x_] := x - f''[x]/f'''[x]
In[66]:= n[x] // FullSimplify
Out[66]= (x (-3 + Cos[x]) + Csc[x] + Tan[x])/(-3 + Cos[x])
In[76]:= n[2.0]
Out[76]= 2.31769
In[77]:= n[n[2.0]]
Out[77]= 2.24088
In[78]:= n[n[n[2.0]]]
Out[78]= 2.23705
```

2
2.318
2.241
2.237

f. Consider the function $g(x) = f(bx)$, where $b > 0$.

i. State a sequence of transformations that map $f'(x)$ to $g'(x)$. (1 mark)

$g'(x) = bf'(bx)$.
So dilation by factor b from x-axis and dilation by factor $\frac{1}{b}$ from the y-axis. (1A)

ii. Find the value(s) of b such that g and $g'(x)$ intersect at least twice for $x \in [0, 2\pi]$. (1 mark)

Play with sliders. From diagram in question we see they are tangent when $b = 1$.
Conclude that two intersections if $b > 1$ (1A)

Question 3 (14 marks)

Consider the following cubic polynomial:

$$p(x) = 2(x - 2)(x^2 - 10x + 28)$$

a. Find the coordinates of the stationary point of $p(x)$ and state its nature. (2 marks)

$(4, 16)$ is a stationary point of inflection. (1A coord, 1A nature).

```
In[97]:= f[x_] := 2 (x - 2) (x^2 - 10 x + 28)
In[98]:= Solve[f'[x] == 0, x]
Out[98]= {{x → 4}, {x → 4}}
In[99]:= f[4]
Out[99]= 16
In[100]:= f''[4]
Out[100]= 0
```

b. Hence, express $p(x)$ in the form $a(x - h)^3 + k$. (1 mark)

$p(x) = 2(x - 4)^3 + 16$ (1A)

```
In[101]:= Solve[a * (x - 4) ^3 + 16 == f[x], a]
Out[101]= {{a → 2}}
```

c. Find the sequence of translations on $p(x)$ that will result in an odd function. (2 marks)

A translation 4 units to the left and a translation 16 units down (1A each).

d.

i. Find the equation of the tangent line of $p(x)$ at the point $x = a$. Provide your answer in terms of a. (2 marks)

Gradient at $x = a$ is $p'(a) = 6(a-4)^2$ (1M). Line tangent has this gradient and passes through $(a, p(a))$.
$y = 6(a-4)^2 x - 4(28 + a^2(a-6))$ (1A).

```
In[102]:= f'[x] // FullSimplify
Out[102]= 6 (-4 + x)^2

In[103]:= TangentLine[f[x], x, a] // FullSimplify
Out[103]= -4 (28 + (-6 + a) a^2) + 6 (-4 + a)^2 x
```

ii. Find the equations of the tangents of $p(x)$ that share the same x-intercept as the cubic. (3 marks)

The tangent in previous part must pass through $(2, 0)$.
Sub in $x = 2$ and equate to zero. $-4(a-5)(a-2)^2 = 0 \implies a = 2, 5$ (1M).
The tangents are
$y = 24x - 48$ and $y = 6x - 12$ (1A each).

```
In[104]:= TangentLine[f[x], x, a] /. x → 2 // FullSimplify
Out[104]= -4 (-5 + a) (-2 + a)^2

In[105]:= Solve[-4 (-5 + a) (-2 + a)^2 == 0, a]
Out[105]= {{a → 2}, {a → 2}, {a → 5}}

In[106]:= TangentLine[f[x], x, 2]
Out[106]= -48 + 24 x

In[107]:= TangentLine[f[x], x, 5]
Out[107]= -12 + 6 x
```

iii. Hence, state the equation of the tangents to the inverse function $p^{-1}(x)$ that share the same y-intercept as the inverse function. (2 marks)

$$y = \frac{x}{24} + 2 \text{ and } y = \frac{x}{6} + 2 \text{ (1A each).}$$

e. Find the acute angle made by the tangents to p and p^{-1} at the point of intersection of these two functions. Give your answer correct to the nearest degree. (2 marks)

Intersect at approx $x = 2.09$. (1M).

Angle is $\left|\arctan(p'(2.09)) - \arctan\left(\frac{1}{p'(2.09)}\right)\right| = 85°$ (1A).

