Write your **student number** in the boxes above. **Letter**

Mathematical Methods ¾

Examination 2 (Tech-Active)

Question and Answer Book - SOLUTIONS

VCE Examination (Term 1 Mock) – April 2025

- Reading time is **15 minutes**
- Writing time is **2 hours**

Materials Supplied

- Question and Answer Book of 23 pages.
- Multiple-Choice Answer Sheet.

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

Student's Full Name: ______________________________

Student's Email: ______________________________

Tutor's Name: ______________________________

Marks (Tutor Only): ____________________

Section A

Instructions

- Answer **all** questions in pencil on the Multiple-Choice Answer Sheet.
- Choose the response that is **correct** or that **best answers** the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1

Learning Objective [1.1.1] Find maximal domain and range.

Consider the function $f : [-2,3] \to R, f(x) = ax - 3$. If the range of f is $[-7,3]$, then the value of a is:

D1

A. 1

B. 2

C. 3

D. 4

Question 2

Learning Objective [1.7.1] Apply Factor Theorem and Remainder Theorem to identify the roots, remainders and find unknown of a function.

Let $p(x) = x^3 - ax + 1$ where $a \in \mathbb{R}$. If the remainder of $p(x)$ when divided by $ax + 8$ is 1, then the value of a is:

D1

$x^3 - a \cdot x + 1 \to p(x)$ — Done

$\text{solve}\left(p\left(\frac{-8}{a}\right) = 1, a\right)$ — $a = 4$

A. 3

B. 4

C. 8

D. -2

Question 3

Learning Objective [1.1.1] Find maximal domain and range.

The maximal domain of the function with rule $f(x) = \frac{1}{\sqrt{\log_e(x)}}$ is:

D1

$\text{domain}\left(\frac{1}{\sqrt{\ln(x)}}, x\right)$ — $1 < x < \infty$

A. $[0, \infty)$

B. $(1, \infty)$

C. $(0, 1)$

D. $(0, \infty)$

Question 4

Learning Objective [1.3.1] Applying x' and y' notation to find transformed points, find interpretation of transformations and altered order of transformations.

Which one of the following statements about transformations is false?

D1

A. A horizontal shift of a function $f(x)$ by 3 units to the right is represented by $f(x+3)$.

B. A reflection of a function $f(x)$ across the x-axis is represented by $-f(x)$.

C. A vertical stretch by a factor of 2 is represented by $2f(x)$.

D. A reflection across the y-axis is represented by $f(-x)$.

Question 5

Learning Objective [2.6.2] Find unknowns for number of solutions.

The equation $x^3 - 2x^2 + 3kx = 0$ has two solutions for:

D1

A. $k = \frac{1}{3}$

B. $k > \frac{1}{3}$

C. $k < \frac{1}{3}$

D. $-\frac{1}{3} < k < \frac{1}{3}$

$\text{solve}(x^3 - 2\cdot x^2 + 3\cdot k\cdot x = 0, x)$

$x = -(\sqrt{1-3\cdot k} - 1) \text{ or } x = \sqrt{1-3\cdot k} + 1 \text{ or } x = 0$

$\text{solve}(-(\sqrt{1-3\cdot k} - 1) = \sqrt{1-3\cdot k} + 1, k)$ $\quad k = \frac{1}{3}$

Question 6

Learning Objective [2.6.1] Find unknowns for general requirements.

D1

The polynomial $p(x) = x^4 - kx + 1$ where $k > 0$ has an inverse on the interval $(-\infty, 1]$ for:

A. $k \geq 1$

B. $k \geq 4$

C. $k \leq 1$

D. $k \leq 4$

$\text{solve}\left(\frac{d}{dx}(p(x)) = 0, x\right)$ $\quad x = \frac{(2\cdot k)^{\frac{1}{3}}}{2}$

$\text{solve}\left(\frac{(2\cdot k)^{\frac{1}{3}}}{2} = 1, k\right)$ $\quad k = 4$

From sliders observe **k>=4**

Question 7

Learning Objective [2.6.1] Find unknowns for general requirements.

The function $f(x) = ax^3 + bx^2 - cx$ has two stationary points for:

D1

A. $a > 0$ and $c < -\frac{b^2}{3a}$.

B. $a > 0$ and $c = -\frac{b^2}{3a}$.

C. $a < 0$ and $c > -\frac{b^2}{3a}$.

D. $a < 0$ and $c < -\frac{b^2}{3a}$.

$\text{solve}\left(\frac{d}{dx}(f(x)) = 0, x\right)$

$x = \frac{\sqrt{3\cdot a\cdot c + b^2} - b}{3\cdot a} \text{ or } x = \frac{-(\sqrt{3\cdot a\cdot c + b^2} + b)}{3\cdot a}$

$\text{solve}(3\cdot a\cdot c + b^2 > 0, c)$

$c > \frac{-b^2}{3\cdot a} \text{ and } a > 0 \text{ or } c < \frac{-b^2}{3\cdot a} \text{ and } a < 0 \text{ or } a = 0 \text{ a}$

D1

Question 8

Learning Objective [1.2.1] Find a new domain to fix composite functions.

