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VCE Mathematical Methods ¾
Differentiation II [0.10]
Workshop

Error Logbook:

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Section A: Recap

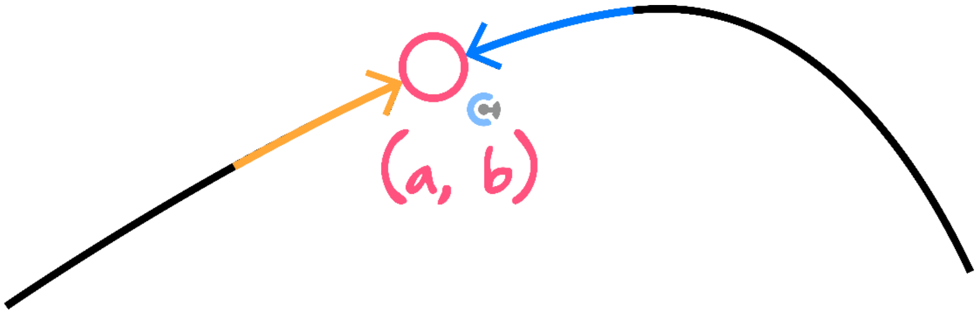
Limits

$$\lim_{x \to a} f(x) = L$$

"The function $f(x)$ approaches L as x approaches a."

- Limit is the value that a function (y-value) approaches as the x-value approaches a value.

Validity of Limits



$$\lim_{x \to a} f(x) \text{ exists if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

- Limit is defined when the left limit equals the right limit.

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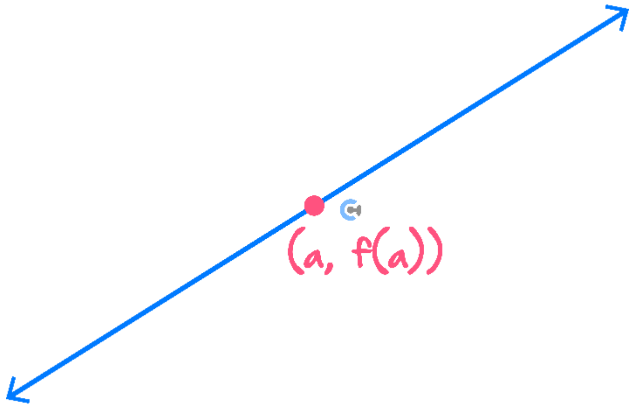
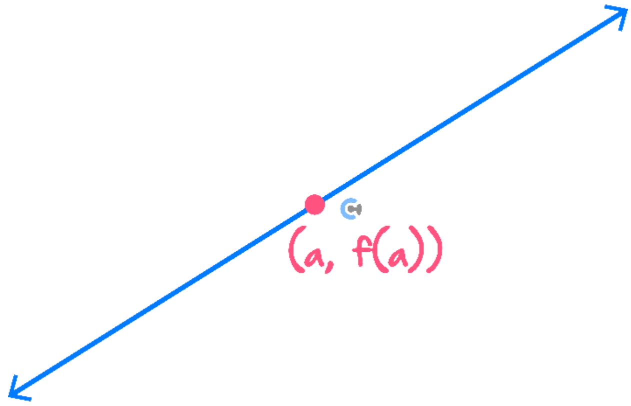
Continuity



- A function f is said to be continuous at a point $x = a$ if:
  1. $f(x)$ is defined at $x = a$.
  2. $\lim_{x \to a} f(x)$ exists.
  3. $\lim_{x \to a} f(x) = f(a)$.

Differentiability



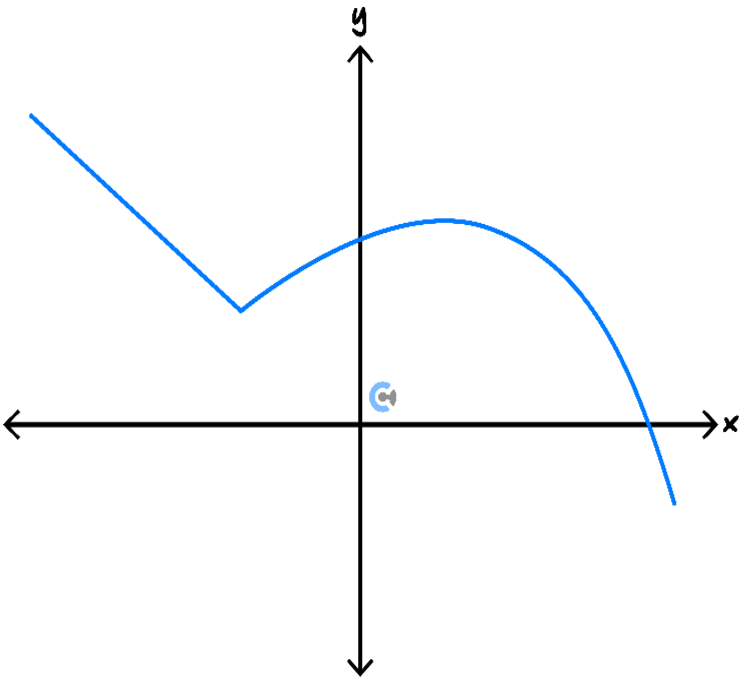
- A function f is said to be differentiable at a point $x = a$ if:
  1. $f(x)$ is continuous at $x = a$.
  2. $\lim_{x \to a} f'(x)$ exists.
     - Limit exists when the left and right limits are the same.
     - Gradient on the LHS and RHS must be the same.

- We **cannot** differentiate:

  1. Discontinuous points.
  2. Sharp points.
  3. Endpoints.

Definition

Finding the Derivative of Hybrid Functions



1. Simply derive each function.
2. Reject the values for x that are not differentiable from the domain.

Definition

Second Derivatives

- The derivative of the derivative.
- To get the second derivative, we can **differentiate the original function twice**.

$$\frac{d^2y}{dx^2} = f''(x)$$

Concavity

- Concave up is when the gradient is increasing.