```
In[135]:= Solve[f[x] == x, x, Reals] // N
Out[135]= {{x → 2.09123}}

In[136]:= Abs[ArcTan[f'[2.0912331809635747`]] -
             ArcTan[1/f'[2.0912331809635747`]]]/ Degree // N
Out[136]= 84.7617
```

Space for Personal Notes

Question 4 (13 marks)

Australia (population: 25 million) has been quarantined from the rest of the world since the discovery of a highly contagious prion. The number of people (in millions) infected with the virus t days after the outbreak begins is:

$$p(t) = \frac{\alpha}{1 + e^{0.15(40-t)}}$$

Where $\alpha \in R$.

a. Given that, eventually, everyone will be infected, show that $\alpha = 25$. (2 marks)

It must be that $\lim_{t\to\infty}(p(t)) = 25$ (1M).

Therefore $\dfrac{\alpha}{1 + \lim_{t\to\infty} e^{0.15(40-t)}} = 25$

$\dfrac{\alpha}{1+0} = 25$

$\implies \alpha = 25$ (1A, logical working leading to conclusion)

```
In[1]:= p[t_] := a / (1 + Exp[0.15 (40 - t)])

In[2]:= Solve[Limit[p[t], t → Infinity] == 25, a]

Out[2]= {{a → 25.}}
```

b. When is $p(t)$ increasing the fastest? Find $p(t)$ at that point. (3 marks)

Increasing the fastest $\implies$ we want to maximise $p(t)$ (1M).

Solve $p''(t) = 0 \implies t = 40$ (1A).

Thus $p(40) = 12.5$. (1A)

```
In[6]:= p[t_] := 25 / (1 + Exp[15 / 100 (40 - t)])

In[9]:= Solve[p''[t] == 0, t, Reals]

Out[9]= {{t → 40}}

In[11]:= p[40] // N

Out[11]= 12.5

In[12]:= Maximize[p'[t], t]

Out[12]= {15/16, {t → 40}}
```

c. The number of people hospitalised is much lower. The number of people hospitalised (in millions), t days after the outbreak begins, is given by:

$$h(t) = ce^{a(t-b)^2}$$

Where $a < 0$ and $b, c > 0$.

i. State $h'(t)$. (1 mark)

$h'(t) = 2ace^{a(t-b)^2}(t-b)$

```
In[17]:= h[t_] := c Exp[a (t - b) ^2]
In[18]:= h'[t]
Out[18]= 2 a c e^(a (-b+t)^2) (-b + t)
```

ii. Show that the maximum occurs at $t = b$ and the maximum value is given by $h(b) = c$. (3 marks)

We solve $h'(t) = 2ace^{a(t-b)^2}(t-b) = 0$.
Note that $2ace^{a(t-b)^2} \neq 0$ because $a, c \neq 0$ and $e^x \neq 0$ for any $x \in \mathbb{R}$. (1M).
Thus only solution to $h'(t) = 0$ is $t = b$ (1A)
Then $h(b) = ce^0 = c$.
This is a maximum because $h''(b) = 2ac < 0$ (1A).

```
In[19]:= h''[b]
Out[19]= 2 a c
```

iii. The maximum number of hospitalisations is 1 million and this occurs 100 days after the outbreak begins. Find the values of b and c. (1 mark)

$b = 100, c = 1$.

```
In[22]:= Solve[h[100] == 1 && h'[100] == 0, Reals]
Out[22]= {{a → 0, c → 1}, {b → 100, c → 1}}
```

iv. Given that the maximum and minimum rate of change, in the number of people hospitalised, occurs $10\sqrt{10}$ days apart, find the value of a. (3 marks)

We solve $h''(t) = 0 \implies t = 100 \pm \frac{1}{\sqrt{2}} \cdot \sqrt{-\frac{1}{a}}$ (1M).

Thus max and min rates of change are $\sqrt{2} \cdot \sqrt{-\frac{1}{a}}$ apart.

We solve $\sqrt{2} \cdot \sqrt{-\frac{1}{a}} = 10\sqrt{10}$ (1M).

$a = -\frac{1}{500} = 0.002$ (1A)