Let B be the domain of f and C be the domain of g. What is the largest domain B such that $g(f(x))$ exists?

A. $\{\, x \in B \mid f(x) \in C \,\}$

B. $B \cap C$

C. B

D. C

We require the range of f to be a subset of the domain of g.
The domain of g is C and the domain of f is B.
In words option A says "x is any number in the domain B such that $f(x)$ is in the domain C".
which means that the range of f is a subset of the domain of g.

Question 9

Learning Objective [2.3.1] Find general derivatives with functional notation.

Let f and g be differentiable functions. The derivative of $f\left(e^{g(x)}\right)$ is: **D1**

A. $e^x e^{g'(x)} f\left(e^{g(x)}\right)$

B. $e^{g'(x)} f'\left(e^{g(x)}\right)$

C. $e^{g(x)} g'(x) f'\left(e^{g(x)}\right)$

D. $e^{g(x)} f'\left(e^{g(x)}\right)$

Question 10

Learning Objective [1.7.4] Identify odd, even functions and correct power functions.

Which of the following statements is false? **D1**

A. The product of two odd functions is an even function.

B. The product of an odd and an even function is an odd function.

C. The composition of two odd functions is an even function.

D. The composition of two even functions is an even function.

$f(x) = x^3$, $g(x) = \sin x$ both odd
$f(g(x)) = (\sin x)^3$ which is still odd

Question 11

Learning Objective [2.4.2] Find minimum and maximum.

The minimum distance between the graph of $y = \sqrt{x}$ and the point $(2, 0)$ is: **D1**

A. $\frac{3}{2}$

B. $2\sqrt{2}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{\sqrt{7}}{2}$

Define $d(x)=\sqrt{(x-2)^2+(\sqrt{x}-0)^2}$	*Done*
fMin$(d(x),x)$	$x=\frac{3}{2}$
$d\left(\frac{3}{2}\right)$	$\frac{\sqrt{7}}{2}$

Question 12

Learning Objective [1.5.3] Find angle between a line and x axis or two lines.

The angle, in degrees, between the lines $y = 2x - 1$ and $y = 3x - \frac{1}{3}$ is closest to: **D1**

