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VCE Mathematical Methods $\frac{3}{4}$
AOS 2 Revision [0.15]
Workshop

Error Logbook:



New Ideas/Concepts	Didn't Read Question
<p>Pg / Q #: _____</p> <p>Notes:</p>	<p>Pg / Q #: _____</p> <p>Notes:</p>
Algebraic/Arithmetic/ Calculator Input Mistake	Working Out Not Detailed Enough
<p>Pg / Q #: _____</p> <p>Notes:</p>	<p>Pg / Q #: _____</p> <p>Notes:</p>

Section A: Cheat Sheets

Cheat Sheet



[2.1.1] - Find the Instantaneous Rate of Change and Average Rate of Change


- The average rate of change of a function $f(x)$ over $x \in [a, b]$ is given by:

 **Average rate of change** = _____

- It is the gradient of the line joining the two points.
- **Instantaneous Rate of Change** is a gradient of a graph at a single point/moment
- **First Principles** derivative definition:


 $f'(x) =$ _____

➤ The Product Rule


-  The derivative of $h(x) = f(x) \times g(x)$ is given by:

$h'(x) =$ _____

➤ The Quotient Rule

-  The derivative of a $h(x) = \frac{f(x)}{g(x)}$ is given by:

$h'(x) =$ _____

-  Always differentiate the top function first.

➤ The Chain Rule

$$y = f(g(x))$$

$$\frac{dy}{dx} = \text{_____}$$

- The **process** for finding derivatives of **composite functions**.

[2.1.2] - Identify the Nature of Stationary Points and Trends (Strictly Increasing and Decreasing)

- Point where the _____ of the function is zero.

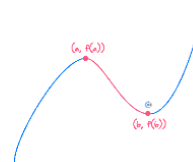
$$f'(x) = 0, \frac{dy}{dx} = 0$$

- We can identify the nature of a stationary point by using the sign table.

x	Less than a	a	Bigger than a
$f'(x)$	Negative	0	Positive
Shape	n - Decreasing curve	Stationary Point	U - Increasing curve

- Find the gradient of the neighboring points.

➤ Strictly Increasing and Strictly Decreasing Functions



Strictly Increasing: $x \in$ _____

Strictly Decreasing: _____

-  Steps:

1. Find the _____.
2. Consider the sign of the _____ between/outside the turning points.



Cheat Sheet

[2.1.3] - Graph Derivative Functions

➤ Steps on Sketching the Derivative Function:

1. Plot x -intercept at the same x -value as the _____ of the original.
2. Consider the trend of the original function and sketch the derivative.

➤ Original is increasing \rightarrow Derivative is _____ the x -axis.

➤ Original is decreasing \rightarrow Derivative is _____ the x -axis

[2.2.1] - Evaluate Limits and Find Points Where the Function is Not Continuous

➤ Limit Definition:

$$\lim_{x \rightarrow a} f(x) = L$$

"The function $f(x)$ approaches L as x approaches a "

➤ Validity of Limit:

$$\lim_{x \rightarrow a} f(x) \text{ exists if } \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

⚙ Limit is defined when the _____ limit equals the _____ limit.

➤ Continuity:

⚙ A function f is said to be continuous at a point $x = a$ if:

1. $f(x)$ is defined at _____.
2. $\lim_{x \rightarrow a} f(x)$ _____.
3. $\lim_{x \rightarrow a} f(x) =$ _____.

[2.2.2] - Apply Differentiability to Find Points Where Functions are Not Differentiable, Domain of the Derivative and Unknowns of a Function

➤ Differentiability:

⚙ A function f is said to be differentiable at a point $x = a$ if:

1. $f(x)$ is continuous at $x = a$.

2. $\lim_{x \rightarrow a} f'(x)$ exists.

⚙ Limit exists when the left and right limits are the same.

⚙ Gradient on the _____ must be the same.

➤ We cannot differentiate:

1. _____
2. _____
3. _____

➤ Finding the Derivative of Hybrid Functions

1. Simply _____ each function.
2. Reject the values for x that are _____ from the domain.

Cheat Sheet



[2.2.3] - Identify Concavity and Find Inflection Points

➤ Second Derivatives

- The derivative of the derivative.
- To get the second derivative, we can **differentiate the original function twice**.

$$\frac{d^2y}{dx^2} = f''(x)$$

➤ Concavity

- Concave up is when the gradient is _____.

$$f''(x) > 0 \rightarrow \text{Concave Up}$$

- Concave down is when the gradient is _____.

$$f''(x) < 0 \rightarrow \text{Concave Down}$$

- _____ is when the gradient is neither increasing nor decreasing.

$$f''(x) = 0 \rightarrow \text{Zero Concavity}$$

➤ Points of Inflection

- A point at which a curve _____ is called a **point of inflection**.

➤ The Second Derivative Test

- Suppose that $f'(a) = 0$ and hence, f has a stationary point at $x = a$. The second derivative test states:

- Concave up gives us local minimum.

$$f''(x) > 0 \rightarrow \text{Local Minimum}$$

- Concave down gives us local maximum.

$$f''(x) < 0 \rightarrow \text{Local Maximum}$$

- Zero concavity gives us stationary point of inflection.

$$f''(x) = 0 \rightarrow \text{Stationary Point of Inflection}$$

[2.3.1] - Find Derivatives with Functional Notation

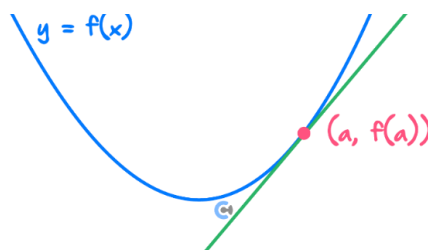
- To derive composite functions like $\sin(f(x))$, apply the _____ rule.

[2.3.2] - Apply Differentiability to Join Two Functions Smoothly

- When two functions join smoothly at a point, the _____ and _____ of each function are both equal at that point.

[2.4.1] - Find Tangents and Normals

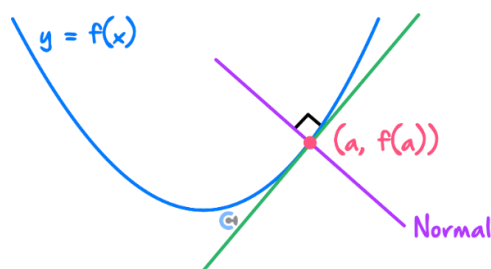
- A **tangent** is a linear line which **just** _____ the curve.
- The gradient of a tangent line has to be equal to the _____ of the curve at the intersection.



$$\text{At } (a, f(a)): \quad m_{\text{tangent}} = \underline{\hspace{2cm}}$$

➤ Normals

- A **normal** is a linear line which is _____ to the tangent.
- The gradient of a normal line has to equal to the _____ of the gradient of the curve at the intersection.



$$\text{At } (a, f(a)): \quad m_{\text{normal}} = \underline{\hspace{2cm}}$$



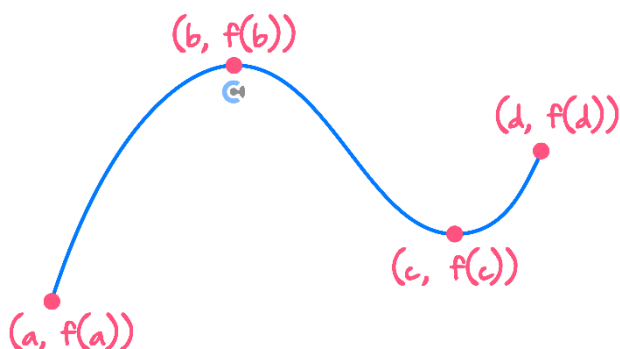
Cheat Sheet

[2.4.2] - Find Minimum and Maximum

➤ Absolute Maximum and Minimum

➤ **Absolute Maxima/Minima** are the overall largest/smallest _____ for the given domain.

➤ They occur at either _____ or _____.



Absolute Min: $f(a)$

Absolute Max: $f(b)$

➤ Steps

1. Find _____ points and _____ points
2. Find the _____ y value for max/min .

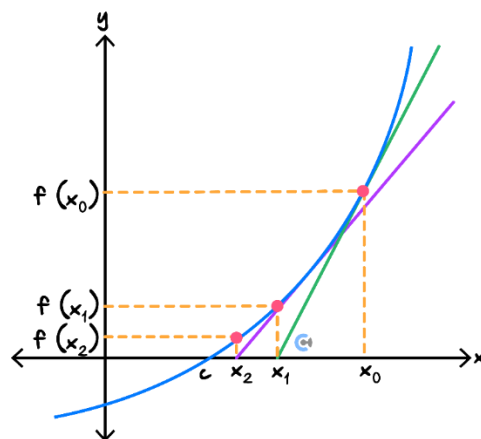
➤ Steps for optimisation

1. Construct a _____ for the subject you want to find the maximum or minimum of.
2. Find its _____ if appropriate.
3. Find its _____ and _____ points.
4. Identify _____ or _____ y value.

[2.4.3] - Apply Newton's Method to Find the Approximation of a Root and Its Limitations

➤ Newton's Method

➤ It is a method of approximating the x -intercept using _____.



$$x_{n+1} = \frac{x_n - f(x_n)/f'(x_n)}{1}$$

➤ Steps

1. Find the _____ at the x value given.
2. Find the _____ of the tangent using iterative formula.
3. Find the next tangent at the $x =$ _____ of the previous tangent.
4. Repeat until the value doesn't change by much.