```
In[23]:= h[t_] := Exp[a (t - 100) ^2]
In[25]:= Solve[h''[t] == 0, t, Reals]
```

Out[25]= $\left\{\left\{t \to 100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a < 0\right\}, \left\{t \to 100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a < 0\right\}\right\}$

In[26]:= $100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} - \left(100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}\right)$

Out[26]= $\sqrt{2}\sqrt{-\frac{1}{a}}$

In[27]:= Solve$\left[\sqrt{2}\sqrt{-\frac{1}{a}} == 10\sqrt{10}, a\right]$

Out[27]= $\left\{\left\{a \to -\frac{1}{500}\right\}\right\}$

Space for Personal

Section C: Tech-Active Solutions – Mathematica

Question Number	Solutions
1a	
1b	In[25]:= `EuclideanDistance[{x, 2}, {10, 7}]` Out[25]= $\sqrt{25+\text{Abs}[-10+x]^2}$ In[26]:= `Assuming[x ≥ 0, Refine[EuclideanDistance[{x, 2}, {10, 7}]]]` Out[26]= $\sqrt{25+(-10+x)^2}$
1c	

1d	In[37]:= $T[x\_] := \sqrt{x\text{^}2+4} + \dfrac{\sqrt{25+(-10+x)^2}}{3}$
	In[38]:= `T'[x]`
	Out[38]= $\dfrac{-10+x}{3\sqrt{25+(-10+x)^2}} + \dfrac{x}{\sqrt{4+x^2}}$
	In[39]:= `Solve[T'[x] == 0, x] // N`
	Out[39]= `{{x → 0.615607}}`
	In[40]:= ``T[0.6156067766766208`]``
	Out[40]= 5.63703
	In[43]:= `Minimize[{T[x], 0 ≤ x ≤ 4}, x] // N`
	Out[43]= `{5.63703, {x → 0.615607}}`

1e	In[19]:= $Tq[x\_] := \dfrac{\sqrt{x\text{^}2+4}}{q} + \dfrac{\sqrt{25+(x-10)\text{^}2}}{3}$
	In[20]:= `Solve[Tq'[4] == 0, q]`
	Out[20]= $\left\{\left\{q \to \sqrt{\frac{61}{5}}\right\}\right\}$
	In[21]:= `Solve[Tq'[4] == 0, q] // N`
	Out[21]= `{{q → 3.49285}}`

2a	In[56]:= `f[x_] := Exp[-Cos[x]]` In[57]:= `f'[x]` Out[57]= $e^{-\text{Cos}[x]}\ \text{Sin}[x]$
2b	In[58]:= `Solve[f[x] == f'[x] && 0 ≤ x ≤ 2 Pi]` Out[58]= $\left\{\left\{x \to \frac{\pi}{2}\right\}, \left\{x \to \frac{\pi}{2}\right\}\right\}$ In[59]:= `f[Pi / 2]` Out[59]= 1

2ci	In[23]:= `NormalLine[f[x], x, a]`
	Out[23]= $e^{-\text{Cos}[a]} + a\, e^{\text{Cos}[a]}\ \text{Csc}[a] - e^{\text{Cos}[a]}\ x\ \text{Csc}[a]$
	In[27]:= $e^{-\text{Cos}[a]} + a\, e^{\text{Cos}[a]}\ \text{Csc}[a] - e^{\text{Cos}[a]}\ x\ \text{Csc}[a]$ `/. x → Pi`
	Out[27]= $e^{-\text{Cos}[a]} + a\, e^{\text{Cos}[a]}\ \text{Csc}[a] - e^{\text{Cos}[a]}\ \pi\ \text{Csc}[a]$
	In[29]:= `Solve[`$e^{-\text{Cos}[a]} + a\, e^{\text{Cos}[a]}\ \text{Csc}[a] - e^{\text{Cos}[a]}\ \pi\ \text{Csc}[a]$ `== 0 && 0 ≤ a ≤ 2 Pi, a] // N`
	Out[29]= `{{a → 1.74605}, {a → 4.53713}}`

2cii	In[30]:= ``NormalLine[f[x], x, 1.7460514603409005`]``
	Out[30]= `2.67997 - 0.853062 x`
	In[31]:= ``NormalLine[f[x], x, 4.537133846838686`]``
	Out[31]= `-2.67997 + 0.853062 x`
	In[35]:= ``Abs[ArcTan[0.8530621074843592`] - ArcTan[-0.8530621074843592`]] / Degree // N``
	Out[35]= 80.9325

2d	In[42]:= `Maximize[f[x] - f'[x], x] // N`
	Out[42]= `{3.5732, {x → 3.71642}}`

2ei	In[60]:= `f''[x] // FullSimplify`
	Out[60]= $e^{-\text{Cos}[x]}\ \left(\text{Cos}[x] + \text{Sin}[x]^2\right)$

2eii	In[65]:= $n[x\_] := x - \frac{f''[x]}{f'''[x]}$
	In[66]:= `n[x] // FullSimplify`
	Out[66]= $\frac{x\ (-3 + \text{Cos}[x]) + \text{Csc}[x] + \text{Tan}[x]}{-3 + \text{Cos}[x]}$
	In[76]:= `n[2.0]`
	Out[76]= 2.31769
	In[77]:= `n[n[2.0]]`
	Out[77]= 2.24088
	In[78]:= `n[n[n[2.0]]]`
	Out[78]= 2.23705

2fi	
2fii	
3a	In[97]:= `f[x_] := 2 (x - 2) (x^2 - 10 x + 28)` In[98]:= `Solve[f'[x] == 0, x]` Out[98]= {{x → 4}, {x → 4}} In[99]:= `f[4]` Out[99]= 16 In[100]:= `f''[4]` Out[100]= 0
3b	In[101]:= `Solve[a * (x - 4) ^3 + 16 == f[x], a]` Out[101]= {{a → 2}}
3c	

3di	In[102]:= `f'[x] // FullSimplify`
	Out[102]= $6\,(-4+x)^2$
	In[103]:= `TangentLine[f[x], x, a] // FullSimplify`
	Out[103]= $-4\left(28+(-6+a)\,a^2\right)+6\,(-4+a)^2\,x$

3dii	In[104]:= `TangentLine[f[x], x, a] /. x → 2 // FullSimplify`
	Out[104]= $-4\,(-5+a)\,(-2+a)^2$
	In[105]:= `Solve[-4 (-5 + a) (-2 + a)^2 == 0, a]`
	Out[105]= {{a → 2}, {a → 2}, {a → 5}}
	In[106]:= `TangentLine[f[x], x, 2]`
	Out[106]= $-48+24\,x$
	In[107]:= `TangentLine[f[x], x, 5]`
	Out[107]= $-12+6\,x$

<table>
<tr><td>3diii</td><td>

