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VCE Mathematical Methods ¾
Differentiation II [0.10]
Workshop

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Section A: Recap

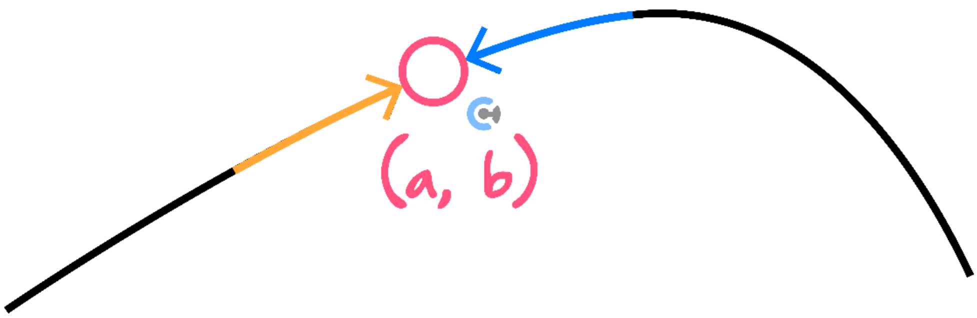
Limits

$$\lim_{x \to a} f(x) = L$$

"The function $f(x)$ approaches L as x approaches a."

- Limit is the value that a function (y-value) approaches as the x-value approaches a value.

Validity of Limits



$$\lim_{x \to a} f(x) \text{ exists if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

- Limit is defined when the left limit equals the right limit.

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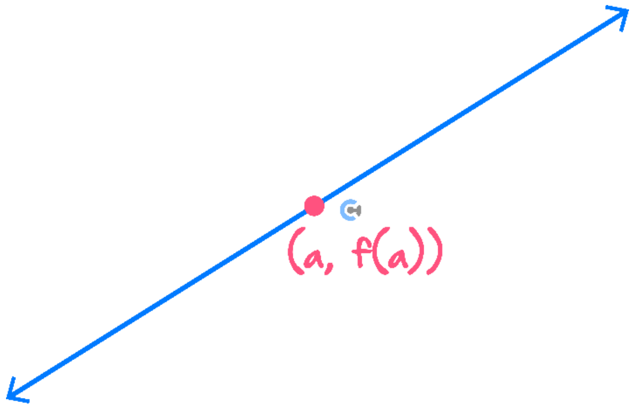
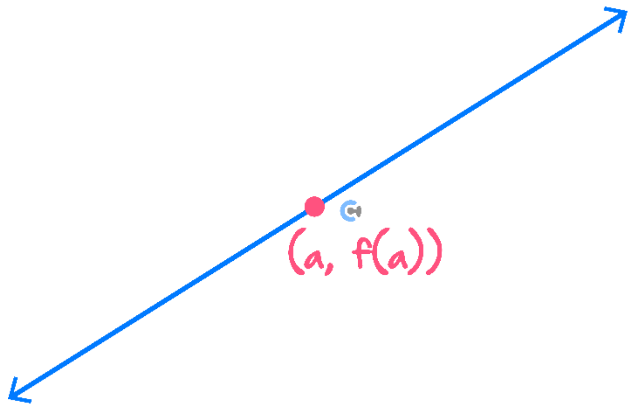
Continuity



- A function f is said to be continuous at a point $x = a$ if:

  1. $f(x)$ is defined at $x = a$.
  2. $\lim_{x \to a} f(x)$ exists.
  3. $\lim_{x \to a} f(x) = f(a)$.

Differentiability



- A function f is said to be differentiable at a point $x = a$ if:

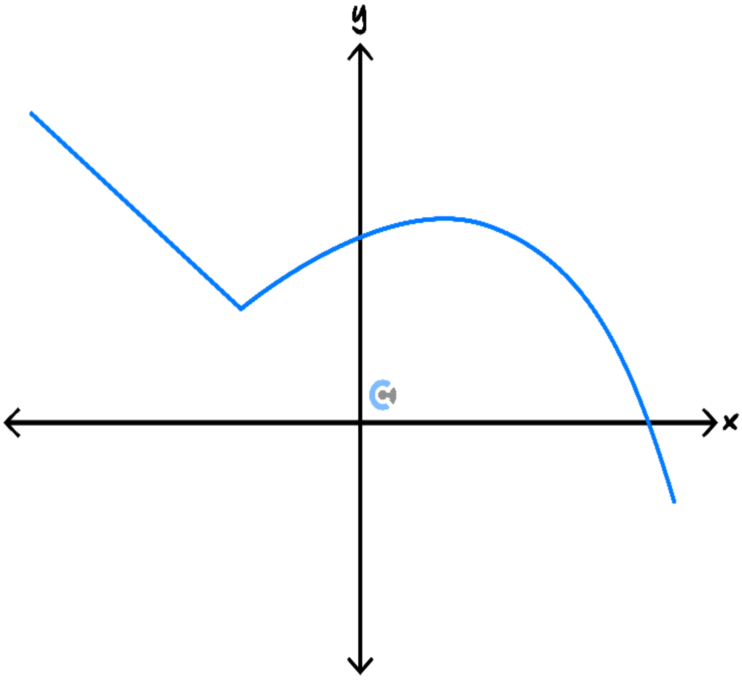
  1. $f(x)$ is continuous at $x = a$.
  2. $\lim_{x \to a} f'(x)$ exists.
     - Limit exists when the left and right limits are the same.
     - Gradient on the LHS and RHS must be the same.

- We **cannot** differentiate:

  1. Discontinuous points.
  2. Sharp points.
  3. Endpoints.

Finding the Derivative of Hybrid Functions

Definition



1. Simply derive each function.
2. Reject the values for x that are not differentiable from the domain.

Second Derivatives

Definition

- The derivative of the derivative.
- To get the second derivative, we can **differentiate the original function twice**.

$$\frac{d^2y}{dx^2} = f''(x)$$

Concavity

- Concave up is when the gradient is increasing.

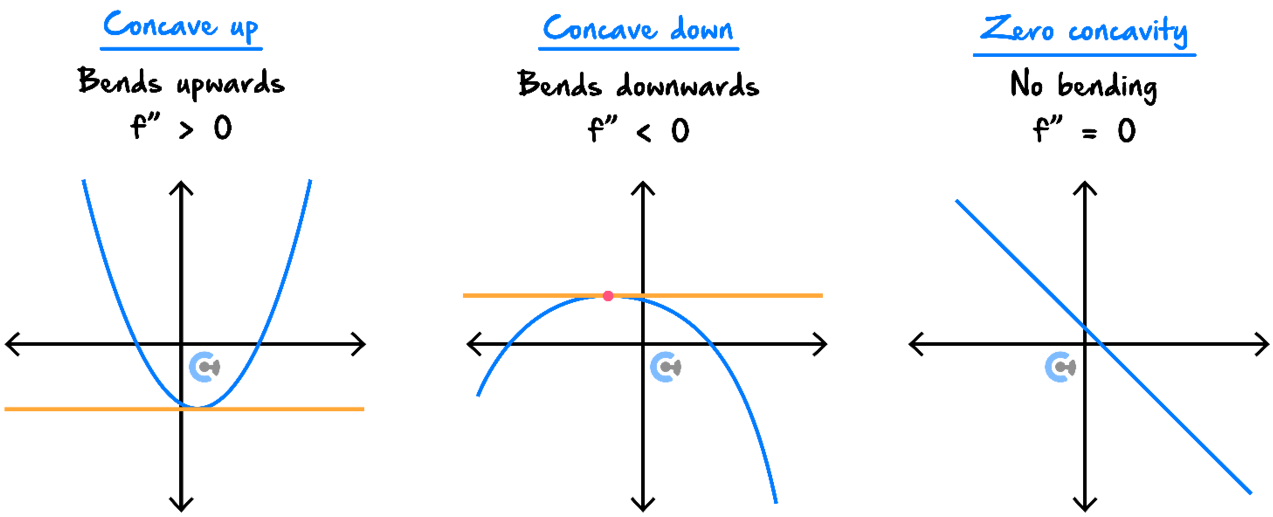
$$f''(x) > 0 \rightarrow \text{Concave Up}$$

- Concave down is when the gradient is decreasing.

$$f''(x) < 0 \rightarrow \text{Concave Down}$$

- Zero concavity is when the gradient is neither increasing nor decreasing.

$$f''(x) = 0 \rightarrow \text{Zero Concavity}$$



Concavity is also linked to how the curve is bent.

