CONTOUREDUCATION

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VCE Mathematical Methods ¾
SAC 1 Revision I [0.16]
Workshop

Error Logbook:

New Ideas/Concepts	Didn't Read Question
Pg / Q #: ________________ Notes:	Pg / Q #: ________________ Notes:
Algebraic/Arithmetic/ Calculator Input Mistake	**Working Out Not Detailed Enough**
Pg / Q #: ________________ Notes:	Pg / Q #: ________________ Notes:

Section A: SAC 1 Success

Welcome to the first SAC 1 workshop!

Context: SAC 1 Workshops

- SAC 1 - 50% of SACs, 20% of the study score.
- Will be running all the way till mid-June.
- After that the workshop will turn into SAC 2 Workshop (Integration).
- Make sure to complete the SAC 1 ~ 8.

SAC 1 50%
SAC 2/3 50%
40%

Successful SAC

overlooked

$$\text{Study Score} = \text{How much you know} \times \underbrace{\text{How much you show}}$$

- Answer everything you know.
- Answer without mistakes.
- Time Management is **key!**

• Time management
• Units / working out

Analogy: Skipping Questions

- Let's say if you were to fight them and win, you get assigned marks.

- Who would you fight first?

skip harder qs, and come back to easier

- Skip the hard question with little marks if it doesn't make sense during the reading time.

SAC Proficiency List

Before the SAC

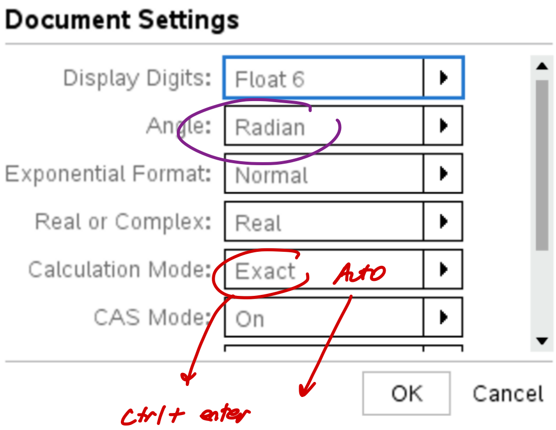
- ☐ Prepare your stationery including a ruler, eraser and your mechanical pencil lead.
- ☐ Skim through the bound reference (if applicable).

* Angle between 2 lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- ☐ Do not speak to other people and lock in.
- ☐ TI & Mathematica Only: Check your Contour UDFS.
- ☐ TI Only: Check technology settings.

Doc → 7 → 2



ctrl + enter

Reading Time

- ☐ **Detailed strategy** on how to exactly solve the question on your technology – Don't just **read**, think about how to solve it and using what technology commands. + Domain + key words

  ↳ concepts?

  "state ..."

  "Define..."

- ☐ Identify questions to **skip.**

  For difficult SACs, it's not necessarily about getting the 100%. It's about getting better than everyone else. While others are stuck on a hard 1-marker question, you can do an easy 3-marker first.

- ☐ Identify questions to **start first** – You don't have to start from Q1!
- ☐ Look for potential pitfalls – **Units, Domain restriction of the unknown, variable and function meaning.**

Writing Time

- ☐ **Circle what the question is asking** for in the question.
- ☐ Spend the first **50%** of the time on all the **easy questions** you identified.
- ☐ Spend the next **25%** of doing the **difficult questions** you left blank.

- Spend the last $\mathbf{25\%}$ of the time on **checking your answers**.
- Check your answer by reading the question again and see if you answered the question.
- Check in the order of

  Domino effect (check a, b, c first) > ***Questions with high marks*** $(\mathbf{3+})$ > *Hard Questions*

- **TI** ONLY: Use new document - **doc $\mathbf{4, 1}$.**

After the SAC

- Think about how each mark loss can be prevented using this proficiency list.
- Think about the big picture and improve the marks -

  Instead of spending 10 minutes on 10c) (1 mark), I should have checked 5a) (3 marks)

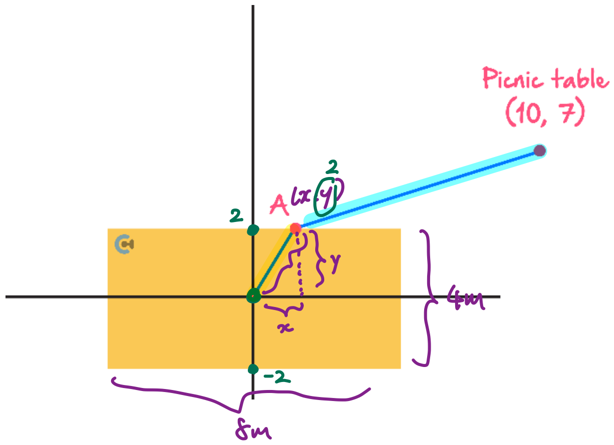
Space for Personal Notes

Section B: SAC Questions – Tech Active (53 Marks)

10min reading
60min

Question 1 (11 marks)

James is in the middle of a rectangular pool with dimensions of 4 m (length) by 8 m (width). Food has just been placed down on the picnic table, so James needs to get out of the pool and arrive at the picnic table as fast as possible. He can swim at a speed of 1 m/s and run at a speed of 3 m/s. James is initially at the origin $(0, 0)$.



a. The point, $A(x, y)$, can be on any point along the horizontal section of the pool in the first quadrant. Find the distance that James needs to swim and state any domain restrictions. (2 marks)

$OA = \sqrt{x^2 + y^2}$, $y = 2$

$= \sqrt{x^2 + 4}$

$x \in (0, 4]$

b. Find the distance that James needs to run and state any domain restrictions. (2 marks)

$A(x, 2)$ $P(10, 7)$

$AP = \sqrt{(x-10)^2 + (2-7)^2}$

$= \sqrt{(x-10)^2 + 25}$, $x \in (0, 4]$

c. Hence, find the total time taken by James to reach the picnic table. (1 mark)

$s = \frac{d}{t}$, $t = \frac{d}{s}$

$t = \underbrace{\frac{\sqrt{x^2+4}}{1}}_{\text{time swimming}} + \underbrace{\frac{\sqrt{(x-10)^2+25}}{3}}_{\text{time running}}$.