➤ **Tolerance:** The maximum difference between x_n and x_{n+1} we can have when we stop the iteration.

We stop when $|x_{n+1} - x_n| < \text{_____}$.

➤ Limitation of Newton's Method

➤ **Terminating Sequence:** Occurs when we hit a _____.

➤ **Approximating a Wrong Root:** Occurs when we start on the _____.

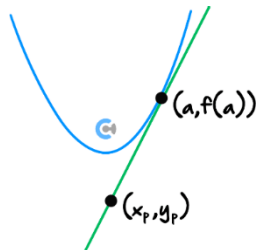
➤ **Oscillating Sequence:** Occurs when we _____ between two values without getting closer to the real root.



Cheat Sheet

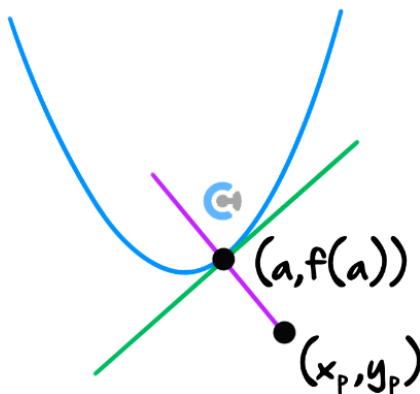
[2.5.1] - Advanced Tangents and Normal Questions

- Finding tangents/normals to functions, which also pass through a given point
- Tangent of $f(x)$ at $x = a$ passes through (x_p, y_p) .



$$f'(a) = \underline{\hspace{2cm}}$$

- Normal of $f(x)$ at $x = a$ passes through (x_p, y_p) .



$$-\frac{1}{f'(a)} = \underline{\hspace{2cm}}$$

[2.5.2] - Advanced Maximum/Minimum Questions

- To find the maximum/minimum instantaneous rate of change, we find the turning point of the _____ function.

[2.6.1] - Advanced Maximum/Minimum Questions

- **Families of Functions:** Functions with an unknown.
- They involve understanding and using transformations.
- They involve the use of sliders/manipulate on CAS/technology.

[2.6.2] - Find Unknowns for Number of Solutions

- For a function to "touch" a line as a tangent:
 - They intersect.

$$f(a) = mx + c$$

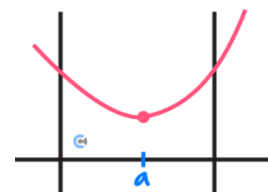
- With the same gradient.

$$f'(a) = \underline{\hspace{2cm}}$$

- We solve these _____.

[2.6.3] - Find Unknowns for Minimum and Maximum

- Minimum/maximum at a turning point:

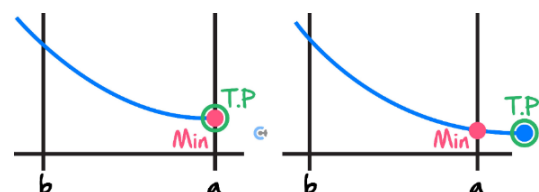


- To achieve minimum/maximum at $x = a$.

$$f'(a) = 0$$

- This is only when $x = a$ is not an _____.

- Minimum/maximum at an endpoint:



- Step 1: Find the value of the unknown such that the turning point occurs at $x = a$.

$$f'(a) = 0$$

- Step 2: Find the value of the unknown such that the turning point occurs after/below $x = a$.

- We can determine whether the unknown can be bigger or smaller than the value achieved in step 1.

$$f'(a \pm \text{something}) = \underline{\hspace{2cm}}$$

Cheat Sheet



[2.7.1] - Evaluate Pseudocode with Conditional Statements and Loops

➤ Assigning Variables:

- To construct algorithms for more mathematical/complex problems, assigning variables will be useful.

$A \leftarrow 3$ assigns the value 3 to the variable A.

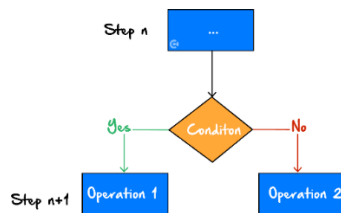
- We can also update our variables using the arrow.

$A \leftarrow A + 3$ assigns the value $A + 3$ to the variable A.

- Since the value of A was already 3, its new value will be 6.

➤ Selections:

- Selections allow us to perform different operations at a given step, depending on a certain condition.



- We are selectively performing an operation.

"If-then"

```
if condition then
    operation
end if
```

- **Allows** us to perform an operation only when a certain condition is met.

"Else"

```
if condition then
    operation 1
else
    operation 2
end if
```

- Provides an opportunity to perform an operation only when a certain condition is met.

"Else-If"

```
if condition 1 then
    operation 1
else if condition 2 then
    operation 2
else
    operation 3
end if
```

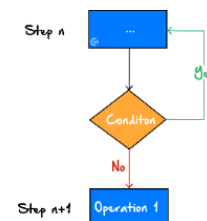
- Provides an opportunity to add multiple pathways, each with different conditions.

➤ Iteration (Loops):

- Iteration (a.k.a. looping) allows us to repeat steps in a Controlled way.

- It is controlled by the condition

- E.g., we only loop when a condition is met.



For loops:

```
for variable from lower bound to upper bound
    condition
    operation
end for
```

- Loops for which a variable increases by one each time it loops.

- The variable gets moved from the lower bound to the upper bound by 1.

- **While loops:** Loops that **do not** change the value of any variable by default.

```
while condition
    operation
end while
```



Cheat Sheet

[2.7.2] - Evaluate and Understand the Pseudocode for Different Implementations of Newton's Method

- A key component of Newton's method is the recursive relationship.

$$x_{n+1} = x_n - \underline{\hspace{2cm}}$$

- Newton's method requires an input function $f(x)$, the derivative $f'(x)$ and an initial value x_0 .
- The number of iterations that Newton's method performs can be limited in our pseudocode.
- The pseudocode can also specify a tolerance for Newton's method where the algorithm terminates if

$$|\underline{\hspace{2cm}}| < \textit{Tolerance}$$

Section B: Exam 1 Questions (32 Marks)

INSTRUCTION:

- **Regular: 26 Marks. 4 Minutes Reading. 36 Minutes Writing.**
- **Extension: 32 Marks. 4 Minutes Reading. 36 Minutes Writing.**



Question 1 (6 marks)

- a. Let $f(x) = \sqrt{\tan(e^{2x})}$, Find $f'(x)$. (2 marks) [2.1.1]

- b. Let $g(x) = \frac{\log_e(3x+1)}{3x+1}$. Find $g'(0)$. (2 marks) [2.1.1]

- c. Find the equation of the normal to $y = x(x - 3)^2$ at the origin $(0, 0)$. (2 marks) **[2.4.1]**

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Question 2 (4 marks)

A tech company is designing a reward system for users who participate in an online task. The reward (in points) that a user receives depends on how long they work on the task, measured in hours. However, to balance effort with efficiency, the company sets the reward function as:

$$R(x) = x \cdot 2^{1-x},$$

where x is the number of hours worked (with $x > 0$), and $R(x)$ is the reward in points

- a. Show that the derivative of the reward function is given by:

$$R'(x) = 2^{1-x}(1 - x \log_e(2)).$$

HINT: First write 2^{1-x} in terms of the exponential function. (2 marks) **[2.1.1]**

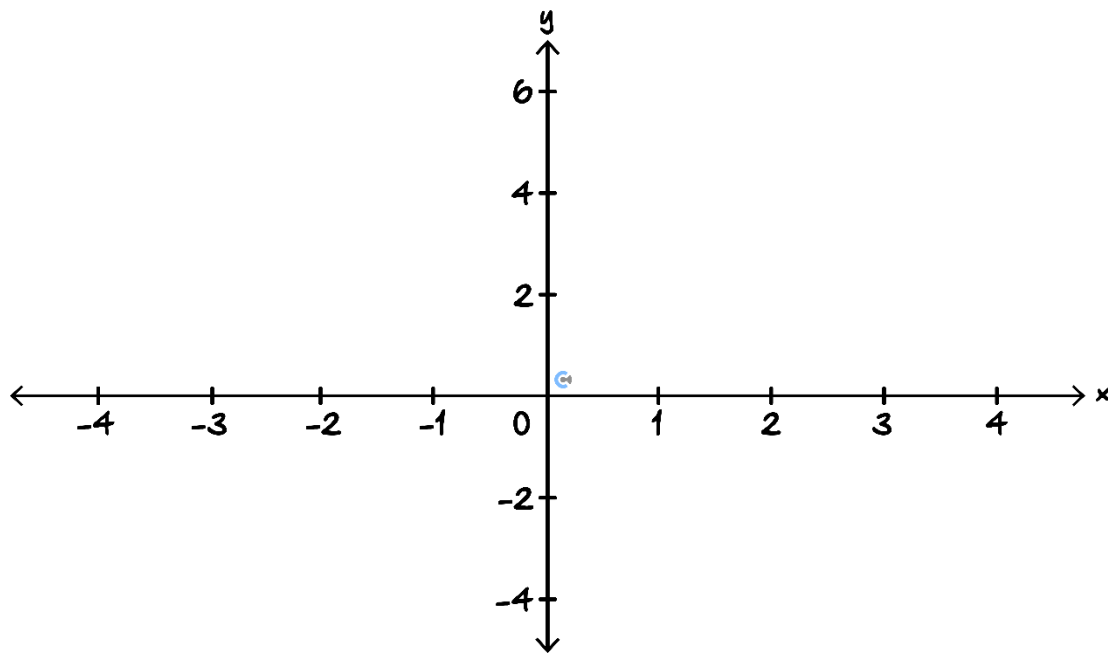
- b. Determine the value of x that maximises the reward. What is the maximum reward value? (2 marks) **[2.4.2]**

Question 3 (4 marks) [2.1.3]

The graph of $f(x)$, where $x \in \mathbb{R}$, has the following properties.