```
In[128]:= Solve[f[y] == x, y]
```

$$\text{Out[128]= } \left\{\left\{y \to 4 + \frac{(-16+x)^{1/3}}{2^{1/3}}\right\}, \left\{y \to 4 - \frac{\left(1 - i\sqrt{3}\right)(-16+x)^{1/3}}{2 \times 2^{1/3}}\right\}, \left\{y \to 4 - \frac{\left(1 + i\sqrt{3}\right)(-16+x)^{1/3}}{2 \times 2^{1/3}}\right\}\right\}$$

$$\text{In[129]:= } \texttt{f1[x\_] := 4 + } \frac{\texttt{Surd[x - 16, 3]}}{\texttt{Surd[2, 3]}}$$

```
In[130]:= TangentLine[f1[x], x, a] /. x → 0
```

$$\text{Out[130]= } 4 - \frac{a}{3 \times 2^{1/3} \sqrt[3]{-16+a}^{2}} + \frac{\sqrt[3]{-16+a}}{2^{1/3}}$$

$$\text{In[131]:= Solve}\left[4 - \frac{a}{3 \times 2^{1/3} \sqrt[3]{-16+a}^{2}} + \frac{\sqrt[3]{-16+a}}{2^{1/3}} == 2, a\right]$$

```
Out[131]= {{a → 0}, {a → 18}}
```

```
In[132]:= TangentLine[f1[x], x, 0]
```

$$\text{Out[132]= } 2 + \frac{x}{24}$$

```
In[133]:= TangentLine[f1[x], x, 18]
```

$$\text{Out[133]= } 2 + \frac{x}{6}$$

</td></tr>
<tr><td>3e</td><td>

```
In[135]:= Solve[f[x] == x, x, Reals] // N
Out[135]= {{x → 2.09123}}
```

$$\text{In[136]:= Abs}\left[\texttt{ArcTan[f'[2.0912331809635747`]]} - \texttt{ArcTan}\left[\frac{1}{\texttt{f'[2.0912331809635747`]}}\right]\right] \Big/ \texttt{Degree // N}$$

```
Out[136]= 84.7617
```

</td></tr>
<tr><td>4a</td><td>

$$\text{In[1]:= } \texttt{p[t\_] := } \frac{a}{\texttt{1 + Exp[0.15 (40 - t)]}}$$