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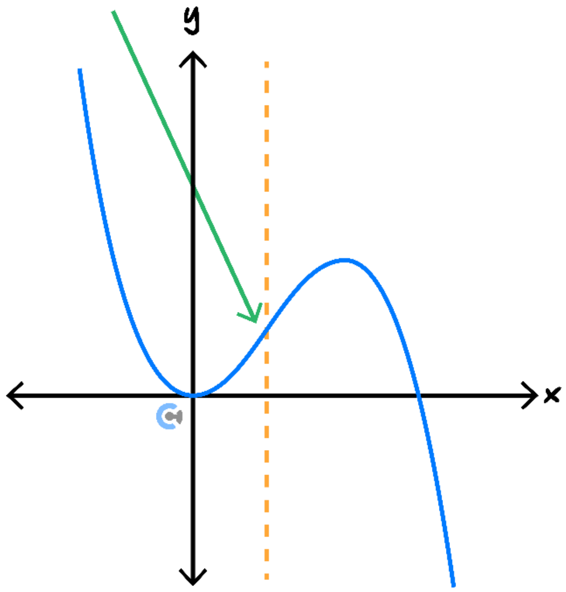
Points of Inflection

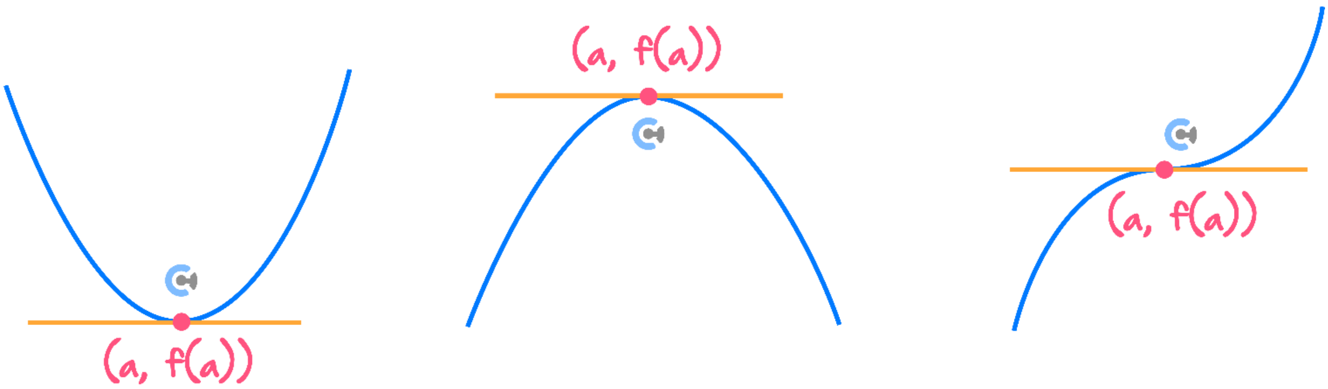
- A point at which a curve **changes concavity** is called a **point of inflection**.



$$f''(x) = 0$$

- Simply, it is when the bending changes.

The Second Derivative Test



- Suppose that $f'(a) = 0$ and hence, f has a stationary point at $x = a$. The second derivative test states:

  - **Concave up gives us a local minimum.**

$$f''(x) > 0 \rightarrow \text{Local Minimum}$$

- Concave down gives us a local maximum.

$$f''(x) < 0 \rightarrow \text{Local Maximum}$$

- Zero concavity gives us a stationary point of inflection.

$$f''(x) = 0 \rightarrow \text{Stationary Point of Inflection}$$

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Section B: Warm Up

Question 1

Evaluate the following limits if they exist, otherwise, explain why they do not exist.

a. $\lim_{x \to 3}(x^2 - 3)$

$$\boxed{\lim_{x \to 3}(x^2 - 3) = 6}$$

b. $\lim_{x \to 0}\left(\frac{x^2 - 2x}{x}\right)$

$$\boxed{\lim_{x \to 0}\frac{x^2 - 2x}{x} = \lim_{x \to 0}(x - 2) = -2}$$

c. $\lim_{x \to 1}\left(\frac{3}{x-1}\right)$

$\lim_{x \to 1^-}\frac{3}{x-1} = -\infty$ and $\lim_{x \to 1^+}\frac{3}{x-1} = \infty$.
So overall limit does not exist since left and right limits differ.

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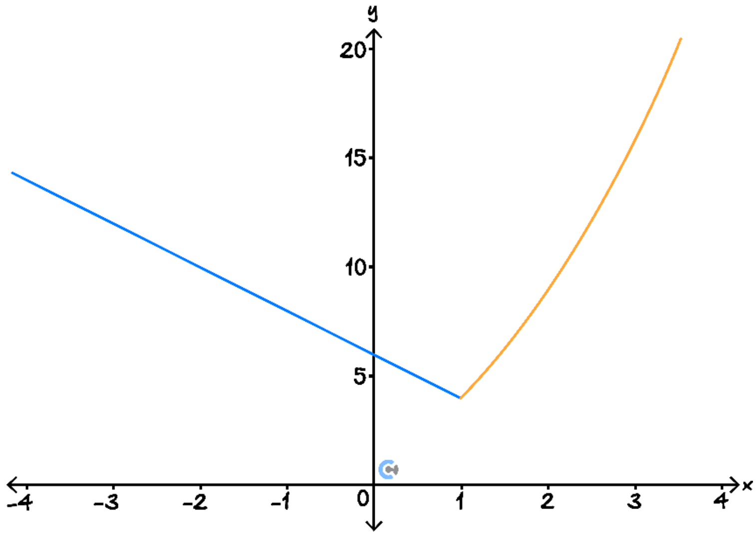
Question 2

a. Consider the function $f(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ 6 - 2x, & x < 1 \end{cases}$ shown in the graph below:



i. State all values of x for which f is continuous.

$\boxed{x \in \mathbb{R}.}$

ii. State all values of x for which f is differentiable.