A. 11.31

B. 0.20

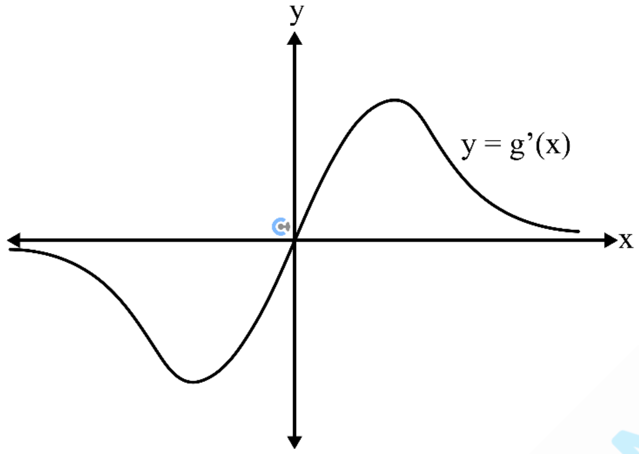
C. 8.13

D. 0.14

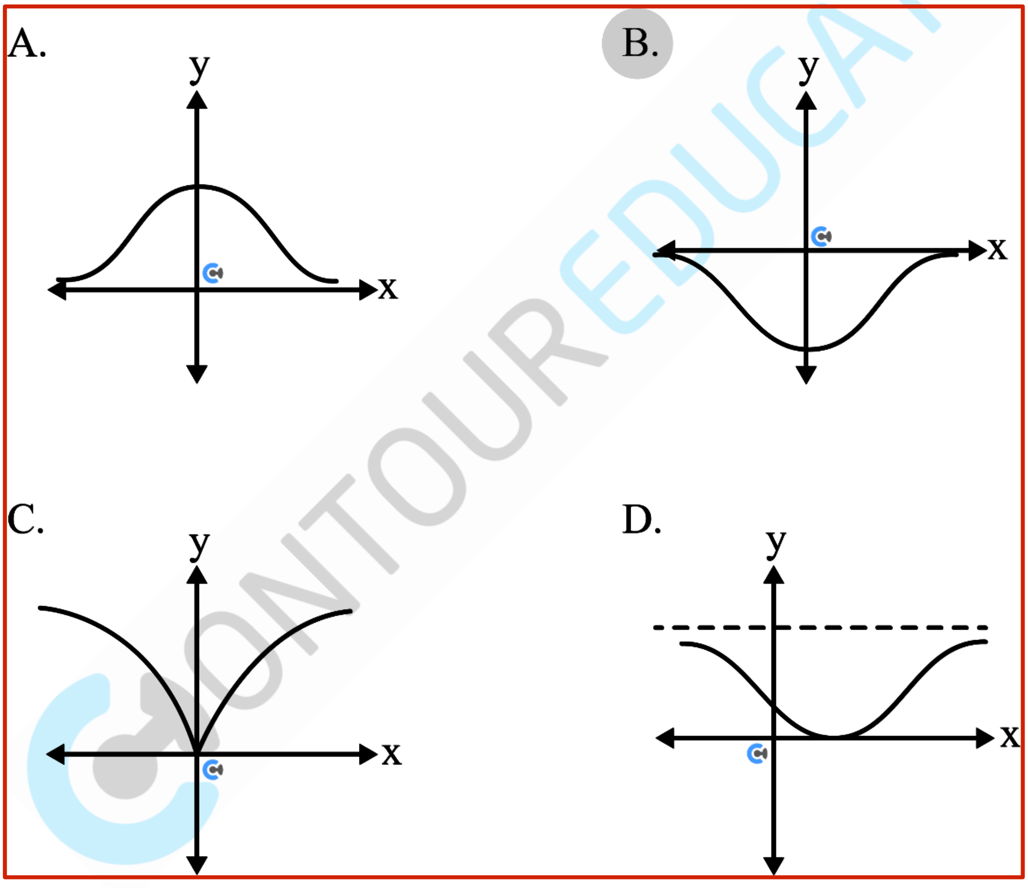
$\left\lvert \tan^{-1}(2) - \tan^{-1}(3) \right\rvert$	8.130102

Question 13 Learning Objective [2.1.3] Graph Derivative Functions.

Part of the graph of $y = g'(x)$ is shown below. **D1**



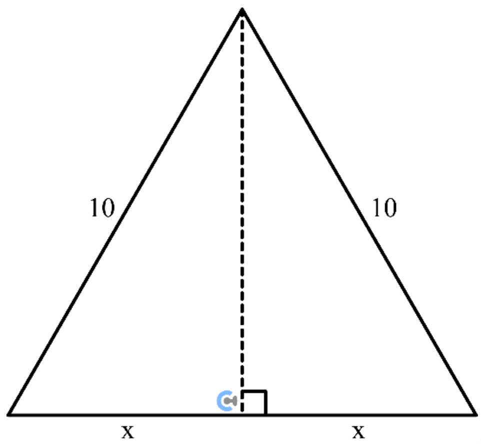
That part of the graph of $y = g(x)$ that corresponds to this graph, could be represented by:



Question 14

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions. **D1**

The value of x that maximises the area of the triangle below is equal to



A. 5

B. $5\sqrt{2}$

C. $\sqrt{20}$

D. 10

$$\text{fMax}\left(\frac{1}{2}\cdot 2\cdot x\cdot \sqrt{100-x^2},x\right) \quad x=5\cdot\sqrt{2}$$

Question 15

Learning Objective [2.6.2] Find unknowns for number of solutions. **D1**

The value of k such that the polynomial $p(x) = x^3 - 3x + k$ has exactly two x-intercepts is equal to

A. $k = 0$

B. $k = 1$

C. $k = 2$

D. $k = 3$

$$\text{solve}\left(x^3-3\cdot x+2=0,x\right) \quad x=-2 \text{ or } x=1$$

Question 16 [2.7]

The following algorithm applies Newton's method using a for loop with 4 iterations. **D1**

Inputs:

$f(x)$, a function of x

$df(x)$, the derivative of $f(x)$

$x0$, an initial estimate

define newton($f(x), df(x), x0$)

for i from 1 to 4

if $df(x0) = 0$ Then

return "Error: Division by zero"

else

$x0 \leftarrow x0 - f(x0) \div df(x0)$

end if

end for

return $x0$

$x - \dfrac{f(x)}{\frac{d}{dx}(f(x))} \rightarrow n(x)$	*Done*
$n(0.5)$	0.6
$n(0.6)$	0.6173913
$n(0.61739130434783)$	0.6180331
$n(0.61803309522068)$	0.618034

The return value of the function newton $(x^3 - 2x + 1, 3x^2 - 2, 0.5)$ is closest to:

A. 0.500

B. 0.618

C. 0.717

D. 0.698

Question 17 Learning Objective [1.2.3] Find the gradient of inverse functions.

