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VCE Mathematical Methods ¾
Differentiation II [0.10]
Workshop Solutions

Error Logbook:

Mistake/Misconception #1		Mistake/Misconception #2	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	
Mistake/Misconception #3		**Mistake/Misconception #4**	
Question #:	Page #:	Question #:	Page #:
Notes:		Notes:	

Section A: Recap

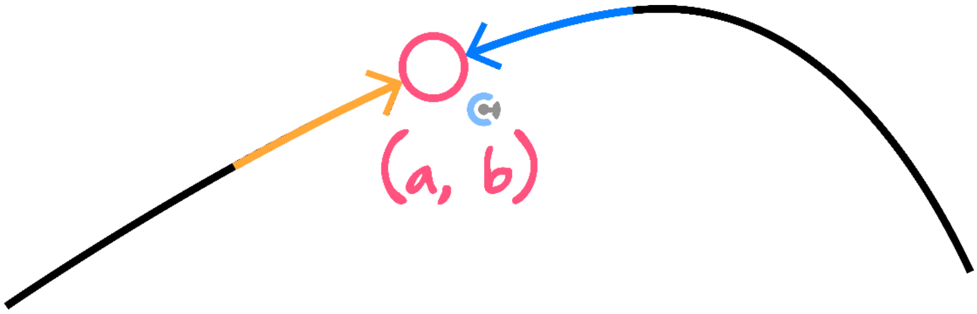
Limits

$$\lim_{x \to a} f(x) = L$$

"The function $f(x)$ approaches L as x approaches a."

- Limit is the value that a function (y-value) approaches as the x-value approaches a value.

Validity of Limits



$$\lim_{x \to a} f(x) \text{ exists if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

- Limit is defined when the left limit equals the right limit.

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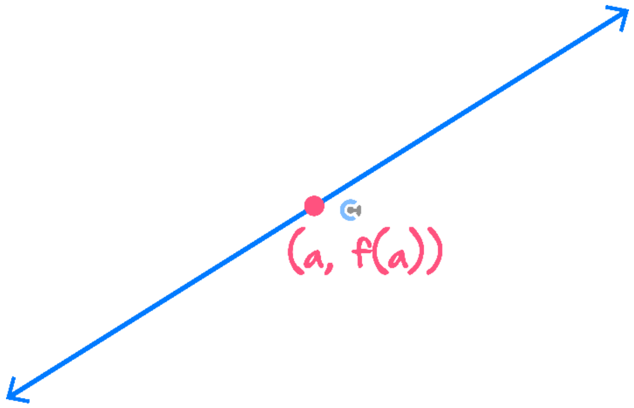
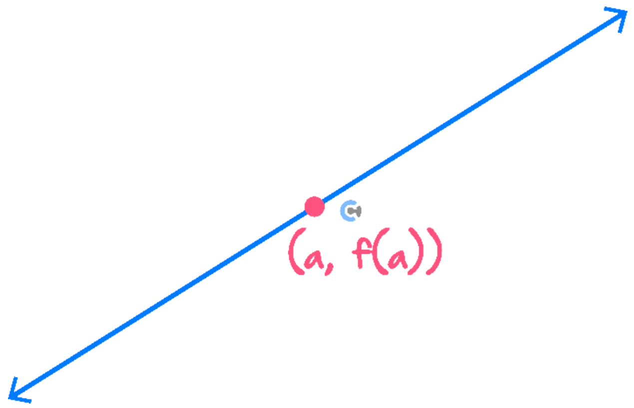
Continuity



- A function f is said to be continuous at a point $x = a$ if:

  1. $f(x)$ is defined at $x = a$.
  2. $\lim_{x \to a} f(x)$ exists.
  3. $\lim_{x \to a} f(x) = f(a)$.

Differentiability

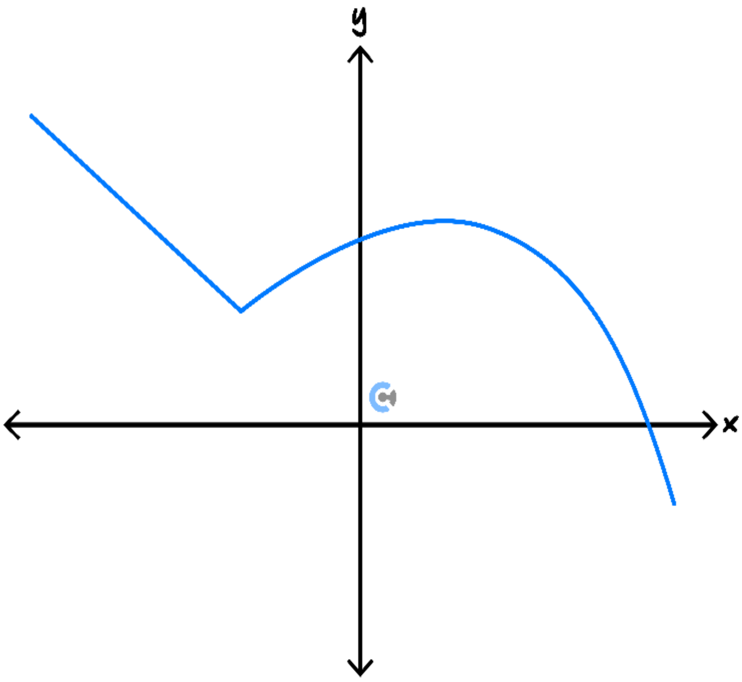


- A function f is said to be differentiable at a point $x = a$ if:

  1. $f(x)$ is continuous at $x = a$.
  2. $\lim_{x \to a} f'(x)$ exists.
     - Limit exists when the left and right limits are the same.
     - Gradient on the LHS and RHS must be the same.

- We **cannot** differentiate:
  1. Discontinuous points.
  2. Sharp points.
  3. Endpoints.

Finding the Derivative of Hybrid Functions



1. Simply derive each function.
2. Reject the values for x that are not differentiable from the domain.

Second Derivatives

- The derivative of the derivative.
- To get the second derivative, we can **differentiate the original function twice**.

$$\frac{d^2y}{dx^2} = f''(x)$$

Concavity

- Concave up is when the gradient is increasing.