min of $t(x)$

d. Hence, or otherwise, find the coordinates of A, such that James minimises the total time taken and states this minimal time taken. Give your answers correct to two decimal places. (3 marks)

$t'(x) = 0$

$x = 0.615606776677...$

$t(0.6156...) \approx \boxed{5.64s}$

$A(0.62, 2)$

~~$t(0.6156) \approx$ y-value.~~

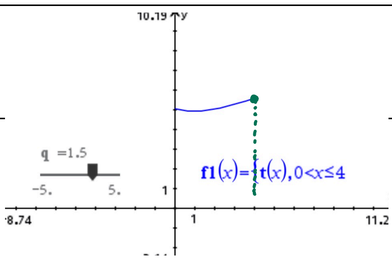
e. James now swims at a speed of q m/s. Find the value(s) of q such that the time it takes James to reach the picnic table is minimised at an endpoint. Give your answer correct to two decimal places. (3 marks)

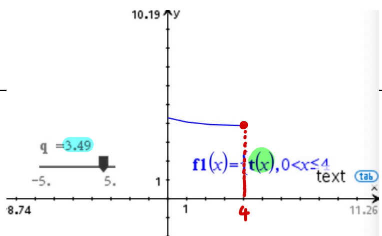
$t(x) = \frac{\sqrt{x^2+4}}{q} + \frac{\sqrt{(x-10)^2+25}}{3}$

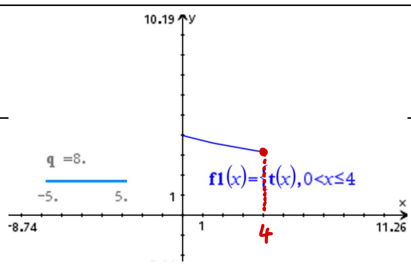
$\boxed{\text{min at } x=4}$ $t'(4) = 0$, solve for q:

$q \approx 3.49$

$\therefore q \geq 3.49$

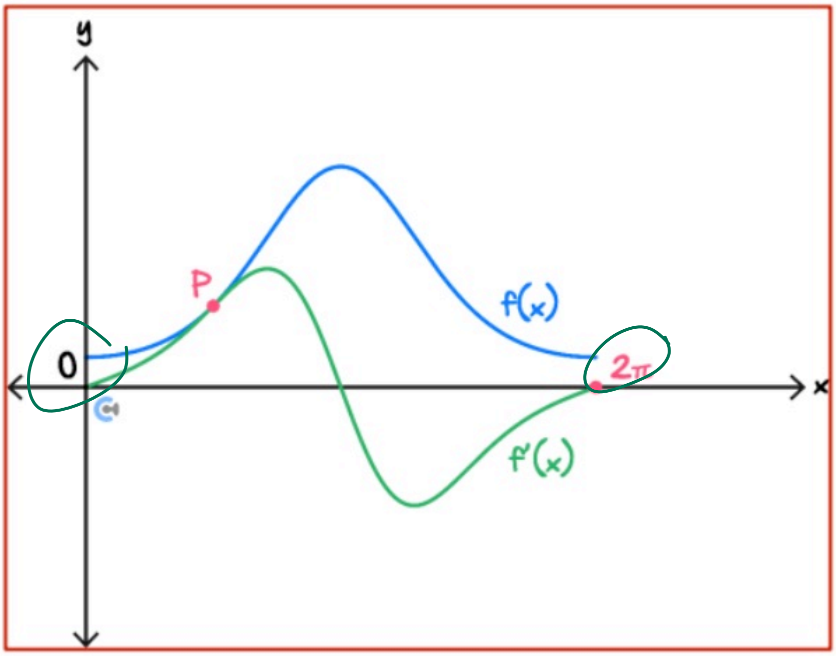






Question 2 (15 marks)

Consider the function $f: [0, 2\pi] \to R, f(x) = e^{-\cos(x)}$. The graph below shows $y = f(x)$ and $y = f'(x)$. The two functions touch at the point P.



a. State $f'(x)$. (1 mark)

$f'(x) = \sin(x)\, e^{-\cos(x)}$

$\sin(x) \cdot e^{-\cos(x)}$

b. Find the exact coordinates of P. (2 marks)

solve $\left(f(x) = f'(x), x\right) \mid 0 \leq x \leq 2\pi$

$x = \frac{\pi}{2}$

$f\left(\frac{\pi}{2}\right) = 1$

$P: \left(\frac{\pi}{2}, 1\right)$

c. A normal line is drawn to the function f when $x = a$, and $a \neq \pi$.

normaline$(f(x), x, a)$

i. Find the two possible values of a such that the normal line passes through the point $(\pi, 0)$. Give your answers correct to two decimal places. (2 marks)

normal at $x=a$:

$$n(x) = \frac{-e^{\cos(a)}x}{\sin(a)} + \frac{e^{-\cos(a)}\left(ae^{2\cos(a)} + \sin(a)\right)}{\sin(a)}$$

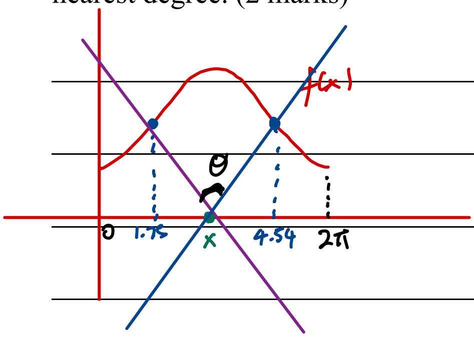
solve$(n(\pi)=0, a) \mid 0 \le a \le 2\pi$

1. strict order
2. Domain

$n(\pi) = 0$, solve for a:

$a \approx 1.75, 4.54$

ii. Find the acute angle made by the two normal lines when they intersect. Give your answer correct to the nearest degree. (2 marks)



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$-0.853063x + 2.67...$

$m_1 =$ Gradient of normal at 1.75

$0.853059x - 2.67...$

$m_2 =$ Gradient of normal at 4.54

solve for θ:

$\theta \approx 1.4... \times \frac{180}{\pi}$

$\approx 81°$

d. Find the maximum vertical distance between the functions $f(x)$ and $f'(x)$ and state the x value for which this maximum distance occurs. Give your answers correct to two decimal places. (2 marks).

let $v(x) = f(x) - f'(x)$

Top Bottom

$v'(x) = 0$

$x \approx 3.72$

$v(3.72...) \approx 3.57$ units

e.