- $f(0) = 3, f(-3) = 0, f(3) = 0$
- $f'(-2) = 0, f'(1) = 0, f'(3) = 0$
- $f'(x) < 0$ for $x \in (1, 3)$ and $f'(x) > 0$ for $x \in (-\infty, -2) \cup (-2, 1) \cup (3, \infty)$.

Sketch a possible graph of $y = f(x)$ on the axes below. Label any axial intercepts with their coordinates.



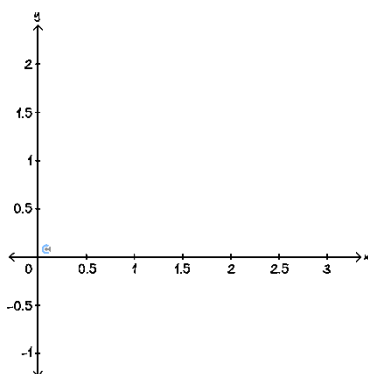
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Question 4 (6 marks)

A skateboard ramp has been built at Contour Park. The cross-section of the ramp is modelled by the function,

$$f: [0, 3] \rightarrow R, f(x) = \frac{1}{1 + 2x}$$

- a.** Sketch the curve of the ramp on the axes below. Label endpoints with coordinates. (2 marks)



- b.** The ramp is supported by a rail which touches the curve at one point and has a gradient of $-\frac{1}{2}$.

- i.** Find the coordinates of the point where the rail meets the ramp. (2 marks) [2.1.1]

- ii.** Find the equation that models the rail and sketch the rail on the axes given in **part a**. (2 marks) [2.4.1]

Question 5 (8 marks)

Consider the function $f(x) = x(x - 1)(3 - x)$.

- a.** Find the x -coordinates of any stationary points of f . (2 marks) **[2.1.2]**

- b.** State the nature of the stationary points with x -values as found in **part a**. (1 mark) **[2.1.2]**

- c. Find the equations of the two tangents to f that pass through the origin. (3 marks) [2.5.1]

- d. **Extension.** The acute angle θ , made by the two tangents to f that pass through the origin, in radians, is given by $\theta = p\pi - \tan^{-1}(q)$, where p is a positive rational number, and q is a positive integer.

Find the values of p and q . (2 marks) [2.5.1]

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Let $f(x) = (x+1)e^{2x}$. Let $f^{(n)}(x)$ denote the n^{th} derivative of f with respect to x , where $n \geq 1$.

Find a formula for $f^{(n)}(x)$ in terms of x and n .

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MM34 [0.15] - AOS 2 Revision - Workshop

Section C: Tech-Active Exam Skills

Calculator Commands: Finding Derivatives

➤ Mathematica

$$f'[x]$$

➤ TI

➤ Shift Minus

$$\frac{d}{dx}(f(x))$$

➤ Casio

➤ Math 2

$$\frac{d}{dx}(f(x))$$



Calculator Commands: Finding Second Derivatives

➤ Mathematica

$$f''[x]$$

$$= D[f[x], \{x, 2\}]$$

➤ TI

➤ Shift Minus

$$\frac{d^2}{dx^2}(f(x))$$

➤ Casio

➤ Math 2

$$\frac{d^2}{dx^2}(f(x))$$



Calculator Commands: Finding Tangents on CAS

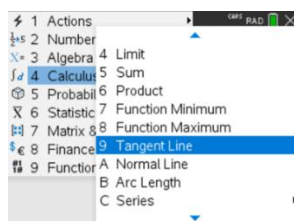
➤ Mathematica

<< SuiteTools`

TangentLine[f[x], x, a]

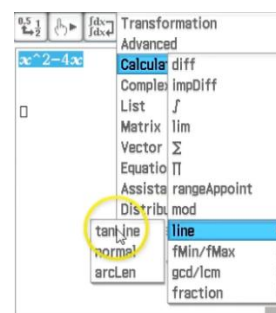
➤ TI-Nspire

➤ Menu 4 9



$$\text{tangentLine}(f(x), x, a)$$

➤ Casio Classpad



$$\text{tangentLine}(f(x), x, a)$$



Calculator Commands: Finding Normals on CAS

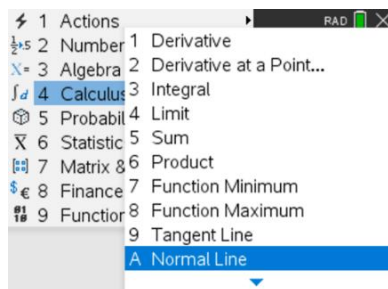
➤ Mathematica

<< SuiteTools`

`NormalLine[f[x], x, a]`

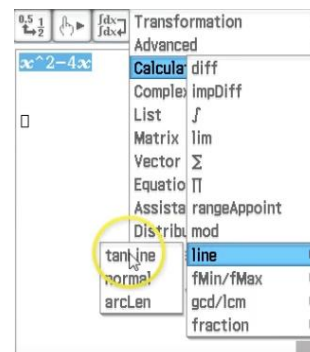
➤ TI-Nspire

Menu 4 A



`normalLine(f(x), x, a)`

➤ Casio Classpad



`normalLine(f(x), x, a)`

Calculator Commands: Finding Absolute Max and Min for $x \in [a, b]$

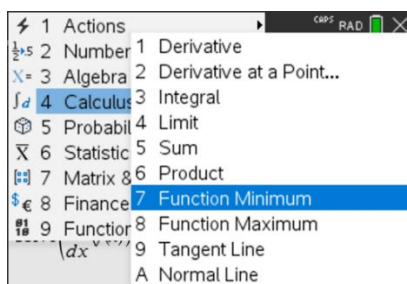
➤ Mathematica

`Maximize[{f[x], a ≤ x ≤ b}, x]`

`Minimize[{f[x], a ≤ x ≤ b}, x]`

➤ TI-Nspire

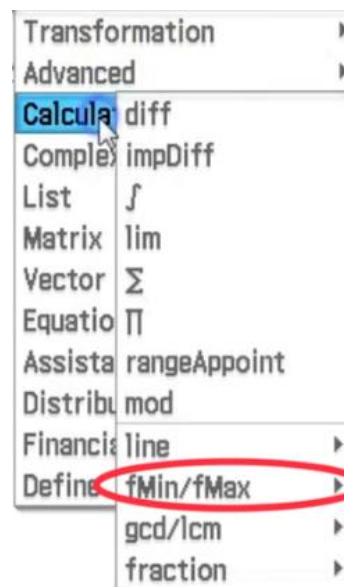
Menu 4 7 and Menu 4 8



`fMax(f(x), x, a, b)`

`fMin(f(x), x, a, b)`

➤ Casio Classpad



`fMax(f(x), x, a, b)`

`fMin(f(x), x, a, b)`



Calculator Commands: Newton's Method on Technology

➤ Consider finding a root to $f(x) = x^3 - 2$ with initial value $x_0 = 1$.

➤ Mathematica.

```
In[531]:= f[x_] := x^3 - 2
In[533]:= n[x_] := x - f[x]/f'[x]
In[534]:= n[x] // Together
Out[534]= 2 (1 + x^3) / (3 x^2)

In[537]:= For[i = 1; x = 1, i < 5, i++, x = 2 (1.0 + x^3) / (3 x^2); Print[x]]
1.33333
1.26389
1.25993
1.25992
```

➤ TI. Define the $n(x)$ function then keep iterating by putting your previous value back into $n(x)$.

Define $f(x)=x^3-2$	Done
$x \frac{f(x)}{\frac{d}{dx}(f(x))}$	$\frac{2 \cdot (x^3+1)}{3 \cdot x^2}$
Define $n(x)=\frac{2 \cdot (x^3+1)}{3 \cdot x^2}$	Done
$n(1)$	1.33333
$n(1.33333333333333)$	1.26389
$n(1.26388888888889)$	1.25993

➤ Classpad.

Use the same method as TI. OR, under Sequences.

Recursive	Explicit
<input checked="" type="checkbox"/> $a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$	
$a_0 = 1$	
<input type="checkbox"/> $b_{n+1} = \square$	
$b_0 = 0$	
<input type="checkbox"/> $c_{n+1} = \square$	
$c_0 = 0$	
n	a_n
1	1.3333
2	1.2639
3	1.2599
4	1.2599
5	1.2599



Calculator Commands: Joining Smoothly

➤ Mathematica

```
f[x_] := One Function
g[x_] := Another Function
Solve[f[x value] == g[x value] && f'[x value] == g'[x value]]
```

➤ TI and Casio

Define each branch as $f(x)$ and $g(x)$.