Let f be a one-to-one differentiable function such that $f(1) = 3$ and $f'(1) = 4$. Let g be the inverse of f. If $g(3) = 1$ and $g'(1) = 2$, then $g'(3)$ is: **D1**

A. 1

B. 4

C. $\frac{1}{2}$

D. $\frac{1}{4}$

$$g'(3) = \frac{1}{f'(g(3))}$$

$$g'(3) = \frac{1}{f'(1)} = \frac{1}{4}$$

Question 18 Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

Consider the family of functions: **D1**

$$f_k(x) = kx - x^2, x \in [0,5]$$

Where k is a real constant. For which of the following ranges of k does the maximum value of $f_k(x)$ occur at an endpoint?

A. $k < 0$ or $k > 10$

B. $k \leq 0$ or $k \geq 10$

C. $0 < k < 10$

D. $k \leq 0$ or $k > 10$

Question 19 Learning Objective [1.5.3] Find angle between a line and x axis or two lines.

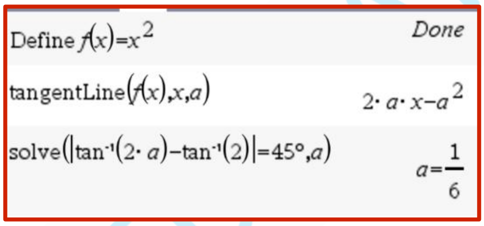
A possible value of a for which the tangent to the curve $f(x) = x^2$ at $x = a$ makes an angle of $45°$ with the line $y = 2x + 1$ is: **D1**

A. $-\frac{1}{6}$

B. $\frac{1}{3}$

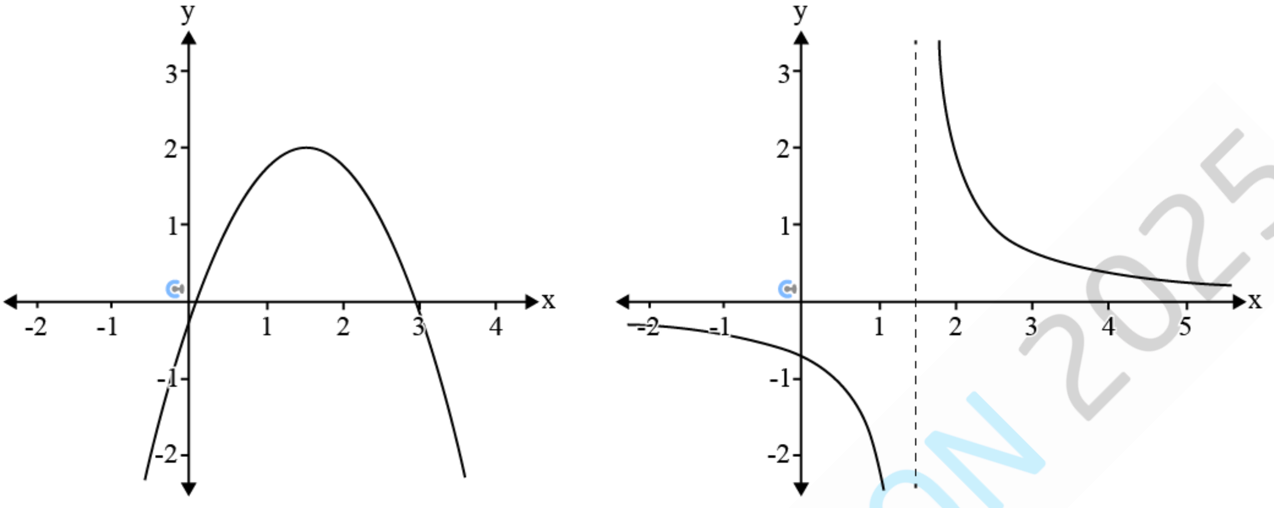
C. $\frac{1}{6}$

D. $-\frac{1}{3}$



Question 20 Learning Objective [1.8.3] Apply shape/graph to solve Number of Solutions questions.

Consider the graphs of two functions f and g, shown on the axes below. **D1**



The turning point of f is at $x = 1.5$ and occurs where $y > 0$.

The vertical asymptote of g is $x = 1.5$ and g has no x-intercepts.

On the interval $x \in [0, 4]$, the number of x-intercepts for the graph of the function h where $h = f' \times g$ is:

A. 0

B. 1

C. 2

D. 3

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.

Question 1 (13 marks)

Consider the function $f(x) = -\frac{x^4}{16} + \frac{x^3}{6} + \frac{15x^2}{8} - 9x + 1$.

a. Find the rule of the derivative of $f(x)$. 1 mark

D1

Learning Objective [2.3.1] Find general derivatives with functional notation.

$f'(x) = -\frac{1}{4}x^3 + \frac{1}{2}x^2 + \frac{15}{4}x - 9.$ [1A]

b. Find the x coordinates of the stationary points of f. 1 mark

D1

Learning Objective [2.3.1] Find general derivatives with functional notation.