```
In[2]:= Solve[Limit[p[t], t → Infinity] == 25, a]
Out[2]= {{a → 25.}}
```

</td></tr>
</table>

4b	In[6]:= `p[t_] :=` $\dfrac{25}{1+\text{Exp}[15/100\ (40-t)]}$
	 In[9]:= `Solve[p''[t] == 0, t, Reals]`
	 Out[9]= `{{t → 40}}`
	 In[11]:= `p[40] // N`
	 Out[11]= `12.5`
	 In[12]:= `Maximize[p'[t], t]`
	 Out[12]= $\left\{\frac{15}{16}, \{t \to 40\}\right\}$

4ci	In[17]:= `h[t_] := c Exp[a (t - b)^2]` In[18]:= `h'[t]` Out[18]= $2\,a\,c\,e^{a\,(-b+t)^2}\,(-b+t)$
4cii	In[19]:= `h''[b]` Out[19]= `2 a c`
4ciii	In[22]:= `Solve[h[100] == 1 && h'[100] == 0, Reals]` Out[22]= `{{a → 0, c → 1}, {b → 100, c → 1}}`
4civ	In[23]:= `h[t_] := Exp[a (t - 100)^2]` In[25]:= `Solve[h''[t] == 0, t, Reals]` Out[25]= $\left\{\left\{t \to 100-\frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a<0\right\}, \left\{t \to 100+\frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a<0\right\}\right\}$ In[26]:= $100+\frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}-\left(100-\frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}\right)$ Out[26]= $\sqrt{2}\sqrt{-\frac{1}{a}}$ In[27]:= $\text{Solve}\left[\sqrt{2}\sqrt{-\frac{1}{a}} == 10\sqrt{10}, a\right]$ Out[27]= $\left\{\left\{a \to -\frac{1}{500}\right\}\right\}$

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Section D: Tech-Active Solutions – Casio

Question Number	Solutions
1a	
1b	$\sqrt{(x-10)^2+(2-7)^2}$ $\sqrt{x^2-20\cdot x+125}$
1c	
1d	Define $T(x)=\sqrt{x^2+4}+\frac{\sqrt{(x-10)^2+25}}{3}$ done solve$\left(\frac{d}{dx}(T(x))=0, x\right)$ $\{x=0.6156067766\}$ T(0.6156067766) 5.637028955 fMin(T(x), x, 0, 4) {MinValue=5.637028955, x=0.6156067766}
1e	Define $Tq(x)=\frac{\sqrt{x^2+4}}{q}+\frac{\sqrt{25+(x-10)^2}}{3}$ done solve$\left(\frac{d}{dx}(Tq(x)=0) \mid x=4, q\right)$ $\left\{q=\frac{\sqrt{305}}{5}\right\}$ solve$\left(\frac{d}{dx}(Tq(x)=0) \mid x=4, q\right)$ $\{q=3.492849839\}$
2a	Define $f(x)=e^{-\cos(x)}$ done $\frac{d}{dx}(f(x))$ $\sin(x)\cdot e^{-\cos(x)}$

2b	$\text{solve}\left(f(x)=\frac{d}{dx}(f(x))\mid 0\le x\le 2\pi, x\right)$ $\left\{x=\frac{\pi}{2}\right\}$ $f\left(\frac{\pi}{2}\right)$ 1
2ci	$\text{normal}(f(x),x,a)$ $\frac{-x\cdot e^{\cos(a)}}{\sin(a)}+\frac{a\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}$ $\text{ans}\mid x=\pi$ $\frac{a\cdot e^{\cos(a)}}{\sin(a)}-\frac{\pi\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}$ $\text{solve}\left(\frac{a\cdot e^{\cos(a)}}{\sin(a)}-\frac{\pi\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}=0\mid 0\le a\le 2\pi, a\right.$ $\{a=1.74605146, a=4.537133847\}$
2cii	$\text{normal}(f(x),x,1.74605146)$ $-0.8530621077\cdot x+2.67997365$ $\text{normal}(f(x),x,4.537133847)$ $0.8530621076\cdot x-2.679973651$ $\lvert\tan^{-1}(-0.8530621077)-\tan^{-1}(0.8530621076)\rvert$ 80.93247628 Change to Deg mode! Make sure to change back to Rad
2d	$\text{fMax}\left(f(x)-\frac{d}{dx}(f(x)),x,0,2\pi\right)$ $\{\text{MaxValue}=3.573200775, x=3.7164\}$
2ei	$\frac{d^2}{dx^2}(f(x))$ $\left((\sin(x))^2+\cos(x)\right)\cdot e^{-\cos(x)}$