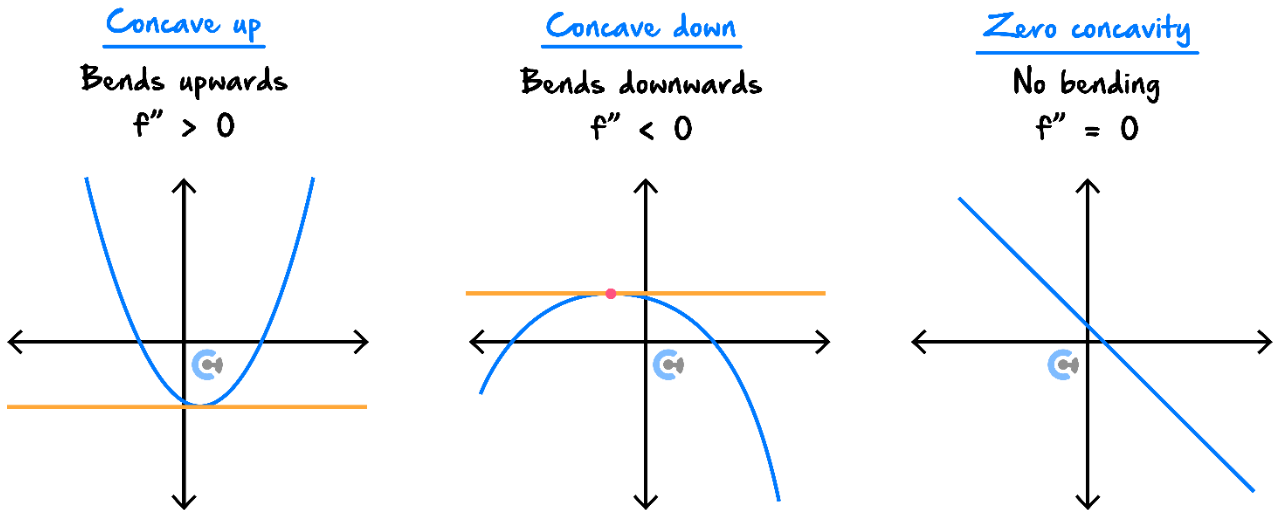
$$f''(x) > 0 \rightarrow \text{Concave Up}$$

- Concave down is when the gradient is decreasing.

$$f''(x) < 0 \rightarrow \text{Concave Down}$$

- Zero concavity is when the gradient is neither increasing nor decreasing.

$$f''(x) = 0 \rightarrow \text{Zero Concavity}$$



Concavity is also linked to how the curve is bent.

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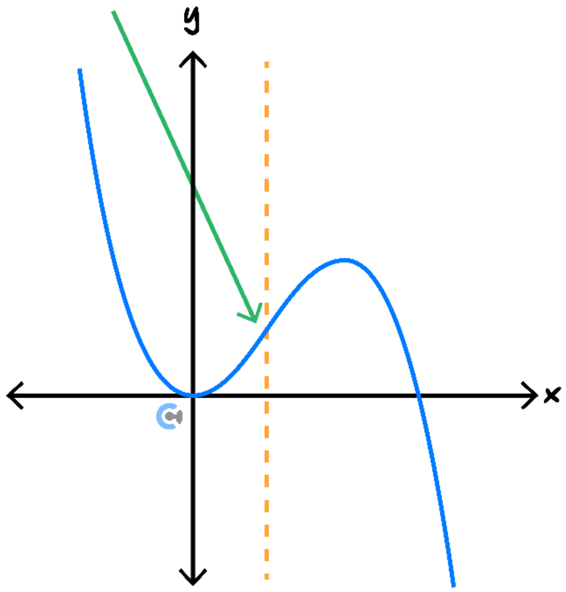
Points of Inflection

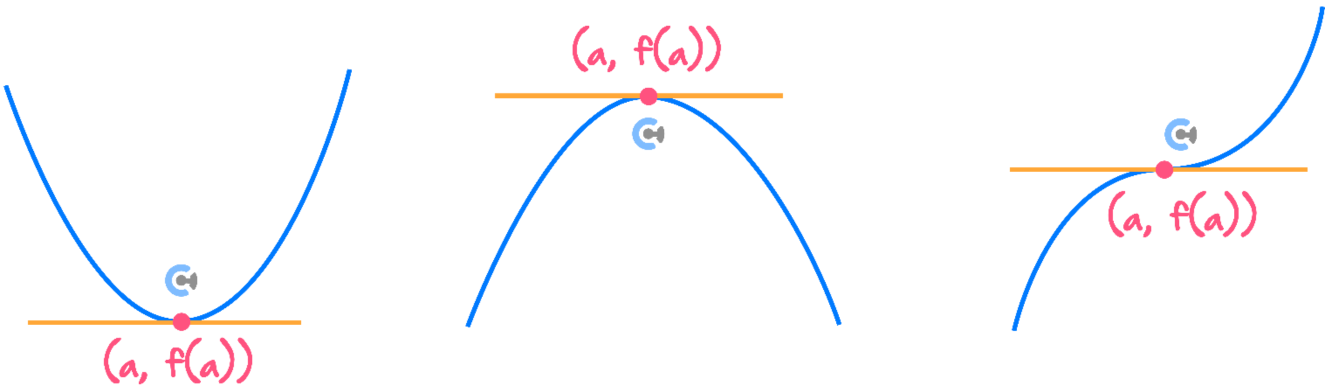
- A point at which a curve **changes concavity** is called a **point of inflection**.



$$f''(x) = 0$$

- Simply, it is when the bending changes.

The Second Derivative Test



- Suppose that $f'(a) = 0$ and hence, f has a stationary point at $x = a$. The second derivative test states:
  - **Concave up gives us a local minimum.**

$$f''(x) > 0 \rightarrow \text{Local Minimum}$$

- Concave down gives us a local maximum.

$$f''(x) < 0 \rightarrow \text{Local Maximum}$$

- Zero concavity gives us a stationary point of inflection.

$$f''(x) = 0 \rightarrow \text{Stationary Point of Inflection}$$

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Section B: Warm Up

Question 1

Evaluate the following limits if they exist, otherwise, explain why they do not exist.

a. $\lim_{x \to 3}(x^2 - 3)$

$$\lim_{x \to 3}(x^2 - 3) = 6$$

b. $\lim_{x \to 0}\left(\frac{x^2-2x}{x}\right)$

$$\lim_{x \to 0}\frac{x^2 - 2x}{x} = \lim_{x \to 0}(x-2) = -2$$

c. $\lim_{x \to 1}\left(\frac{3}{x-1}\right)$

$\lim_{x \to 1^-}\frac{3}{x-1} = -\infty$ and $\lim_{x \to 1^+}\frac{3}{x-1} = \infty$.
So overall limit does not exist since left and right limits differ.

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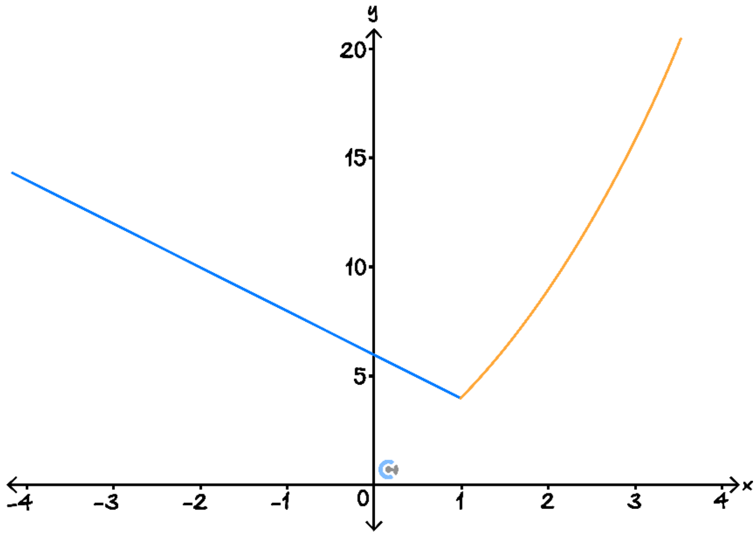
Question 2

a. Consider the function $f(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ 6 - 2x, & x < 1 \end{cases}$ shown in the graph below:



i. State all values of x for which f is continuous.