$x = -4$ *or* $x = 3$ **1A**

$\text{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$ $x=-4$ or $x=3$

c. State the values of x where the function $f(x)$ is strictly decreasing. **D1** 1 mark

$\text{solve}\left(\frac{d}{dx}(f(x))\leq 0,x\right)$ $x\geq -4$

Learning Objective [2.1.2] Identify the nature of stationary points and trend (Strictly increasing and decreasing).

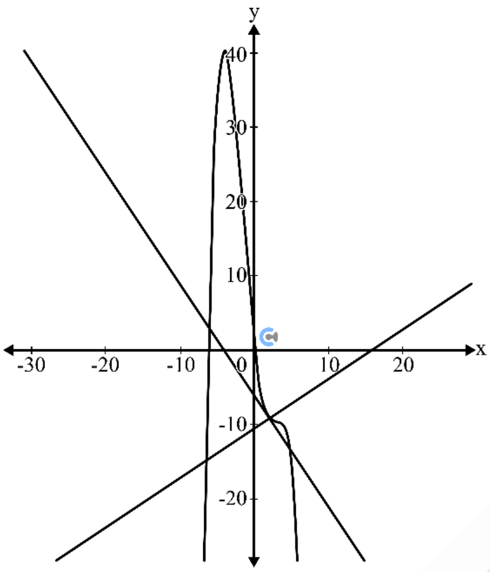
$x \geq -4$ **1A**

d. State the nature of the stationary points of $f(x)$. **D1** 2 marks

Local maximum at $x = -4$ **1A**
Stationary point of inflection at $x = 3$ **1A**

Learning Objective [2.1.2] Identify the nature of stationary points and trend (Strictly increasing and decreasing).

The diagram below shows part of the graph of $y = f(x)$, the tangent to the graph at $x = 2$, and a straight line drawn perpendicular to that tangent at $x = 2$. The equation of the tangent at $x = 2$ is $y = -\frac{3x}{2} - \frac{37}{6}$.



e. Using Newton's method with $x_0 = 2$, find the next iterate x_1. **D1** 1 mark

Learning Objective [2.4.3] Apply newton's method to find the approximation of a root and its limitations.

$$-\frac{3x_1}{2} - \frac{37}{6} = 0 \implies x_1 = -\frac{37}{9}. \text{ (1A)}$$

f. Find the y-axis intercept of the tangent to the graph of f at $x = 2$. **D1** 1 mark

Learning Objective [2.4.1] Find tangents and normals.

$(0, -\frac{37}{6})$ (1A)

g. Find the equation of the line perpendicular to the graph of f at $x = 2$. Hence, find the coordinates of the y-intercept of a line perpendicular to the graph of f at $x = 2$. **D1** 2 marks

Learning Objective [2.4.1] Find tangents and normals.

normalLine(f(x),x=2) $\quad \frac{2\cdot x}{3} - \frac{21}{2}$

$y = \frac{2x}{3} - \frac{21}{2}$. **1A**

$\left(0, -\frac{21}{2}\right)$ **1A**

h. Find the area of the triangle formed by the y-axis intercepts of both the tangent line and the line perpendicular to it at $x = 2$, and the point $(2, f(2))$. **D1** 2 marks

Learning Objective [2.4.1] Find tangents and normals.

$$A = \frac{1}{2} \times \left(-\frac{37}{6} - \left(-\frac{21}{2}\right)\right) \times 2 \quad [1M]$$

$$= \frac{13}{3} \quad [1A]$$

i. The tangent to the graph of f at another point to the left is parallel to the tangent at $x = 2$. Find the coordinates of this point. **D1** 2 marks

Learning Objective [1.5.2] Find parallel and perpendicular lines.

$f'(x) = -\frac{3}{2}$ 1M

$x = -\sqrt{15}$ since x < 2

$\left(-\sqrt{15}, \frac{13\sqrt{15}}{2} + \frac{241}{16}\right)$ 1A

Do not write in this area.

Question 2 (18 marks)

A new rollercoaster at Funland theme park is modelled by the height function:

$$h(x) = \begin{cases} -0.5x^2 + 4x + 2, & 0 \le x \le 5, \\ ax + b, & 5 < x \le 10, \end{cases}$$

where $h(x)$ gives height (in metres) above ground when the car is x metres from the station.

a. State the initial height. **D1** 1 mark

Learning Objective [2.1.1] Find instantaneous rate of change and average rate of change.

$h(0) = 2\ m$ **1A**

b. Find the average rate of change in height from $x = 0$ to $x = 5$. **D1** 2 marks

Learning Objective [2.1.1] Find instantaneous rate of change and average rate of change.