2eii	Define $n(x)=x-\frac{\frac{d^2}{dx^2}(f(x))}{\frac{d^3}{dx^3}(f(x))}$ done $n(x)$ $x-\frac{(\sin(x))^2+\cos(x)}{(\sin(x))^3+3\cdot\cos(x)\cdot\sin(x)-\sin(x)}$ $n(2.0)$ 2.317694099 $n(\text{ans})$ 2.240875415 $n(\text{ans})$ 2.237046622
2fi	

2fii	
3a	Define p(x)=2(x−2)(x^2−10x+28) done $\text{solve}\left(\frac{d}{dx}(p(x))=0, x\right)$ {x=4} p(4) 16 $\frac{d^2}{dx^2}(p(x))\|x=4$ 0
3b	solve(a*(x−4)^3+16=p(x),a {a=2}
3c	
3di	tanLine(p(x),x,a) $2\cdot(a^2-10\cdot a+28)\cdot(a-2)+x\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))-a\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))$ simplify(ans) $-4\cdot a^3+6\cdot a^2\cdot x+24\cdot a^2-48\cdot a\cdot x+96\cdot x-112$ collect(ans,x $(6\cdot a^2-48\cdot a+96)\cdot x-4\cdot a^3+24\cdot a^2-112$
3dii	tanLine(p(x),x,a) $2\cdot(a^2-10\cdot a+28)\cdot(a-2)+x\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))-a\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))$ simplify(ans) $-4\cdot a^3+6\cdot a^2\cdot x+24\cdot a^2-48\cdot a\cdot x+96\cdot x-112$ collect(ans,x $(6\cdot a^2-48\cdot a+96)\cdot x-4\cdot a^3+24\cdot a^2-112$ ans|x=2 $-4\cdot a^3+24\cdot a^2+2\cdot(6\cdot a^2-48\cdot a+96)-112$ solve(ans=0,a {a=2,a=5} tanLine(p(x),x,2) $24\cdot x-48$ tanLine(p(x),x,5) $6\cdot x-12$
3diii	
3e	solve(p(x)=x,x {x=2.091233181} Define $dp(x)=\frac{d}{dx}(p(x))$ done $\left\|\tan^{-1}(dp(2.091233181))-\tan^{-1}\left(\frac{1}{dp(2.091233181)}\right)\right\|$ 84.76166868 Change to Deg then change back to Rad

4a	Define $p(t)=\frac{a}{1+e^{0.15(40-t)}}$ done solve($\lim_{t\to\infty}(p(t))=25, a$ $\{a=25\}$
4b	Define $p(t)=\frac{25}{1+e^{0.15(40-t)}}$ done solve($\frac{d^2}{dt^2}(p(t))=0, t$) $\{t=40\}$ p(40) 12.5 fMax$\left(\frac{d}{dt}(p(t)), t, -\infty, \infty\right)$ {MaxValue=0.9375, t=40}
4ci	Define h(t)=c*e^(a*(t-b)^2) done $\frac{d}{dt}(h(t))$ $-2\cdot a\cdot c\cdot(b-t)\cdot e^{a\cdot(b-t)^2}$
4cii	$\frac{d^2}{dt^2}(h(t))\|t=b$ $2\cdot a\cdot c$
4ciii	$\left\{\begin{array}{l} h(100)=1 \\ \frac{d}{dt}(h(t))=0\|t=100 \end{array}\right\|_{b,c}$ {b=100, c=1}
4civ	solve($\frac{d^2}{dt^2}(h(t))=0, t$) $\left\{t=\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}, t=\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\}$ $\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|$ $\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|$ simplify($\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|$) $\left\|\frac{1}{a}\right\|\cdot\sqrt{-2\cdot a}$ solve($\left\|\frac{1}{a}\right\|\cdot\sqrt{-2\cdot a}=10\sqrt{10}, a$ $\left\{a=-\frac{1}{500}\right\}$

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Section E: Tech-Active Solutions - TI

Question Number	Solutions
1a	
1b	
1c	
1d	*methods_func\analysed*$(t(x),x,0,4)$ ▸ Start Point: $\begin{bmatrix} 0 & \frac{5\cdot\sqrt{5}}{3}+2 \end{bmatrix}$ ▸ End Point: $\begin{bmatrix} 4 & \frac{\sqrt{61}}{3}+2\cdot\sqrt{5} \end{bmatrix}$ ▸ Maximal Domain: $0\le x\le 4$ ▸ Asymptotes: (2) Define $t(x)=\sqrt{x^2+4}+\frac{1}{3}\cdot\sqrt{25+(x-10)^2}$ *Done* $y=\frac{10}{3}-\frac{4\cdot x}{3}$ (Oblique) $y=\frac{4\cdot x}{3}-\frac{10}{3}$ (Oblique) ▸ No x −Intercepts Found ▸ Vertical Intercept: $\begin{bmatrix} 0 & \frac{5\cdot\sqrt{5}}{3}+2 \end{bmatrix}$ ▸ Derivative: $\frac{x-10}{3\cdot\sqrt{x^2-20\cdot x+125}}+\frac{x}{\sqrt{x^2+4}}$ ▸ No Inflection Points Found ▸ Stationary Point: $\begin{bmatrix} 0.615607 & 5.63703 \end{bmatrix}$ (Local min.)