TI: Define its derivative as $df(x)$ and $dg(x)$.

Casio: Define them as different names.

Solve $f(a) = g(a)$ and $df(a) = dg(a)$ simultaneously.



Calculator Commands: Using Sliders/Manipulate on CAS

➤ Mathematica

```
Manipulate[Plot[function, {x, xmin, xmax}],
{unknown, lowerbound, upperbound}]
```

NOTE: The function **must** be typed out instead of using its saved name.

➤ TI-Nspire

☐ $f1(x)=function\ with\ unknown$

Create Sliders

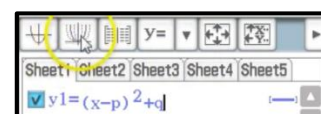
Create a slider for:

☒ unknown

OK Cancel

unknown = type any num
-5.00000 5.00000

➤ Casio Classpad



Space for Personal Notes



Calculator Commands: Stationary Point

- ALWAYS sketch the graph first to get an idea of the nature of the stationary point.
- The turning points for a function $f(x)$ can be found by solving $f'(x) = 0$ and subbing the result into f .
- **Example:** Find the turning point for $f(x) = e^{-x^2+2x}$.
- TI:

Define $f(x) = e^{-x^2+2 \cdot x}$ Done

solve $\left(\frac{d}{dx}(f(x)) = 0, x \right)$ $x=1$

$f(1)$ e

- Casio:

define f(x) = e^{-x^2+2x}	done
solve($\frac{d}{dx}(f(x))=0, x$)	$\{x=1\}$
f(1)	e

- Mathematica:

In[4]:= `f[x_] := Exp[-x^2 + 2 x]`

In[5]:= `Solve[f'[x] == 0 && y == f[x], Reals]`

Out[5]= `{{x -> 1, y -> e}}`



Calculator Commands: Finding Tangents/Normals Which Pass Through a Point

- Suppose we want to find the equation of a tangent/normal to the graph of $f(x)$ that passes through the point $P(x_1, y_1)$.
- Steps:
 1. Find the equation of the tangent to $f(x)$ at arbitrary point $x = a$.
 2. Let this tangent line be $t(x)$.
 3. Solve the equation $t(x_1) = y_1$ to find possible value(s) of a .
 4. Find the equation of the tangent at $x = a$.
- A similar procedure for the normal line.
- **Example:** Find the equation of a tangent to $f(x) = x^3 - 2x$ that passes through the point $(0, 2)$.

```

In[564]:= f[x_] := x^3 - 2 x
In[565]:= TangentLine[f[x], {x, a}]
Out[565]= -2 a^3 + (-2 + 3 a^2) x

In[566]:= t[x_] := -2 a^3 + (-2 + 3 a^2) x
In[568]:= Solve[t[0] == 2, a, Reals]
Out[568]= {{a -> -1}}

In[570]:= t[x] /. a -> -1
Out[570]= 2 + x

In[571]:= TangentLine[f[x], {x, -1}]
Out[571]= 2 + x
  
```

Space for Personal Notes

Section D: Exam 2 Questions (33 Marks)

INSTRUCTION:

- **Regular: 30 Marks. 5 Minutes Reading. 40 Minutes Writing.**
- **Extension: 34 Marks. 5 Minutes Reading. 36 Minutes Writing.**



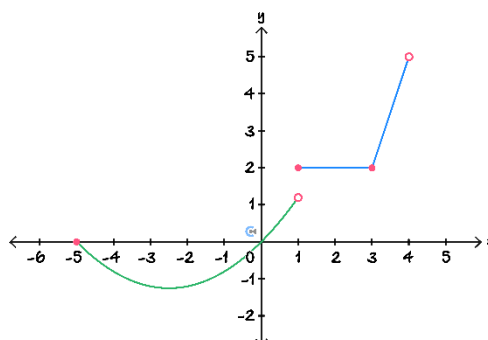
Question 7 (1 mark) [2.3.1]

The derivative of $g(x)e^{2x}$, with respect to x is:

- A. $2e^{2x}(g(x) + g'(x))$
- B. $e^{2x}(2g(x) + g'(x))$
- C. $g'(x)e^{2x}$
- D. $xg'(x) + e^{2x}g(x)$

Question 8 (1 mark) [2.2.2]

The graph of the hybrid function $y = h(x)$ is shown below.



Hence, $h(x)$ is:

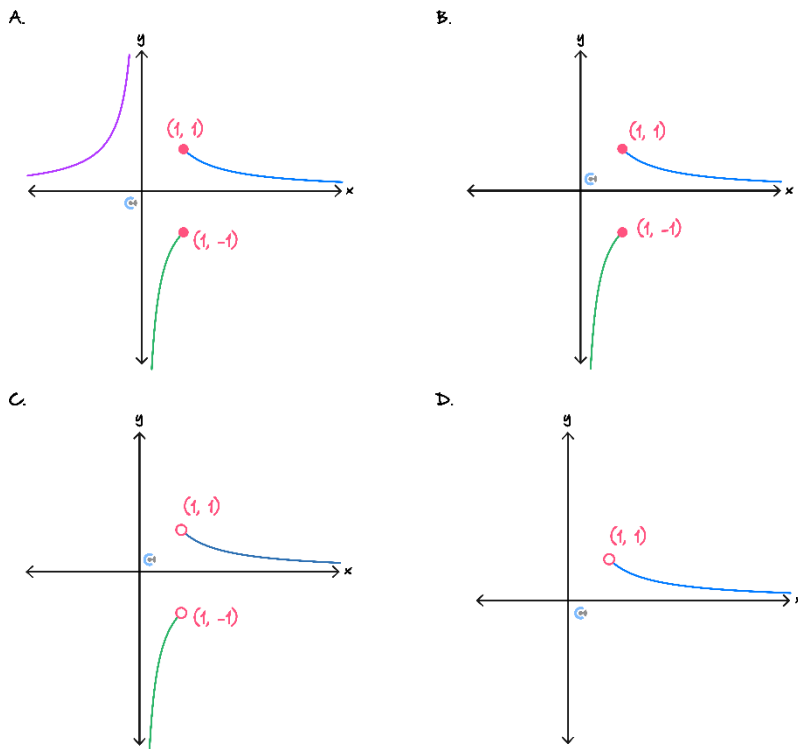
- A. Not differentiable at $x = 1$ and $x = 4$ but is differentiable at $x = -5$ and $x = 3$.
- B. Not differentiable at $x = 1$, $x = 3$ and $x = 4$ but is differentiable at $x = -5$.
- C. Not differentiable at $x = -5$, $x = 1$ and $x = 4$ but is differentiable at $x = 3$.
- D. Not differentiable at $x = -5$, $x = 1$, $x = 3$ and $x = 4$.

Question 9 (1 mark) [2.1.3]

Consider the hybrid function:

$$f(x) = \begin{cases} \log_e(x), & x > 1 \\ -\log_e(x), & x < 1 \end{cases}$$

The graph of $f'(x)$ could be :



Question 10 (1 mark) [2.1.1]

Consider the hybrid function:

$$h(x) = \begin{cases} \frac{\sin(3x)}{x}, & x \neq 0 \\ 3, & x = 0 \end{cases}$$

Which of the following statements is true about the continuity and differentiability of $h(x)$ at $x = 0$?

- A. $h(x)$ is continuous but not differentiable at $x = 0$.
- B. $h(x)$ is continuous and differentiable at $x = 0$.
- C. $h(x)$ is not continuous at $x = 0$.
- D. $h(x)$ is not continuous but is differentiable at $x = 0$.

Question 11 (1 mark)

Let $f(x) = e^{-b^2x^2}$, where $b > 0$. $f(x)$ is concave down for:

- A. $-b < x < b$
- B. $-\frac{1}{\sqrt{b}} < x < \frac{1}{\sqrt{b}}$
- C. $-\frac{1}{\sqrt{2b}} < x < \frac{1}{\sqrt{2b}}$
- D. $-\sqrt{b} < x < \sqrt{b}$

Question 12 (1 mark)

Consider the hybrid function:

$$h(x) = \begin{cases} -ax + 3, & x \leq 1 \\ x^2 - bx + 4, & x > 1 \end{cases}$$

Where $a, b \in \mathbb{R}$, h is smooth continuous for all $x \in \mathbb{R}$ if:

- A. $a = 2$ and $b = 4$
- B. $a = -2$ and $b = 4$
- C. $a = -2$ and $b = -4$
- D. $a = 1$ and $b = 4$

Question 13 (1 mark) [2.6.2]

Let $a \in \mathbb{R}$. The graphs of $y = e^{-x} + a$ and $y = 2 - x$, intersect exactly once when:

- A. $a = -2$
- B. $a = -1$
- C. $a = 1$
- D. $a = 2$

Question 14 (1 mark) [2.6.3]

The function $f(x) = x^3 \log_e(x - k)$ has a local minimum when $x = -2$. The value of k is closest to:

- A. -3.12
- B. -3.54
- C. -2.53
- D. -3.71

Question 15 (1 mark) [2.7.1] [2.7.2]

An implementation of Newton's method is shown below.

```

define newton( $f(x), x_0, n, tol$ ) :
     $df(x) \leftarrow$  the derivative of  $f(x)$ 
     $i \leftarrow 0$ 
     $x_n \leftarrow x_0$ 
    while  $i < n$  do
        if  $df(x_n) = 0$  then
            return "Error: Division by zero"
        end if
         $x_{n+1} \leftarrow x_n - \frac{f(x_n)}{df(x_n)}$ 
        if  $-tol < x_{n+1} - x_n < tol$  then
            return  $x_{n+1}$ 
        end if
         $x_n \leftarrow x_{n+1}$ 
         $i \leftarrow i + 1$ 
    end while
    return  $x_n$ 

```

What is the value of i when the function $\text{Newton}(x^2 - 5, 5, 10, 0.001)$ finishes running?