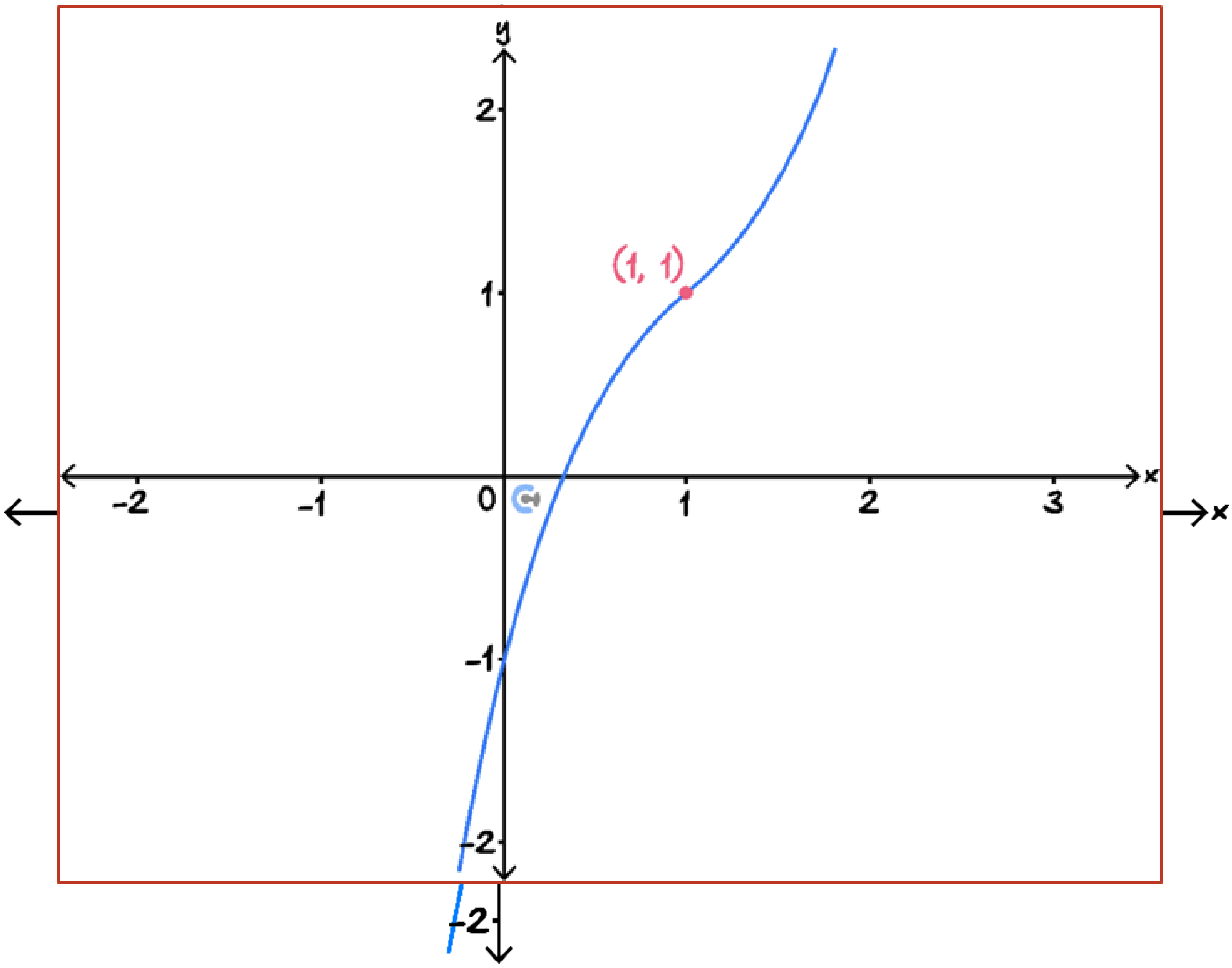
$\boxed{x \in \mathbb{R} \setminus \{1\}.}$

b. The function $g(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$ is continuous and differentiable for all $x \in \mathbb{R}$. Determine the values of a and b.

Continuous: $4 = a + b$.
Differentiable: $4 = a$.
So $a = 4$ and $b = 0$.

Question 3

a. Consider the graph of the cubic polynomial shown below:



i. Circle the point of inflection.

ii. Is the graph concave up or concave down after the point of inflection?

Concave up.

b. Consider the polynomial function $f(x) = x^3 - 3x^2 + 2$.

i. Determine the coordinates of any stationary points of f.

$f'(x) = 3x^2 - 6x = 0 \implies 3x(x-2) = 0 \implies x = 0, 2.$
Stationary points at $(0, 2)$ and $(2, -2)$

ii. Use the second derivative to determine the nature of any stationary points of f.

$f''(x) = 6x - 6$. $f''(0) = -6 < 0$ and $f''(2) = 6 > 0$.
Therfore $(0, 2)$ is a local maximum and $(2, -2)$ is a local minimum.

iii. Determine when f is concave down.

$f''(x) < 0 \implies x < 1.$

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Section C: Exam 1 Questions (22 Marks)

INSTRUCTION: 22 Marks. 27 Minutes Writing.

Question 4 (9 marks)

Consider the piecewise function:

1) Contin (Join)

2) $\lim_{x \to a} f(x)$ (Smooth)

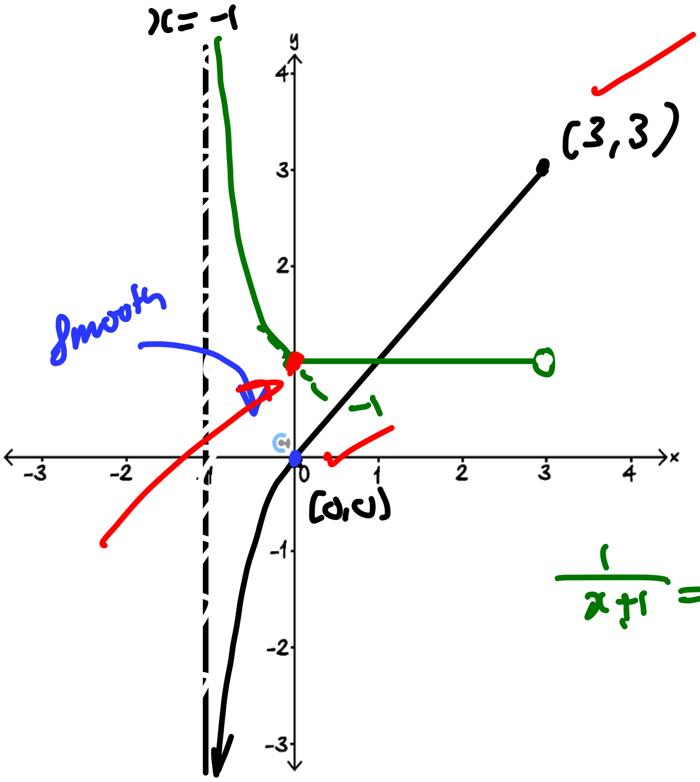
$$f : R \to R, f(x) = \begin{cases} \log_e(x+1), & -1 < x < 0 \\ ax + b, & 0 \le x \le 3 \end{cases}$$

$\frac{1}{x+1}$ → a

a. Determine the values of a and b such that the graph of f is differentiable at $x = 0$. (2 marks)

Join	Smooth
$\log_e(1) = b$	$\frac{1}{0+1} = a$
$b = 0$	$a = 1$

b. Hence, sketch the function f for $-1 < x \le 3$ on the axes below, labelling all key coordinates and asymptotes with their equations. (2 marks)



$\frac{1}{x+1} \Rightarrow \frac{-1}{(x+1)^2} = -1.$

c. Calculate $f'(x)$ and state its domain. (2 marks)

$$f'(x) = \begin{cases} \frac{1}{x+1}, & -1 < x < 0 \\ 1, & 0 \le x < 3 \end{cases}$$

Can't diff endpoint

Domain $f' = (-1, 3)$

d. Hence, sketch the graph of $y = f'(x)$ on the same axes provided in **part b.** (2 marks)

e. State the values of x for which $f''(x)$ is defined. (1 mark)

Sharp point of f' at $x = 0$

Dom $f' = (-1, 3)$

Dom $f'' = (-1, 3) \setminus \{0\}$.