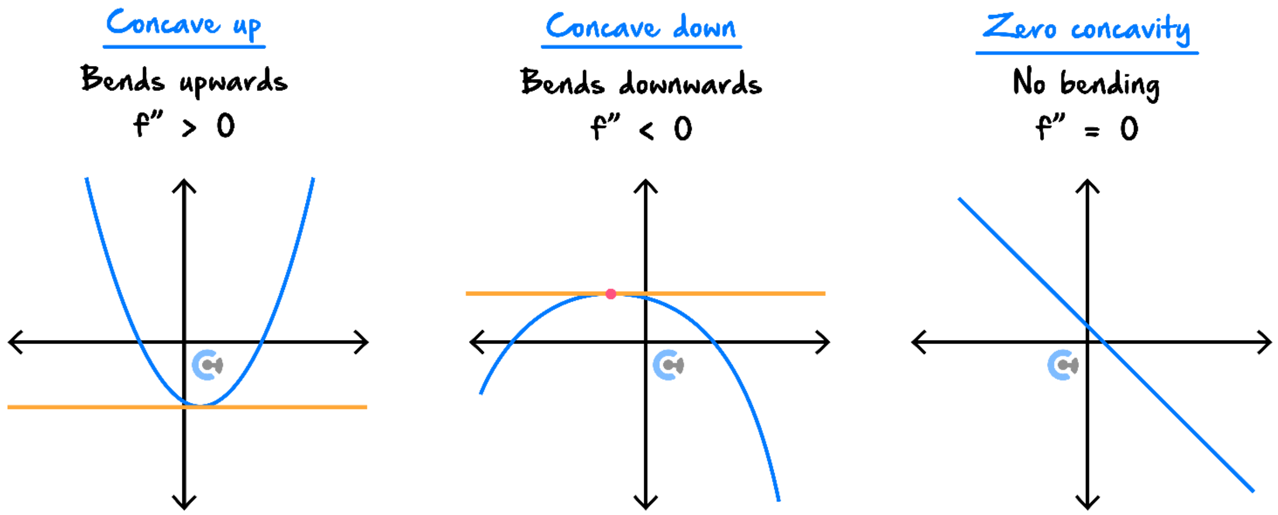
$$f''(x) > 0 \rightarrow \text{Concave Up}$$

- Concave down is when the gradient is decreasing.

$$f''(x) < 0 \rightarrow \text{Concave Down}$$

- Zero concavity is when the gradient is neither increasing nor decreasing.

$$f''(x) = 0 \rightarrow \text{Zero Concavity}$$



Concavity is also linked to how the curve is bent.

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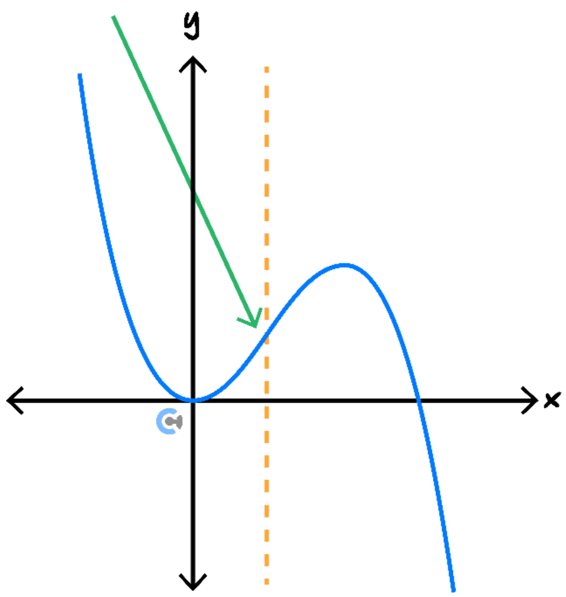
Points of Inflection

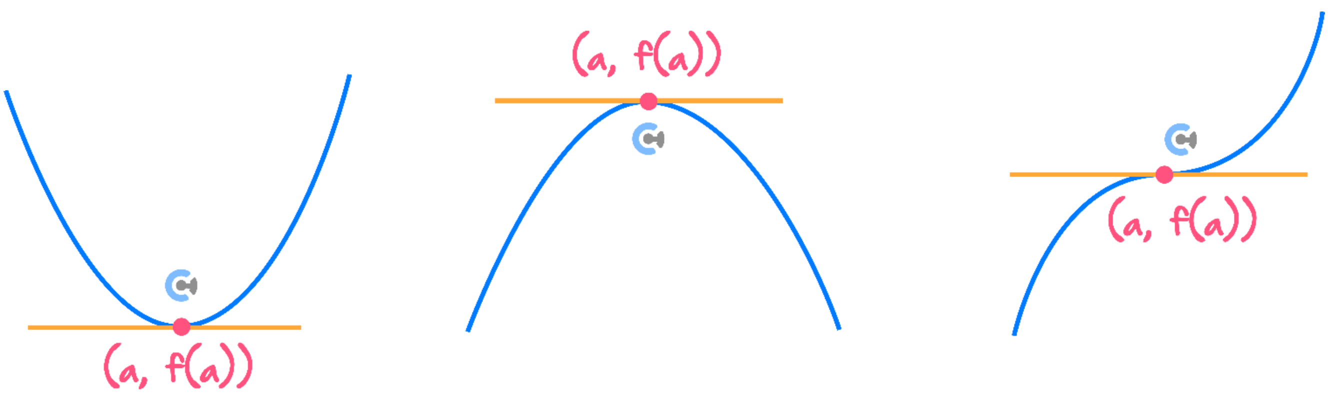
- A point at which a curve **changes concavity** is called a **point of inflection**.



$$f''(x) = 0$$

- Simply, it is when the bending changes.