$\frac{h(5)-h(0)}{5-0}$ $\frac{3}{2}$

$\frac{h(5)-h(0)}{5-0}$ **1M** $= \frac{3}{2}$. **1A**

c. Determine the maximum height reached for $0 \le x \le 5$. **D1** 2 marks

Learning Objective [2.4.2] Find minimum and maximum.

fMax$(-0.5\cdot x^2+4\cdot x+2,x,0,5)$ x=4

$h(4)$ 10

$\frac{d}{dx}(-0.5x^2 + 4x + 2) = 0$ **1M**

$x = 4$

$Max\ height = h(4) = 10$ **1A**

It is known that the two pieces join smoothly at $x = 5$.

d. Find the gradient of the rollercoaster at the joining point $P(5, h(5))$. **D1** 2 marks

Learning Objective [2.1.1] Find instantaneous rate of change and average rate of change.

$\frac{d}{dx}(-0.5x^2 + 4x + 2) = -x + 4$ **1M**

$\frac{dh_1}{dx}|_{x=5} = -5 + 4 = -1$ **1A**

e. Show that the conditions for smooth joining give $a = -1$ and $b = 14.5$. **D2** 2 marks

Learning Objective [2.3.2] Apply differentiability to join two functions smoothly.

Require $h_1(5) = h_2(5)$ and $h_2'(5) = h_1'(5)$ **1M**

$h_1(5) = h_2(5)$.

$h_1(5) = -0.5(5)^2 + 4(5) + 2 = -12.5 + 20 + 2 = 9.5$.

$h_2(x) = ax + b$

$5a + b = 9.5$.(Equation 1)

$h_2'(5) = h_1'(5)$

$a = -1$.(Equation 2)

$5(-1) + b = 9.5 \Rightarrow -5 + b = 9.5 \Rightarrow b = 14.5$.

1M for solving a, b

f. Find the concavity of h at $x = 3$. **D1** 2 marks

Learning Objective [2.2.3] Identify concavity and find inflection points.

$\frac{d^2h_1}{dx^2} = \frac{d}{dx}(-x + 4) = -1$. **1M**

Hence concave down **1A**

g. Find the total horizontal distance (in metres) over which the rollercoaster remains at or above $8\,m$ in height. 2 marks

Learning Objective [2.4.2] Find minimum and maximum.

D1

$h(x) \geq 8$ **1M**

solve$(h(x)\geq 8, x)$

$2 \leq x \leq \frac{13}{2}$

$\frac{13}{2} - 2 = 4.5\,m$ **1A**

After safety testing, ride engineer Sam wants to make the coaster "more thrilling" by vertically stretching every height by a factor $k > 0$. The transformed height function is $h^*(x) = k\,h(x)$.

h. State the single transformation (in terms of k) mapping h to h^*. 1 mark

D2

Learning Objective [1.3.3] Find transformations from transformed function (Reverse Engineering).

Dilation by a factor of k from the x-axis **1A**

i. Find the value of k such that the new maximum height is $15\,m$. 1 mark

D1

$10 * k = 15$

$k = 1.5$ **1A**

Learning Objective [1.4.3] Apply transformations of functions to find its domain, range, transformed points and tangents.

Sam now wants to install a horizontal support beam at height of 12 metres which joins two points of the rollercoaster.

j. Determine all values of k, correct to three decimal places, for the support beam to be longer than 5 metres. 3 marks

D2

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

Let $f(x) = k(-0.5x^2 + 4x + 2)$ and $g(x) = k(-x + 14.5)$.

For beam to be 5 metres we require that $f(x) = g(x+5)$ and $f(x) = 12$

Solving yields $k = 1.566$. [1M]

We also need that $g(10) \leq 12 \implies k \leq 2.667$. [1M]

Therefore $1.566 < k \leq 2.667$. [1A]

Question 3 (16 marks)

Consider the polynomial $p(x) = x^3 - kx^2$ where $k \in [0, \infty)$.

a. Find the coordinate(s) of the stationary point(s) of $p(x)$ in terms of k. 2 marks

D1

Learning Objective [2.3.1] Find general derivatives with functional notation.

$p'(x) = 0$ and $y = p(x)$ 1M

$(0,0)$ and $\left(\frac{2k}{3}, \frac{-4k^3}{27}\right)$ 1A

$\text{solve}\left(\frac{d}{dx}(p(x))=0 \text{ and } y=p(x), x, y\right)$

$x=\frac{2 \cdot k}{3} \text{ and } y=\frac{-4 \cdot k^3}{27} \text{ or } x=0 \text{ and } y=0$

b. Find the values of k such that there are two stationary points. 1 mark

D1

Learning Objective [2.6.2] Find unknowns for number of solutions.

$k > 0$ 1A

c. State the value of k such that $p(x)$ is an odd function. 1 mark

D1

Learning Objective [1.7.4] Identify odd, even functions and correct power functions.

$k = 0$ 1A

d. Suppose $k = 1$. Find the exact value(s) of d for which the graph of $y = p(x) + d$ has:

i. Exactly one x-intercept. 1 mark

D1

Learning Objective [2.6.2] Find unknowns for number of solutions.