$x \in \mathbb{R}$.

ii. State all values of x for which f is differentiable.

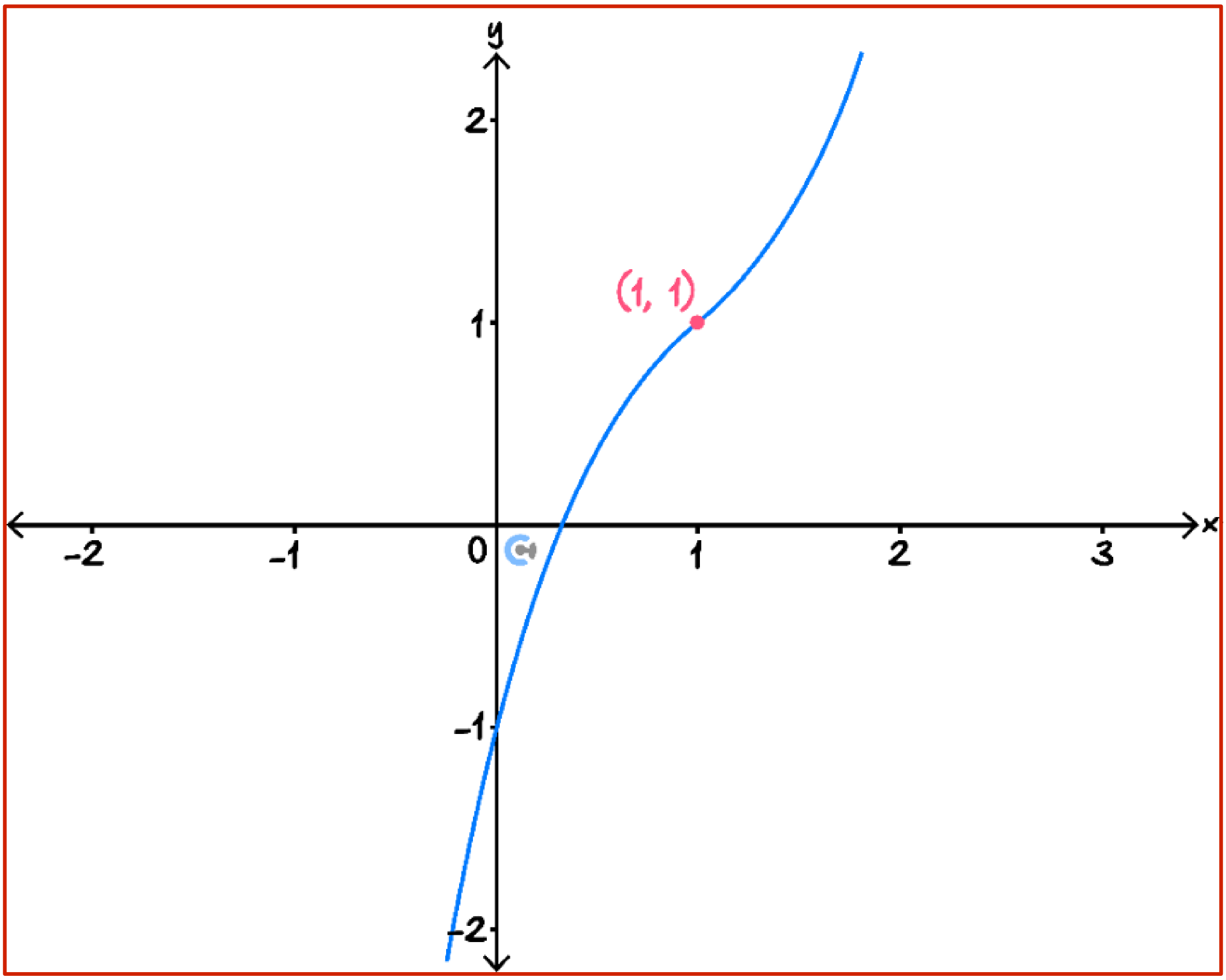
$x \in \mathbb{R} \setminus \{1\}$.

b. The function $g(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$ is continuous and differentiable for all $x \in \mathbb{R}$. Determine the values of a and b.

Continuous: $4 = a + b$.
Differentiable: $4 = a$.
So $a = 4$ and $b = 0$.

Question 3

a. Consider the graph of the cubic polynomial shown below:



i. Circle the point of inflection.

ii. Is the graph concave up or concave down after the point of inflection?

Concave up.

b. Consider the polynomial function $f(x) = x^3 - 3x^2 + 2$.

i. Determine the coordinates of any stationary points of f.

$f'(x) = 3x^2 - 6x = 0 \implies 3x(x-2) = 0 \implies x = 0, 2.$
Stationary points at $(0, 2)$ and $(2, -2)$

ii. Use the second derivative to determine the nature of any stationary points of f.

$f''(x) = 6x - 6.$ $f''(0) = -6 < 0$ and $f''(2) = 6 > 0.$
Therfore $(0, 2)$ is a local maximum and $(2, -2)$ is a local minimum.

iii. Determine when f is concave down.

$f''(x) < 0 \implies x < 1.$

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Section C: Exam 1 Questions (22 Marks)

INSTRUCTION: 22 Marks. 27 Minutes Writing.

Question 4 (9 marks)

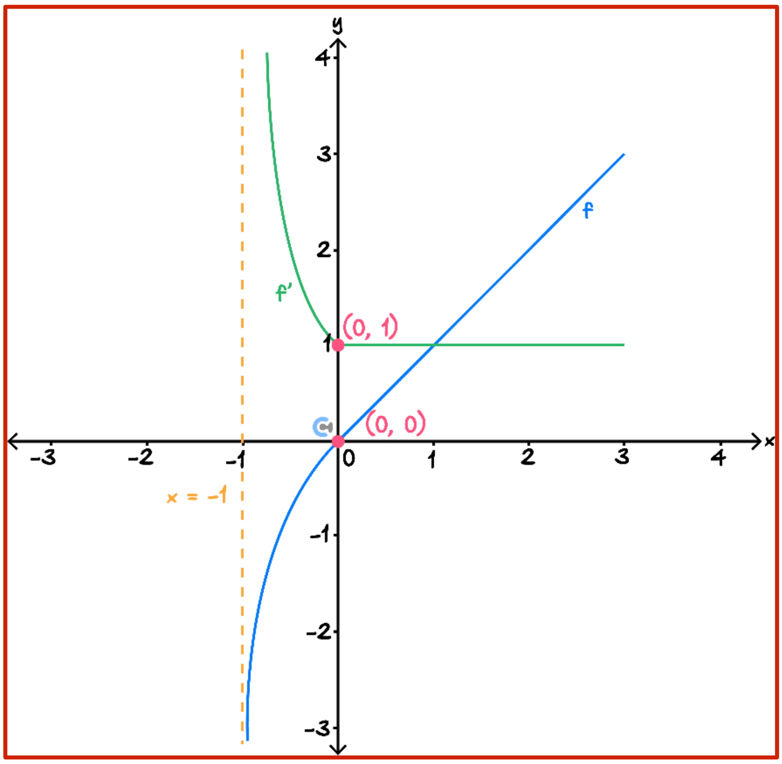
Consider the piecewise function:

$$f: R \to R, f(x) = \begin{cases} \log_e(x+1), & -1 < x < 0 \\ ax + b, & 0 \le x \le 3 \end{cases}$$

a. Determine the values of a and b such that the graph of f is differentiable at $x = 0$. (2 marks)

```
In[4]:= f[x_] := Log[x+1]
In[2]:= g[x_] := a*x+b
In[5]:= Solve[f[0] == g[0] && f'[0] == g'[0]]
Out[5]= {{a → 1, b → 0}}
```

b. Hence, sketch the function f for $-1 < x \le 3$ on the axes below, labelling all key coordinates and asymptotes with their equations. (2 marks)



c. Calculate $f'(x)$ and state its domain. (2 marks)

$$f'(x) = \begin{cases} \dfrac{1}{x+1}, & -1 < x < 0 \\ 1, & 0 \leq x < 3 \end{cases}$$

d. Hence, sketch the graph of $y = f'(x)$ on the same axes provided in **part b**. (2 marks)

e. State the values of x for which $f''(x)$ is defined. (1 mark)

$$x \in (-1, 0) \cup (0, 3)$$

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Question 5 (7 marks)

Consider the function $g(x) = (x-1)^3 e^{2x}$.

a. Show that $g'(x) = e^{2x}\,(x-1)^2(2x+1)$. (2 marks)

Make use of the product and chain rules.

$g'(x) = 3(x-1)^2e^{2x} + 2e^{2x}(x-1)^3 = e^{2x}(x-1)^2(3+2(x-1)) = e^{2x}(x-1)^2(2x+1)$

b. Find the coordinates of any stationary points of g. (2 marks)

$g'(x) = 0 \implies x = -\frac{1}{2}, 1.$

$g\left(-\frac{1}{2}\right) = -\left(\frac{3}{2}\right)^3 e^{-1} = -\frac{27}{8}e^{-1}$ and $g(1) = 0$.

Therefore stationary points $\left(-\frac{1}{2}, -\frac{27}{8e}\right)$ and $(1,0)$.

It is known that $g''(x) = 2e^{2x}(2x^3 - 3x + 1)$.

c. Use the second derivative to determine the nature of any stationary points of g. (2 marks)

$g''\left(-\frac{1}{2}\right) = 2e^{-1}\left(1 + \frac{3}{2} - 2 \times \frac{1}{8}\right) = 2e^{-1} \times \frac{9}{4} = \frac{9}{2e} > 0.$

Therefore $\left(-\frac{1}{2}, -\frac{27}{8}e\right)$ is a local minimum.

$g''(1) = 2e^2(1-3+2) = 0$. Therefore $(1,0)$ is a stationary point of inflection.

d. Evaluate $\lim_{x \to -\infty} g(x)$. (1 mark)

$\lim_{x \to -\infty} g(x) = 0.$
This is because e^{2x} grows much faster than x^3.

Question 6 (6 marks)

Consider the function $f : [-4, 1] \to R, f(x) = x^2 e^x$.

a. Find $f'(x)$. (1 mark)

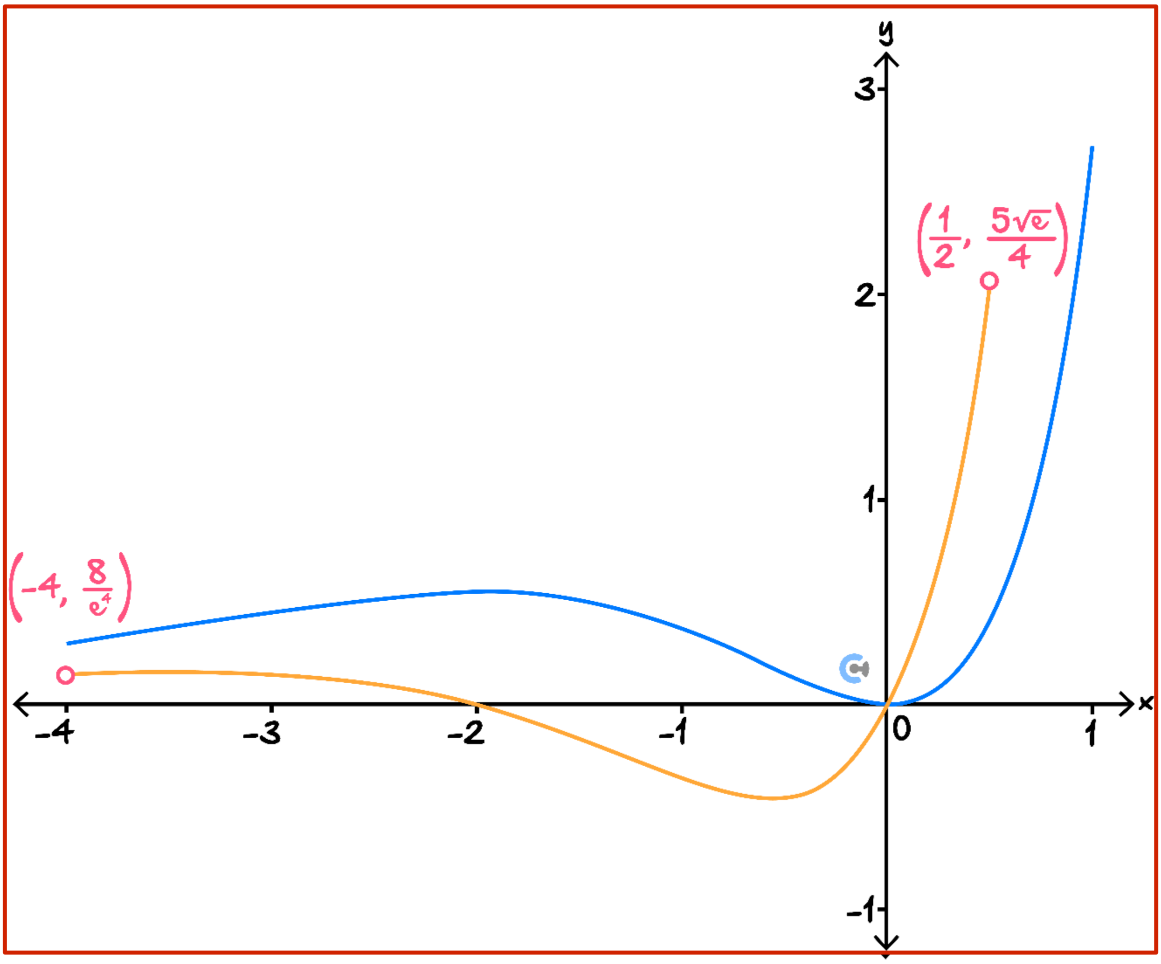
$$f'(x) = e^x x^2 + 2e^x x = e^x x(x+2)$$

b. Hence, find the exact coordinates of any stationary points of f. (2 marks)

$f'(x) = 0 \implies x = -2, 0.$
$f(-2) = 4e^{-2}$ and $f(0) = 0.$
Therefore stationary points at $(-2, 4e^{-2})$ and $(0, 0)$.

c. The graph of $y = f(x)$ is shown in the axes below. Sketch the graph of $y = f'(x)$ for $x \in \left(-4, \frac{1}{2}\right)$ on the axes below. Label the endpoints with their exact coordinates.