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Question 5 (7 marks)

Consider the function $g(x) = (x-1)^3 e^{2x}$.

a. Show that $g'(x) = e^{2x}(x-1)^2(2x+1)$. (2 marks)

$g'(x) = 3(x-1)^2 e^{2x} + (x-1)^3 \cdot e^{2x} \cdot 2$

$= e^{2x} \cdot (x-1)^2 \cdot [3 + 2(x-1)]$

$= e^{2x} \cdot (x-1)^2 \cdot (2x+1)$

b. Find the coordinates of any stationary points of g. (2 marks)

$g'(x) = 0$

$x - 1 = 0$. $\quad 2x + 1 = 0$

$x = 1$, $\quad x = \frac{-1}{2}$.

$\therefore (1, 0)$, $\left(\frac{-1}{2}, \frac{-27}{8e}\right)$

It is known that $g''(x) = 2e^{2x}(2x^3 - 3x + 1)$.

c. Use the second derivative to determine the nature of any stationary points of g. (2 marks)

$g''(1) = 2e^2 \cdot (2 - 3 + 1)$

$= 2e^2 \cdot (0) = 0$

$g'' = 0$

$\therefore$ S.P.I

$g''(-\frac{1}{2}) = 2e^{-1} \cdot (2 \cdot \frac{-1}{8} + \frac{3}{2} + 1)$

$= 2e^{-1} \cdot (\frac{-1}{4} + \frac{3}{2} + 1)$

$= 2e^{-1} \cdot (\frac{9}{4}) = +$

$\therefore$ local Min

$\lim_{x\to\infty}\left(\frac{x}{x^2+4}\right)=0.$

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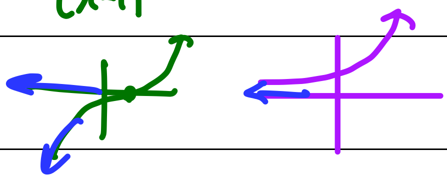
d. Evaluate $\lim_{x\to-\infty} g(x)$. (1 mark)

$\lim_{x\to-\infty}\left((x-1)^3 e^{2x}\right)=0.$

"Who is faster?"

$e^{2x} > (x-1)^3.$

$(x-1)^3$



Question 6 (6 marks)

Consider the function $f : [-4, 1] \to R, f(x) = x^2 e^x$.

a. Find $f'(x)$. (1 mark)

Rule + Down.

$f'(x) = 2xe^x + x^2e^x$

$x \in (-4, 1)$

b. Hence, find the exact coordinates of any stationary points of f. (2 marks)

$f'(x) = 2xe^x + x^2e^x = 0$

$e^x(2x + x^2) = 0$

$x(x+2) = 0$

$x = 0, -2.$

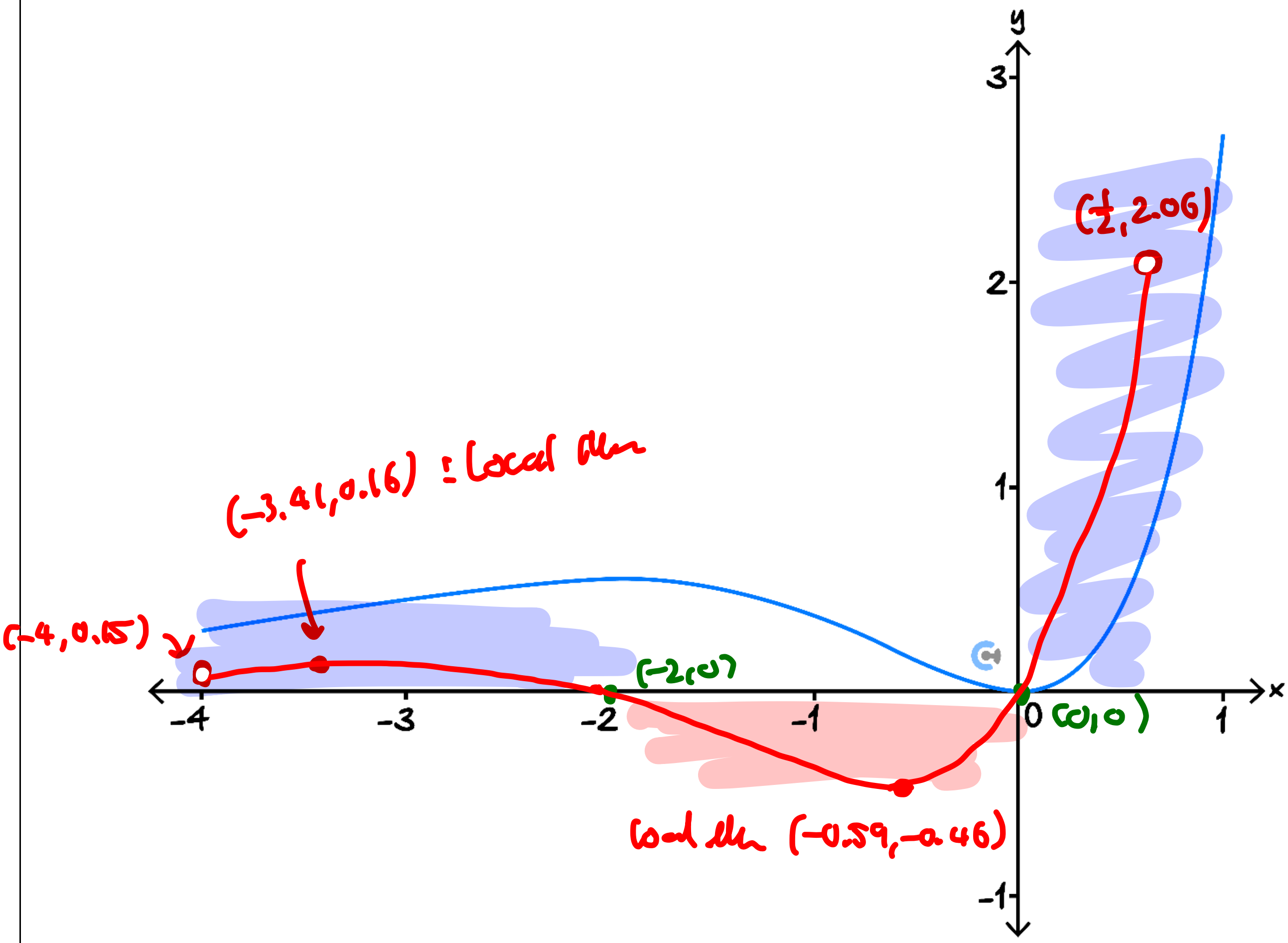
$\therefore (0,0), (-2, 4e^{-2})$

$f'(x)=0.$

$x=0, -2$

c. The graph of $y = f(x)$ is shown in the axes below. Sketch the graph of $y = f'(x)$ for $x \in \left(-4, \frac{1}{2}\right)$ on the axes below. Label the endpoints with their exact coordinates.

Use the fact that the local maximum of $y = f'(x)$ occurs at approximately $(-3.41, 0.16)$, the local minimum occurs at approximately $(-0.59, -0.46)$, $f'(-4) \approx 0.15$ and $f'\left(\frac{1}{2}\right) \approx 2.06$. (3 marks)



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Section D: Tech Active Exam Skills

Calculator Commands: Finding Derivatives

Mathematica	TI-Nspire	Casio
`f'[x]` `D[f[x], x]`	Shift Minus. $\frac{d}{dx}(f(x))$	Math 2. $\frac{d}{dx}(f(x))$

Calculator Commands: Finding Second Derivatives

Mathematica	TI-Nspire	Casio
`f''[x]` `D[f[x], {x, 2}]`	Shift Minus. $\frac{d^2}{dx^2}(f(x))$	Math 2. $\frac{d^2}{dx^2}(f(x))$

Calculator Commands: Stationary Point

- ALWAYS sketch the graph first to get an idea of the nature of the stationary point.
- The turning points for a function $f(x)$ can be found by solving $f'(x) = 0$ and subbing the result into f.
- **Example:** Find the turning point for $f(x) = e^{-x^2+2x}$.