$d > \frac{4}{27}$ or $d < 0$ 1A

ii. Exactly three x-intercepts. 1 mark

D1

Learning Objective [2.6.2] Find unknowns for number of solutions.

$0 < d < \frac{4}{27}$ 1A

e. Find the value of k for which the tangent to the function p at $x = 1$ hits the x-axis at an angle of 60 degrees. 2 marks

D1

Learning Objective [1.5.3] Find angle between a line and x axis or two lines.

$p'(1) = \tan(60°)$ **1M**

$k = \frac{3-\sqrt{3}}{2}$ **1A**

f. Find the value of k for which the graph of $y = p(x)$ has a tangent of $y = x - 5$. 3 marks

D1

Learning Objective [2.4.1] Find tangents and normals.

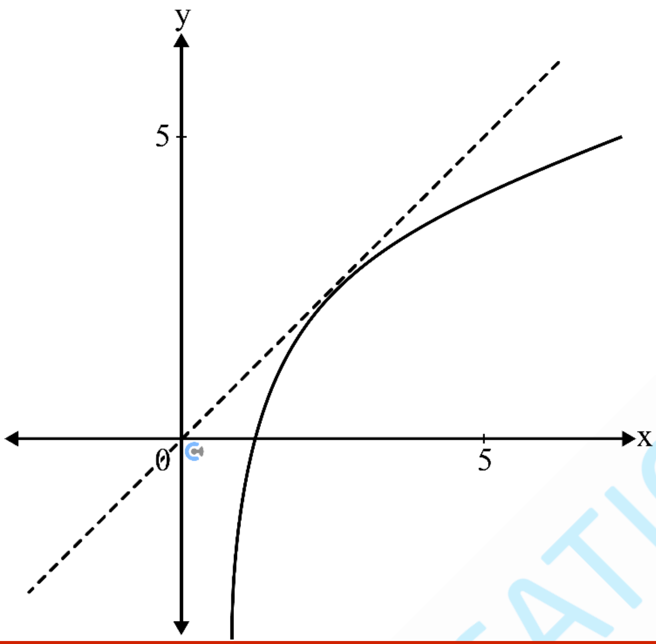
We solve the equations

$$p(x) = x - 5 \quad (1M)$$
$$p'(x) = 1 \quad (1M)$$

We get $x = 2, k = \frac{11}{4}$.

Thus $k = \frac{11}{4}$ (1A, also accept $k = 2.75$)

A new function is defined by composing the function $\log_e(x)$ with the derivative of $p(x)$. Let $f:(c,\infty)\to\mathbb{R}, f(x)=\log_e(p'(x))$ be a one-to-one function. A section of the function is shown below.



Learning Objective [1.2.1] Find a new domain to fix composite functions.

g. Find the smallest possible value of c in terms of k. 2 marks

D1

$\text{solve}\left(\frac{d}{dx}(p(x))>0,x\right)|k\geq 0$

$x>\frac{2\cdot k}{3}$ and $k\geq 0$ or $x<0$ and $k\geq 0$ **1M** for making p'(x)>0

$c=\frac{2k}{3}$ **1A**

h. Find the value of k correct to three decimal places for which the line $y=x$ is tangent to the graph of $y=f(x)$. 2 marks

D1

Learning Objective [2.6.2] Find unknowns for number of solutions.

$\begin{cases} f(x)=x \\ f'(x)=1 \end{cases}$ for x and k **1M**

$x=2.530, k=1.314$ **1A**

i. Hence, or otherwise, find the values of k for which the functions f and f^{-1} have two points of intersection. Give your answer correct to three decimal places. 1 mark

D1

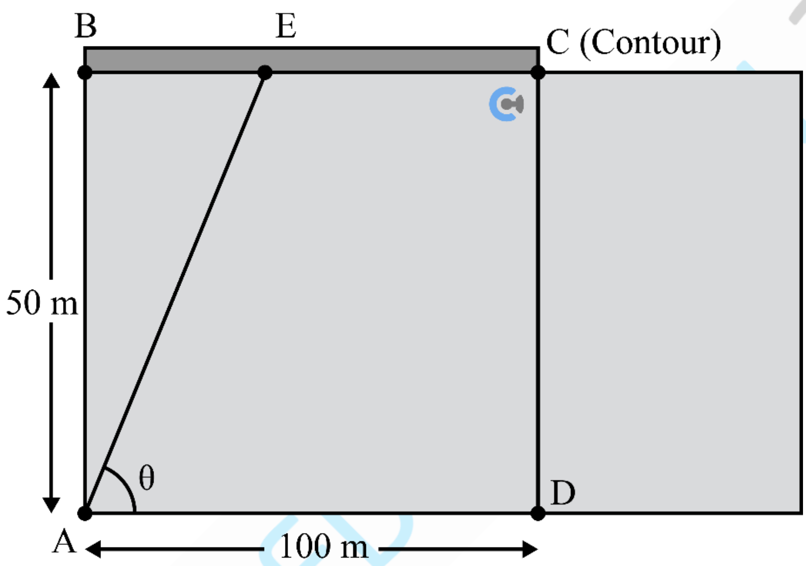
Learning Objective [2.6.2] Find unknowns for number of solutions.