Use the fact that the local maximum of $y = f'(x)$ occurs at approximately $(-3.41, 0.16)$, the local minimum occurs at approximately $(-0.59, -0.46)$, $f'(-4) \approx 0.15$ and $f'\left(\frac{1}{2}\right) \approx 2.06$. (3 marks)



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Section D: Tech Active Exam Skills

Calculator Commands: Finding Derivatives

Mathematica	TI-Nspire	Casio
`f'[x]` `D[f[x], x]`	Shift Minus. $\frac{d}{dx}(f(x))$	Math 2. $\frac{d}{dx}(f(x))$

Calculator Commands: Finding Second Derivatives

Mathematica	TI-Nspire	Casio
`f''[x]` `D[f[x], {x, 2}]`	Shift Minus. $\frac{d^2}{dx^2}(f(x))$	Math 2. $\frac{d^2}{dx^2}(f(x))$

Calculator Commands: Stationary Point

- ALWAYS sketch the graph first to get an idea of the nature of the stationary point.
- The turning points for a function $f(x)$ can be found by solving $f'(x) = 0$ and subbing the result into f.
- **Example:** Find the turning point for $f(x) = e^{-x^2+2x}$.

Mathematica	TI-Nspire	Casio Classpad
In[4]:= `f[x_] := Exp[-x^2 + 2 x]` In[5]:= `Solve[f'[x] == 0 && y == f[x], Reals]` Out[5]= `{{x → 1, y → e}}`	Define $f(x)=e^{-x^2+2\cdot x}$ — Done $\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$ — $x=1$ $f(1)$ — e	define $f(x) = e^{-x^2+2x}$ — done $\text{solve}(\frac{d}{dx}(f(x))=0, x)$ — $\{x=1\}$ $f(1)$ — e

Section E: Exam 2 Questions (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.

Question 7 (1 mark)

Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x+1}$. If $h(x) = f(x)g(x)$, then the maximal domain of the derivative of h is:

A. $[-1, 3]$

B. $(-1, 3)$

C. $[-1, \infty)$

D. $(-\infty, -1) \cup (3, \infty)$

Question 8 (1 mark)

Which one of the following functions is differentiable for all real values of x?

A. $f(x) = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$

B. $f(x) = \begin{cases} 8x + 4, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

C. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

D. $f(x) = \begin{cases} 4x + 1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

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Question 9 (1 mark)

Consider the following function:

$$f(x) = \begin{cases} 0, & x < 0 \\ \sin(x), & x \geq 0 \end{cases}$$

Which of the following statements is true?

A. The function is continuous at $x = 0$ and differentiable at $x = 0$.

B. The function is continuous at $x = 0$ and not differentiable at $x = 0$.

C. The function is not continuous at $x = 0$ and differentiable at $x = 0$.

D. The function is not continuous at $x = 0$ and not differentiable at $x = 0$.

Question 10 (1 mark)

If $f(x) = \begin{cases} 2x^2 - 2, & -5 \leq x \leq 0 \\ 3x - 2, & 0 < x \leq 10 \end{cases}$, the domain of $f'(x)$ is:

A. $[-5, 10]$

B. $R\backslash\{0\}$

C. $(-5, 10)$

D. $(-5, 0) \cup (0, 10)$

Question 11 (1 mark)

Assume that the functions below are defined on their maximal domain, which of the following functions is differentiable at $x = 3$?

A. $f(x) = \sqrt{x - 3}$

B. $f(x) = \log_e(x - 3)$

C. $f(x) = (x - 2)^3$

D. $f(x) = \frac{1}{x-3}$

Question 12 (1 mark)

Let f be a function with a domain R such that $f'(4) = 0$ and $f'(x) < 0$ when $x \neq 4$. At $x = 4$, the graph of f has a:

A. Local minimum.

B. Local maximum.

C. Gradient of 4.

D. Stationary point of inflection.

Question 13 (1 mark)

The cubic function $R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b, and c are positive constants, have no stationary points when:

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

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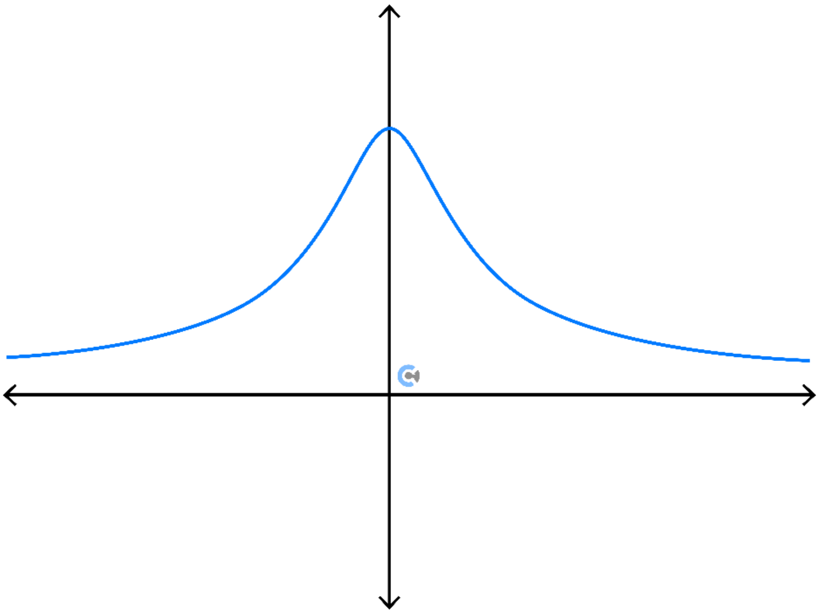
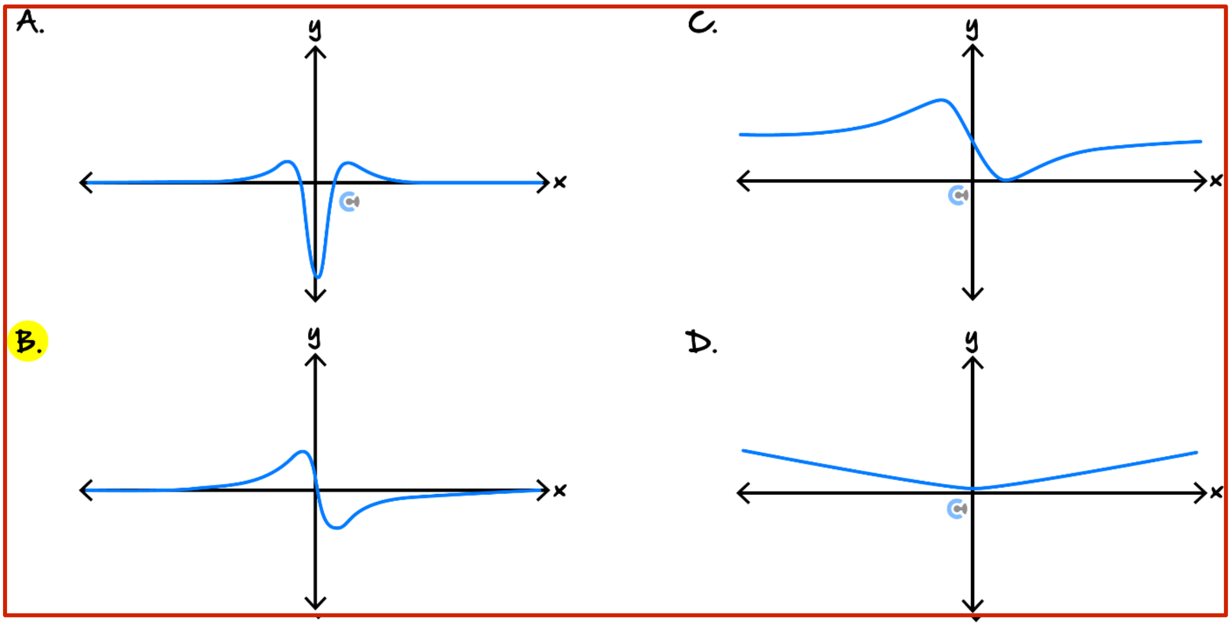
Question 14 (1 mark)