The Second Derivative Test



- Suppose that $f'(a) = 0$ and hence, f has a stationary point at $x = a$. The second derivative test states:

  - **Concave up gives us a local minimum.**

$$f''(x) > 0 \rightarrow \text{Local Minimum}$$

- Concave down gives us a local maximum.

$$f''(x) < 0 \rightarrow \text{Local Maximum}$$

- Zero concavity gives us a stationary point of inflection.

$$f''(x) = 0 \rightarrow \text{Stationary Point of Inflection}$$

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Section B: Warm Up

Question 1

Evaluate the following limits if they exist, otherwise, explain why they do not exist.

a. $\lim_{x \to 3}(x^2 - 3)$

b. $\lim_{x \to 0}\left(\frac{x^2-2x}{x}\right)$

c. $\lim_{x \to 1}\left(\frac{3}{x-1}\right)$

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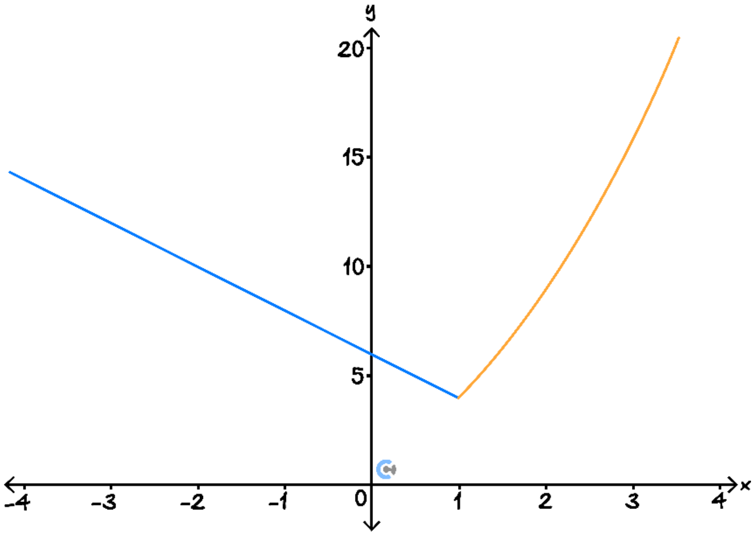
Question 2

a. Consider the function $f(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ 6 - 2x, & x < 1 \end{cases}$ shown in the graph below:



i. State all values of x for which f is continuous.

---

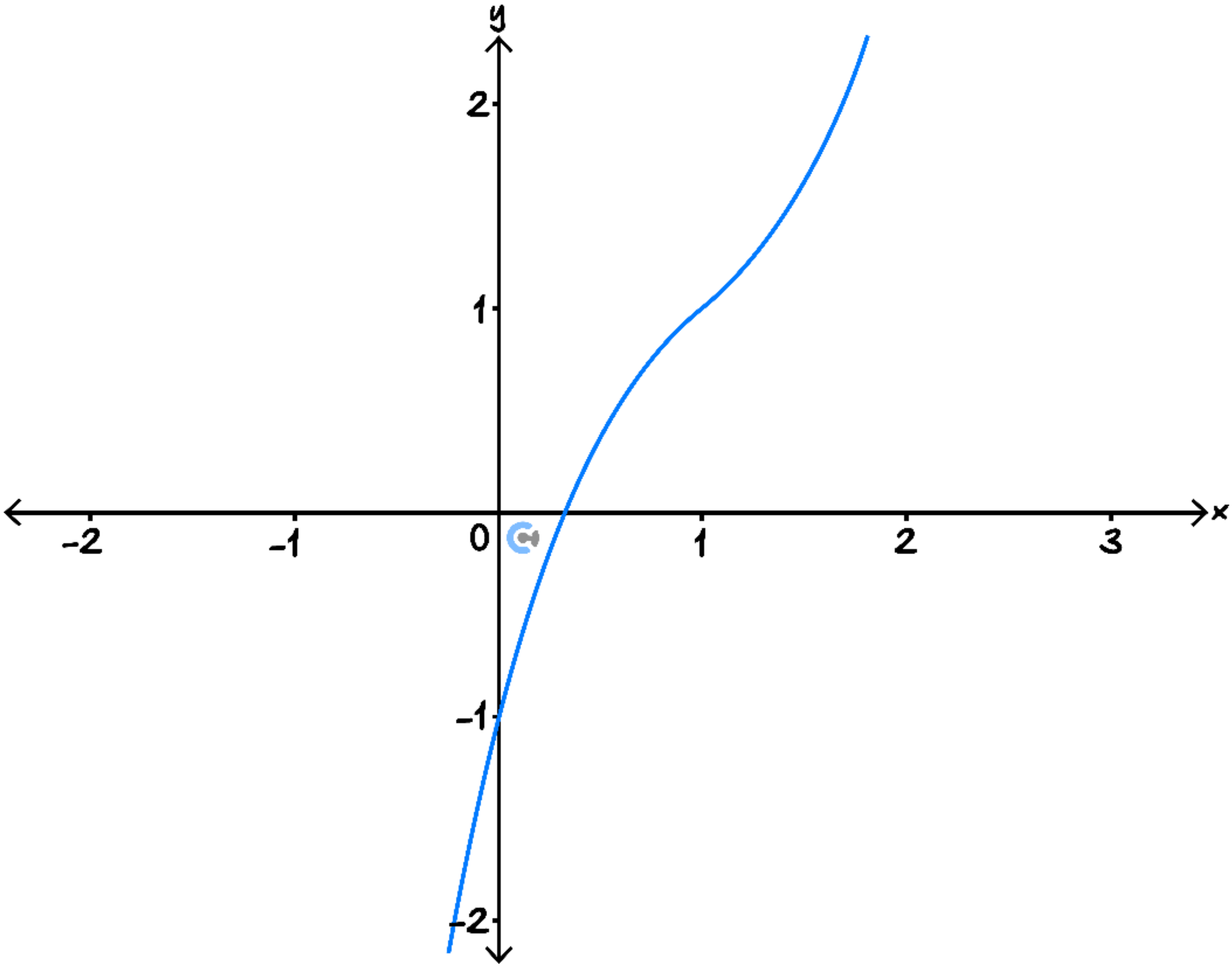
ii. State all values of x for which f is differentiable.

---

b. The function $g(x) = \begin{cases} x^2 + 2x + 1, & x \geq 1 \\ ax + b, & x < 1 \end{cases}$ is continuous and differentiable for all $x \in \mathbb{R}$. Determine the values of a and b.