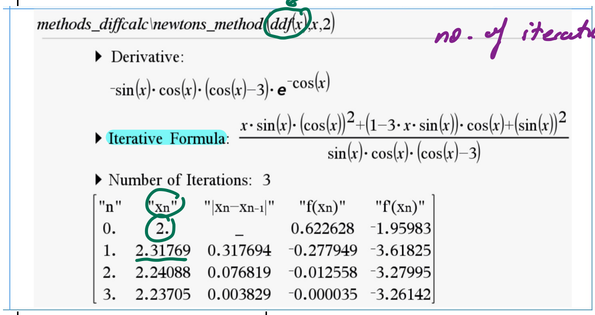
T.P of $f(x)$, $f'(x)=0$

T.P of $f'(x)$, $f''(x)=0$

i. Write down an equation that can be solved using Newton's method to approximate the x-value for a turning point of $f'(x)$. (2 marks)

$f''(x)=0$, $e^{-\cos(x)}\left(\cos(x)+\sin^2(x)\right)=0$

ii. Find an approximate x-value for a turning point of $f'(x)$ by completing 3 iterations of Newton's method with $x_0 = 2$, in the table below. Write your answers correctly to three decimal places. (2 marks)



no. of iterations: 3

	2
	2.318
	2.241
	2.237

$g'(x) = bf'(bx)$

f. Consider the function $g(x) = f(bx)$, where $b > 0$.

OLD: $y = f'(x)$

NEW: $y' = bf'(bx')$

i. State a sequence of transformations that map $f'(x)$ to $g'(x)$. (1 mark)

$f'(x) \rightarrow g'(x)$

$f'(x) \rightarrow bf'(bx)$

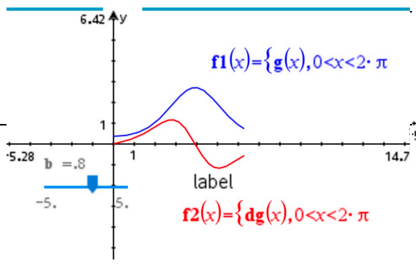
1. Dil of b from x-axis

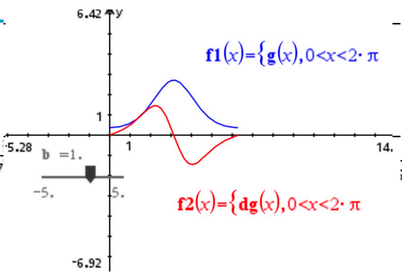
2. Dil of $\frac{1}{b}$ from y-axis

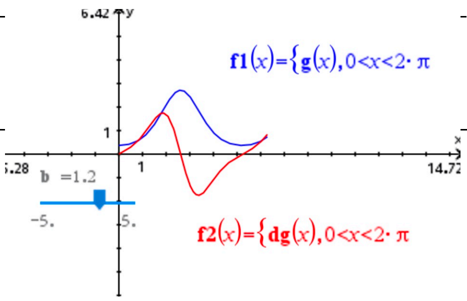
ii. Find the value(s) of b such that g and $g'(x)$ intersect at least twice for $x \in [0, 2\pi]$. (1 mark)

Sliders:

$b>1$







Question 3 (14 marks)

Consider the following cubic polynomial:

$$p(x) = 2(x-2)(x^2 - 10x + 28)$$

a. Find the coordinates of the stationary point of $p(x)$ and state its nature. (2 marks)

$p'(x) = 0$, $x = 4$

$p(4) = 16$

$(4, 16)$ S.P.I

Ø

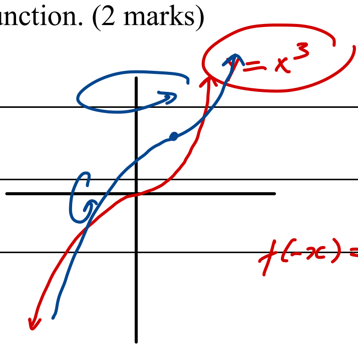
b. Hence, express $p(x)$ in the form $a(x-h)^3 + k$. (1 mark)

(h, k) S.P.O.I.

$2(x-4)^3 + 16$

c. Find the sequence of translations on $p(x)$ that will result in an odd function. (2 marks)

1. Translate 4 in negative x-axis
2. Translate 16 in negative y-axis



$f(-x) = -f(x)$

d. tangentLine(p(x), x, a)

i. Find the equation of the tangent line of $p(x)$ at the point $x = a$. Provide your answer in terms of a. (2 marks)

$p'(a) = 6(a^2 - 8a + 16)$ (1)

$t(x) = y = (6(a^2 - 8a + 16))x - 4(a^3 - 6a^2 + 28)$ (1)

ii. Find the equations of the tangents of $p(x)$ that share the same x-intercept as the cubic. (3 marks)

$p(x) = 0$

$\boxed{x = 2}$

tangent has x-int at $x = 2$.

solve$(t(2) = 0, a)$

$a = 2, 5$

tangentLine(p(x), x, 2)

$y = 24x - 48$

tangentLine(p(x), x, 5)

$y = 6x - 12$

iii. Hence, state the equation of the tangents to the inverse function $p^{-1}(x)$ that share the same y-intercept as the inverse function. (2 marks)

Everyth is inversed! from part ii)

Answers: inverse tangents from part ii)

$x = 24y - 48$

$\boxed{y = \frac{x + 48}{24}}$

$x = 6y - 12$

$\boxed{y = \frac{x + 12}{6}}$

e. Find the acute angle made by the tangents to p and p^{-1} at the point of intersection of these two functions. Give your answer correct to the nearest degree. (2 marks)

$p(x) = x$, $\boxed{x = 2.09}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

m_1 = tangent to p at $x = 2.09$ $= p'(2.09)$

m_2 = tangent to p^{-1} at $x = 2.09$ $= \frac{1}{p'(2.09)}$

solve for θ

$\boxed{\theta \approx 85^\circ}$

Space for Personal Notes

$m = \frac{dy}{dx}$

$m_I = \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$

Question 4 (13 marks)

Australia (population: 25 million) has been quarantined from the rest of the world since the discovery of a highly contagious prion. The number of people (in millions) infected with the virus t days after the outbreak begins is:

$$p(t) = \frac{\alpha}{1 + e^{0.15(40-t)}}$$

Where $\alpha \in R$.

