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VCE Mathematical Methods $\frac{3}{4}$
SAC 1 Revision I [0.16]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
Pg / Q #: _____ Notes:	Pg / Q #: _____ Notes:

Section A: SAC 1 Success

Welcome to the first SAC 1 workshop!

Context: SAC 1 Workshops

➤ SAC 1: 50% of SACs, 20% of the study score.

➤ Will be running all the way till mid-June.

➤ After that the workshop will turn into SAC 2 Workshop (Integration).

➤ Make sure to complete the SAC 1 ~ 8. (Monday)

(Online - Sac Drop)

1) ~~Top~~ ppl who got 40+
in your school

2) Top 2

Successful SAC

Study Score = How much you know × How much you show ^{Overlooked}

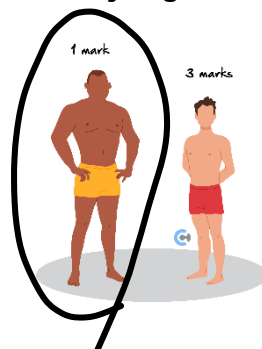
➤ Answer everything you know.

➤ Answer without mistakes.

➤ Time Management is key!

Analogy: Skipping Questions

➤ Let's say if you were to fight them and win, you get assigned marks.



➤ Who would you fight first?

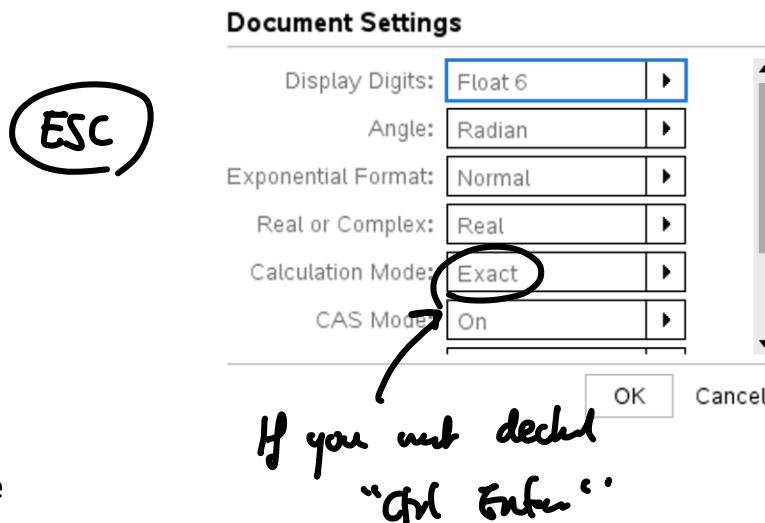
➤ Skip the hard question with little marks if it doesn't make sense during the reading time.



SAC Proficiency List

Before the SAC

- ☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).
- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.



Reading Time

- ☐ **Detailed strategy** on how to **exactly solve the question on your technology** - Don't just read, think about how to solve it and using what technology commands.
- ☐ Identify questions to skip.

For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** - You don't have to start from Q1!
- ☐ Look for potential pitfalls - Units, Domain restriction of the unknown, variable and function meaning.

Writing Time

- ☐ Circle what the question is asking for in the question.
- ☒ Spend the first 50% of the time on all the **easy questions** you identified.
- ☐ Spend the next 25% of doing the **difficult questions** you left blank.

- ☐ Spend the last 25% of the time on **checking your answers**.
- ☐ Check your answer by reading the question again and see if you answered the question.
- ☐ Check in the order of

Domino effect (check a, b, c first) > *Questions with high marks* (3+) > *Hard Questions*

- ☐ **TI ONLY:** Use new document - **doc 4, 1**.

After the SAC

- ☐ Think about how each mark loss can be prevented using this proficiency list.
- ☐ Think about the big picture and improve the marks -

Instead of spending 10 minutes on 10c) (1 mark), I should have checked 5a) (3 marks)

Space for Personal Notes

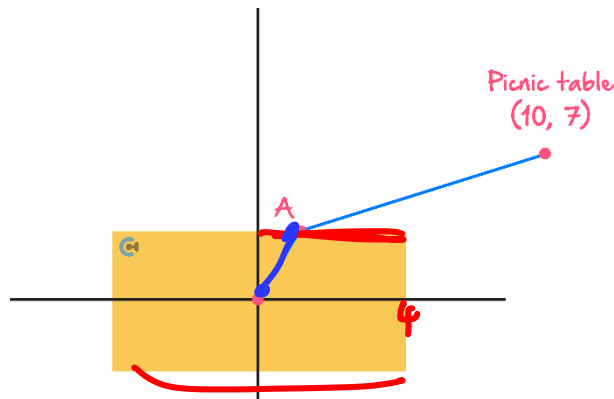
Section B: SAC Questions - Tech Active (53 Marks)

15 min

75 min

Question 1 (11 marks)

James is in the middle of a rectangular pool with dimensions of 4 m (length) by 8 m (width). Food has just been placed down on the picnic table, so James needs to get out of the pool and arrive at the picnic table as fast as possible. He can swim at a speed of 1 m/s and run at a speed of 3 m/s. James is initially at the origin (0, 0).