Question 3

a. Consider the graph of the cubic polynomial shown below:



i. Circle the point of inflection.

ii. Is the graph concave up or concave down after the point of inflection?

b. Consider the polynomial function $f(x) = x^3 - 3x^2 + 2$.

i. Determine the coordinates of any stationary points of f.

ii. Use the second derivative to determine the nature of any stationary points of f.

iii. Determine when f is concave down.

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Section C: Exam 1 Questions (22 Marks)

INSTRUCTION: 22 Marks. 27 Minutes Writing.

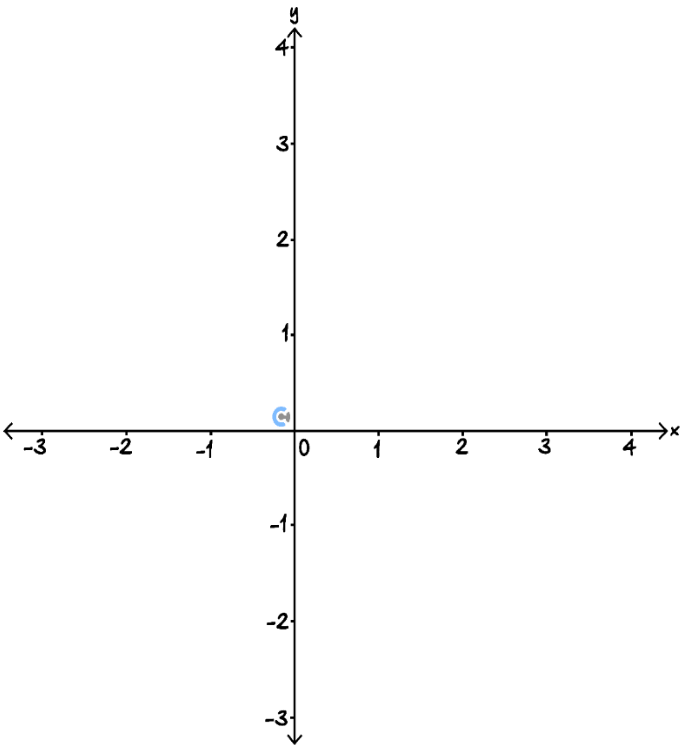
Question 4 (9 marks)

Consider the piecewise function:

$$f : R \to R, f(x) = \begin{cases} \log_e(x+1), & -1 < x < 0 \\ ax + b, & 0 \le x \le 3 \end{cases}$$

a. Determine the values of a and b such that the graph of f is differentiable at $x = 0$. (2 marks)

b. Hence, sketch the function f for $-1 < x \le 3$ on the axes below, labelling all key coordinates and asymptotes with their equations. (2 marks)



c. Calculate $f'(x)$ and state its domain. (2 marks)

d. Hence, sketch the graph of $y = f'(x)$ on the same axes provided in **part b**. (2 marks)

e. State the values of x for which $f''(x)$ is defined. (1 mark)

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Question 5 (7 marks)

Consider the function $g(x) = (x-1)^3 e^{2x}$.

a. Show that $g'(x) = e^{2x}(x-1)^2(2x+1)$. (2 marks)

b. Find the coordinates of any stationary points of g. (2 marks)

It is known that $g''(x) = 2e^{2x}(2x^3 - 3x + 1)$.

c. Use the second derivative to determine the nature of any stationary points of g. (2 marks)

d. Evaluate $\lim_{x \to -\infty} g(x)$. (1 mark)

Question 6 (6 marks)

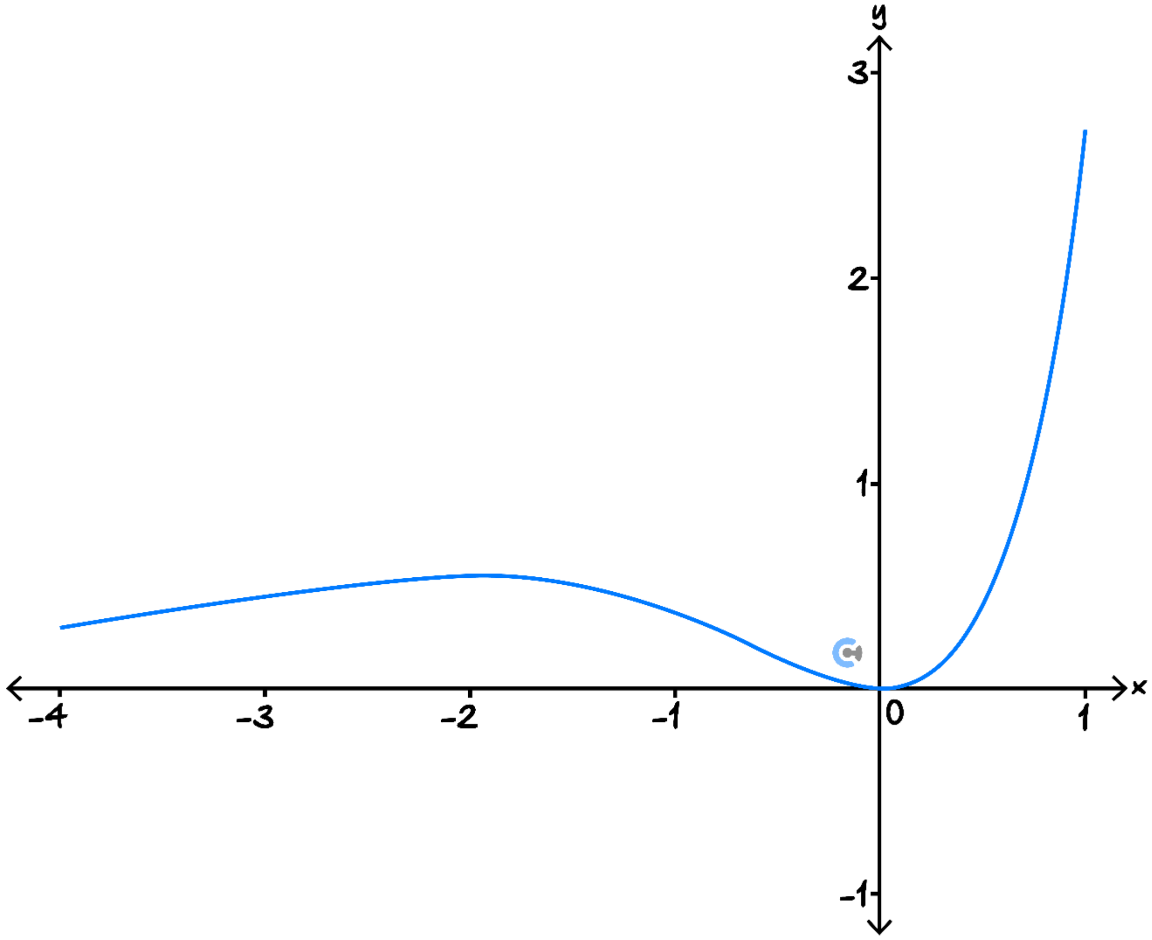
Consider the function $f : [-4, 1] \to R, f(x) = x^2 e^x$.