1e		Define $tq(x)=\frac{\sqrt{x^2+4}}{q}+\frac{1}{3}\cdot\sqrt{25+(x-10)^2}$ *Done* Define $dtq(x)=\frac{d}{dx}(tq(x))$ *Done* solve$(dtq(4)=0,q)$ $q=3.49285$
2a		Define $f(x)=e^{-\cos(x)}$ *Done* Define $df(x)=\frac{d}{dx}(f(x))$ *Done* $df(x)$ $\sin(x)\cdot e^{-\cos(x)}$
2b		*methods_func\intersectd*$(f(x),df(x),x,0,2\cdot\pi)$ ▸ Intersection Points: (1) $\begin{bmatrix}\frac{\pi}{2} & 1\end{bmatrix}$
2ci		normalLine$(f(x),x,a)$ $\frac{e^{-\cos(a)}\cdot(a\cdot e^{2\cdot\cos(a)}+\sin(a))}{\sin(a)}-\frac{e^{\cos(a)}\cdot x}{\sin(a)}$ solve(normalLine$(f(x),x,a)=0,x)$ $x=e^{-2\cdot\cos(a)}\cdot(a\cdot e^{2\cdot\cos(a)}+\sin(a))$ solve$(e^{-2\cdot\cos(a)}\cdot(a\cdot e^{2\cdot\cos(a)}+\sin(a))=\pi,a)\vert 0\le a\le 2\cdot\pi$ $a=1.74605$ or $a=3.14159$ or $a=4.53713$
2cii		normalLine$(f(x),x,1.74605)$ $2.67997-0.853063\cdot x$ normalLine$(f(x),x,4.53713)$ $0.853059\cdot x-2.67995$ $\tan^{-1}(0.853063)-\tan^{-1}(-0.853063)$ 1.41254 $\frac{1.4125392159322\cdot 180}{\pi}$ 80.9325

2d	$methods\_func\backslash analysed(f(x)-df(x),x,0,2\cdot\pi)$ ▶ Start Point: $\begin{bmatrix}0 & e^{-1}\end{bmatrix}$ ▶ End Point: $\begin{bmatrix}2\cdot\pi & e^{-1}\end{bmatrix}$ ▶ Maximal Domain: $0\le x\le 2\cdot\pi$ ▶ x−Intercept: $\begin{bmatrix}\frac{\pi}{2} & 0\end{bmatrix}$ ▶ Vertical Intercept: $\begin{bmatrix}0 & e^{-1}\end{bmatrix}$ ▶ Derivative: $(-\cos(x)-\sin(x)\cdot(\sin(x)-1))\cdot e^{-\cos(x)}$ ▶ Inflection Points: (2) $\begin{bmatrix}2.9021 & 2.01513\end{bmatrix}$ (Increasing) $\begin{bmatrix}4.48161 & 2.48069\end{bmatrix}$ (Decreasing) ▶ Stationary Points: (2) $\begin{bmatrix}1.5708 & 0.\end{bmatrix}$ (Local min.) $\begin{bmatrix}3.71642 & 3.5732\end{bmatrix}$ (Local max.)
2ei	Define $ddf(x)=\frac{d}{dx}(df(x))$ *Done* $ddf(x)$ $\quad (\cos(x)+(\sin(x))^2)\cdot e^{-\cos(x)}$
2eii	$methods\_diffcalc\backslash newtons\_method(ddf(x),x,2)$ ▶ Derivative: $^{-}\sin(x)\cdot\cos(x)\cdot(\cos(x)-3)\cdot e^{-\cos(x)}$ ▶ Iterative Formula: $\frac{x\cdot\sin(x)\cdot(\cos(x))^2+(1-3\cdot x\cdot\sin(x))\cdot\cos(x)+(\sin(x))^2}{\sin(x)\cdot\cos(x)\cdot(\cos(x)-3)}$ ▶ Number of Iterations: 3 $\begin{bmatrix}\text{"n"} & \text{"xn"} & \text{"\|xn−xn-1\|"} & \text{"f(xn)"} & \text{"f'(xn)"} \\ 0. & 2. & \_ & 0.622628 & ^{-}1.95983 \\ 1. & 2.31769 & 0.317694 & ^{-}0.277949 & ^{-}3.61825 \\ 2. & 2.24088 & 0.076819 & ^{-}0.012558 & ^{-}3.27995 \\ 3. & 2.23705 & 0.003829 & ^{-}0.000035 & ^{-}3.26142\end{bmatrix}$

2fi	
2fii	
3a	Define $p(x)=2\cdot(x-2)\cdot(x^2-10\cdot x+28)$ *Done* $methods\_func\backslash analyse(p(x),x)$ ▸ Start Point: $[-\infty \quad -\infty]$ ▸ End Point: $[\infty \quad \infty]$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ x−Intercept: $[2 \quad 0]$ ▸ Vertical Intercept: $[0 \quad -112]$ ▸ Derivative: $6\cdot x^2-48\cdot x+96$ ▸ Inflection Point: $[4 \quad 16]$ (Stationary) ▸ Stationary Point: $[4 \quad 16]$ (Inflection)