- a. The point, $A(x, y)$, can be on any point along the horizontal section of the pool in the first quadrant. Find the distance that James needs to swim and state any domain restrictions. (2 marks)

$$\sqrt{x^2 + 4} \text{ m}$$

$$x \in (0, 4]$$

- b. Find the distance that James needs to run and state any domain restrictions. (2 marks)

$$\sqrt{25 + (x - 10)^2} \text{ m}$$

$$x \in (0, 4]$$

- c. Hence, find the total time taken by James to reach the picnic table. (1 mark)

$\frac{\text{dist}}{\text{speed}}$

$$T = \frac{\sqrt{x^2 + 4}}{1} + \frac{\sqrt{25 + (x-10)^2}}{3} \text{ sec}$$

$\downarrow$
g

- d. Hence, or otherwise, find the coordinates of A, such that James minimises the total time taken and states this minimal time taken. Give your answers correct to two decimal places. (3 marks)

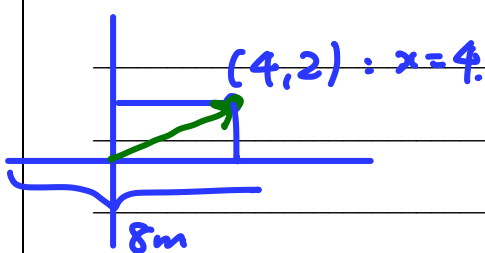
$$\frac{dT}{dx} = 0 \quad (1M)$$

$$x = 0.615607 \quad (1A) \quad \therefore (0.62, 2) \text{ is } A$$

$$T(0.615607) = 5.64 \text{ sec} \quad (1A)$$

- e. James now swims at a speed of q m/s. Find the value(s) of q such that the time it takes James to reach the picnic table is minimised at an endpoint. Give your answer correct to two decimal places. (3 marks)

$$T = \frac{\sqrt{x^2 + 4}}{q} + \frac{\sqrt{25 + (x-10)^2}}{3} \quad (1M)$$



$$T'(4) = 0$$

$$q = 3.49 \quad (1A)$$

$\therefore$ min time

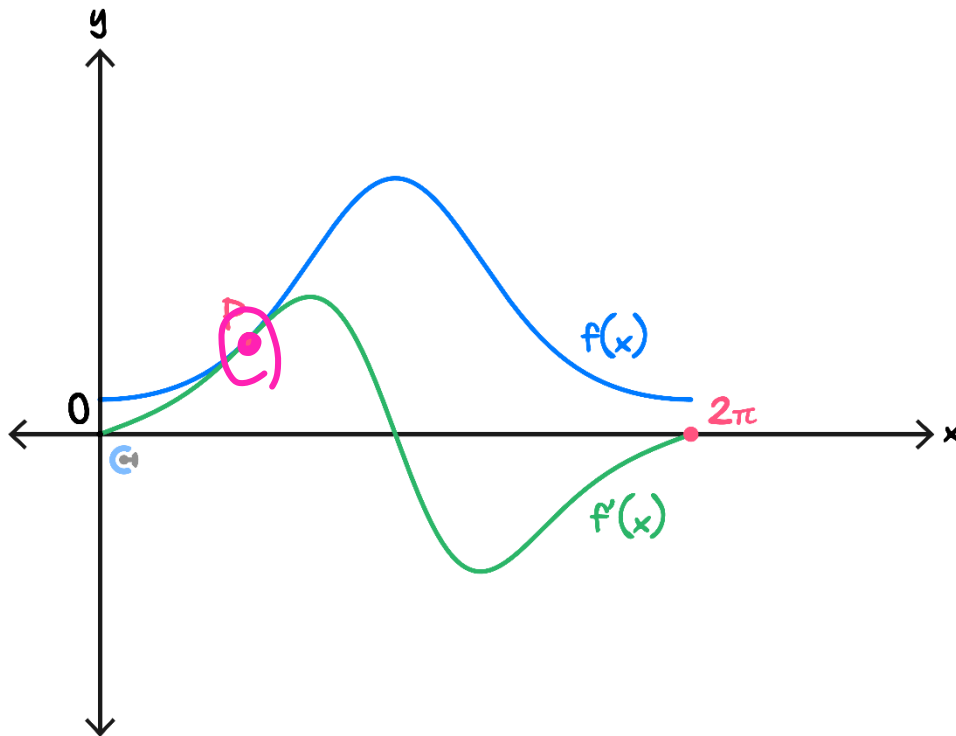
when $x = 4$
(or after $x = 4$)

$$q \geq 3.49 \quad (1A)$$

$$T'(4.1) = 0, \quad q = 3.5..$$

Question 2 (15 marks)

Consider the function $f: [0, 2\pi] \rightarrow \mathbb{R}, f(x) = e^{-\cos(x)}$. The graph below shows $y = f(x)$ and $y = f'(x)$. The two functions touch at the point P .



- a. State $f'(x)$. (1 mark)

$$e^{-\cos(x)} \sin(x)$$

- b. Find the exact coordinates of P . (2 marks)

$$P: f'(x) = f(x) \quad (1M)$$

$$x = \frac{\pi}{2}$$

$$P: \left(\frac{\pi}{2}, 1 \right) \quad (1M)$$

c. A normal line is drawn to the function f when $x = a$, and $a \neq \pi$.

If your tech only gives 1 ans, restriction do it again w $a > \pi$

- i. Find the two possible values of a such that the normal line passes through the point $(\pi, 0)$. Give your answers correct to two decimal places. (2 marks)

IM { $y = e^{-\cos(a)} + \frac{a e^{\cos(a)}}{\sin(a)} - \frac{e^{\cos(a)}}{\sin(a)} \cdot x$ Step 1) Find norml at $x=a$

∴ normal line

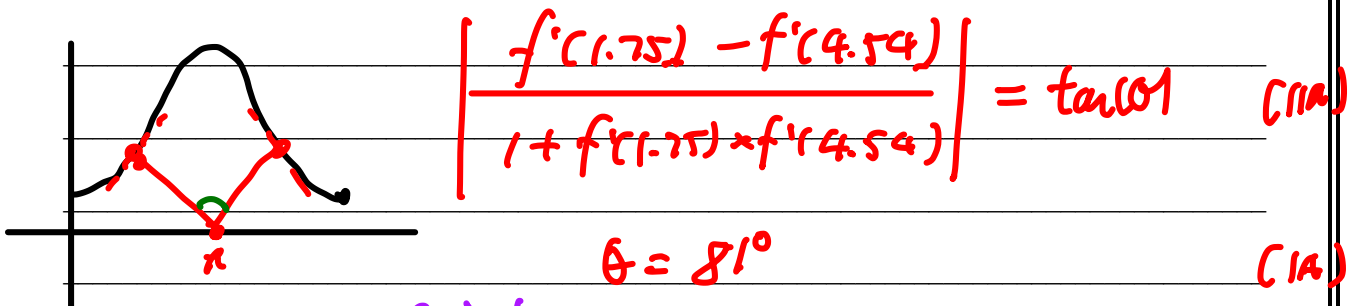
$T(4a)$

Also: (Calc → line)

$0 = e^{-\cos(a)} + \dots - \dots \cdot \pi$

1A { $a = 1.75, 4.54$ Step 2) Sub $(\pi, 0)$

- ii. Find the acute angle made by the two normal lines when they intersect. Give your answer correct to the nearest degree. (2 marks)



Subject

- d. Find the maximum vertical distance between the functions $f(x)$ and $f'(x)$ and state the x value for which this maximum distance occurs. Give your answers correct to two decimal places. (2 marks).