Mathematica

```
In[4]:= f[x_] := Exp[-x^2 + 2 x]
In[5]:= Solve[f'[x] == 0 && y == f[x], Reals]
Out[5]= {{x → 1, y → e}}
```

TI-Nspire

Define $f(x)=e^{-x^2+2\cdot x}$	Done
$\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$	$x=1$
$f(1)$	e

Casio Classpad

define $f(x) = e^{-x^2+2x}$	done
$\text{solve}(\frac{d}{dx}(f(x))=0, x)$	$\{x=1\}$
$f(1)$	e

Section E: Exam 2 Questions (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.

Question 7 (1 mark)

Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x+1}$. If $h(x) = f(x)g(x)$, then the maximal domain of the derivative of h is:

Ended and

A. $[-1, 3]$

Ti: domain$(\sqrt{3-x} \times \sqrt{x+1}, x)$

B. $(-1, 3)$

$x \in [-1, 3]$

C. $[-1, \infty)$

D. $(-\infty, -1) \cup (3, \infty)$

Max Func Dom$[\sqrt{\ } \times \sqrt{\ }, x]$

Question 8 (1 mark)

Which one of the following functions is differentiable for all real values of x?

A. $f(x) = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$

B. $f(x) = \begin{cases} 8x + 4, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

C. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

D. $f(x) = \begin{cases} 4x + 1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

4

$2(2x+1)\cdot 2$

$= 2(1)\cdot 2 = 4$

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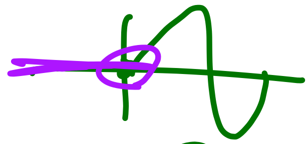
Question 9 (1 mark)

TI:

CASIO:

Consider the following function:

$$f(x) = \begin{cases} 0, & x < 0 \\ \sin(x), & x \geq 0 \end{cases}$$



Which of the following statements is true?

A. The function is continuous at $x = 0$ and differentiable at $x = 0$.

B. The function is continuous at $x = 0$ and not differentiable at $x = 0$.

C. The function is not continuous at $x = 0$ and differentiable at $x = 0$.

D. The function is not continuous at $x = 0$ and not differentiable at $x = 0$.

Question 10 (1 mark)

$4x$

If $f(x) = \begin{cases} 2x^2 - 2, & -5 \leq x \leq 0 \\ 3x - 2, & 0 < x \leq 10 \end{cases}$ the domain of $f'(x)$ is:

3

Differentiate at $x = 0$

1) Join: $-2 = -2$

2) Smooth: $0 = 3$

A. $[-5, 10]$

B. $R\backslash\{0\}$

C. $(-5, 10)$

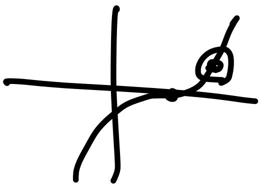
D. $(-5, 0) \cup (0, 10)$

Question 11 (1 mark)

Assume that the functions below are defined on their maximal domain, which of the following functions is differentiable at $x = 3$?

A. $f(x) = \sqrt{x - 3}$

B. $f(x) = \log_e(x - 3)$



C. $f(x) = (x - 2)^3$

D. $f(x) = \frac{1}{x-3}$

Question 12 (1 mark)

Let f be a function with a domain R such that $f'(4) = 0$ and $f'(x) < 0$ when $x \neq 4$. At $x = 4$, the graph of f has a:

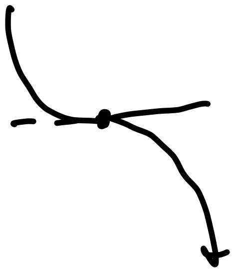
dec

A. Local minimum.

B. Local maximum.

C. Gradient of 4.

D. Stationary point of inflection.



Question 13 (1 mark)

The cubic function $R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b, and c are positive constants, have no stationary points when:

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

Solve for stat pts.

$\text{solve}\left(\frac{d}{dx}(a*x^3 - b*x^2 + c*x) = 0, x\right)$

$x = \frac{\sim \pm \sqrt{b^2 - 3ac}}{\sim}$ ← discriminant

$b^2 - 3ac < 0$

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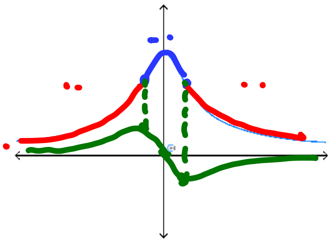
Question 14 (1 mark)

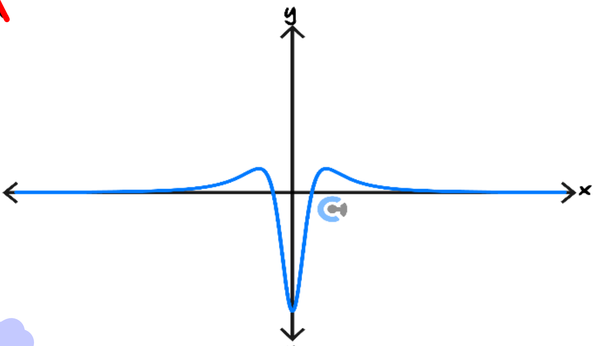
The graph of $y = f(x)$ is shown below. Which of the graphs could correspond to $y = f'(x)$?



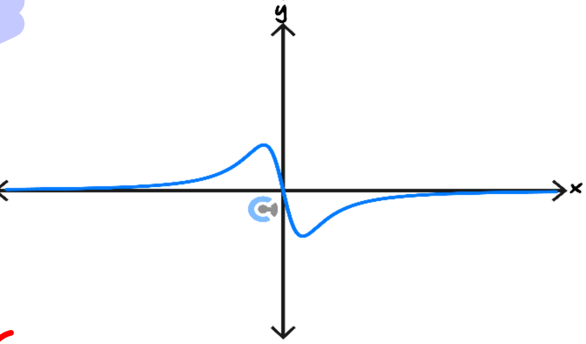
f: Inflech

f': ~~Stating~~ Pos turning

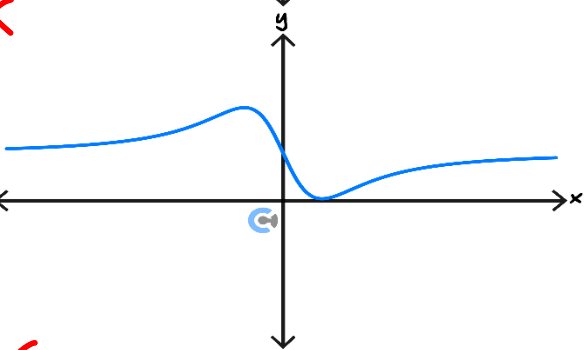
A.



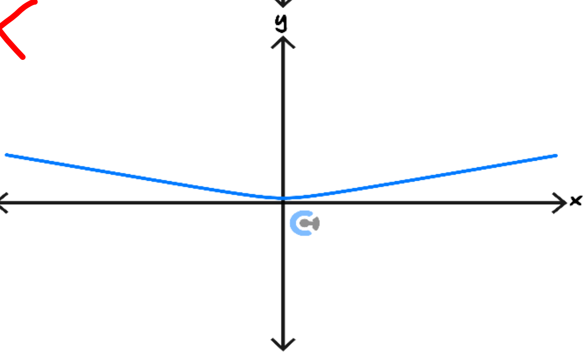
B.