- A. 2
- B. 3
- C. 4
- D. 5

Question 16 (1 mark) [2.1.3]

Consider the polynomial function that is continuous and smooth for all $x \in \mathbb{R}$ and has the following features:

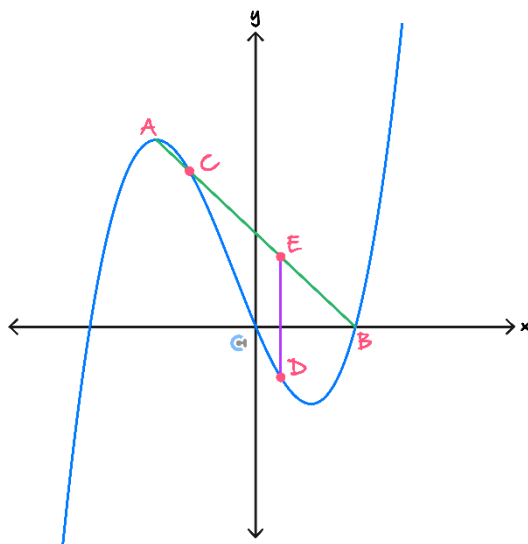
- ▶ $f'(x) = 0, x \in \{2, 7, 10\}$
- ▶ $f'(x) < 0, x \in (-\infty, 2) \cup (2, 7) \cup (10, \infty)$
- ▶ $f'(x) > 0, x \in (7, 10)$

Which of the following statements is true about $f(x)$?

- A. $f(x)$ has a stationary point of inflection at $x = 2$, a local maximum at $x = 7$, and a local minimum at $x = 10$.
- B. $f(x)$ has x -intercepts at $x = 2, x = 7$, and $x = 10$.
- C. $f(x)$ has a stationary point of inflection at $x = 2$, a local minimum at $x = 7$, and a local maximum at $x = 10$.
- D. $f(x)$ has x -intercepts at $x = 2$ and $x = 7$, and a local maximum between $x = 7$ and $x = 10$.

Question 17 (14 marks)

The graph of $f(x) = \frac{9x^3}{16} + \frac{3x^2}{4} - \frac{15x}{4}$ is shown below:



- a. Write down the coordinates of point B. (1 mark)

- b.** Write down the coordinates of the turning point at point A . (1 mark) [2.1.2]

- c.** Let L be the line joining points A and B .

- i.** Show that the equation of the line L is $2y + 3x = 6$. (2 marks)

- ii.** The line L passes through the graph of $f(x)$ at point C . Write down the coordinates of point C . (1 mark)

- iii.** Find the distance AC . (1 mark)

- iv. Extension.** The line L undergoes a transformation T such that it still passes through the point B , but is now tangent to the graph of f at a point P , where P has an x -coordinate less than zero. Give the coordinates of point P and describe the transformation T . (3 marks)

The vertical line segment DE joins the graph of $f(x)$ and the line joining points A and B . We wish to maximise the length of the line segment DE .

- d.** Write down an expression for the length DE in terms of x . (1 mark) [2.5.2]

- e.** Determine the value of x , correct to two decimal places, for which the length DE is a maximum and determine the maximum length of the line segment DE , correct to two decimal places. (You do not have to verify that this value gives the maximum length for the line DE). (2 marks) [2.5.2]

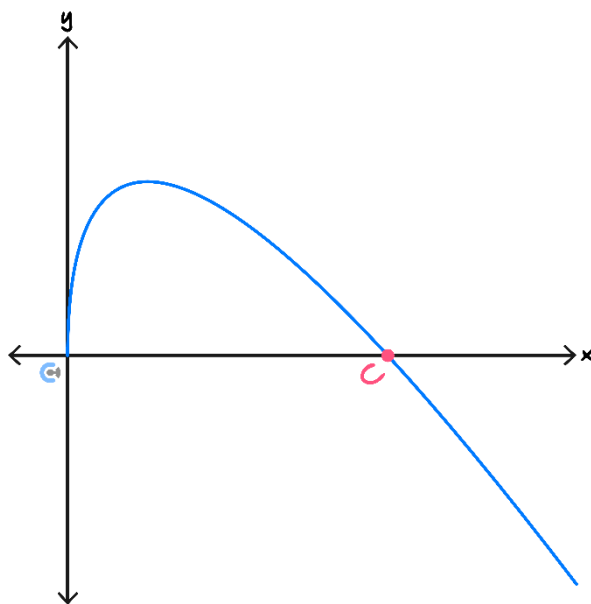
- f. Newton's method is used to find a solution to the equation $f(x) = 2$, with $x_0 = 3$. Complete the table below for the values of x_1, x_2 and x_3 . Give your answers correct to three decimal places. (2 marks) [2.4.3]

x_0	3
x_1	
x_2	
x_3	

Question 18 (10 marks)

Consider the family of functions $f_a: [0, \infty) \rightarrow \mathbb{R}$ which is defined by $f_a(x) = 6a\sqrt{x} - x$, where a is a real number and $a > 0$.

Part of the graph of f_a is shown below.



- a. Find c in terms of a , where $f_a(c) = 0$ and $c \neq 0$. (2 marks) [2.6.1]

- b.** Determine the interval over which f_a is strictly decreasing. (2 marks) [2.1.2]

- c.** Show that the equation of the tangent to the graph f_a at the point $(c, 0)$ is : (2 marks) [2.4.1]

$$y = -\frac{1}{2}x + 18a^2$$

- d.** The function $f_a(x)$ is transformed to form $g_a(x)$, where $g_a(x)$ is defined as

$$g_a(x) = f_a(x) - b$$

Find the value of b , in terms of a , such that the tangent drawn to the curve of $g_a(x)$ at $x = c$ passes through the origin. (2 marks) [2.6.1]

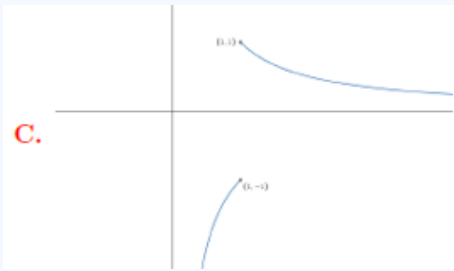
Let $h_a: [0, d] \rightarrow \mathbb{R}$, $h_a(x) = f(x)$, where d is chosen as large as possible and such that h_a is a one-to-one function and $a > 0$.

- e. State the coordinates of any points of intersection between h_a and its inverse function h_a^{-1} . Give your answer in terms of a where appropriate. (1 mark)

- f. Find the angle made by tangents to h_a and h_a^{-1} at points where the respective curves intersect each other. (1 mark) [2.5.1]

Space for Personal Notes

Section E: Tech-Active Solutions - Mathematica

Question Number	Solutions
7	$e^{2x}(2g(x) + g'(x))$ <pre> In[4]:= D[g[x]*Exp[2 x], x] // Factor Out[4]= e^{2 x} (2 g[x] + g'[x]) </pre>
8	<p>Not differentiable at $x = -5$, $x = 1$, $x = 3$ and $x = 3$.</p> <p>Not differentiable at endpoints/points of discontinuity.</p>
9	
10	<p>$h(x)$ is continuous and differentiable at $x = 0$.</p> <pre> In[5]:= Limit[Sin[3 x]/x, x -> 0] Out[5]= 3 In[6]:= Limit[D[Sin[3 x]/x, x], x -> 0] Out[6]= 0 </pre>
11	$-\frac{1}{\sqrt{2b}} < x < \frac{1}{\sqrt{2b}}$ <pre> In[61]:= f[x_] := Exp[-b^2 x^2] In[62]:= Solve[f'[x] == 0] Out[62]:= {{b -> 0}, {x -> -1/(sqrt(2) b)}, {x -> 1/(sqrt(2) b)}} In[63]:= Reduce[f''[x] < 0] Out[63]:= (b ∈ ℝ && b ≠ 0 && x == 0) (b ∈ ℝ && b ≠ 0 && -sqrt(1/b^2)/sqrt(2) < x < sqrt(1/b^2)/sqrt(2)) </pre>