$t \to \infty$

a. Given that, eventually, everyone will be infected, show that $\alpha = 25$. (2 marks)

Button left to Book.

$\lim_{t \to \infty} (p(t)) = \alpha$.

$\therefore \boxed{\alpha = 25}$

(As total pop is 25 mil)

max increase/gradient

b. When is $p(t)$ increasing the fastest? Find $p(t)$ at that point. (3 marks)

$p''(t) = 0$

$t = 40$

$p(40) = 12.5$

c. The number of people hospitalised is much lower. The number of people hospitalised (in millions), t days after the outbreak begins, is given by:

$$h(t) = ce^{a(t-b)^2}$$

Where $a < 0$ and $b, c > 0$.

i. State $h'(t)$. (1 mark)

$h'(t) = 2ace^{a(t-b)^2} \cdot (t-b)$

ii. Show that the maximum occurs at $t = b$ and the maximum value is given by $h(b) = c$. (3 marks)

$h'(t) = 0$

$\underbrace{2ac}_{}\underbrace{e^{a(t-b)^2}}_{\neq 0} \cdot (t-b) = 0$

$a < 0$, $c > 0$

$\therefore \; t - b = 0$

connected $\boxed{t = b}$

$h(b) = ce^{a(b-b)^2}$

$= c \cdot e^{a \cdot 0}$

$= c \cdot e^0$

$= c$

iii. The maximum number of hospitalisations is 1 million and this occurs 100 days after the outbreak begins. Find the values of b and c. (1 mark)

$h(100) = 1$

$b = 100$

$c = 1$

$h'(t)$

iv. Given that the maximum and minimum rate of change, in the number of people hospitalised, occurs $10\sqrt{10}$ days apart, find the value of a. (3 marks)

$h''(t)=0$

$h''(t)=0 \quad | \quad a<0$

$t = 100 \pm \sqrt{\frac{1}{-2a}}$

$\left(100+\sqrt{\frac{1}{-2a}}\right) - \left(100-\sqrt{\frac{1}{-2a}}\right) = 10\sqrt{10}$

solve for a:

$a = \frac{-1}{500}$

Space for Personal Notes

Section C: Tech-Active Solutions – Mathematica

Question Number	Solutions
1a	
1b	In[25]:= `EuclideanDistance[{x, 2}, {10, 7}]` Out[25]= $\sqrt{25 + \text{Abs}[-10 + x]^2}$ In[26]:= `Assuming[x ≥ 0, Refine[EuclideanDistance[{x, 2}, {10, 7}]]]` Out[26]= $\sqrt{25 + (-10 + x)^2}$
1c	
1d	In[37]:= $T[x\_] := \sqrt{x\text{^}2 + 4} + \frac{\sqrt{25 + (-10 + x)^2}}{3}$ In[38]:= `T'[x]` Out[38]= $\frac{-10 + x}{3\sqrt{25 + (-10 + x)^2}} + \frac{x}{\sqrt{4 + x^2}}$ In[39]:= `Solve[T'[x] == 0, x] // N` Out[39]= `{{x → 0.615607}}` In[40]:= `T[0.6156067766766208`]` Out[40]= `5.63703` In[43]:= `Minimize[{T[x], 0 ≤ x ≤ 4}, x] // N` Out[43]= `{5.63703, {x → 0.615607}}`

1e	In[19]:= $Tq[x\_] := \frac{\sqrt{x\text{^}2 + 4}}{q} + \frac{\sqrt{25 + (x - 10)\text{^}2}}{3}$
	In[20]:= `Solve[Tq'[4] == 0, q]`
	Out[20]= $\left\{\left\{q \to \sqrt{\frac{61}{5}}\right\}\right\}$
	In[21]:= `Solve[Tq'[4] == 0, q] // N`
	Out[21]= `{{q → 3.49285}}`

2a	`In[56]:= f[x_] := Exp[-Cos[x]]`
	`In[57]:= f'[x]`
	Out[57]= $e^{-\text{Cos}[x]}\,\text{Sin}[x]$

2b	`In[58]:= Solve[f[x] == f'[x] && 0 ≤ x ≤ 2 Pi]`
	Out[58]= $\{\{x \to \frac{\pi}{2}\}, \{x \to \frac{\pi}{2}\}\}$
	`In[59]:= f[Pi / 2]`
	Out[59]= 1

2ci	`In[23]:= NormalLine[f[x], x, a]`
	Out[23]= $e^{-\text{Cos}[a]} + a\,e^{\text{Cos}[a]}\,\text{Csc}[a] - e^{\text{Cos}[a]}\,x\,\text{Csc}[a]$
	In[27]:= $e^{-\text{Cos}[a]} + a\,e^{\text{Cos}[a]}\,\text{Csc}[a] - e^{\text{Cos}[a]}\,x\,\text{Csc}[a]$ /. x → Pi
	Out[27]= $e^{-\text{Cos}[a]} + a\,e^{\text{Cos}[a]}\,\text{Csc}[a] - e^{\text{Cos}[a]}\,\pi\,\text{Csc}[a]$
	In[29]:= Solve[$e^{-\text{Cos}[a]} + a\,e^{\text{Cos}[a]}\,\text{Csc}[a] - e^{\text{Cos}[a]}\,\pi\,\text{Csc}[a]$ == 0 && 0 ≤ a ≤ 2 Pi, a] // N
	`Out[29]= {{a → 1.74605}, {a → 4.53713}}`