3b	
3c	
3di	$methods\_diffcalc\backslash tangent\_line(p(x),x,a)$ ▸ Derivative: $6\cdot x^2-48\cdot x+96$ ▸ Gradient: $6\cdot a^2-48\cdot a+96$ ▸ Passes Through: $[a \quad 2\cdot(a-2)\cdot(a^2-10\cdot a+28)]$ ▸ x−Intercept: $\left[\frac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)} \quad 0\right]$ ▸ Vertical Intercept: $[0 \quad -4\cdot(a^3-6\cdot a^2+28)]$ ▸ Tangent Line: $6\cdot(a^2-8\cdot a+16)\cdot x-4\cdot(a^3-6\cdot a^2+28)$

3dii	$\text{solve}(p(x)=0,x)$ $\quad x=2$ $\text{solve}(6\cdot(a^2-8\cdot a+16)\cdot x-4\cdot(a^3-6\cdot a^2+28)=$ ▸ $\quad x=\dfrac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)}$ ⚠ $\text{solve}\left(\dfrac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)}=2,a\right)$ $\text{tangentLine}(p(x),x,\{2,5\})$ $\quad \{24\cdot x-48,6\cdot x-12\}$
3diii	$\text{solve}(y=24\cdot x-48,x)$ $\quad x=\dfrac{y}{24}+2$ $\text{solve}(y=6\cdot x-12,x)$ $\quad x=\dfrac{y}{6}+2$
3e	$\text{solve}(p(x)=x,x)$ $\quad x=2.09123$ $\text{Define } dp(x)=\dfrac{d}{dx}(p(x))$ $\quad \textit{Done}$ $2\cdot\left(\tan^{-1}(dp(2.09123))-\dfrac{\pi}{4}\right)$ $\quad 1.47937$ $\dfrac{1.4793705022705\cdot 180}{\pi}$ $\quad 84.7617$
4a	$\text{Define } p(t)=\dfrac{a}{1+\mathbf{e}^{0.15\cdot(40-t)}}$ $\quad \textit{Done}$ $\lim\limits_{t\to\infty}(p(t))$ $\quad a$

4b	Define $dp(t)=\frac{d}{dt}(p(t))$ Done $methods\_func\backslash analyse(dp(t),t)$ ▶ Start Point: $[-\infty \quad 0.]$ ▶ End Point: $[\infty \quad 0.]$ ▶ Maximal Domain: $-\infty<t<\infty$ ▶ Asymptote: $y=0.$ (Horizontal) ▶ No t –Intercepts Found ▶ Vertical Intercept: $[0 \quad 0.009249]$ ▶ Derivative: $\frac{-226.923\cdot(1.16183)^t\cdot((1.16183)^t-403.429)}{((1.16183)^t+403.429)^3}$ ▶ Inflection Points: (2) $[31.221 \quad 0.625001]$ (Increasing) $[48.7809 \quad 0.625001]$ (Decreasing) ▶ Stationary Point: $[40.001 \quad 0.937501]$ (Local max.) $p(40.001)$ 12.5009
4ci	Define $h(t)=c\cdot \mathbf{e}^{a\cdot(t-b)^2}$ Done Define $dh(t)=\frac{d}{dt}(h(t))$ Done $dh(t)$ $\left(2\cdot a\cdot \mathbf{e}^{a\cdot b^2}\cdot c\cdot t-2\cdot a\cdot \mathbf{e}^{a\cdot b^2}\cdot b\cdot c\right)\cdot \mathbf{e}^{a\cdot t^2-}$ ▸ $\text{factor}(h(t),t)$ $c\cdot \mathbf{e}^{a\cdot t^2-2\cdot a\cdot b\cdot t+a\cdot b^2}$

4cii		$\text{solve}(dh(t)=0,t)$ $\quad t=b \text{ or } a\cdot c=0$ $h(b)$ $\quad c$
4ciii		$\text{solve}(h(100)=1 \text{ and } dh(100)=0,b,c)$ $b=100 \text{ and } c=1$
4civ		$\text{Define } ddh(t)=\frac{d}{dt}(dh(t))$ $\quad \text{Done}$ $\text{solve}(ddh(t)=0,t)$ $t=\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a} \text{ or } t=\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a} \text{ or }$ $\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a}-\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a}$ $\quad \frac{-\sqrt{2}}{\sqrt{-a}}$ $\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a}-\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a}$ $\quad \frac{\sqrt{2}}{\sqrt{-a}}$ $\text{solve}\left(\frac{\sqrt{2}}{\sqrt{-a}}=10\cdot\sqrt{10},a\right)$ $\quad a=\frac{-1}{500}$

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