C.



D.

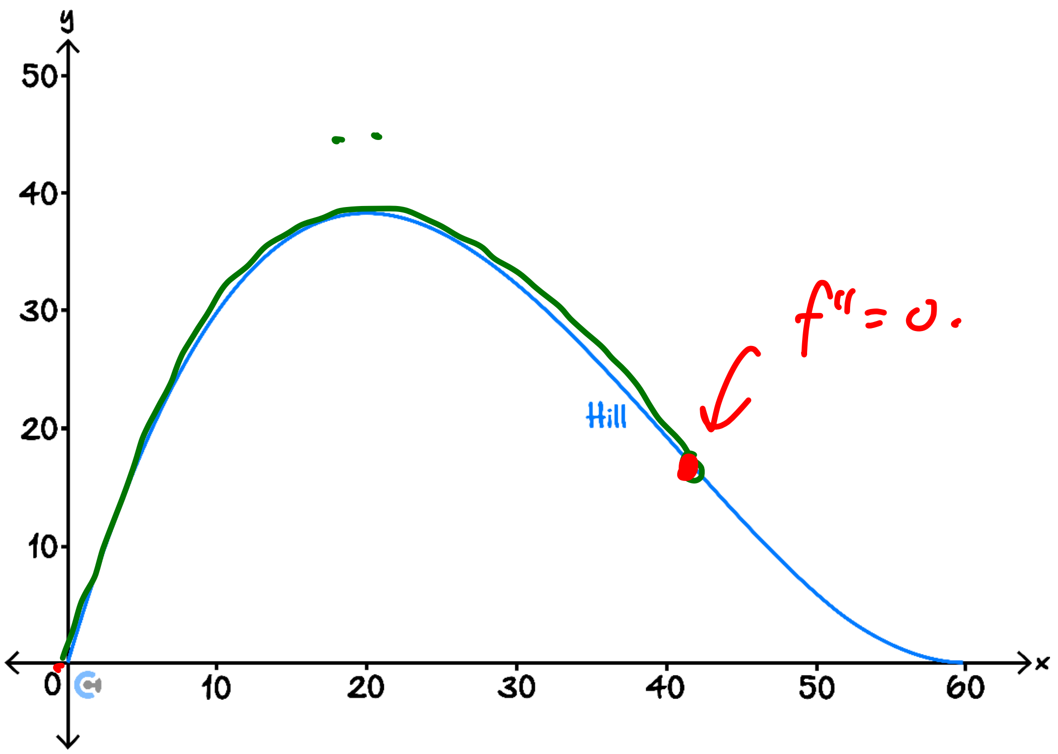


Question 15 (12 marks)

The Billboa Adventure fun camp is constructing a zip line to help attract more schools to choose them to host their school camps. The zipline is to be constructed above a hill on the property. The hill is modelled by:

$$y = \frac{3x(x-60)^2}{2500}, \qquad x \in [0, 60]$$

Where x is the horizontal distance, in metres, from an origin and y is the height, in metres, above this origin.



a. Find $\frac{dy}{dx}$. (2 marks)

$\frac{dy}{dx} = \frac{9(x-20)(x-60)}{2500} = \frac{9(x^2-80x+1200)}{2500}$ | $x \in (0, 60)$.

b. State the set of values for which the **gradient** of the hill is strictly decreasing. (1 mark)

$\frac{d^2y}{dx^2} = 0$

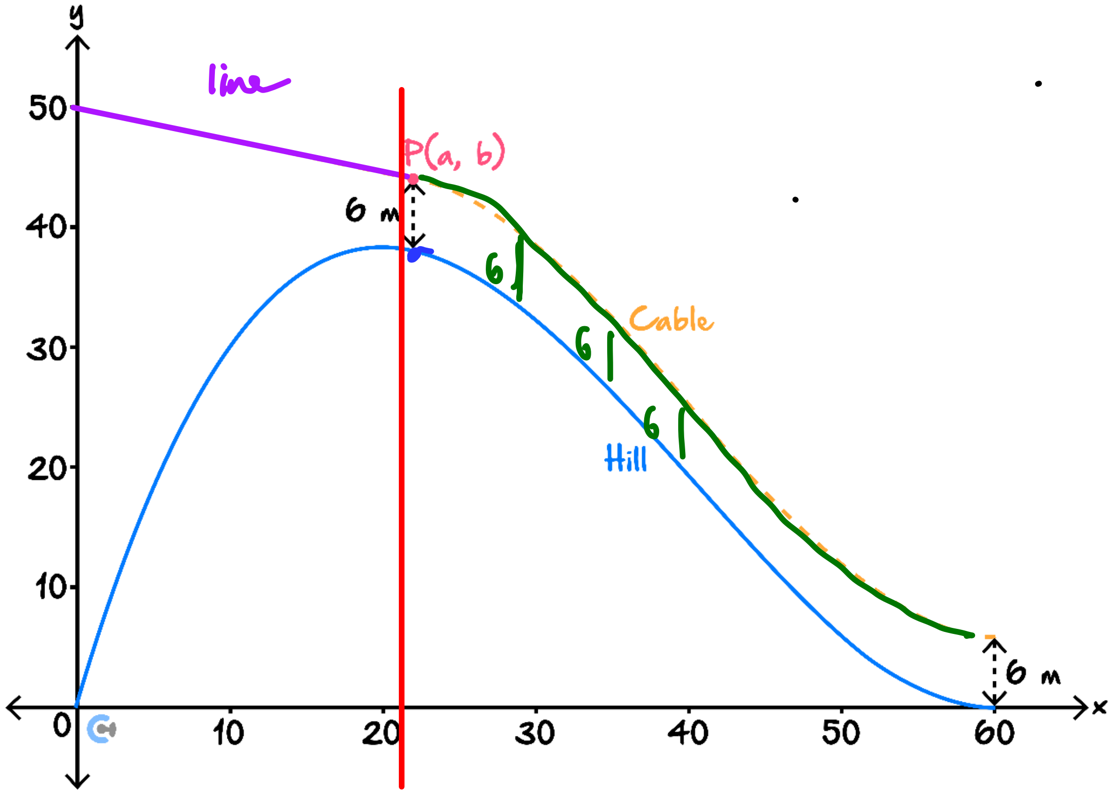
$x = 40.$

$x \in [0, 40]$

Not differentiable

Gradient is decreasing → concave down

The cable for the zip line is connected to a platform at the origin at a height of 50 metres and is straight for $0 \leq x \leq a$, where $20 \leq a \leq 30$. The straight section joins the curved section at $P(a, b)$. The cable is exactly $6\ m$ vertically above the hill from $a \leq x \leq 60$, as shown in the graph below:



c. State the rule, in terms of x, for the height of the cable above the horizontal axis for $x \in [a, 60]$. (1 mark)