The graph of $y = f(x)$ is shown below. Which of the graphs could correspond to $y = f'(x)$?





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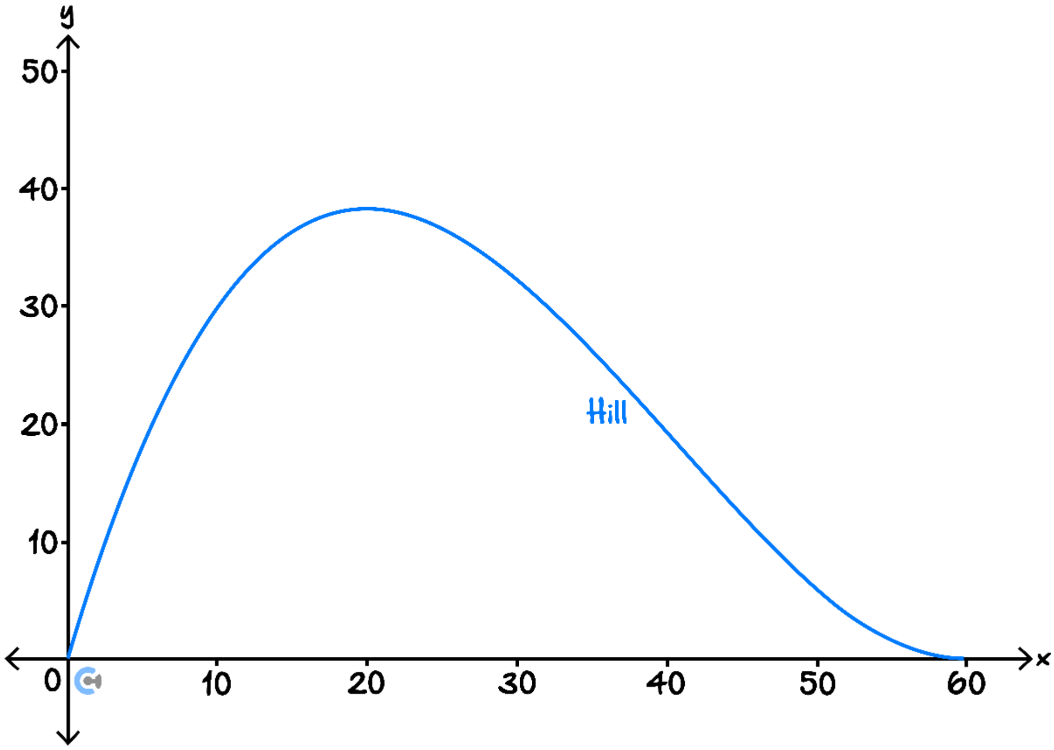
Question 15 (12 marks)

The Billboa Adventure fun camp is constructing a zip line to help attract more schools to choose them to host their school camps. The zipline is to be constructed above a hill on the property. The hill is modelled by:

$$y = \frac{3x(x-60)^2}{2500}, \qquad x \in [0, 60]$$

Where x is the horizontal distance, in metres, from an origin and y is the height, in metres, above this origin.



a. Find $\frac{dy}{dx}$. (2 marks)

Let $f(x) = \frac{3x(x-60)^2}{2500}$.

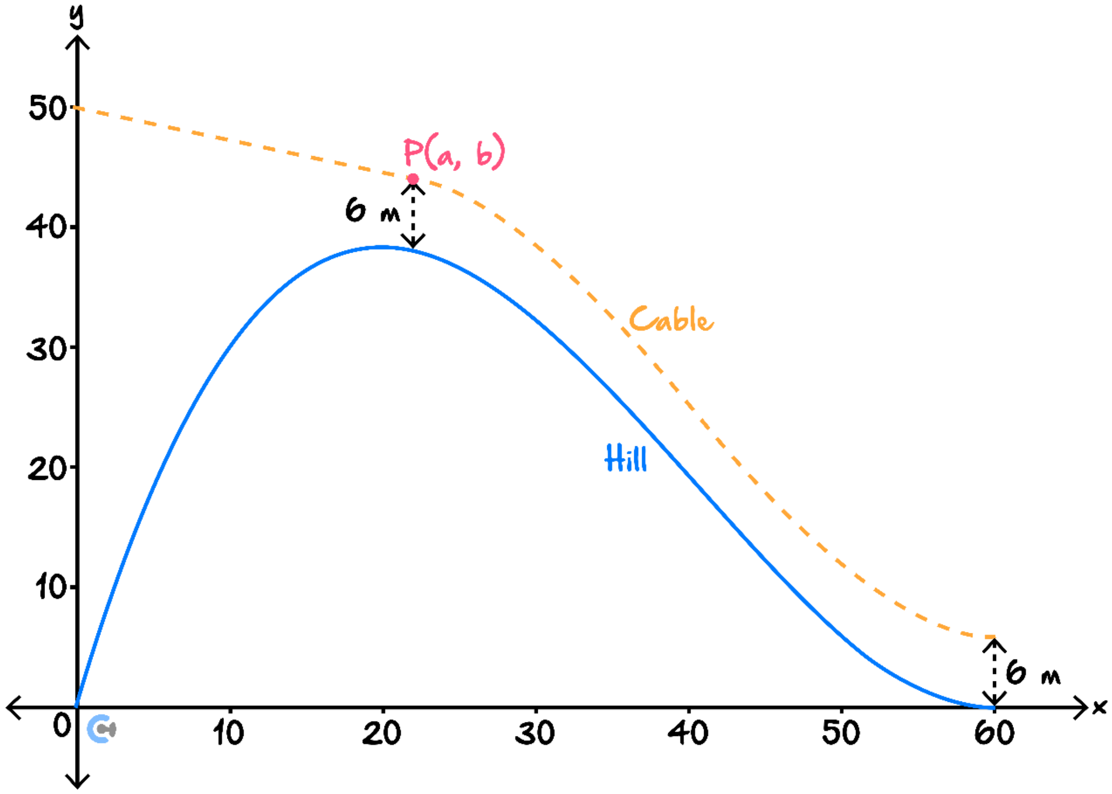
Then $f'(x) = \frac{9\left(x^2 - 80x + 1200\right)}{2500} = \frac{9(x-20)(x-60)}{2500}$

b. State the set of values for which the **gradient** of the hill is strictly decreasing. (1 mark)

Note that the question is asking when the gradient hill is strictly decreasing **not** when the hill is strictly decreasing.