1)

$v(x) = f(x) - f'(x)$ (1M)

$v'(x) = 0$ (1A)

$x = 3.72$

$v(3.72) = 3.57$

e.

let $f'' = g$

- i. Write down an equation that can be solved using Newton's method to approximate the x -value for a turning point of $f'(x)$. (2 marks)

$f''(x) = 0$

$g(x) = 0$

$x - \frac{g(x)}{g'(x)} = 0$

$x - \frac{f''(x)}{f'''(x)} = 0$

check on CAS.

1M: Newton's

1A: Correct

- ii. Find an approximate x -value for a turning point of $f'(x)$ by completing 3 iterations of Newton's method with $x_0 = 2$, in the table below. Write your answers correctly to three decimal places. (2 marks).

x_0	2
x_1	2.318
x_2	2.241
x_3	2.237

$f'' \neq 0$
fine

- f. Consider the function $g(x) = f(bx)$, where $b > 0$.

- i. State a sequence of transformations that map $f'(x)$ to $g'(x)$. (1 mark)

$f'(x)$ $g(x) = b \cdot f(bx)$

all b from x axis

all b from y axis.

- ii. Find the value(s) of b such that g and $g'(x)$ intersect at least twice for $x \in [0, 2\pi]$. (1 mark)

Sketch 2
Monotonic

$b > 1$

bc originally
we hit once.

Question 3 (14 marks)

Consider the following cubic polynomial:

$$p(x) = 2(x - 2)(x^2 - 10x + 28)$$

- a. Find the coordinates of the stationary point of $p(x)$ and state its nature. (2 marks)

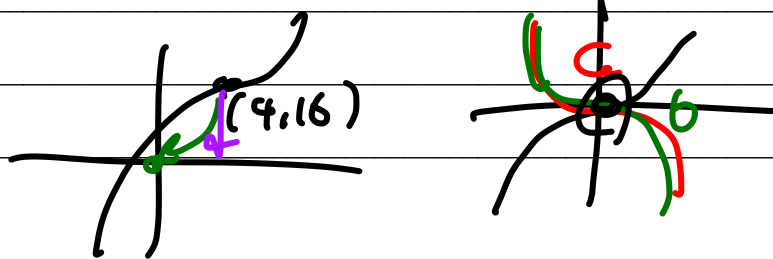
(4, 16) .

s.p. $\mathbb{Z}$

- b. Hence, express $p(x)$ in the form $a(x - h)^3 + k$. (1 mark)

$$2(x - 4)^3 + 16 .$$

- c. Find the sequence of translations on $p(x)$ that will result in an odd function. (2 marks)



translate 4 units in the
negative direction of the x axis

translate 16 units in the
negative direction of the y axis

d.

- i. Find the equation of the tangent line of $p(x)$ at the point $x = a$. Provide your answer in terms of a . (2 marks)

$$f'(a) = m_T = 6(a-4)^2 \quad (1M)$$

$$y = 6(a-4)^2 x - 4(28 + a^2(a-6)) \quad (1A)$$

- ii. Find the equations of the tangents of $p(x)$ that share the same x-intercept as the cubic. (3 marks)

$$p(x) = 0$$

$$x = 2 \quad (2, 0) \quad \downarrow \quad (1M)$$

$$6(a-4)^2 x - 4(28 + a^2(a-6)) = 0 \quad \downarrow \quad (1M)$$

$$a = 2, 3$$

$$y = 24x - 428$$

$$y = 6x + 12 \quad \downarrow \quad (1A)$$

- iii. Hence, state the equation of the tangents to the inverse function $p^{-1}(x)$ that share the same y-intercept as the inverse function. (2 marks)

Function is even

$\Rightarrow$ answer is also even

$$y = \frac{x}{24} + 2$$

$$y = \frac{x}{6} + 2 \quad (1A \text{ each})$$

- e. Find the acute angle made by the tangents to p and p^{-1} at the point of intersection of these two functions. Give your answer correct to the nearest degree. (2 marks)

$$p(x) = x$$

$$x = 2.09$$

inner gradient = reciprocal

$$\left| \frac{p'(2.09) - \frac{1}{p'(2.09)}}{1 + p'(2.09) \cdot \frac{1}{p'(2.09)}} \right| = \tan(\theta)$$

Space for Personal Notes

$$84.76$$

$$\approx 85^\circ \quad (1A)$$

Question 4 (13 marks)

Australia (population: 25 million) has been quarantined from the rest of the world since the discovery of a highly contagious prion. The number of people (in millions) infected with the virus t days after the outbreak begins is:

$$p(t) = \frac{\alpha}{1 + e^{0.15(40-t)}}$$

Where $\alpha \in \mathbb{R}$.

- a. Given that eventually, everyone will be infected, find the value of α . (2 marks)

$$\lim_{t \rightarrow \infty} p(t) = \alpha$$

$$\alpha = 25$$

- b. When is $p(t)$ increasing the fastest? Find $p(t)$ at that point. (3 marks)

$$\text{Max } p'(t)$$

$$p''(t) = 0 \quad (1M)$$

$$t = 40 \text{ days} \quad (1M)$$

$$p(40) = 12.5 \quad (1M)$$

$$12,500,000$$

"detached"
Feel free
to skip
=> Ready
time

Ready time

Find
"pop" we
approach

- c. The number of people hospitalised is much lower. The number of people hospitalised (in millions), t days after the outbreak begins, is given by:

$$h(t) = ce^{a(t-b)^2}$$

Where $a < 0$ and $b, c > 0$.