a. Find $f'(x)$. (1 mark)

b. Hence, find the exact coordinates of any stationary points of f. (2 marks)

c. The graph of $y = f(x)$ is shown in the axes below. Sketch the graph of $y = f'(x)$ for $x \in \left(-4, \frac{1}{2}\right)$ on the axes below. Label the endpoints with their exact coordinates.

Use the fact that the local maximum of $y = f'(x)$ occurs at approximately $(-3.41, 0.16)$, the local minimum occurs at approximately $(-0.59, -0.46)$, $f'(-4) \approx 0.15$ and $f'\left(\frac{1}{2}\right) \approx 2.06$. (3 marks)



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Section D: Tech Active Exam Skills

Calculator Commands: Finding Derivatives

- **Mathematica**

```
f'[x]
D[f[x], x]
```

- **TI-Nspire**
  - Shift Minus.

$$\frac{d}{dx}(f(x))$$

- **Casio**
  - Math 2.

$$\frac{d}{dx}(f(x))$$

Calculator Commands: Finding Second Derivatives

- **Mathematica**

```
f''[x]
D[f[x], {x, 2}]
```

- **TI-Nspire**
  - Shift Minus.

$$\frac{d^2}{dx^2}(f(x))$$

- **Casio**
  - Math 2.

$$\frac{d^2}{dx^2}(f(x))$$

Calculator Commands: Stationary Point

- ALWAYS sketch the graph first to get an idea of the nature of the stationary point.
- The turning points for a function $f(x)$ can be found by solving $f'(x) = 0$ and subbing the result into f.
- **Example:** Find the turning point for $f(x) = e^{-x^2+2x}$.

- **Mathematica**

```
In[4]:= f[x_] := Exp[-x^2 + 2 x]
In[5]:= Solve[f'[x] == 0 && y == f[x], Reals]
Out[5]= {{x → 1, y → e}}
```

- **TI-Nspire**

Define $f(x)=e^{-x^2+2\cdot x}$	*Done*
solve$\left(\frac{d}{dx}(f(x))=0,x\right)$	$x=1$
$f(1)$	e

- **Casio Classpad**

define $f(x) = e^{-x^2+2x}$	done
solve$\left(\frac{d}{dx}(f(x))=0, x\right)$	$\{x=1\}$
$f(1)$	e

Section E: Exam 2 Questions (20 Marks)

INSTRUCTION: 20 Marks. 24 Minutes Writing.

Question 7 (1 mark)

Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x+1}$. If $h(x) = f(x)g(x)$, then the maximal domain of the derivative of h is:

A. $[-1, 3]$

B. $(-1, 3)$

C. $[-1, \infty)$

D. $(-\infty, -1) \cup (3, \infty)$

Question 8 (1 mark)

Which one of the following functions is differentiable for all real values of x?

A. $f(x) = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$

B. $f(x) = \begin{cases} 8x+4, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

C. $f(x) = \begin{cases} 2x+1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

D. $f(x) = \begin{cases} 4x+1, & x < 0 \\ (2x+1)^2, & x \geq 0 \end{cases}$

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Question 9 (1 mark)

Consider the following function:

$$f(x) = \begin{cases} 0, & x < 0 \\ \sin(x), & x \geq 0 \end{cases}$$

Which of the following statements is true?

A. The function is continuous at $x = 0$ and differentiable at $x = 0$.

B. The function is continuous at $x = 0$ and not differentiable at $x = 0$.

C. The function is not continuous at $x = 0$ and differentiable at $x = 0$.

D. The function is not continuous at $x = 0$ and not differentiable at $x = 0$.

Question 10 (1 mark)

If $f(x) = \begin{cases} 2x^2 - 2, & -5 \leq x \leq 0 \\ 3x - 2, & 0 < x \leq 10 \end{cases}$, the domain of $f'(x)$ is:

A. $[-5, 10]$

B. $R\backslash\{0\}$

C. $(-5, 10)$

D. $(-5, 0) \cup (0, 10)$

Question 11 (1 mark)

Assume that the functions below are defined on their maximal domain, which of the following functions is differentiable at $x = 3$?

A. $f(x) = \sqrt{x - 3}$

B. $f(x) = \log_e(x - 3)$

C. $f(x) = (x - 2)^3$

D. $f(x) = \frac{1}{x-3}$

Question 12 (1 mark)

Let f be a function with a domain R such that $f'(4) = 0$ and $f'(x) < 0$ when $x \neq 4$. At $x = 4$, the graph of f has a:

A. Local minimum.

B. Local maximum.

C. Gradient of 4.

D. Stationary point of inflection.

Question 13 (1 mark)

The cubic function $R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b, and c are positive constants, have no stationary points when:

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

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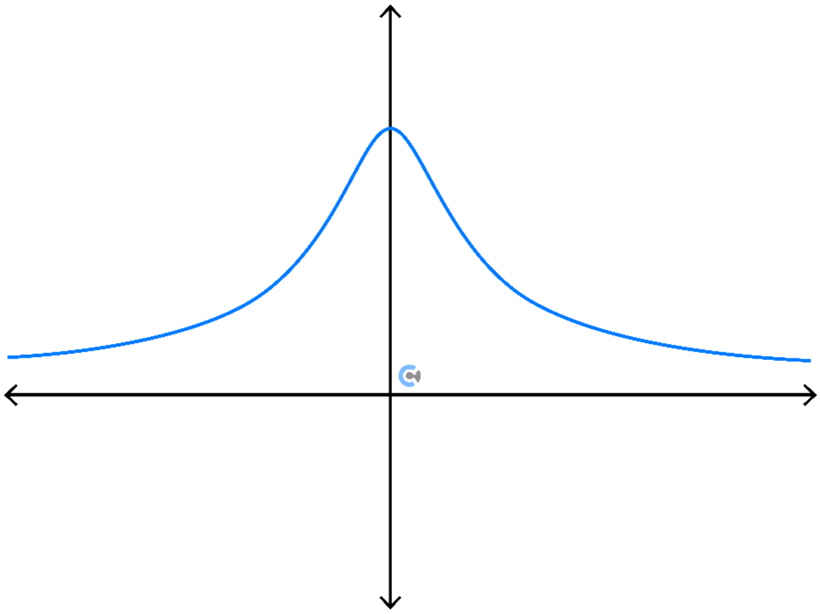
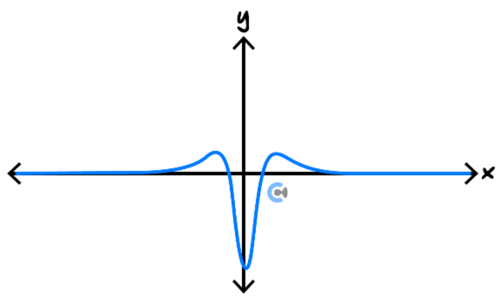
Question 14 (1 mark)

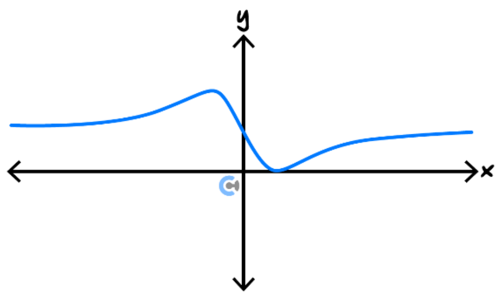
The graph of $y = f(x)$ is shown below. Which of the graphs could correspond to $y = f'(x)$?



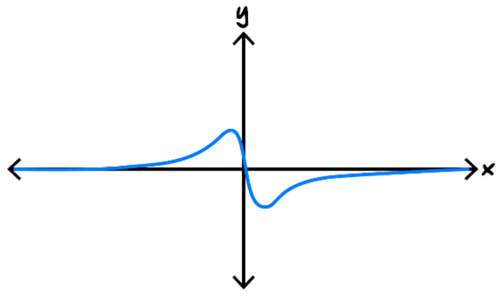
A.



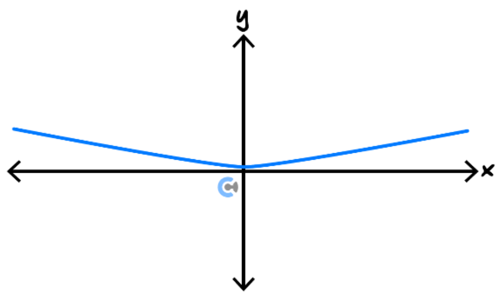
C.



B.



D.



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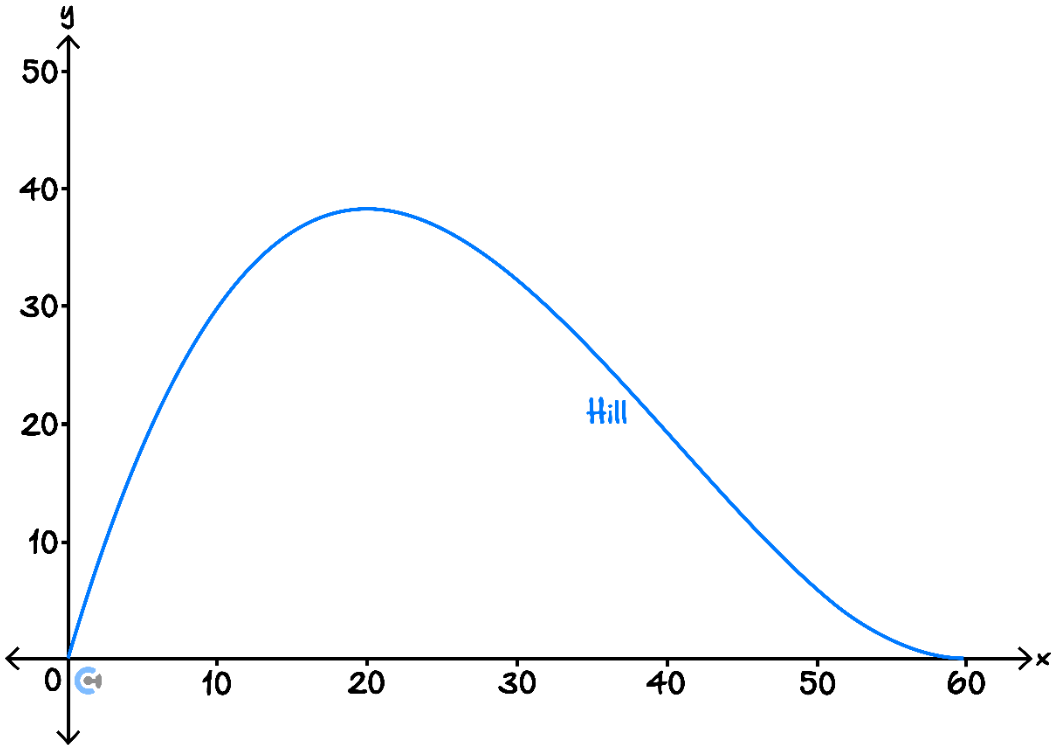
Question 15 (12 marks)

The Billboa Adventure fun camp is constructing a zip line to help attract more schools to choose them to host their school camps. The zipline is to be constructed above a hill on the property. The hill is modelled by:

$$y = \frac{3x(x-60)^2}{2500}, \qquad x \in [0, 60]$$

Where x is the horizontal distance, in metres, from an origin and y is the height, in metres, above this origin.