2cii	``In[30]:= NormalLine[f[x], x, 1.746051460340900 5`]``
	`Out[30]= 2.67997 - 0.853062 x`
	``In[31]:= NormalLine[f[x], x, 4.537133846838686`]``
	`Out[31]= -2.67997 + 0.853062 x`
	``In[35]:= Abs[ArcTan[0.8530621074843592`] - ArcTan[-0.8530621074843592`]] / Degree // N``
	`Out[35]= 80.9325`

2d	`In[42]:= Maximize[f[x] - f'[x], x] // N` `Out[42]= {3.5732, {x → 3.71642}}`

2ei	`In[60]:= f''[x] // FullSimplify`
	Out[60]= $e^{-\text{Cos}[x]}\,(\text{Cos}[x] + \text{Sin}[x]^2)$

2eii	In[65]:= n[x_] := $x - \frac{f''[x]}{f'''[x]}$
	`In[66]:= n[x] // FullSimplify`
	Out[66]= $\frac{x\,(-3 + \text{Cos}[x]) + \text{Csc}[x] + \text{Tan}[x]}{-3 + \text{Cos}[x]}$
	`In[76]:= n[2.0]`
	`Out[76]= 2.31769`
	`In[77]:= n[n[2.0]]`
	`Out[77]= 2.24088`
	`In[78]:= n[n[n[2.0]]]`
	`Out[78]= 2.23705`

2fi	
2fii	
3a	In[97]:= f[x_] := 2 (x − 2) (x^2 − 10 x + 28) In[98]:= Solve[f'[x] == 0, x] Out[98]= {{x → 4}, {x → 4}} In[99]:= f[4] Out[99]= 16 In[100]:= f''[4] Out[100]= 0
3b	In[101]:= Solve[a * (x − 4) ^3 + 16 == f[x], a] Out[101]= {{a → 2}}
3c	

3di	In[102]:= f'[x] // FullSimplify
	Out[102]= $6\,(-4+x)^2$
	In[103]:= TangentLine[f[x], x, a] // FullSimplify
	Out[103]= $-4\left(28+(-6+a)\,a^2\right)+6\,(-4+a)^2\,x$

3dii	In[104]:= TangentLine[f[x], x, a] /. x → 2 // FullSimplify
	Out[104]= $-4\,(-5+a)\,(-2+a)^2$
	In[105]:= Solve[$-4\,(-5+a)\,(-2+a)^2$ == 0, a]
	Out[105]= {{a → 2}, {a → 2}, {a → 5}}
	In[106]:= TangentLine[f[x], x, 2]
	Out[106]= $-48+24\,x$
	In[107]:= TangentLine[f[x], x, 5]
	Out[107]= $-12+6\,x$

3diii	In[128]:= `Solve[f[y] == x, y]` Out[128]= $\left\{\left\{y \to 4 + \frac{(-16+x)^{1/3}}{2^{1/3}}\right\}, \left\{y \to 4 - \frac{(1 - i\sqrt{3})(-16+x)^{1/3}}{2 \times 2^{1/3}}\right\}, \left\{y \to 4 - \frac{(1 + i\sqrt{3})(-16+x)^{1/3}}{2 \times 2^{1/3}}\right\}\right\}$ In[129]:= `f1[x_] := 4 + Surd[x - 16, 3]/Surd[2, 3]` In[130]:= `TangentLine[f1[x], x, a] /. x → 0` Out[130]= $4 - \frac{a}{3 \times 2^{1/3} \sqrt[3]{-16+a}^2} + \frac{\sqrt[3]{-16+a}}{2^{1/3}}$ In[131]:= Solve$\left[4 - \frac{a}{3 \times 2^{1/3} \sqrt[3]{-16+a}^2} + \frac{\sqrt[3]{-16+a}}{2^{1/3}} == 2, a\right]$ Out[131]= $\{\{a \to 0\}, \{a \to 18\}\}$ In[132]:= `TangentLine[f1[x], x, 0]` Out[132]= $2 + \frac{x}{24}$ In[133]:= `TangentLine[f1[x], x, 18]` Out[133]= $2 + \frac{x}{6}$

3e	In[135]:= `Solve[f[x] == x, x, Reals] // N`
	Out[135]= $\{\{x \to 2.09123\}\}$
	In[136]:= Abs$\left[\text{ArcTan}[f'[2.0912331809635747`]] - \text{ArcTan}\left[\frac{1}{f'[2.0912331809635747`]}\right]\right]$ / Degree // N
	Out[136]= 84.7617

4a	In[1]:= $p[t\_] := \frac{a}{1 + \text{Exp}[0.15\,(40 - t)]}$ In[2]:= `Solve[Limit[p[t], t → Infinity] == 25, a]` Out[2]= $\{\{a \to 25.\}\}$

4b	In[6]:= $p[t\_] := \frac{25}{1 + Exp[15/100\ (40 - t)]}$
	In[9]:= Solve[p''[t] == 0, t, Reals]
	Out[9]= {{t → 40}}
	In[11]:= p[40] // N
	Out[11]= 12.5
	In[12]:= Maximize[p'[t], t]
	Out[12]= $\left\{\frac{15}{16}, \{t \to 40\}\right\}$

4ci	In[17]:= h[t_] := c Exp[a (t - b) ^2] In[18]:= h'[t] Out[18]= $2\,a\,c\,e^{a\,(-b+t)^2}\,(-b+t)$
4cii	In[19]:= h''[b] Out[19]= 2 a c
4ciii	In[22]:= Solve[h[100] == 1 && h'[100] == 0, Reals] Out[22]= {{a → 0, c → 1}, {b → 100, c → 1}}
4civ	In[23]:= h[t_] := Exp[a (t - 100) ^2] In[25]:= Solve[h''[t] == 0, t, Reals] Out[25]= $\left\{\left\{t \to 100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a < 0\right\}, \left\{t \to 100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} \text{ if } a < 0\right\}\right\}$ In[26]:= $100 + \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}} - \left(100 - \frac{\sqrt{-\frac{1}{a}}}{\sqrt{2}}\right)$ Out[26]= $\sqrt{2}\sqrt{-\frac{1}{a}}$ In[27]:= Solve$\left[\sqrt{2}\sqrt{-\frac{1}{a}} == 10\sqrt{10}, a\right]$ Out[27]= $\left\{\left\{a \to -\frac{1}{500}\right\}\right\}$