12	$a = 2 \text{ and } b = 4$ <pre>In[80]:= h1[x_] := -a x + 3 In[81]:= h2[x_] := x^2 - b x + 4 In[82]:= Solve[h1[1] == h2[1] && h1'[1] == h2'[1]] Out[82]= {{b -> 2 + a}}</pre>																								
13	$a = 1$ <pre>In[86]:= Solve[f[x] == g[x] && f'[x] == g'[x], Reals] Out[86]= {{a -> 1, x -> 0}}</pre>																								
14	-3.54 <pre>In[90]:= f[x_] := x^3 Log[x - k] In[91]:= Solve[f'[-2] == 0, k] // N Out[91]= {{k -> -3.5412}}</pre>																								
15	4 The algorithm terminates in its 4th iteration. $i = 4$ Mathematica code used found here: https://pastebin.com/SXKL89qE <pre>In[106]:= NewtonMethod[x^2 - 5, 5, 10, 0.001]</pre> <table><thead><tr><th>Iteration</th><th>x_n</th><th>x_{n-1}</th><th>$x_{n-1} - x_n$</th></tr></thead><tbody><tr><td>0</td><td>5.000000</td><td>3.000000</td><td>2.000000</td></tr><tr><td>1</td><td>3.000000</td><td>2.333333</td><td>0.666667</td></tr><tr><td>2</td><td>2.333333</td><td>2.238095</td><td>0.095238</td></tr><tr><td>3</td><td>2.238095</td><td>2.23607</td><td>0.00203</td></tr><tr><td>4</td><td>2.23607</td><td>2.23607</td><td>$0. \times 10^{-6}$</td></tr></tbody></table>	Iteration	x_n	x_{n-1}	$ x_{n-1} - x_n $	0	5.000000	3.000000	2.000000	1	3.000000	2.333333	0.666667	2	2.333333	2.238095	0.095238	3	2.238095	2.23607	0.00203	4	2.23607	2.23607	$0. \times 10^{-6}$
Iteration	x_n	x_{n-1}	$ x_{n-1} - x_n $																						
0	5.000000	3.000000	2.000000																						
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3	2.238095	2.23607	0.00203																						
4	2.23607	2.23607	$0. \times 10^{-6}$																						
16	<p>$f(x)$ has a stationary point of inflection at $x = 2$, a local minimum at $x = 7$, and a local maximum at $x = 10$.</p> <p>Stationary point at $x = 2$ but sign of derivative does not change, hence stationary point of inflection. Stationary point at $x = 7$ and derivative sign changes from negative to positive therefore local minimum. Stationary point at $x = 10$ and derivative sign changes from positive to negative hence local maximum.</p>																								

17 (a)	$(2, 0)$ <pre>In[111]:= f[x_] := 9 x^3 / 16 + 3 x^2 / 4 - 15 x / 4 In[112]:= Solve[f[x] == 0, x] Out[112]= {{x -> -10/3}, {x -> 0}, {x -> 2}}</pre>
17 (b)	$(-2, 6)$ <pre>In[114]:= Solve[f'[x] == 0 && y == f[x]] Out[114]= {{x -> -2, y -> 6}, {x -> 10/9, y -> -200/81}}</pre>
17 (c)(i)	<p>Gradient = $-\frac{6}{4} = -\frac{3}{2}$ (1M) Line passes through $(2, 0)$. Thus</p> $y - 0 = -\frac{3}{2}(x - 2)$ $\implies 2y + 3x = 6 \quad (1A)$
17(c)(ii)	$\left(-\frac{4}{3}, 5\right)$ <pre>In[116]:= Solve[c[x] == f[x] && y == f[x]] Out[116]= {{x -> -2, y -> 6}, {x -> -4/3, y -> 5}, {x -> 2, y -> 0}}</pre>
17(c)(iii)	$\frac{\sqrt{13}}{3}$ <pre>In[117]:= EuclideanDistance[{-4/3, 5}, {-2, 6}] Out[117]= sqrt(13)/3</pre>

17(c)(iv)

Since the point B is fixed, the transformation that we look for should be a dilation by factor k from the x -axis.

We find that the tangent to f when $x = -\frac{5}{3}$ passes through B (1M).

So P has coordinates $\left(-\frac{5}{3}, \frac{275}{48}\right)$ (1A)

The transformation that L undergoes is a dilation by factor $\frac{25}{24}$ from the x -axis. (1A)

```

In[25]:= TangentLine[f[x], x, a]
Out[25]:= -3/8 a^2 (2 + 3 a) + (-15/4 + 3/2 a + 27/16 a^2) x

In[26]:= -3/8 a^2 (2 + 3 a) + (-15/4 + 3/2 a + 27/16 a^2) x /. x -> 2
Out[26]:= -3/8 a^2 (2 + 3 a) + 2 (-15/4 + 3/2 a + 27/16 a^2)

In[27]:= Solve[-3/8 a^2 (2 + 3 a) + 2 (-15/4 + 3/2 a + 27/16 a^2) == 0, a]
Out[27]:= {{a -> -5/3}, {a -> 2}, {a -> 2}}

In[28]:= f[-5/3]
Out[28]:= 275/48

In[29]:= m = -275/48 / (-5/3 + 2)
Out[29]:= -25/16

In[37]:= -25/16 / (-3/2)
Out[37]:= 25/24

```

17(d)

$$\left(3 - \frac{3x}{2}\right) - f(x) = -\frac{9x^3}{16} - \frac{3x^2}{4} + \frac{9x}{4} + 3$$

```
In[118]:= c[x] - f[x]
```

```
Out[118]:= 3 + 9 x / 4 - 3 x^2 / 4 - 9 x^3 / 16
```

17(e)

$$x \approx 0.79 \text{ (1A)}$$

$$\text{Max distance} \approx 4.03 \text{ (1A)}$$

```
In[119]:= Maximize[{c[x] - f[x], -2 <= x <= 2}, x] // N
```

```
Out[119]:= {4.03211, {x -> 0.792837}}
```

17(f)

x_0	0
x_1	2.455
x_2	2.293
x_3	2.278


	<pre> In[183]:= g[x_] := f[x] - 2 In[184]:= n[x_] := x - $\frac{g[x]}{g'[x]}$ In[185]:= n[3.0] Out[185]= 2.4549 In[186]:= n[n[3.0]] Out[186]= 2.29296 In[187]:= n[n[n[3.0]]] Out[187]= 2.27825 </pre>
18(a)	<p>We must solve $6a\sqrt{c} - c = 0$ (1M) $c = 36a^2$ (1A)</p> <pre> In[31]:= f[x_] := 6 a \sqrt{x} - x In[32]:= Solve[f[c] == 0, c] </pre> <p>... Solve: There may be values of the parameters for which some or all solutions are not valid.</p> <pre> Out[32]= {{c -> 0}, {c -> 36 a^2}} </pre>
18(b)	<p>We find the turning point. Solve $f'_a(x) = \frac{3a}{\sqrt{x}} - 1 = 0$ (1M) $\Rightarrow x = 9a^2$. Therefore strictly decreasing on $[9a^2, \infty)$ (1A)</p> <pre> In[33]:= f'[x] Out[33]= $-1 + \frac{3a}{\sqrt{x}}$ In[34]:= Solve[f'[x] == 0, x] </pre> <p>... Solve: There may be values of the parameters for which some or all solutions are not valid.</p> <pre> Out[34]= {{x -> 9 a^2}} </pre>

18(c)	$f'_a(c) = \frac{3a}{\sqrt{c}} - 1 = \frac{3a}{6a} - 1 = -\frac{1}{2} \text{ (1M).}$ <p>Tangent passes through $(36a^2, 0)$ and with gradient $-\frac{1}{2}$.</p> <p>Thus $y = -\frac{1}{2}(x - 36a^2) = -\frac{1}{2}x + 18a^2$ (1A)</p> <pre> In[87]:= TangentLine[f[x], x, 36 a^2] // Expand Out[87]= 18 a \sqrt{a^2} - x + \frac{\sqrt{a^2} x}{2 a} In[88]:= Assuming[a > 0, Refine[18 a \sqrt{a^2} - x + \frac{\sqrt{a^2} x}{2 a}]] Out[88]= 18 a^2 - \frac{x}{2} </pre>
18(d)	$g'_a(c) = f'_a(c) = -\frac{1}{2}.$ $g_a(c) = f_a(c) - b = -b.$ <p>The tangent is $y + b = -\frac{1}{2}x + 18a^2$ (1M)</p> <p>To pass through $(0, 0)$ must have $b = 18a^2$. (1A)</p> <pre> In[94]:= Assuming[a > 0, Refine[TangentLine[f[x] - b, x, 36 a^2]]] // Expand Out[94]= 18 a^2 - b - \frac{x}{2} In[96]:= Solve[0 == 18 a^2 - b - \frac{x}{2} /. x -> 0, b] Out[96]= {{b -> 18 a^2}} </pre>
18(e)	$d = 9a^2$ <p>$(0, 0)$ and $(9a^2, 9a^2)$ (1A)</p>
18(f)	<p>The tangents are vertical/horizontal lines. Thus 90° or $\frac{\pi}{2}$ (1A)</p>