- i. State $h'(t)$. (1 mark)

$$h'(t) = 2ace^{a(t-b)^2} \cdot (t-b)$$

- ii. Show that the maximum occurs at $t = b$ and the maximum value is given by $h(b) = c$. (3 marks)

$$2ace^{a(t-b)^2} \cdot (t-b) = 0$$

$$t-b = 0$$

$$\text{as } e^x \neq 0$$

$$a \neq 0$$

$$c \neq 0$$

$$\underline{t = b}$$

$$h(b) = c \cdot e^{a(b-b)^2}$$

$$= c \cdot e^0$$

$$= c$$

- iii. The maximum number of hospitalisations is 1 million and this occurs 100 days after the outbreak begins. Find the values of b and c . (1 mark)

$$h(b) = 1 = c$$

$$b = 100$$

$$h'(100) = 0$$

$$h(100) = 1$$

- iv. Given that the maximum and minimum ^{..}rate of change^{..} in the number of people hospitalised, occurs $10\sqrt{10}$ days apart, find the value of a . (3 marks)

$h' = \text{subject}$

$$(\text{subject}' = 0) \quad h''(t) = 0 \quad | \quad \underline{a < 0}$$

$$t = 100 \pm \sqrt{\frac{1}{-2a}}$$

1 M

$$|\text{Max } t - \text{Min } t| = 2\sqrt{\frac{1}{-2a}} = 10\sqrt{10} \quad 1 \text{ M.}$$

Space for Personal Notes

$$a = -\frac{1}{500}$$

1 A

Section C: Tech-Active Solutions – Mathematica

Question Number	Solutions
1a	
1b	<pre> In[25]:= EuclideanDistance[{x, 2}, {10, 7}] Out[25]= $\sqrt{25 + \text{Abs}[-10 + x]^2}$ In[26]:= Assuming[x ≥ 0, Refine[EuclideanDistance[{x, 2}, {10, 7}]]] Out[26]= $\sqrt{25 + (-10 + x)^2}$ </pre>
1c	
1d	<pre> In[37]:= T[x_] := $\sqrt{x^2 + 4} + \frac{\sqrt{25 + (-10 + x)^2}}{3}$ In[38]:= T'[x] Out[38]= $\frac{-10 + x}{3 \sqrt{25 + (-10 + x)^2}} + \frac{x}{\sqrt{4 + x^2}}$ In[39]:= Solve[T'[x] == 0, x] // N Out[39]= {{x → 0.615607}} In[40]:= T[0.6156067766766208`] Out[40]= 5.63703 In[43]:= Minimize[{T[x], 0 ≤ x ≤ 4}, x] // N Out[43]= {5.63703, {x → 0.615607}} </pre>
1e	<pre> In[19]:= Tq[x_] := $\frac{\sqrt{x^2 + 4}}{q} + \frac{\sqrt{25 + (x - 10)^2}}{3}$ In[20]:= Solve[Tq'[4] == 0, q] Out[20]= {{q → $\sqrt{\frac{61}{5}}$}} In[21]:= Solve[Tq'[4] == 0, q] // N Out[21]= {{q → 3.49285}} </pre>

2a	<pre>In[56]:= f[x_] := Exp[-Cos[x]] In[57]:= f'[x] Out[57]= e^{-Cos[x]} Sin[x]</pre>
2b	<pre>In[58]:= Solve[f[x] == f'[x] && 0 ≤ x ≤ 2 Pi] Out[58]= {{x → $\frac{\pi}{2}$}, {x → $\frac{\pi}{2}$}} In[59]:= f[Pi/2] Out[59]= 1</pre>
2ci	<pre>In[23]:= NormalLine[f[x], x, a] Out[23]= e^{-Cos[a]} + a e^{Cos[a]} Csc[a] - e^{Cos[a]} x Csc[a] In[27]:= e^{-Cos[a]} + a e^{Cos[a]} Csc[a] - e^{Cos[a]} x Csc[a] /. x → Pi Out[27]= e^{-Cos[a]} + a e^{Cos[a]} Csc[a] - e^{Cos[a]} π Csc[a] In[29]:= Solve[e^{-Cos[a]} + a e^{Cos[a]} Csc[a] - e^{Cos[a]} π Csc[a] == 0 && 0 ≤ a ≤ 2 Pi, a] // N Out[29]= {{a → 1.74605}, {a → 4.53713}}</pre>
2cii	<pre>In[30]:= NormalLine[f[x], x, 1.7460514603409005`] Out[30]= 2.67997 - 0.853062 x In[31]:= NormalLine[f[x], x, 4.537133846838686`] Out[31]= -2.67997 + 0.853062 x In[35]:= Abs[ArcTan[0.8530621074843592`] - ArcTan[-0.8530621074843592`]] / Degree // N Out[35]= 80.9325</pre>
2d	<pre>In[42]:= Maximize[f[x] - f'[x], x] // N Out[42]= {3.5732, {x → 3.71642}}</pre>
2ei	<pre>In[60]:= f'''[x] // FullSimplify Out[60]= e^{-Cos[x]} (Cos[x] + Sin[x]²)</pre>
2eii	<pre>In[65]:= n[x_] := x - $\frac{f''[x]}{f'''[x]}$ In[66]:= n[x] // FullSimplify Out[66]= $\frac{x(-3 + \cos[x]) + \csc[x] + \tan[x]}{-3 + \cos[x]}$ In[76]:= n[2.0] Out[76]= 2.31769 In[77]:= n[n[2.0]] Out[77]= 2.24088 In[78]:= n[n[n[2.0]]] Out[78]= 2.23705</pre>