Hill $+6$

$$-\frac{3x(x-60)^2}{2500}+6.$$

Plural

d. Find the values of x for which the gradient of the cable is equal to the average gradient of the hill for $x \in [20, 60]$. (3 marks)

$$\frac{y(60)-y(20)}{60-20} = \frac{dy}{dx}$$

$$x = \frac{20}{3}(6 \pm \sqrt{3})$$

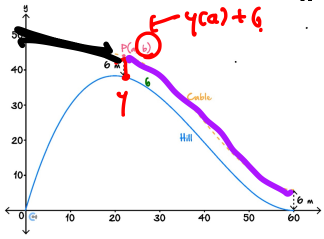
$$= 40 \pm \frac{20\sqrt{3}}{3}.$$

>20

✓

e. The gradients of the straight and curved sections of the cable approach the same value at $x = a$, so there is a continuous and smooth join at P. Differentiate

i. State the gradient of the cable at P $\boxed{\text{in terms of } a.}$ (1 mark)



$$\frac{6-50}{a-0} = \frac{y(a)+6-50}{a}$$

$$= \frac{3a^2}{2500} - \frac{18a}{125} - \frac{44}{a} + \frac{108}{25}$$

OR: $\frac{dy}{dx}\Big|_{x=a}$

$$= \frac{9a^2}{2500} - \frac{36}{125}a + \frac{108}{25}$$

ii. Find the coordinates of P, with each value correct to two decimal places. (3 marks)

$$\frac{3a^2}{2500} - \frac{18a}{125} - \frac{44}{a} + \frac{108}{25} = \frac{9a^2}{2500} - \frac{36}{125}a + \frac{108}{25}$$

$a = 21.9506$

$(21.95, \underbrace{44.13}_{y(a)+6})$

iii. Find the value of the gradient at P, correct to one decimal place. (1 mark)

-0.3

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Section F: Extension Exam 1 (10 Marks)

INSTRUCTION: 10 Marks. 10 Minutes Writing.

Question 16 (10 marks)

Consider the function $f(x) = x^2 e^{-x^2}$.

a. Show that $f(x) = f(-x)$ and hence, state whether f is an even or an odd function. (1 mark)

$f(-x) = (-x)^2 e^{-(-x)^2} = x^2 e^{-x^2} = f(x)$.
f is an even function.

b.

i. Find $f'(x)$. (1 mark)

$f'(x) = 2xe^{-x^2} + x^2(-2x)e^{-x^2} = 2xe^{-x^2} - 2x^3e^{-x^2} = 2xe^{-x^2}(1 - x^2)$

ii. Hence, determine the coordinates for any stationary points of f. (2 marks)

$f'(x) = 0 \implies x = 0$ or $1 - x^2 = 0$. So $x = -1, 0, 1$.
$f(0) = 0$ and $f(-1) = f(1) = e^{-1} = \dfrac{1}{e}$.
So stationary points $(-1, e^{-1}), (0, 0)$ and $(1, e^{-1})$

c.

i. Show that the second derivative of f is given by $f''(x) = 2e^{-x^2}(2x^4 - 5x^2 + 1)$. (2 marks)

$f'(x) = 2xe^{-x^2}(1 - x^2)$ and so using the product rule:

$$\begin{aligned} f''(x) &= (2e^{-x^2} - 4x^2e^{-x^2})(1 - x^2) - 4x^2e^{-x^2} \\ &= 2e^{-x^2} - 2x^2e^{-x^2} - 4x^2e^{-x^2} + 4x^4e^{-x^2} - 4x^2e^{-x^2} \\ &= 2e^{-x^2} - 10x^2e^{-x^2} + 4x^4e^{-x^2} \\ &= 2e^{-x^2}(2x^4 - 5x^2 + 1) \end{aligned}$$

ii. Use the second derivative to determine the nature of the stationary points found in **part b. ii.** (2 marks)

$f''(-1) = f''(1) = 2e^{-1}(2 - 5 + 1) = -\frac{4}{e} < 0$. Therefore $(-1, e^{-1})$ and $(1, e^{-1})$ are local maximums.
$f''(0) = 2$. Therefore $(0, 0)$ is a local minimum.

iii. Determine how many points of inflection f has. (2 marks)

$f''(x) = 0 \implies 2x^4 - 5x^2 + 1 = 0$. Let $g(x) = 2x^4 - 5x^2 + 1$. Let $a = x^2$ then

$$2a^2 - 5a + 1 = 0 \implies a = \frac{5 \pm \sqrt{17}}{4}$$

Note that both these solutions are greater than zero, hence g has four roots. Thus f has four points of inflection.

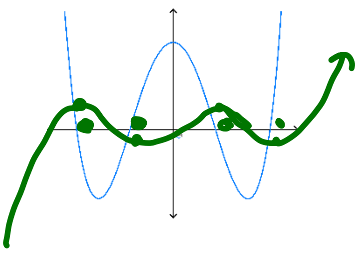
Space for Personal Notes

Section G: Extension Exam 2 (18 Marks)

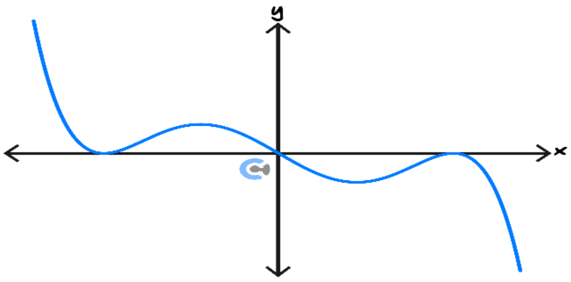
INSTRUCTION: 18 Marks. 18 Minutes Writing.

Question 17 (1 mark) Antidiff

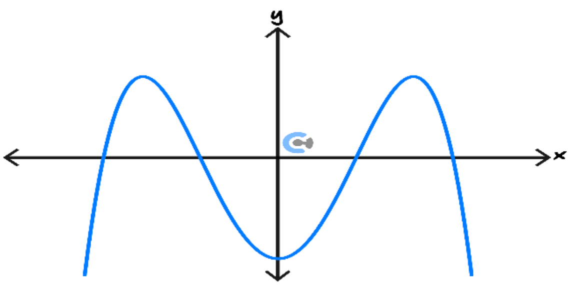
The graph of $y = f'(x)$ is shown below. The graph of $y = f(x)$ could be:



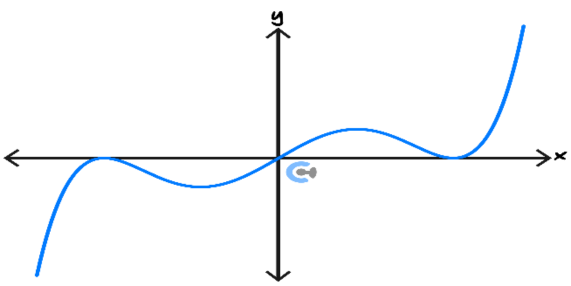
A.



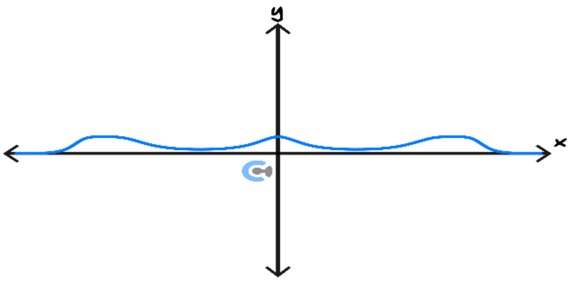
B.



C.



D.