Section F: Tech-Active Solutions - Casio

Question Number	Solutions
7	$\frac{d}{dx} (g(x) \times e^{2x})$ $\frac{d}{dx} (g(x)) \cdot e^{2 \cdot x} + 2 \cdot e^{2 \cdot x} \cdot g(x)$ <p>factor (ans)</p> $\left(\frac{d}{dx} (g(x)) + 2 \cdot g(x) \right) \cdot e^{2 \cdot x}$
8	Not differentiable at $x = -5, x = 1, x = 3$ and $x = 3$.
9	C
10	<p>$h(x)$ is continuous and differentiable at $x = 0$.</p> $\lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{x} \right)$ <p style="text-align: right;">3</p> $\lim_{x \rightarrow 0} \left(\frac{d}{dx} \left(\frac{\sin(3x)}{x} \right) \right)$ <p style="text-align: right;">0</p>
11	$\text{solve} \left(\frac{d^2}{dx^2} (e^{-b^2 x x^2}) = 0, x \right)$ $\left\{ x = \frac{-\sqrt{2}}{2 \cdot b}, x = \frac{\sqrt{2}}{2 \cdot b} \right\}$ $\text{solve} \left(\frac{d^2}{dx^2} (e^{-b^2 x x^2}) < 0 \mid b > 0 \right)$ $\left\{ (4 \cdot b^4 \cdot x^2 - 2 \cdot b^2) \cdot e^{-b^2 \cdot x^2} < 0 \right\}$ <p>Inequality does not work :(</p>

12	<pre> Define f(x)=-a*x+3 done Define g(x)=x^2-b*x+4 done Define m(x)=d/dx(f(x)) done Define n(x)=d/dx(g(x)) done {f(1)=g(1) m(1)=n(1) a, b {a=b-2, b=b} </pre>
13	<pre> Define f(x)=e^-x+a done Define g(x)=2-x done {f(x)=g(x) d/dx(f(x))=d/dx(g(x)) x, a {x=0, a=1} </pre>
14	<pre> -3.54 ----- solve(d/dx(f(x))=0 x=-2, k) {k=-3.541202191} </pre>
15	<pre> Define f(x)=x^2-5 done Define n(x)=x-d/dx(f(x)) done n(5) 3 n(3) 2.333333333 n(2.333333333) 2.238095238 n(2.238095238) 2.236068896 </pre>

16	<p>$f(x)$ has a stationary point of inflection at $x = 2$, a local minimum at $x = 7$, and a local maximum at $x = 10$.</p> <p>Stationary point at $x = 2$ but sign of derivative does not change, hence stationary point of inflection. Stationary point at $x = 7$ and derivative sign changes from negative to positive therefore local minimum. Stationary point at $x = 10$ and derivative sign changes from positive to negative hence local maximum.</p>
17 (a)	<p>Define $f(x) = 9x^3/16 + 3x^2/4$ </p> <p>done</p> <p>solve $f(x) = 0, x$</p> $\left\{ x=0, x=2, x=-\frac{10}{3} \right\}$
17 (b)	$\left\{ \begin{array}{l} \frac{d}{dx}(f(x)) = 0 \\ y = f(x) \end{array} \right _{x,y}$ $\left\{ \{x=-2, y=6\}, \left\{ x=\frac{10}{9}, y=-\frac{20}{81} \right\} \right\}$
17 (c)(i)	<p>Gradient $= -\frac{6}{4} = -\frac{3}{2}$ (1M) Line passes through $(2, 0)$. Thus</p> $y - 0 = -\frac{3}{2}(x - 2)$ $\Rightarrow 2y + 3x = 6 \quad (1A)$
17(c)(ii)	$\left\{ \begin{array}{l} 2y+3x=6 \\ y=f(x) \end{array} \right _{x,y}$ $\left\{ \{x=2, y=0\}, \left\{ x=-\frac{4}{3}, y=5 \right\} \right\}$

17(c)(iii)	$\sqrt{(-2 - (-4/3))^2 + (6-5)^2}$ $\frac{\sqrt{13}}{3}$
17(c)(iv)	<pre>tanLine(f(x), x, a) 9*a^3+x*(27*a^2+3*a-15)-a solve(ans=0 x=2, a) {a=2, a=-5/3} f(-5/3) 275/48 m=-275/48 / (5/3+2) m=-25/16 -25/16 / -3/2 25/24</pre>
17(d)	<pre>Define c(x)=3-3x/2 done c(x)-f(x) -9*x^3/16-3*x^2/4+9*x/4+3</pre>
17(e)	<pre>fMax(c(x)-f(x), x, -2, 2) {MaxValue=4.032107349, x=0.7928365251}</pre>
17(f)	<pre>Define n(x)=x-(f(x)-2)/(d/dx(f(x)-2)) done n(3) 2.454901961 n(2.454901961) 2.292958821 n(2.292958821) 2.278251116</pre>

18(a)	<p>Define $f(x) = 6a\sqrt{x} - x$</p> <p>done</p> <p>solve($f(c) = 0, c$)</p> <p>$\{c = 0, c = 36 \cdot a^2\}$</p> <p>-</p>
18(b)	<p>solve($\frac{d}{dx}(f(x)) = 0, x$)</p> <p>$\{x = 9 \cdot a^2\}$</p>
18(c)	<p>tanLine($f(x), x, 36a^2$)</p> <p>$-36 \cdot a^2 \cdot \left(\frac{a}{2 \cdot a } - 1\right) - 36 \cdot a^2 + 36 \cdot a^2$</p> <p>ans $a > 0$</p> <p>$18 \cdot a^2 - \frac{x}{2}$</p>
18(d)	<p>tanLine($f(x) - b, x, 36a^2$)</p> <p>$-36 \cdot a^2 \cdot \left(\frac{a}{2 \cdot a } - 1\right) - 36 \cdot a^2 + 36 \cdot a^2$</p> <p>ans $a > 0$</p> <p>$18 \cdot a^2 - \frac{x}{2} - b$</p> <p>solve($ans = 0 \mid x = 0, b$)</p> <p>$\{b = 18 \cdot a^2\}$</p>
18(e)	<p>$d = 9a^2$</p> <p>$(0, 0)$ and $(9a^2, 9a^2)$ (1A)</p>
18(f)	<p>The tangents are vertical/horizontal lines. Thus 90° or $\frac{\pi}{2}$ (1A)</p>

Section G: Tech-Active Solutions - TI

Question Number	Solutions
7	$e^{2x}(2g(x) + g'(x))$ <p>©Shortcut: [shift][−] for derivative</p> $\text{factor}\left(\frac{d}{dx}(g(x) \cdot e^{2 \cdot x})\right)$ $\left(\frac{d}{dx}(g(x)) + 2 \cdot g(x)\right) \cdot e^{2 \cdot x}$
8	<p>Not differentiable at $x = -5, x = 1, x = 3$ and $x = 3$.</p> <p>Not differentiable at endpoints/points of discontinuity.</p>
9	C
10	<p>$h(x)$ is continuous and differentiable at $x = 0$.</p> <p>©Shortcut: [menu][4][4] for limit</p> <p>Define $g(x) = \frac{\sin(3 \cdot x)}{x}$ Done</p> <p>Define $dg(x) = \frac{d}{dx}(g(x))$ Done</p> <p>$\lim_{x \rightarrow 0} (g(x))$ 3</p> <p>$\lim_{x \rightarrow 0} (dg(x))$ 0</p>

11	<div>$-\frac{1}{\sqrt{2b}} < x < \frac{1}{\sqrt{2b}}$</div> <div>©Tip: Use template button under [del] key for second derivative template.</div> <div>Define $f(x)=e^{-b^2 \cdot x^2}$ Done</div> <div>$\text{solve}\left(\frac{d^2}{dx^2}(f(x)) < 0, x\right) b > 0$</div> <div>$\frac{-\sqrt{2}}{2 \cdot b} < x < \frac{\sqrt{2}}{2 \cdot b} \text{ and } b > 0$</div>									
12	<div>$a = 2 \text{ and } b = 4$</div> <div>$\text{\textbackslash solve_smooth}(-a \cdot x + 3, x^2 - b \cdot x + 4, x, 1, \{a, b\})$</div> <div><div>▶ Left Derivative: $-a$</div><div>▶ Right Derivative: $2 \cdot x - b$</div><div><table><tr><td>"At x=1:"</td><td>"Left Func."</td><td>"Right Func."</td></tr><tr><td>"Value:"</td><td>$3 - a$</td><td>$5 - b$</td></tr><tr><td>"Gradient:"</td><td>$-a$</td><td>$2 - b$</td></tr></table></div><div>▶ Solutions: $a = \mathbf{c1} - 2$ and $b = \mathbf{c1}$</div></div>	"At x=1:"	"Left Func."	"Right Func."	"Value:"	$3 - a$	$5 - b$	"Gradient:"	$-a$	$2 - b$
"At x=1:"	"Left Func."	"Right Func."								
"Value:"	$3 - a$	$5 - b$								
"Gradient:"	$-a$	$2 - b$								
13	<div>$a = 1$</div> <div>$\text{methods_diffcalc}\text{\textbackslash solve_touch}(e^{-x} + a, 2 - x, x, a)$</div> <div><div>▶ Derivative 1: $-e^{-x}$</div><div>▶ Derivative 2: -1</div><div>▶ Equating functions and derivatives.</div><div>▶ Solutions: $x = 0$ and $a = 1$</div></div>									

14


-3.54

Define $f(x) = x^3 \cdot \ln(x-k)$

Done

Define $df(x) = \frac{d}{dx}(f(x))$

Done

 solve($df(-2)=0, k$)

$k = -3.5412$

15

4

Number of Iterations:

OK

Cancel

methods_diffcalc\newtons_method($x^2-5, x, 5$)

► Derivative: $2 \cdot x$

► Iterative Formula: $\frac{x^2+5}{2 \cdot x}$

► Number of Iterations: 10

"n"	"xn"	"f(xn)"	"f'(xn)"
0.	5.	20.	10.
1.	3.	4.	6.
2.	2.33333	0.444444	4.66667
3.	2.2381	0.00907	4.47619
4.	2.23607	0.000004	4.47214
5.	2.23607	9.E-13	4.47214
6.	2.23607	0.	4.47214
7.	2.23607	0.	4.47214

Note that on the 4th step, the program computes the 5th x-value and compares to the 4th x-value. Since this difference is less than the tolerance, the program terminates on the 4th step.