2fi	
2fii	
3a	<pre> In[97]:= f[x_] := 2 (x - 2) (x^2 - 10 x + 28) In[98]:= Solve[f'[x] == 0, x] Out[98]= {{x -> 4}, {x -> 4}} In[99]:= f[4] Out[99]= 16 In[100]:= f''[4] Out[100]= 0 </pre>
3b	<pre> In[101]:= Solve[a * (x - 4)^3 + 16 == f[x], a] Out[101]= {{a -> 2}} </pre>
3c	
3di	<pre> In[102]:= f'[x] // FullSimplify Out[102]= 6 (-4 + x)^2 In[103]:= TangentLine[f[x], x, a] // FullSimplify Out[103]= -4 (28 + (-6 + a) a^2) + 6 (-4 + a)^2 x </pre>
3dii	<pre> In[104]:= TangentLine[f[x], x, a] /. x -> 2 // FullSimplify Out[104]= -4 (-5 + a) (-2 + a)^2 In[105]:= Solve[-4 (-5 + a) (-2 + a)^2 == 0, a] Out[105]= {{a -> 2}, {a -> 2}, {a -> 5}} In[106]:= TangentLine[f[x], x, 2] Out[106]= -48 + 24 x In[107]:= TangentLine[f[x], x, 5] Out[107]= -12 + 6 x </pre>

3diii	<pre> In[128]:= Solve[f[y] == x, y] Out[128]= {{y -> 4 + (-16 + x)^(1/3)/2^(1/3)}, {y -> 4 - (1 - I*sqrt(3))(-16 + x)^(1/3)/(2*2^(1/3))}, {y -> 4 - (1 + I*sqrt(3))(-16 + x)^(1/3)/(2*2^(1/3))}} In[129]:= f1[x_] := 4 + Surd[x - 16, 3]/Surd[2, 3] In[130]:= TangentLine[f1[x], x, a] /. x -> 0 Out[130]= 4 - a/(3*2^(1/3)*sqrt[3]{-16 + a}^2) + sqrt[3]{-16 + a}/2^(1/3) In[131]:= Solve[4 - a/(3*2^(1/3)*sqrt[3]{-16 + a}^2) + sqrt[3]{-16 + a}/2^(1/3) == 2, a] Out[131]= {{a -> 0}, {a -> 18}} In[132]:= TangentLine[f1[x], x, 0] Out[132]= 2 + x/24 In[133]:= TangentLine[f1[x], x, 18] Out[133]= 2 + x/6 </pre>
3e	<pre> In[135]:= Solve[f[x] == x, x, Reals] // N Out[135]= {{x -> 2.09123}} In[136]:= Abs[ArcTan[f'[2.0912331809635747`]] -            ArcTan[1/f'[2.0912331809635747`]]]/Degree // N Out[136]= 84.7617 </pre>
4a	<pre> In[1]:= p[t_] := a/(1 + Exp[0.15 (40 - t)]) In[2]:= Solve[Limit[p[t], t -> Infinity] == 25, a] Out[2]= {{a -> 25.}} </pre>

4b	<pre> In[6]:= p[t_] := $\frac{25}{1 + \text{Exp}[15 / 100 (40 - t)]}$ In[9]:= Solve[p'[t] == 0, t, Reals] Out[9]= {{t -> 40}} In[11]:= p[40] // N Out[11]= 12.5 In[12]:= Maximize[p'[t], t] Out[12]= $\left\{ \frac{15}{16}, \{t \rightarrow 40\} \right\}$ </pre>
4ci	<pre> In[17]:= h[t_] := c Exp[a (t - b)^2] In[18]:= h'[t] Out[18]= 2 a c e^{a (-b+t)^2} (-b + t) </pre>
4cii	<pre> In[19]:= h''[b] Out[19]= 2 a c </pre>
4ciii	<pre> In[22]:= Solve[h[100] == 1 && h'[100] == 0, Reals] Out[22]= {{a -> 0, c -> 1}, {b -> 100, c -> 1}} </pre>
4civ	<pre> In[23]:= h[t_] := Exp[a (t - 100)^2] In[25]:= Solve[h''[t] == 0, t, Reals] Out[25]= {{t -> $100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}$ if a < 0}, {t -> $100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}$ if a < 0}} In[26]:= $100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} - \left(100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \right)$ Out[26]= $\sqrt{2} \sqrt{-\frac{1}{a}}$ In[27]:= Solve[$\sqrt{2} \sqrt{-\frac{1}{a}} == 10 \sqrt{10}, a]$ Out[27]= {{a -> $-\frac{1}{500}$}} </pre>

Space for Personal Notes

Section D: Tech-Active Solutions - Casio

Question Number	Solutions
1a	
1b	$\sqrt{(x-10)^2 + (2-7)^2}$ $\sqrt{x^2 - 20x + 125}$
1c	
1d	<pre> Define T(x)=√x^2+4+$\frac{\sqrt{(x-10)^2+25}}{3}$ done solve($\frac{d}{dx}$(T(x))=0, x {x=0.6156067766} T(0.6156067766) 5.637028955 fMin(T(x), x, 0, 4) {MinValue=5.637028955, x=0.6156067766} </pre>
1e	<pre> Define Tq(x)=$\frac{\sqrt{x^2+4}}{q} + \frac{\sqrt{25+(x-10)^2}}{3}$ done solve($\frac{d}{dx}$(Tq(x)=0) x=4, q) {q=$\frac{\sqrt{305}}{5}$} solve($\frac{d}{dx}$(Tq(x)=0) x=4, q) {q=3.492849839} □ </pre>
2a	<pre> Define f(x)=e^{-cos(x)} done $\frac{d}{dx}$(f(x)) sin(x) • e^{-cos(x)} </pre>