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Section D: Tech-Active Solutions – Casio

Question Number	Solutions
1a	
1b	$\sqrt{(x-10)^\wedge 2+(2-7)^\wedge 2}$ $\sqrt{x^2-20\cdot x+125}$
1c	
1d	Define $T(x)=\sqrt{x^\wedge 2+4}+\frac{\sqrt{(x-10)^\wedge 2+25}}{3}$ done solve$\left(\frac{d}{dx}(T(x))=0, x\right.$ $\{x=0.6156067766\}$ $T(0.6156067766)$ 5.637028955 fMin$(T(x), x, 0, 4)$ $\{\text{MinValue}=5.637028955, x=0.6156067766\}$
1e	Define $Tq(x)=\frac{\sqrt{x^\wedge 2+4}}{q}+\frac{\sqrt{25+(x-10)^\wedge 2}}{3}$ done solve$\left(\frac{d}{dx}(Tq(x)=0)\mid x=4, q\right)$ $\left\{q=\frac{\sqrt{305}}{5}\right\}$ solve$\left(\frac{d}{dx}(Tq(x)=0)\mid x=4, q\right)$ $\{q=3.492849839\}$
2a	Define $f(x)=e^{-\cos(x)}$ done $\frac{d}{dx}(f(x))$ $\sin(x)\cdot e^{-\cos(x)}$

2b		solve$\left(f(x)=\frac{d}{dx}(f(x)) \mid 0\le x\le 2\pi, x\right)$ $\left\{x=\frac{\pi}{2}\right\}$ $f\left(\frac{\pi}{2}\right)$ 1
2ci		normal$(f(x), x, a)$ $\frac{-x\cdot e^{\cos(a)}}{\sin(a)}+\frac{a\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}$ ans$\mid x=\pi$ $\frac{a\cdot e^{\cos(a)}}{\sin(a)}-\frac{\pi\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}$ solve$\left(\frac{a\cdot e^{\cos(a)}}{\sin(a)}-\frac{\pi\cdot e^{\cos(a)}}{\sin(a)}+e^{-\cos(a)}=0 \mid 0\le a\le 2\pi, a\right)$ $\{a=1.74605146, a=4.537133847\}$
2cii		normal$(f(x), x, 1.74605146)$ $-0.8530621077\cdot x+2.67997365$ normal$(f(x), x, 4.537133847)$ $0.8530621076\cdot x-2.679973651$ $\lvert\tan^{-1}(-0.8530621077)-\tan^{-1}(0.8530621076)\rvert$ 80.93247628 Change to Deg mode! Make sure to change back to Rad
2d		fMax$\left(f(x)-\frac{d}{dx}(f(x)), x, 0, 2\pi\right)$ $\{\text{MaxValue}=3.573200775, x=3.7164\}$
2ei		$\frac{d^2}{dx^2}(f(x))$ $\left((\sin(x))^2+\cos(x)\right)\cdot e^{-\cos(x)}$
2eii		Define $n(x)=x-\frac{\frac{d^2}{dx^2}(f(x))}{\frac{d^3}{dx^3}(f(x))}$ done $n(x)$ $x-\frac{(\sin(x))^2+\cos(x)}{(\sin(x))^3+3\cdot\cos(x)\cdot\sin(x)-\sin(x)}$ $n(2.0)$ 2.317694099 $n(\text{ans})$ 2.240875415 $n(\text{ans})$ 2.237046622
2fi		

2fii	
3a	Define p(x)=2(x−2)(x^2−10x+28) done solve($\frac{d}{dx}$(p(x))=0, x {x=4} p(4) 16 $\frac{d^2}{dx^2}$(p(x))|x=4 0
3b	solve(a*(x−4)^3+16=p(x), a {a=2}
3c	
3di	tanLine(p(x), x, a) $2\cdot(a^2-10\cdot a+28)\cdot(a-2)+x\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))-a\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))$ simplify(ans) $-4\cdot a^3+6\cdot a^2\cdot x+24\cdot a^2-48\cdot a\cdot x+96\cdot x-112$ collect(ans, x $(6\cdot a^2-48\cdot a+96)\cdot x-4\cdot a^3+24\cdot a^2-112$
3dii	tanLine(p(x), x, a) $2\cdot(a^2-10\cdot a+28)\cdot(a-2)+x\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))-a\cdot(2\cdot(a^2-10\cdot a+28)+2\cdot(a-2)\cdot(2\cdot a-10))$ simplify(ans) $-4\cdot a^3+6\cdot a^2\cdot x+24\cdot a^2-48\cdot a\cdot x+96\cdot x-112$ collect(ans, x $(6\cdot a^2-48\cdot a+96)\cdot x-4\cdot a^3+24\cdot a^2-112$ ans|x=2 $-4\cdot a^3+24\cdot a^2+2\cdot(6\cdot a^2-48\cdot a+96)-112$ solve(ans=0, a {a=2, a=5} tanLine(p(x), x, 2) $24\cdot x-48$ tanLine(p(x), x, 5) $6\cdot x-12$
3diii	
3e	solve(p(x)=x, x {x=2.091233181} Define dp(x)=$\frac{d}{dx}$(p(x)) done $\left\|\tan^{-1}(dp(2.091233181))-\tan^{-1}\left(\frac{1}{dp(2.091233181)}\right)\right\|$ 84.76166868 Change to Deg then change back to Rad