Look at a graph of $f'(x)$ and note that the gradient is not defined at $x = 0$ to get the answer $x \in (0, 40]$.

The cable for the zip line is connected to a platform at the origin at a height of 50 metres and is straight for $0 \leq x \leq a$, where $20 \leq a \leq 30$. The straight section joins the curved section at $P(a, b)$. The cable is exactly 6 m vertically above the hill from $a \leq x \leq 60$, as shown in the graph below:



c. State the rule, in terms of x, for the height of the cable above the horizontal axis for $x \in [a, 60]$. (1 mark)

$$h(x) = f(x) + 6 = \frac{3x(x-60)^2}{2500} + 6$$

d. Find the values of x for which the gradient of the cable is equal to the average gradient of the hill for $x \in [20, 60]$. (3 marks)

Average gradient $= \dfrac{f(60) - f(20)}{60 - 20} = -\dfrac{24}{25}$.

Solve $h'(x) = -\dfrac{24}{25} \implies x = \dfrac{20}{3}(6 \pm \sqrt{3})$

e. The gradients of the straight and curved sections of the cable approach the same value at $x = a$, so there is a continuous and smooth join at P.

i. State the gradient of the cable at P, in terms of a. (1 mark)

$$\frac{h(a)-50}{a} == \frac{3a^2}{2500} - \frac{18a}{125} - \frac{44}{a} + \frac{108}{25} = \frac{3a^3 - 360a^2 + 10800a - 110000}{2500a}$$

ii. Find the coordinates of P, with each value correct to two decimal places. (3 marks)

We must simply solve $h'(a) = \dfrac{h(a)-50}{a} \implies a = 21.9506$.
Then $h(21.9506) = 44.1349$.
Correct to two decimal places P is at $(21.95, 44.13)$.

iii. Find the value of the gradient at P, correct to one decimal place. (1 mark)

$$h'(21.9506) = -0.3$$

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Section F: Extension Exam 1 (10 Marks)

INSTRUCTION: 10 Marks. 10 Minutes Writing.

Question 16 (10 marks)

Consider the function $f(x) = x^2 e^{-x^2}$.

a. Show that $f(x) = f(-x)$ and hence, state whether f is an even or an odd function. (1 mark)

$f(-x) = (-x)^2 e^{-(-x)^2} = x^2 e^{-x^2} = f(x)$.
f is an even function.

b.

i. Find $f'(x)$. (1 mark)

$f'(x) = 2xe^{-x^2} + x^2(-2x)e^{-x^2} = 2xe^{-x^2} - 2x^3e^{-x^2} = 2xe^{-x^2}(1 - x^2)$

ii. Hence, determine the coordinates for any stationary points of f. (2 marks)

$f'(x) = 0 \implies x = 0$ or $1 - x^2 = 0$. So $x = -1, 0, 1$.
$f(0) = 0$ and $f(-1) = f(1) = e^{-1} = \dfrac{1}{e}$.
So stationary points $(-1, e^{-1})$, $(0, 0)$ and $(1, e^{-1})$

c.

i. Show that the second derivative of f is given by $f''(x) = 2e^{-x^2}(2x^4 - 5x^2 + 1)$. (2 marks)

$f'(x) = 2xe^{-x^2}(1 - x^2)$ and so using the product rule:

$$\begin{aligned} f''(x) &= (2e^{-x^2} - 4x^2e^{-x^2})(1 - x^2) - 4x^2e^{-x^2} \\ &= 2e^{-x^2} - 2x^2e^{-x^2} - 4x^2e^{-x^2} + 4x^4e^{-x^2} - 4x^2e^{-x^2} \\ &= 2e^{-x^2} - 10x^2e^{-x^2} + 4x^4e^{-x^2} \\ &= 2e^{-x^2}(2x^4 - 5x^2 + 1) \end{aligned}$$

ii. Use the second derivative to determine the nature of the stationary points found in **part b. ii.** (2 marks)

$f''(-1) = f''(1) = 2e^{-1}(2 - 5 + 1) = -\frac{4}{e} < 0$. Therefore $(-1, e^{-1})$ and $(1, e^{-1})$ are local maximums.
$f''(0) = 2$. Therefore $(0, 0)$ is a local minimum.

iii. Determine how many points of inflection f has. (2 marks)

$f''(x) = 0 \implies 2x^4 - 5x^2 + 1 = 0$. Let $g(x) = 2x^4 - 5x^2 + 1$. Let $a = x^2$ then

$$2a^2 - 5a + 1 = 0 \implies a = \frac{5 \pm \sqrt{17}}{4}$$

Note that both these solutions are greater than zero, hence g has four roots. Thus f has four points of inflection.

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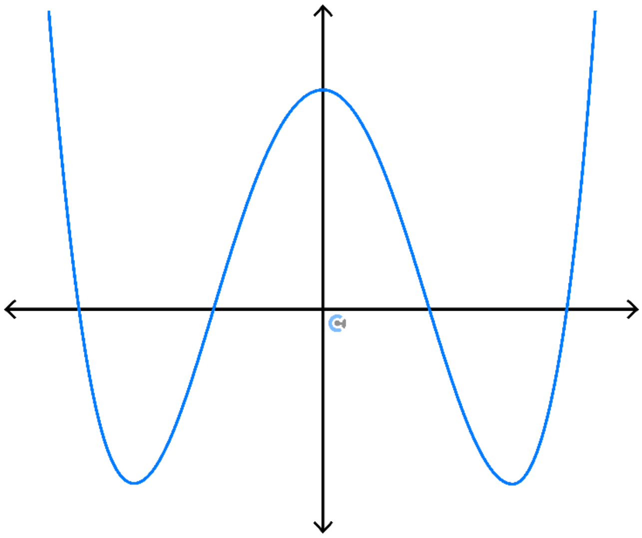
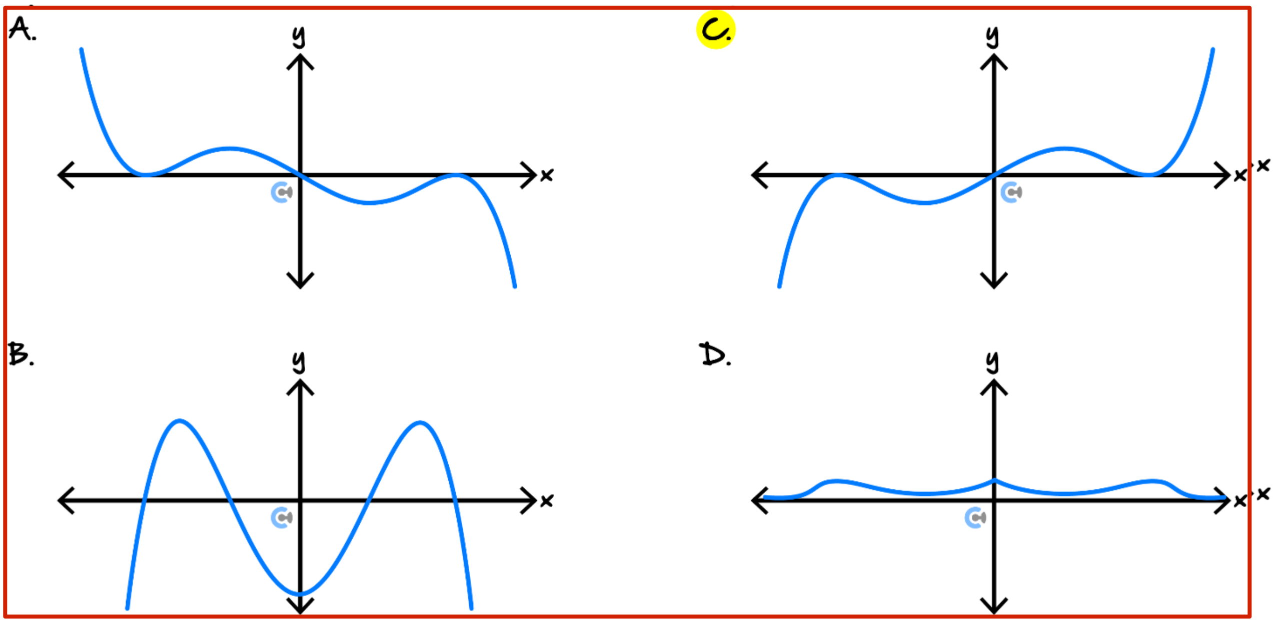
Section G: Extension Exam 2 (18 Marks)