2b	$\text{solve}(f(x) = \frac{d}{dx}(f(x)) \mid 0 \leq x \leq 2\pi, x)$ $\left\{x = \frac{\pi}{2}\right\}$ $f\left(\frac{\pi}{2}\right)$ 1
2ci	normal(f(x), x, a) $\frac{-x \cdot e^{\cos(a)}}{\sin(a)} + \frac{a \cdot e^{\cos(a)}}{\sin(a)} + e^{-\cos(a)}$ ans x=π $\frac{a \cdot e^{\cos(a)}}{\sin(a)} - \frac{\pi \cdot e^{\cos(a)}}{\sin(a)} + e^{-\cos(a)}$ $\text{solve}\left(\frac{a \cdot e^{\cos(a)}}{\sin(a)} - \frac{\pi \cdot e^{\cos(a)}}{\sin(a)} + e^{-\cos(a)} = 0 \mid 0 \leq a \leq 2\pi, a\right)$ {a=1.74605146, a=4.537133847}
2cii	normal(f(x), x, 1.74605146) -0.8530621077·x+2.67997365 normal(f(x), x, 4.537133847) 0.8530621076·x-2.679973651 tan ⁻¹ (-0.8530621077)-tan ⁻¹ (0.8530621076) 80.93247628 Change to Deg mode! Make sure to change back to Rad
2d	$\text{fMax}\left(f(x) - \frac{d}{dx}(f(x)), x, 0, 2\pi\right)$ {MaxValue=3.573200775, x=3.7164}
2ei	$\frac{d^2}{dx^2}(f(x))$ $((\sin(x))^2 + \cos(x)) \cdot e^{-\cos(x)}$
2eii	Define n(x)= $x - \frac{\frac{d^2}{dx^2}(f(x))}{\frac{d^3}{dx^3}(f(x))}$ done n(x) $x - \frac{(\sin(x))^2 + \cos(x)}{(\sin(x))^3 + 3 \cdot \cos(x) \cdot \sin(x) - \sin(x)}$ n(2.0) 2.317694099 n(ans) 2.240875415 n(ans) 2.237046622
2fi	

2fii	
3a	<pre> Define p(x)=2(x-2)(x^2-10x+28) done solve(d/dx(p(x))=0,x {x=4} p(4) 16 d^2/dx^2(p(x)) x=4 0 </pre>
3b	<pre> solve(a*(x-4)^3+16=p(x),a {a=2} </pre>
3c	
3di	<pre> tanLine(p(x),x,a) 2*(a^2-10*a+28)*(a-2)+x*(2*(a^2-10*a+28)+2*(a-2)*(2*a-10))-a*(2*(a^2-10*a+28)+2*(a-2)*(2*a-10)) simplify(ans) -4*a^3+6*a^2*x+24*a^2-48*a*x+96*x-112 collect(ans,x (6*a^2-48*a+96)*x-4*a^3+24*a^2-112 </pre>
3dii	<pre> tanLine(p(x),x,a) 2*(a^2-10*a+28)*(a-2)+x*(2*(a^2-10*a+28)+2*(a-2)*(2*a-10))-a*(2*(a^2-10*a+28)+2*(a-2)*(2*a-10)) simplify(ans) -4*a^3+6*a^2*x+24*a^2-48*a*x+96*x-112 collect(ans,x (6*a^2-48*a+96)*x-4*a^3+24*a^2-112 ans x=2 -4*a^3+24*a^2+2*(6*a^2-48*a+96)-112 solve(ans=0,a {a=2,a=5} tanLine(p(x),x,2) 24*x-48 tanLine(p(x),x,5) 6*x-12 </pre>
3diii	
3e	<pre> solve(p(x)=x,x {x=2.091233181} Define dp(x)=d/dx(p(x)) done tan^-1(dp(2.091233181))-tan^-1(1/dp(2.091233181)) 84.76166868 Change to Deg then change back to Rad </pre>

4a	Define $p(t) = \frac{a}{1 + e^{0.15(40-t)}}$ solve($\lim_{t \rightarrow \infty} (p(t)) = 25, a$)	done {a=25}
4b	Define $p(t) = \frac{25}{1 + e^{0.15(40-t)}}$ solve($\frac{d^2}{dt^2} (p(t)) = 0, t$) p(40) fMax($\frac{d}{dt} (p(t)), t, -\infty, \infty$)	done {t=40} 12.5 {MaxValue=0.9375, t=40}
4ci	Define $h(t) = c \cdot e^{a \cdot (t-b)^2}$ $\frac{d}{dt} (h(t))$	done $-2 \cdot a \cdot c \cdot (b-t) \cdot e^{a \cdot (b-t)^2}$
4cii	$\frac{d^2}{dt^2} (h(t)) t=b$	$2 \cdot a \cdot c$
4ciii	$\left\{ \begin{array}{l} h(100) = 1 \\ \frac{d}{dt} (h(t)) = 0 t=100 \end{array} \right\} b, c$	{b=100, c=1}
4civ	solve($\frac{d^2}{dt^2} (h(t)) = 0, t$) $\left\{ t = \frac{2 \cdot a \cdot b - \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a}, t = \frac{2 \cdot a \cdot b + \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} \right\}$ $\left \frac{2 \cdot a \cdot b + \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} - \frac{2 \cdot a \cdot b - \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} \right$ $\left \frac{2 \cdot a \cdot b + \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} - \frac{2 \cdot a \cdot b - \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} \right$ simplify($\left \frac{2 \cdot a \cdot b + \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} - \frac{2 \cdot a \cdot b - \sqrt{2} \cdot \sqrt{-a}}{2 \cdot a} \right$) $\left \frac{1}{a} \right \cdot \sqrt{-2 \cdot a}$ solve($\left \frac{1}{a} \right \cdot \sqrt{-2 \cdot a} = 10\sqrt{10}, a$) $\left\{ a = -\frac{1}{500} \right\}$	

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Section E: Tech-Active Solutions - TI