Question 18 (1 mark)

Let $f : \mathbb{R} \to \mathbb{R}$,

quartic

$$f(x) = (x-2)(x-3)(4x^2 + ax + 6)$$

Where a is a real number. If f has p stationary points, the possible values of p are:

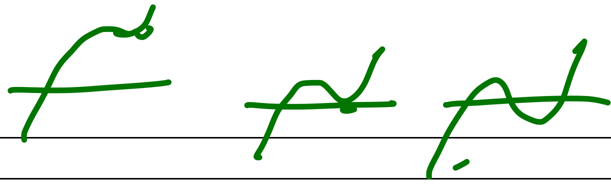
f': cubic = 0

A. $p \in \{1, 2, 3, 4\}$

B. $p \in \{0, 1, 2, 3\}$

C. $p \in \{1, 2\}$

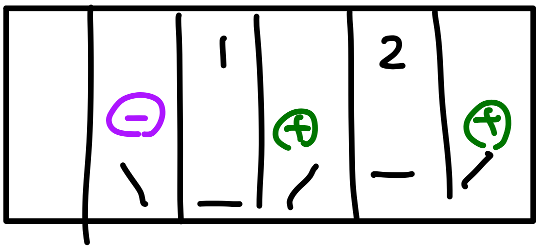
D. $p \in \{1, 2, 3\}$



Question 19 (1 mark)

If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that:

- $f'(x) = 0$ at $x = 1$ and $x = 2$.
- $f'(x) > 0$ at $x > 2$ and $1 < x < 2$.
- $f'(x) < 0$ at $x < 1$.



Which of the following statements is **correct**?

A. The graph has a stationary point of inflection at $x = 2$ and a maximum at $x = 1$.

B. The graph has a stationary point of inflection at $x = 2$ and a minimum at $x = 1$.

C. The graph has a stationary point of inflection at $x = 1$ and a minimum at $x = 2$.

D. The graph has a minimum at $x = 1$ and a maximum at $x = 2$.

Space for Personal Notes

Question 20 (1 mark)

Let $f^{(n)}(x)$ be the n^{th} derivative of $f(x)$. That is, it has been differentiated n times, where $n = 2p$ and p is a positive integer. If $f(x) = xe^{-x^3}$, how many axial intercepts does $f^{(n)}(x)$ have?

A. n

B. $2n$

C. $2n + 1$

D. $n + 1$

$n=2$: $f''(x) = (3x^3-4)\cdot x^2 \cdot 3e^{-x^3}$. 2 soln.

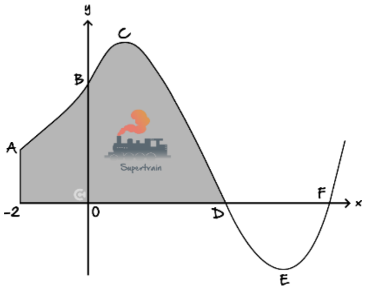
$n=4$: $f''''(x) = 0$, 3 soln + y int = 4 soln

$n=6$:

```
Solve[f''''''[x] == 0, x, Reals]
풀이 함수              실수 영역
{{x → 0}, {x → 0.508…}, {x → 1.02…}, {x → 1.49…}, {x → 1.96…}}
```

Question 21 (14 marks)

A part of the track for Tim's model train follows the curve passing through A, B, C, D, E and F shown:



Tim designed it by putting axes on the drawing as shown. The track is made up of two curves, one to the left of the y-axis and the other to the right.

B is the point $(0, 7)$. The curve from B to F is part of the graph of $f(x) = px^3 + qx^2 + rx + s$ where p, q, r and s are constants and $f'(0) = 4.25$.

a. Show that $s = 7$ and $r = 4.25$. (2 marks)

$f(0) = s = 7$.

$f'(x) = 3px^2 + 2qx + r$

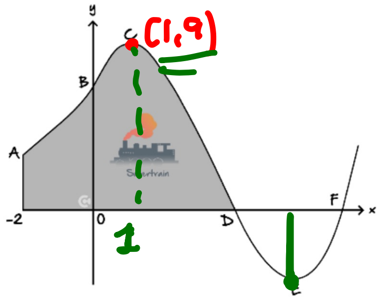
$f'(0) = r = 4.25$

$C\ (1, 9)$ is the furthest point reached by the track in the positive y direction.

b. Use this information to write two equations involving p and q. (2 marks)

$f(1) = 9 \rightarrow p + q + 4.25 + 7 = 9$

$f'(1) = 0 \rightarrow 3p + 2q + 4.25 = 0$



The values of p and q are $p = 0.25$ and $q = -2.5$, respectively. So, $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$.

c.

i. Find the exact coordinates of D and F. (1 mark)

D: $(4, 0)$. F: $(7, 0)$

ii. Find the greatest distance that the track is from the x-axis, when it is below the x-axis, correct to two decimal places. (1 mark)

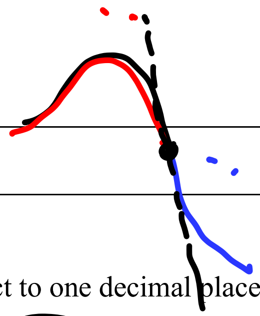
$f'(x) = 0$ $\quad f(\sim) = -3.70.$

$x = 1, \sim$

3.70 units

d.

i. Find the value of x for which $f''(x) = 0$. (1 mark)



$x = \frac{10}{3}$

ii. Hence, find the steepest decline of f between C and D, correct to one decimal place. (1 mark)

$f'\left(\frac{10}{3}\right) = -\frac{49}{12} \approx -4.1$

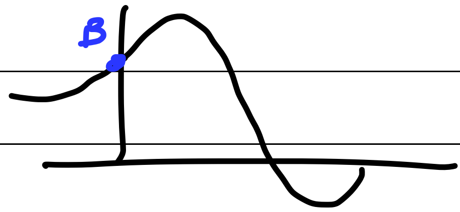
The curve from A to B is part of the graph with the equation $g(x) = \frac{a}{1-bx}$, where a and b are positive real constants. The track passes smoothly from one section of the track to the other at B (that is, the gradients of the curves are equal at B).

e. Find the exact values of a and b. (3 marks)

Join) $g(0) = f(c)$

Smooth) $g'(0) = f'(0)$

$a = 7, \quad b = \frac{12}{28}.$



Tim adds a new part to the track when $x = 8$. Call the point on the track when $x = 8$, G.
This new section of the track is modelled by the function $h(x) = m \cdot k^{-(x-8)}$ for $x \geq 8$ and $m, k > 0$.

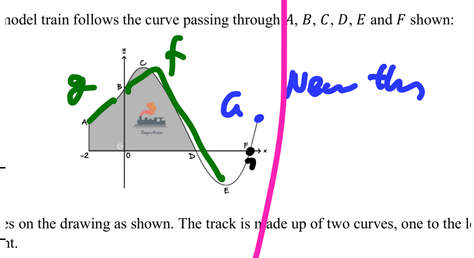
Restriction

f.

i. Find the value of m so that the track is continuous at G. (1 mark)

$h(8) = f(8)$

$m = 9.$



TI & CASIO Graph: Try each k (Sliders)

Reflection

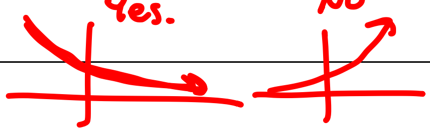
ii. It is known that $\lim_{x\to\infty} h(x) = 0$. Find the possible values of k. (1 mark)

solve $\left(\lim_{x\to\infty}\left(9 \cdot k^{-(x-8)}\right) = 0, k\right)$

$\log(k) > 0$

$k > 1$

Yes. No



```
f[x_] := 9 * k^-(x-8)

Limit[f[x], x → Infinity]
|극한                |무한대

0 if Log[k] > 0
```

solve.

iii. Find the value of k if $h(x)$ passes through the point $(10,4)$. (1 mark)

$9 \cdot k^{-(10-8)} = 4$

$k = 3/2$ $\quad k \neq -\frac{3}{2}$ as $k > 0$

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