INSTRUCTION: 18 Marks. 18 Minutes Writing.

Question 17 (1 mark)

The graph of $y = f'(x)$ is shown below. The graph of $y = f(x)$ could be:





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Question 18 (1 mark)

Let $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = (x-2)(x-3)(4x^2 + ax + 6)$$

Where a is a real number. If f has p stationary points, the possible values of p are:

A. $p \in \{1, 2, 3, 4\}$

B. $p \in \{0, 1, 2, 3\}$

C. $p \in \{1, 2\}$

D. $p \in \{1, 2, 3\}$

Question 19 (1 mark)

If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that:

- $f'(x) = 0$ at $x = 1$ and $x = 2$.
- $f'(x) > 0$ at $x > 2$ and $1 < x < 2$.
- $f'(x) < 0$ at $x < 1$.

Which of the following statements is **correct**?

A. The graph has a stationary point of inflection at $x = 2$ and a maximum at $x = 1$.

B. The graph has a stationary point of inflection at $x = 2$ and a minimum at $x = 1$.

C. The graph has a stationary point of inflection at $x = 1$ and a minimum at $x = 2$.

D. The graph has a minimum at $x = 1$ and a maximum at $x = 2$.

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Question 20 (1 mark)

Let $f^{(n)}(x)$ be the n^{th} derivative of $f(x)$. That is, it has been differentiated n times, where $n = 2p$ and p is a positive integer. If $f(x) = xe^{-x^3}$, how many axial intercepts does $f^{(n)}(x)$ have?

A. n

B. $2n$

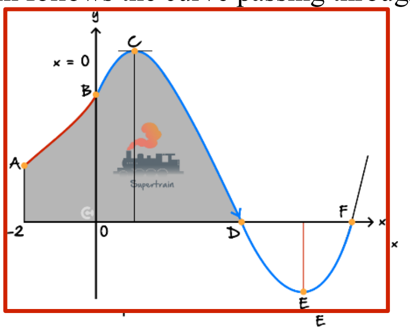
C. $2n + 1$

D. $n + 1$

```
In[28]:= f[x_] := x * Exp[-x^2]
In[29]:= Table[Length[Reduce[D[f[x], {x, 2 p}] == 0, x, Reals]], {p, 1, 30}]
Out[29]= {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31,
  33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61}
```

Question 21 (14 marks)

A part of the track for Tim's model train follows the curve passing through A, B, C, D, E and F shown:



Tim designed it by putting axes on the drawing as shown. The track is made up of two curves, one to the left of the y-axis and the other to the right.

B is the point $(0, 7)$. The curve from B to F is part of the graph of $f(x) = px^3 + qx^2 + rx + s$ where p, q, r and s are constants and $f'(0) = 4.25$.

a. Show that $s = 7$ and $r = 4.25$. (2 marks)

Show that $s = 7$ and $r = 4.25$. (2 marks)

$f(0) = 0 + 0 + 0 + s = 7$

$f'(x) = 3px^2 + 2qx + r$

$\therefore s = 7$

$f'(0) = r = 4.25$

$C\ (1, 9)$ is the furthest point reached by the track in the positive y direction.

b. Use this information to write two equations involving p and q. (2 marks)

$f(1) = 9 \rightarrow p + q + 4.25 + 7 = 9$

$f'(1) = 0. \rightarrow 3p + 2q + 4.25 = 0$

The values of p and q are $p = 0.25$ and $q = -2.5$, respectively. So, $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$.

c.

i. Find the exact coordinates of D and F. (1 mark)

$D: (4, 0)$ $F: (7, 0)$

ii. Find the greatest distance that the track is from the x-axis, when it is below the x-axis, correct to two decimal places. (1 mark)

$$f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$$
$$f'(x) = 4.25 - 5x + 0.75x^2$$
$$f'(x) = 0 => x = 1, 5.67$$
$$Stationary\ points\ at\ x = 1\ and\ x = 5.67$$
$$at\ x = 1\ we\ have\ maximum\ value.$$
$$At\ x = 5.67\ we\ have\ minimum\ value\ at\ E.$$
$$f(5.67) = -3.70$$
$$Depth = 3.70$$

d.

i. Find the value of x for which $f''(x) = 0$. (1 mark)

$$x = \frac{10}{3}$$

ii. Hence, find the steepest decline of f between C and D, correct to one decimal place. (1 mark)

$$f'\left(\frac{10}{3}\right) = -\frac{49}{12} \approx -4.1$$

The curve from A to B is part of the graph with the equation $g(x) = \frac{a}{1-bx}$, where a and b are positive real constants. The track passes smoothly from one section of the track to the other at B (that is, the gradients of the curves are equal at B).

e. Find the exact values of a and b. (3 marks)

g & f are joinly smoothly at $x=a$

$g(a) = f(a)$. $g'(a) = f'(a)$ ✓

$a = 7$, $b = \frac{17}{28}$ ✓ ✓

Tim adds a new part to the track when $x = 8$. Call the point on the track when $x = 8$, G.
This new section of the track is modelled by the function $h(x) = m \cdot k^{-(x-8)}$ for $x \geq 8$ and $m, k > 0$.

f.

i. Find the value of m so that the track is continuous at G. (1 mark)

$m = 9$

ii. It is known that $\lim_{x \to \infty} h(x) = 0$. Find the possible values of k. (1 mark)

$k > 1$

iii. Find the value of k if $h(x)$ passes through the point $(10,4)$. (1 mark)

$k = \frac{3}{2}$

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