Question Number	Solutions
1a	
1b	
1c	
1d	<i>methods_func\analysed(t(x),x,0,4)</i> ▶ Start Point: $\left[0 \quad \frac{5 \cdot \sqrt{5}}{3} + 2 \right]$ ▶ End Point: $\left[4 \quad \frac{\sqrt{61}}{3} + 2 \cdot \sqrt{5} \right]$ ▶ Maximal Domain: $0 \leq x \leq 4$ ▶ Asymptotes: (2) Define $t(x) = \sqrt{x^2 + 4} + \frac{1}{3} \cdot \sqrt{25 + (x-10)^2}$ Done $y = \frac{10}{3} - \frac{4 \cdot x}{3}$ (Oblique) $y = \frac{4 \cdot x}{3} - \frac{10}{3}$ (Oblique) ▶ No x -Intercepts Found ▶ Vertical Intercept: $\left[0 \quad \frac{5 \cdot \sqrt{5}}{3} + 2 \right]$ ▶ Derivative: $\frac{x-10}{3 \cdot \sqrt{x^2 - 20 \cdot x + 125}} + \frac{x}{\sqrt{x^2 + 4}}$ ▶ No Inflection Points Found ▶ Stationary Point: $[0.615607 \quad 5.63703] \text{ (Local min.)}$

1e	Define $tq(x) = \frac{\sqrt{x^2+4}}{q} + \frac{1}{3} \cdot \sqrt{25+(x-10)^2}$ Done Define $dtq(x) = \frac{d}{dx}(tq(x))$ Done ⚠ solve($dtq(4)=0,q$) $q=3.49285$
2a	Define $f(x) = e^{-\cos(x)}$ Done Define $df(x) = \frac{d}{dx}(f(x))$ Done $df(x)$ $\sin(x) \cdot e^{-\cos(x)}$
2b	$methods_funcintersectd(f(x),df(x),x,0,2 \cdot \pi)$ ► Intersection Points: (1) $\left[\begin{array}{cc} \frac{\pi}{2} & 1 \end{array} \right]$
2ci	normalLine($f(x),x,a$) $\frac{e^{-\cos(a)} \cdot (a \cdot e^{2 \cdot \cos(a)} + \sin(a))}{\sin(a)} - \frac{e^{\cos(a)} \cdot x}{\sin(a)}$ solve(normalLine($f(x),x,a$)=0,x) $x = e^{-2 \cdot \cos(a)} \cdot (a \cdot e^{2 \cdot \cos(a)} + \sin(a))$ ⚠ solve($e^{-2 \cdot \cos(a)} \cdot (a \cdot e^{2 \cdot \cos(a)} + \sin(a)) = \pi, a$) $0 \leq a \leq 2 \cdot \pi$ $a = 1.74605$ or $a = 3.14159$ or $a = 4.53713$
2cii	normalLine($f(x),x,1.74605$) $2.67997 - 0.853063 \cdot x$ normalLine($f(x),x,4.53713$) $0.853059 \cdot x - 2.67995$ $\tan^{-1}(0.853063) - \tan^{-1}(-0.853063)$ 1.41254 $\frac{1.4125392159322 \cdot 180}{\pi}$ 80.9325

2d	methods_func\analysed(f(x)-df(x),x,0,2·π) ▶ Start Point: $\begin{bmatrix} 0 & e^{-1} \end{bmatrix}$ ▶ End Point: $\begin{bmatrix} 2\cdot\pi & e^{-1} \end{bmatrix}$ ▶ Maximal Domain: $0\leq x\leq 2\cdot\pi$ ▶ x-Intercept: $\begin{bmatrix} \frac{\pi}{2} & 0 \end{bmatrix}$ ▶ Vertical Intercept: $\begin{bmatrix} 0 & e^{-1} \end{bmatrix}$ ▶ Derivative: $\left(-\cos(x)-\sin(x)\cdot\left(\sin(x)-1\right)\right)\cdot e^{-\cos(x)}$ ▶ Inflection Points: (2) $\begin{bmatrix} 2.9021 & 2.01513 \end{bmatrix}$ (Increasing) $\begin{bmatrix} 4.48161 & 2.48069 \end{bmatrix}$ (Decreasing) ▶ Stationary Points: (2) $\begin{bmatrix} 1.5708 & 0. \end{bmatrix}$ (Local min.) $\begin{bmatrix} 3.71642 & 3.5732 \end{bmatrix}$ (Local max.)																									
2ei	Define $ddf(x)=\frac{d}{dx}(df(x))$ Done $ddf(x) \qquad \left(\cos(x)+(\sin(x))^2\right)\cdot e^{-\cos(x)}$																									
2eii	methods_diffcalc\newtons_method(ddf(x),x,2) ▶ Derivative: $-\sin(x)\cdot\cos(x)\cdot\left(\cos(x)-3\right)\cdot e^{-\cos(x)}$ ▶ Iterative Formula: $\frac{x\cdot\sin(x)\cdot\left(\cos(x)\right)^2+\left(1-3\cdot x\cdot\sin(x)\right)\cdot\cos(x)+\left(\sin(x)\right)^2}{\sin(x)\cdot\cos(x)\cdot\left(\cos(x)-3\right)}$ ▶ Number of Iterations: 3 <table><tr><th>"n"</th><th>"Xn"</th><th>" Xn-Xn-1 "</th><th>"f(Xn)"</th><th>"f'(Xn)"</th></tr><tr><td>0.</td><td>2.</td><td>—</td><td>0.622628</td><td>-1.95983</td></tr><tr><td>1.</td><td>2.31769</td><td>0.317694</td><td>-0.277949</td><td>-3.61825</td></tr><tr><td>2.</td><td>2.24088</td><td>0.076819</td><td>-0.012558</td><td>-3.27995</td></tr><tr><td>3.</td><td>2.23705</td><td>0.003829</td><td>-0.000035</td><td>-3.26142</td></tr></table>	"n"	"Xn"	" Xn-Xn-1 "	"f(Xn)"	"f'(Xn)"	0.	2.	—	0.622628	-1.95983	1.	2.31769	0.317694	-0.277949	-3.61825	2.	2.24088	0.076819	-0.012558	-3.27995	3.	2.23705	0.003829	-0.000035	-3.26142
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3.	2.23705	0.003829	-0.000035	-3.26142																						