$0 \leq k < 1.314$ **1A**

Question 4 (13 marks)

The Late Contour Students, on their way to the Contour Term 1 Exam, find themselves at point A on the Southwest corner of a rectangular car park that is $50\ m$ wide. The students need to get to point C at the Contour Centre, $100\ m$ to the east and on the edge of the car park.

Point B is directly to the north of point A and on the edge of the car park as shown. There is a normal footpath running along the far side BC of the car park. They need to get from A to C. They can travel at a speed of 2 metres per second on the footpath, but only at a speed of 1 metre per second through the car park. Note that the diagram is **NOT** to scale.



a. Calculate the time taken to travel directly from A through the car park to B, and then from B to C using the footpath. 1 mark

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

D1

$A \to B: = \frac{50}{1} = 50\ s$

$B \to C: = \frac{100}{2} = 50\ s$

Note speed=distance/time

$50\ s + 50\ s = 100$ seconds **1A**

b. Calculate the time taken, to the nearest second, to travel directly from A to C through the car park. 2 marks

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

D1

$AC = \sqrt{(100)^2 + (50)^2} = \sqrt{10000 + 2500} = \sqrt{12500} \approx 111.8$ m. **1M**

$$\frac{111.8 \text{ m}}{1 \text{ m/s}} = 111.8 \text{ s}.$$

Answer 112 seconds **1 A**

The Late Contour Students realise that they will not make it in time for the exam if they were to travel either of the ways calculated in **part a.** and **part b.** They decide to travel from point A to E, which is x m horizontally away from B, and travel from E to C via the footpath to optimise their time.

c. Show that the total time taken for the Late Contour Students to go from A to C can be modelled by: 2 marks

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

D2

$$T(x) = \sqrt{x^2 + 2500} + \frac{100 - x}{2}$$

$AE = \sqrt{x^2 + 50^2} = \sqrt{x^2 + 2500}$ **1M**

$T_{AE} = \frac{\sqrt{x^2 + 2500}}{1} = \sqrt{x^2 + 2500}$ **1M**

AC=100 so EC= $100 - x$

$T_{EC} = \frac{100 - x}{2}$ **1M**

$T(x) = T_{AE} + T_{EC} = \sqrt{x^2 + 2500} + \frac{100 - x}{2}$

d. State the appropriate domain of $T(x)$. 1 mark

D1

Learning Objective [1.1.1] - Find maximal domain and range.

$x \in (0, 100)$. (1A)
Endpoints not included because a) they realise that they will be late so cannot go from A to B which is the case when $x = 0$. b) it is stated they travel along the footpath so $x \neq 100$.

e. Write the equation for $\frac{dT}{dx}$, and hence find the value of x required to minimise the time taken to travel from A to C correct to one decimal place. 2 marks

D1

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

$\frac{dT}{dx} = \frac{x}{\sqrt{x^2+2500}} - \frac{1}{2}$ **1A**

$\frac{dT}{dx} = 0$

$x = \sqrt{\frac{2500}{3}} \approx 28.9$ m. **1A**

f. Calculate the minimum time taken to travel from A to C, to the nearest second. 1 mark

D1

Learning Objective [2.5.2] Advanced Maximum/Minimum Questions.

93s **1A**

It is known that the Contour Term 1 Exam is starting in 80 seconds. The students decide to increase their speed on the footpath to be $a\ m/s$ instead of $2\ m/s$, while their speed through the car park stays the same.

g. Find the minimum value of a, correct to two decimal places, such that they can make it to the Contour Centre before the start of the Contour Term 1 Exam. 4 marks

D2

Learning Objective [2.6.3] Find unknowns for minimum and maximum.

$Tnew(x) = \sqrt{x^2 + 2500} + \frac{100-x}{a}$ **1M**

$$\begin{cases} Tnew(x) = 80, \ \mathbf{1M} \\ \frac{dTnew}{dx} = 0 \ \mathbf{1M} \end{cases}$$

$x = 17.34$ m, $a = 3.05$ m/s

$a = 3.05$ m/s **1A**

Define $t(x) = \sqrt{x^2+2500} + \frac{100-x}{a}$ Done

$\text{solve}\left(\begin{cases} t(x)=80 \\ \frac{d}{dx}(t(x))=0 \end{cases}, \{x,a\}\right)$

$x=17.33611$ and $a=3.052596$