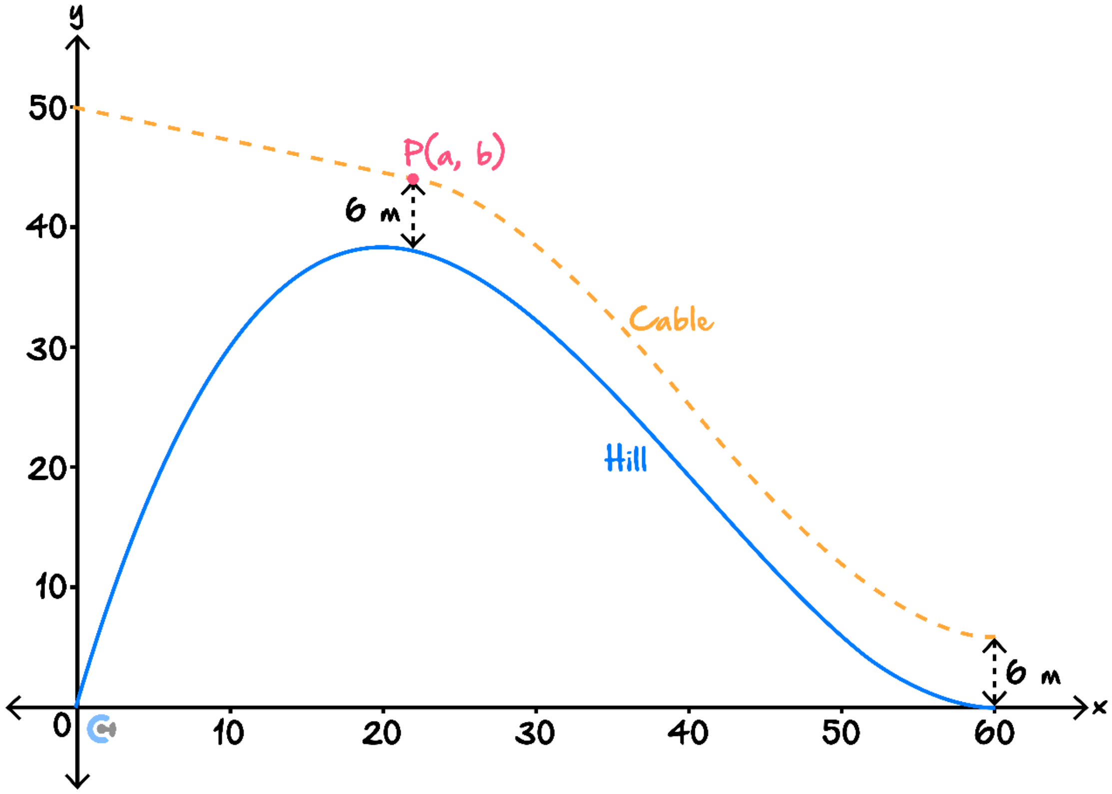


a. Find $\frac{dy}{dx}$. (2 marks)

---

b. State the set of values for which the **gradient** of the hill is strictly decreasing. (1 mark)

The cable for the zip line is connected to a platform at the origin at a height of 50 metres and is straight for $0 \leq x \leq a$, where $20 \leq a \leq 30$. The straight section joins the curved section at $P(a, b)$. The cable is exactly 6 m vertically above the hill from $a \leq x \leq 60$, as shown in the graph below:



c. State the rule, in terms of x, for the height of the cable above the horizontal axis for $x \in [a, 60]$. (1 mark)

---

d. Find the values of x for which the gradient of the cable is equal to the average gradient of the hill for $x \in [20, 60]$. (3 marks)

---

e. The gradients of the straight and curved sections of the cable approach the same value at $x = a$, so there is a continuous and smooth join at P.

i. State the gradient of the cable at P, in terms of a. (1 mark)

ii. Find the coordinates of P, with each value correct to two decimal places. (3 marks)

iii. Find the value of the gradient at P, correct to one decimal place. (1 mark)

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Section F: Extension Exam 1 (10 Marks)

INSTRUCTION: 10 Marks. 10 Minutes Writing.

Question 16 (10 marks)

Consider the function $f(x) = x^2 e^{-x^2}$.

a. Show that $f(x) = f(-x)$ and hence, state whether f is an even or an odd function. (1 mark)

b.

i. Find $f'(x)$. (1 mark)

ii. Hence, determine the coordinates for any stationary points of f. (2 marks)

c.

i. Show that the second derivative of f is given by $f''(x) = 2e^{-x^2}(2x^4 - 5x^2 + 1)$. (2 marks)

ii. Use the second derivative to determine the nature of the stationary points found in **part b. ii.** (2 marks)

iii. Determine how many points of inflection f has. (2 marks)

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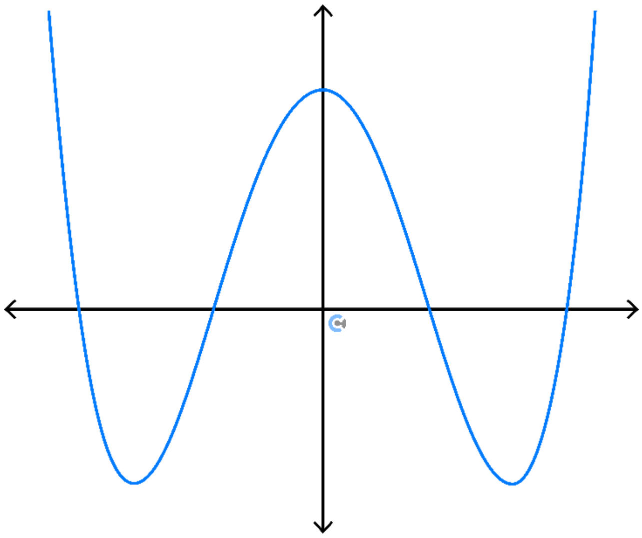
Section G: Extension Exam 2 (18 Marks)

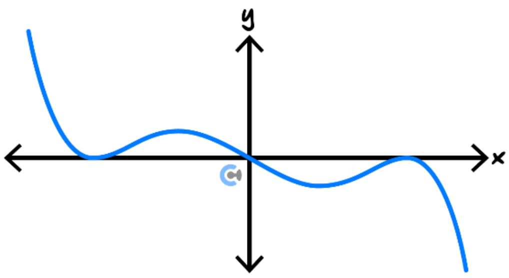
INSTRUCTION: 18 Marks. 18 Minutes Writing.