2fi	
2fii	
3a	Define $p(x)=2 \cdot (x-2) \cdot (x^2-10 \cdot x+28)$ Done <i>methods_func\analyse(p(x),x)</i> ▶ Start Point: $[-\infty \quad -\infty]$ ▶ End Point: $[\infty \quad \infty]$ ▶ Maximal Domain: $-\infty < x < \infty$ ▶ x-Intercept: $[2 \quad 0]$ ▶ Vertical Intercept: $[0 \quad -112]$ ▶ Derivative: $6 \cdot x^2 - 48 \cdot x + 96$ ▶ Inflection Point: $[4 \quad 16]$ (Stationary) ▶ Stationary Point: $[4 \quad 16]$ (Inflection)
3b	
3c	
3di	<i>methods_diffcalc\tangent_line(p(x),x,a)</i> ▶ Derivative: $6 \cdot x^2 - 48 \cdot x + 96$ ▶ Gradient: $6 \cdot a^2 - 48 \cdot a + 96$ ▶ Passes Through: $\left[a \quad 2 \cdot (a-2) \cdot (a^2 - 10 \cdot a + 28) \right]$ ▶ x-Intercept: $\left[\frac{2 \cdot (a^3 - 6 \cdot a^2 + 28)}{3 \cdot (a^2 - 8 \cdot a + 16)} \quad 0 \right]$ ▶ Vertical Intercept: $[0 \quad -4 \cdot (a^3 - 6 \cdot a^2 + 28)]$ ▶ Tangent Line: $6 \cdot (a^2 - 8 \cdot a + 16) \cdot x - 4 \cdot (a^3 - 6 \cdot a^2 + 28)$

3dii	$\text{solve}(p(x)=0,x)$ $x=2$ $\text{solve}\left(6 \cdot (a^2 - 8 \cdot a + 16) \cdot x - 4 \cdot (a^3 - 6 \cdot a^2 + 28) = \frac{2 \cdot (a^3 - 6 \cdot a^2 + 28)}{3 \cdot (a^2 - 8 \cdot a + 16)}\right)$ $\text{solve}\left(\frac{2 \cdot (a^3 - 6 \cdot a^2 + 28)}{3 \cdot (a^2 - 8 \cdot a + 16)} = 2, a\right)$ $\text{tangentLine}(p(x), x, \{2, 5\})$ $\{24 \cdot x - 48, 6 \cdot x - 12\}$
3diii	$\text{solve}(y=24 \cdot x - 48, x)$ $x = \frac{y}{24} + 2$ $\text{solve}(y=6 \cdot x - 12, x)$ $x = \frac{y}{6} + 2$
3e	$\text{solve}(p(x)=x,x)$ $x=2.09123$ $\text{Define } dp(x) = \frac{d}{dx}(p(x))$ <i>Done</i> $2 \cdot \left(\tan^{-1}(dp(2.09123)) - \frac{\pi}{4}\right)$ 1.47937 $\frac{1.4793705022705 \cdot 180}{\pi}$ 84.7617
4a	$\text{Define } p(t) = \frac{a}{1 + e^{0.15 \cdot (40-t)}}$ <i>Done</i> $\lim_{t \rightarrow \infty} (p(t))$ a

4b

Define $dp(t) = \frac{d}{dt}(p(t))$ Done
 $methods_func \backslash analyse(dp(t), t)$

- ▶ Start Point: $[-\infty \ 0.]$
- ▶ End Point: $[\infty \ 0.]$
- ▶ Maximal Domain: $-\infty < t < \infty$
- ▶ Asymptote: $y=0$. (Horizontal)
- ▶ No t -Intercepts Found
- ▶ Vertical Intercept: $[0 \ 0.009249]$
- ▶ Derivative:

$$\frac{-226.923 \cdot (1.16183)^t \cdot ((1.16183)^t - 403.429)}{((1.16183)^t + 403.429)^3}$$

- ▶ Inflection Points: (2)
 - $[31.221 \ 0.625001]$ (Increasing)
 - $[48.7809 \ 0.625001]$ (Decreasing)
- ▶ Stationary Point:
 - $[40.001 \ 0.937501]$ (Local max.)

$$p(40.001) \qquad 12.5009$$

4ci

Define $h(t) = c \cdot e^{a \cdot (t-b)^2}$ Done

Define $dh(t) = \frac{d}{dt}(h(t))$ Done

$$dh(t)$$

$$\left(2 \cdot a \cdot e^{a \cdot b^2} \cdot c \cdot t - 2 \cdot a \cdot e^{a \cdot b^2} \cdot b \cdot c \right) \cdot e^{a \cdot t^2} \rightarrow$$

$$factor(h(t), t) \qquad c \cdot e^{a \cdot t^2 - 2 \cdot a \cdot b \cdot t + a \cdot b^2}$$

4cii	solve($dh(t)=0,t$) $t=b$ or $a \cdot c=0$ $h(b)$ c
4ciii	solve($h(100)=1$ and $dh(100)=0,b,c$) $b=100$ and $c=1$
4civ	Define $ddh(t)=\frac{d}{dt}(dh(t))$ Done solve($ddh(t)=0,t$) $t=\frac{\sqrt{-2 \cdot a}+2 \cdot a \cdot b}{2 \cdot a}$ or $t=\frac{-(\sqrt{-2 \cdot a}-2 \cdot a \cdot b)}{2 \cdot a}$ or $\frac{\sqrt{-2 \cdot a}+2 \cdot a \cdot b}{2 \cdot a}-\frac{-(\sqrt{-2 \cdot a}-2 \cdot a \cdot b)}{2 \cdot a}$ $\frac{-\sqrt{2}}{\sqrt{-a}}$ $\frac{-(\sqrt{-2 \cdot a}-2 \cdot a \cdot b)}{2 \cdot a}-\frac{\sqrt{-2 \cdot a}+2 \cdot a \cdot b}{2 \cdot a}$ $\frac{\sqrt{2}}{\sqrt{-a}}$ solve($\frac{\sqrt{2}}{\sqrt{-a}}=10 \cdot \sqrt{10},a$) $a=\frac{-1}{500}$

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