4a	Define $p(t)=\frac{a}{1+e^{0.15(40-t)}}$ done solve$(\lim_{t\to\infty}(p(t))=25, a$ $\{a=25\}$
4b	Define $p(t)=\frac{25}{1+e^{0.15(40-t)}}$ done solve$(\frac{d^2}{dt^2}(p(t))=0, t)$ $\{t=40\}$ $p(40)$ 12.5 fMax$\left(\frac{d}{dt}(p(t)), t, -\infty, \infty\right)$ {MaxValue=0.9375, t=40}
4ci	Define h(t)=c*e^(a*(t−b)^2) done $\frac{d}{dt}(h(t))$ $-2\cdot a\cdot c\cdot(b-t)\cdot e^{a\cdot(b-t)^2}$
4cii	$\frac{d^2}{dt^2}(h(t))\|t=b$ $2\cdot a\cdot c$
4ciii	$\begin{cases} h(100)=1 \\ \frac{d}{dt}(h(t))=0 \mid t=100 \end{cases}\Bigg\|_{b,c}$ $\{b=100, c=1\}$
4civ	solve$(\frac{d^2}{dt^2}(h(t))=0, t)$ $\left\{t=\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}, t=\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\}$ $\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|$ $\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|$ simplify$\left(\left\|\frac{2\cdot a\cdot b+\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}-\frac{2\cdot a\cdot b-\sqrt{2}\cdot\sqrt{-a}}{2\cdot a}\right\|\right)$ $\left\|\frac{1}{a}\right\|\cdot\sqrt{-2\cdot a}$ solve$\left(\left\|\frac{1}{a}\right\|\cdot\sqrt{-2\cdot a}=10\sqrt{10}, a\right.$ $\left\{a=-\frac{1}{500}\right\}$

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Section E: Tech-Active Solutions - TI

Question Number	Solutions
1a	
1b	
1c	
1d	$methods\_func\backslash analysed(t(x),x,0,4)$ ▸ Start Point: $\left[0 \quad \frac{5\cdot\sqrt{5}}{3}+2\right]$ ▸ End Point: $\left[4 \quad \frac{\sqrt{61}}{3}+2\cdot\sqrt{5}\right]$ ▸ Maximal Domain: $0\le x\le 4$ ▸ Asymptotes: (2) Define $t(x)=\sqrt{x^2+4}+\frac{1}{3}\cdot\sqrt{25+(x-10)^2}$ *Done* $y=\frac{10}{3}-\frac{4\cdot x}{3}$ (Oblique) $y=\frac{4\cdot x}{3}-\frac{10}{3}$ (Oblique) ▸ No x –Intercepts Found ▸ Vertical Intercept: $\left[0 \quad \frac{5\cdot\sqrt{5}}{3}+2\right]$ ▸ Derivative: $\frac{x-10}{3\cdot\sqrt{x^2-20\cdot x+125}}+\frac{x}{\sqrt{x^2+4}}$ ▸ No Inflection Points Found ▸ Stationary Point: $[0.615607 \quad 5.63703]$ (Local min.)

1e		Define $tq(x)=\frac{\sqrt{x^2+4}}{q}+\frac{1}{3}\cdot\sqrt{25+(x-10)^2}$ *Done* Define $dtq(x)=\frac{d}{dx}(tq(x))$ *Done* ⚠ solve$(dtq(4)=0,q)$ $q=3.49285$
2a		Define $f(x)=e^{-\cos(x)}$ *Done* Define $df(x)=\frac{d}{dx}(f(x))$ *Done* $df(x)$ $\sin(x)\cdot e^{-\cos(x)}$
2b		*methods_func\intersectd*$(f(x),df(x),x,0,2\cdot\pi)$ ▸ Intersection Points: (1) $\begin{bmatrix}\frac{\pi}{2} & 1\end{bmatrix}$
2ci		normalLine$(f(x),x,a)$ $\frac{e^{-\cos(a)}\cdot\left(a\cdot e^{2\cdot\cos(a)}+\sin(a)\right)}{\sin(a)}-\frac{e^{\cos(a)}\cdot x}{\sin(a)}$ solve(normalLine$(f(x),x,a)=0,x)$ $x=e^{-2\cdot\cos(a)}\cdot\left(a\cdot e^{2\cdot\cos(a)}+\sin(a)\right)$ ⚠ solve$\left(e^{-2\cdot\cos(a)}\cdot\left(a\cdot e^{2\cdot\cos(a)}+\sin(a)\right)=\pi,a\right)\mid 0\le a\le 2\cdot\pi$ $a=1.74605$ or $a=3.14159$ or $a=4.53713$
2cii		normalLine$(f(x),x,1.74605)$ $2.67997-0.853063\cdot x$ normalLine$(f(x),x,4.53713)$ $0.853059\cdot x-2.67995$ $\tan^{-1}(0.853063)-\tan^{-1}(-0.853063)$ 1.41254 $\frac{1.4125392159322\cdot 180}{\pi}$ 80.9325

2d	$methods\_func\backslash analysed(f(x)-df(x),x,0,2\cdot\pi)$ ▶ Start Point: $\begin{bmatrix}0 & e^{-1}\end{bmatrix}$ ▶ End Point: $\begin{bmatrix}2\cdot\pi & e^{-1}\end{bmatrix}$ ▶ Maximal Domain: $0\le x\le 2\cdot\pi$ ▶ x −Intercept: $\begin{bmatrix}\frac{\pi}{2} & 0\end{bmatrix}$ ▶ Vertical Intercept: $\begin{bmatrix}0 & e^{-1}\end{bmatrix}$ ▶ Derivative: $(-\cos(x)-\sin(x)\cdot(\sin(x)-1))\cdot e^{-\cos(x)}$ ▶ Inflection Points: (2) $\begin{bmatrix}2.9021 & 2.01513\end{bmatrix}$ (Increasing) $\begin{bmatrix}4.48161 & 2.48069\end{bmatrix}$ (Decreasing) ▶ Stationary Points: (2) $\begin{bmatrix}1.5708 & 0.\end{bmatrix}$ (Local min.) $\begin{bmatrix}3.71642 & 3.5732\end{bmatrix}$ (Local max.)
2ei	Define $ddf(x)=\frac{d}{dx}(df(x))$ *Done* $ddf(x)$ $\quad (\cos(x)+(\sin(x))^2)\cdot e^{-\cos(x)}$
2eii	$methods\_diffcalc\backslash newtons\_method(ddf(x),x,2)$ ▶ Derivative: $^{-}\sin(x)\cdot\cos(x)\cdot(\cos(x)-3)\cdot e^{-\cos(x)}$ ▶ Iterative Formula: $\dfrac{x\cdot\sin(x)\cdot(\cos(x))^2+(1-3\cdot x\cdot\sin(x))\cdot\cos(x)+(\sin(x))^2}{\sin(x)\cdot\cos(x)\cdot(\cos(x)-3)}$ ▶ Number of Iterations: 3 $\begin{bmatrix}\text{"n"} & \text{"xn"} & \text{"\|xn−xn-1\|"} & \text{"f(xn)"} & \text{"f'(xn)"} \\ 0. & 2. & \_ & 0.622628 & ^{-}1.95983 \\ 1. & 2.31769 & 0.317694 & ^{-}0.277949 & ^{-}3.61825 \\ 2. & 2.24088 & 0.076819 & ^{-}0.012558 & ^{-}3.27995 \\ 3. & 2.23705 & 0.003829 & ^{-}0.000035 & ^{-}3.26142\end{bmatrix}$