16

$f(x)$ has a stationary point of inflection at $x = 2$, a local minimum at $x = 7$, and a local maximum at $x = 10$.

Stationary point at $x = 2$ but sign of derivative does not change, hence stationary point of inflection.

Stationary point at $x = 7$ and derivative sign changes from negative to positive therefore local minimum.

Stationary point at $x = 10$ and derivative sign changes from positive to negative hence local maximum.

17 (a)

$$\text{Define } f(x) = \frac{9 \cdot x^3}{16} + \frac{3 \cdot x^2}{4} - \frac{15 \cdot x}{4} \quad \text{Done}$$

`methods_func\analyse(f(x),x)`

► Start Point: $[-\infty \quad -\infty]$

► End Point: $[\infty \quad \infty]$

► Maximal Domain: $-\infty < x < \infty$

► x -Intercepts: (3)

$\left[\begin{array}{cc} -10 & 0 \end{array} \right], \left[\begin{array}{cc} 0 & 0 \end{array} \right],$

$\left[\begin{array}{cc} 2 & 0 \end{array} \right]$

► Vertical Intercept: $\left[\begin{array}{cc} 0 & 0 \end{array} \right]$

17 (b)

Note: Analyse program from above continued...

► Derivative:

$$\frac{27 \cdot x^2}{16} + \frac{3 \cdot x}{2} - \frac{15}{4}$$

► Inflection Point:

$$\left[\frac{-4}{9} \quad \frac{143}{81} \right] \text{ (Decreasing)}$$

► Stationary Points: (2)

$$[-2 \quad 6] \text{ (Local max.)}$$

$$\left[\frac{10}{9} \quad \frac{-200}{81} \right] \text{ (Local min.)}$$

17 (c)(i)

Define $a = [-2 \quad 6]$

Done

Define $b = [2 \quad 0]$

Done

methods_misc *Vlinear_info*(a, b)

► Point 1: $[-2 \quad 6]$

► Point 2: $[2 \quad 0]$

► Midpoint: $[0 \quad 3]$

► Distance: $2 \cdot \sqrt{13}$

► Gradient: $\frac{-3}{2}$

► Perp. Bisector: $y = \frac{2 \cdot x}{3} + 3$

► Linear Equation: $y = 3 - \frac{3 \cdot x}{2}$

► x-Intercept: $[2 \quad 0]$

► y-Intercept: $[0 \quad 3]$

17(c)(ii)

Note: We use the linear equation output from the linear_info program above. To quickly define the linear function, start by typing Define l(x)=. Then to up to the linear_info program and highlight the desired part. This can be done by holding shift while pressing the arrow keys. Pressing enter will copy the highlighted selection down to the current line.

$$\text{Define } l(x) = 3 - \frac{3 \cdot x}{2} \quad \text{Done}$$

```
methods_func\intersect(f(x),l(x),x)
```

► Intersection Points: (3)

$$\begin{bmatrix} -2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{4}{3} & 5 \end{bmatrix} |$$

$$\begin{bmatrix} 2 & 0 \end{bmatrix}$$

17(c)(iii)

Note: To quickly define the point, start by typing Define c=, then go up to the output of the intersect program and highlight the given point. This can be done by holding shift while pressing the arrow keys. Pressing enter will copy the highlighted selection down to the current line.

$$\text{Define } c = \begin{bmatrix} -\frac{4}{3} & 5 \end{bmatrix} \quad \text{Done}$$

```
methods_misc\linear_info(a,c)
```

► Point 1: $\begin{bmatrix} -2 & 6 \end{bmatrix}$

► Point 2: $\begin{bmatrix} -\frac{4}{3} & 5 \end{bmatrix} |$

► Midpoint: $\begin{bmatrix} -\frac{5}{3} & \frac{11}{2} \end{bmatrix}$

► Distance: $\frac{\sqrt{13}}{3}$

17(c)(iv)

methods_diffcalc\solve_touch(f(x),k,l(x),x,k)

► Derivative 1: $\frac{27 \cdot x^2}{16} + \frac{3 \cdot x}{2} - \frac{15}{4}$

► Derivative 2: $\frac{-3 \cdot k}{2}$

► Equating functions and derivatives.

► Solutions:

► Solutions:

$x = \frac{-5}{3}$ and $k = \frac{25}{24}$ or $x = 2$ and $k = -4$

Done

$$f\left(\frac{-5}{3}\right)$$

$$\frac{275}{48}$$

Since the point B on the x-axis is fixed, we apply a dilation from the x-axis by k and solve for when the line touches the function, that is, it has the same gradient at the point of intersection with the function. Subbing this x-value back into the function gives the y-coordinate.

17(d)

$$l(x) - f(x)$$

$$\frac{-9 \cdot x^3}{16} - \frac{3 \cdot x^2}{4} + \frac{9 \cdot x}{4} + 3$$

17(e)

methods_func\analysed(l(x)-f(x),x,-2,2)

- ▶ Start Point: $[-2. \ 0.]$
- ▶ End Point: $[2. \ 0.]$
- ▶ Maximal Domain: $-2. \leq x \leq 2.$
- ▶ x -Intercepts: (3.)
 $[-2. \ 0.], [-1.33333 \ 0.],$
 $[2. \ 0.]$
- ▶ Vertical Intercept: $[0. \ 3.]$
- ▶ Derivative:
 $-1.6875 \cdot x^2 - 1.5 \cdot x + 2.25$
- ▶ Inflection Point:
 $[-0.444444 \ 1.90123]$ (Increasing)
- ▶ Stationary Points: (2.)
 $[-1.68173 \ -0.229638]$ (Local min.)
 $[0.792837 \ 4.03211]$ (Local max.)

17(f)

Number of Iterations:

OK

Cancel

methods_diffcalc\newtons_method(f(x)-2,x,3)

► Derivative:

$$\frac{27 \cdot x^2}{16} + \frac{3 \cdot x}{2} - \frac{15}{4}$$

► Iterative Formula: $\frac{2 \cdot (9 \cdot x^3 + 6 \cdot x^2 + 16)}{3 \cdot (x+2) \cdot (9 \cdot x - 10)}$

► Number of Iterations: 3

"n"	"x _n "	"f(x _n)"	"f'(x _n)"
0.	3.	8.6875	15.9375
1.	2.4549	1.63597	10.1021
2.	2.29296	0.125924	8.56174
3.	2.27825	0.000997	8.42622

18(a)

DelVar a

Done

Define $f(x) = 6 \cdot a \cdot \sqrt{x} - x$

Done

solve($f(x)=0,x$) $x=36 \cdot a^2$ and $a \geq 0$ or $x=0$

Note: Remember to delete the variable a since we have already used it in the previous question. It is best practice to insert a new problem page [doc][4][1] to avoid conflicting variables.

18(b)

Define $df(x) = \frac{d}{dx}(f(x))$

Done

$$\text{solve}(df(x) < 0, x) | a > 0 \quad x > 9 \cdot a^2 \text{ and } a > 0$$

18(c)

$$\text{methods_diffcalc tangent_line}(f(x), x, 36 \cdot a^2)$$

▶ Derivative: $\frac{3 \cdot a}{\sqrt{x}} - 1$

▶ Gradient: $\frac{\text{sign}(a)}{2} - 1$

▶ Passes Through:

$$\begin{bmatrix} 36 \cdot a^2 & 36 \cdot a \cdot |a| - 36 \cdot a^2 \end{bmatrix}$$

▶ x-Intercept: $\begin{bmatrix} \frac{-36 \cdot a \cdot |a|}{\text{sign}(a) - 2} & 0 \end{bmatrix}$

▶ Vertical Intercept: $\begin{bmatrix} 0 & 18 \cdot a \cdot |a| \end{bmatrix}$

▶ Tangent Line:

$$\frac{(\text{sign}(a) - 2) \cdot x}{2} + 18 \cdot a \cdot |a|$$

$$\frac{(\text{sign}(a) - 2) \cdot x}{2} + 18 \cdot a \cdot |a| | a > 0 \quad 18 \cdot a^2 - \frac{x}{2}$$

Note: Since $a > 0$, we can replace $\text{sign}(a)$ with 1 in our working out.

18(d)	<div>Define $t(x) = 18 \cdot a^2 - \frac{x}{2}$ Done</div> <div>solve($0 = t(0) - b, b$) $b = 18 \cdot a^2$</div>
18(e)	<div>solve($f(x) = x, x$) $x = 9 \cdot a^2$ and $a \geq 0$ or $x = 0$</div>
18(f)	<div>The tangents are vertical/horizontal lines. Thus 90° or $\frac{\pi}{2}$ (1A)</div>



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VCE Mathematical Methods $\frac{3}{4}$

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