Question 17 (1 mark)

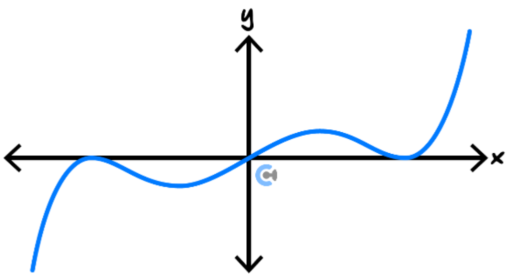
The graph of $y = f'(x)$ is shown below. The graph of $y = f(x)$ could be:



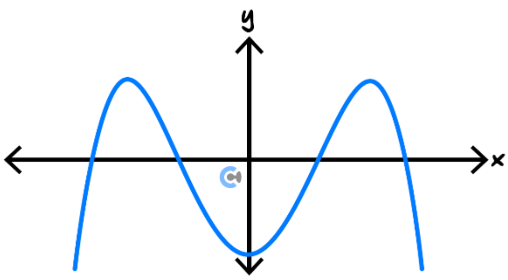
A.



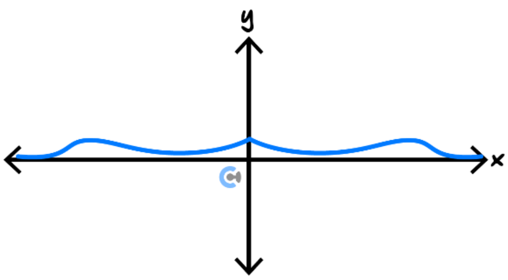
C.



B.



D.



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Question 18 (1 mark)

Let $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = (x-2)(x-3)(4x^2 + ax + 6)$$

Where a is a real number. If f has p stationary points, the possible values of p are:

A. $p \in \{1, 2, 3, 4\}$

B. $p \in \{0, 1, 2, 3\}$

C. $p \in \{1, 2\}$

D. $p \in \{1, 2, 3\}$

Question 19 (1 mark)

If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that:

- $f'(x) = 0$ at $x = 1$ and $x = 2$.
- $f'(x) > 0$ at $x > 2$ and $1 < x < 2$.
- $f'(x) < 0$ at $x < 1$.

Which of the following statements is **correct**?

A. The graph has a stationary point of inflection at $x = 2$ and a maximum at $x = 1$.

B. The graph has a stationary point of inflection at $x = 2$ and a minimum at $x = 1$.

C. The graph has a stationary point of inflection at $x = 1$ and a minimum at $x = 2$.

D. The graph has a minimum at $x = 1$ and a maximum at $x = 2$.

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Question 20 (1 mark)

Let $f^{(n)}(x)$ be the n^{th} derivative of $f(x)$. That is, it has been differentiated n times, where $n = 2p$ and p is a positive integer. If $f(x) = xe^{-x^3}$, how many axial intercepts does $f^{(n)}(x)$ have?

A. n

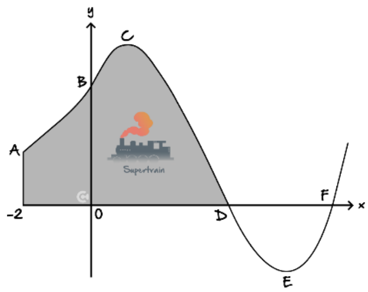
B. $2n$

C. $2n + 1$

D. $n + 1$

Question 21 (14 marks)

A part of the track for Tim's model train follows the curve passing through A, B, C, D, E and F shown:



Tim designed it by putting axes on the drawing as shown. The track is made up of two curves, one to the left of the y-axis and the other to the right.

B is the point $(0, 7)$. The curve from B to F is part of the graph of $f(x) = px^3 + qx^2 + rx + s$ where p, q, r and s are constants and $f'(0) = 4.25$.

a. Show that $s = 7$ and $r = 4.25$. (2 marks)

$C\ (1, 9)$ is the furthest point reached by the track in the positive y direction.

b. Use this information to write two equations involving p and q. (2 marks)

The values of p and q are $p = 0.25$ and $q = -2.5$, respectively. So, $f(x) = 0.25x^3 - 2.5x^2 + 4.25x + 7$.

c.

i. Find the exact coordinates of D and F. (1 mark)

ii. Find the greatest distance that the track is from the x-axis, when it is below the x-axis, correct to two decimal places. (1 mark)

d.

i. Find the value of x for which $f''(x) = 0$. (1 mark)

ii. Hence, find the steepest decline of f between C and D, correct to one decimal place. (1 mark)

The curve from A to B is part of the graph with the equation $g(x) = \frac{a}{1-bx}$, where a and b are positive real constants. The track passes smoothly from one section of the track to the other at B (that is, the gradients of the curves are equal at B).

e. Find the exact values of a and b. (3 marks)

Tim adds a new part to the track when $x = 8$. Call the point on the track when $x = 8$, G.
This new section of the track is modelled by the function $h(x) = m \cdot k^{-(x-8)}$ for $x \geq 8$ and $m, k > 0$.

f.

i. Find the value of m so that the track is continuous at G. (1 mark)

ii. It is known that $\lim_{x \to \infty} h(x) = 0$. Find the possible values of k. (1 mark)

iii. Find the value of k if $h(x)$ passes through the point $(10,4)$. (1 mark)

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