2fi	
2fii	
3a	Define $p(x)=2\cdot(x-2)\cdot(x^2-10\cdot x+28)$ *Done* *methods_func\analyse*$(p(x),x)$ ▸ Start Point: $[-\infty \quad -\infty]$ ▸ End Point: $[\infty \quad \infty]$ ▸ Maximal Domain: $-\infty<x<\infty$ ▸ x−Intercept: $[2 \quad 0]$ ▸ Vertical Intercept: $[0 \quad -112]$ ▸ Derivative: $6\cdot x^2-48\cdot x+96$ ▸ Inflection Point: $[4 \quad 16]$ (Stationary) ▸ Stationary Point: $[4 \quad 16]$ (Inflection)
3b	
3c	
3di	*methods_diffcalc\tangent_line*$(p(x),x,a)$ ▸ Derivative: $6\cdot x^2-48\cdot x+96$ ▸ Gradient: $6\cdot a^2-48\cdot a+96$ ▸ Passes Through: $[a \quad 2\cdot(a-2)\cdot(a^2-10\cdot a+28)]$ ▸ x−Intercept: $\left[\frac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)} \quad 0\right]$ ▸ Vertical Intercept: $[0 \quad -4\cdot(a^3-6\cdot a^2+28)]$ ▸ Tangent Line: $6\cdot(a^2-8\cdot a+16)\cdot x-4\cdot(a^3-6\cdot a^2+28)$

3dii	$\text{solve}(p(x)=0,x)$ $x=2$ $\text{solve}(6\cdot(a^2-8\cdot a+16)\cdot x-4\cdot(a^3-6\cdot a^2+28)=$ ▸ $x=\dfrac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)}$ ⚠ $\text{solve}\left(\dfrac{2\cdot(a^3-6\cdot a^2+28)}{3\cdot(a^2-8\cdot a+16)}=2,a\right)$ $\text{tangentLine}(p(x),x,\{2,5\})$ $\{24\cdot x-48,6\cdot x-12\}$
3diii	$\text{solve}(y=24\cdot x-48,x)$ $x=\dfrac{y}{24}+2$ $\text{solve}(y=6\cdot x-12,x)$ $x=\dfrac{y}{6}+2$
3e	$\text{solve}(p(x)=x,x)$ $x=2.09123$ Define $dp(x)=\dfrac{d}{dx}(p(x))$ *Done* $2\cdot\left(\tan^{-1}(dp(2.09123))-\dfrac{\pi}{4}\right)$ 1.47937 $\dfrac{1.4793705022705\cdot 180}{\pi}$ 84.7617
4a	Define $p(t)=\dfrac{a}{1+e^{0.15\cdot(40-t)}}$ *Done* $\lim\limits_{t\to\infty}(p(t))$ a

4b	Define $dp(t)=\frac{d}{dt}(p(t))$ *Done* $methods\_func\backslash analyse(dp(t),t)$ ▶ Start Point: $[-\infty \quad 0.]$ ▶ End Point: $[\infty \quad 0.]$ ▶ Maximal Domain: $-\infty<t<\infty$ ▶ Asymptote: $y=0.$ (Horizontal) ▶ No t –Intercepts Found ▶ Vertical Intercept: $[0 \quad 0.009249]$ ▶ Derivative: $\frac{-226.923\cdot (1.16183)^{t}\cdot ((1.16183)^{t}-403.429)}{((1.16183)^{t}+403.429)^{3}}$ ▶ Inflection Points: (2) $[31.221 \quad 0.625001]$ (Increasing) $[48.7809 \quad 0.625001]$ (Decreasing) ▶ Stationary Point: $[40.001 \quad 0.937501]$ (Local max.) $p(40.001)$ 12.5009
4ci	Define $h(t)=c\cdot e^{a\cdot (t-b)^{2}}$ *Done* Define $dh(t)=\frac{d}{dt}(h(t))$ *Done* $dh(t)$ $\left(2\cdot a\cdot e^{a\cdot b^{2}}\cdot c\cdot t-2\cdot a\cdot e^{a\cdot b^{2}}\cdot b\cdot c\right)\cdot e^{a\cdot t^{2}-}$ ▸ $\text{factor}(h(t),t)$ $c\cdot e^{a\cdot t^{2}-2\cdot a\cdot b\cdot t+a\cdot b^{2}}$

4cii	$\text{solve}(dh(t)=0,t)$	$t=b$ or $a\cdot c=0$
	$h(b)$	c
4ciii	$\text{solve}(h(100)=1 \text{ and } dh(100)=0,b,c)$	$b=100$ and $c=1$
4civ	$\text{Define } ddh(t)=\frac{d}{dt}(dh(t))$	Done
	$\text{solve}(ddh(t)=0,t)$	$t=\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a}$ or $t=\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a}$ or ▸
	$\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a}-\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a}$	$\frac{-\sqrt{2}}{\sqrt{-a}}$
	$\frac{-(\sqrt{-2\cdot a}-2\cdot a\cdot b)}{2\cdot a}-\frac{\sqrt{-2\cdot a}+2\cdot a\cdot b}{2\cdot a}$	$\frac{\sqrt{2}}{\sqrt{-a}}$
	$\text{solve}\left(\frac{\sqrt{2}}{\sqrt{-a}}=10\cdot\sqrt{10},a\right)$	$a=\frac{-1}{500}$

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