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VCE Mathematical Methods ½
Exponentials [5.1]
Workbook Solutions

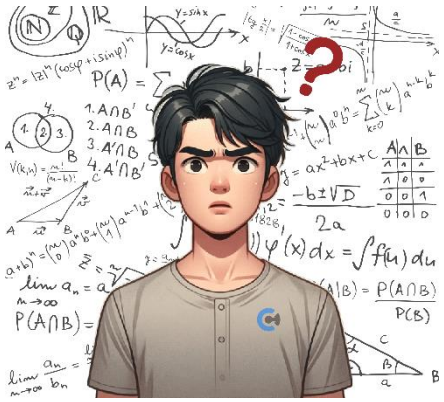
Outline:

Section A: Basics of Exponentials

Sub-Section: Introduction to Exponentials

Context: Sam's Problem

- Sam decides to multiply his weight by 1.1 each time he eats a chocolate bar.



- Sam, over easter, loses control and eats 100 chocolates.
- Now, Sam has a problem other than his weight.
- How does he multiply 1.1 hundred times?
- How does he represent that in a concise way?

Exponentials

Definition

$$base \times \cdots \times base = base^{power}$$

- Exponentiation is a _____ stacked multiplication _____.
- The power represents the _____ number of bases we are multiplying _____.

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Question 1 Walkthrough.

Solve the following equation for x.

$$4^x = 64$$

$$x = 3$$

NOTE: To solve the power, think about how many bases you need to get 64.

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Question 2

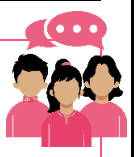
Solve the following equations for x.

a. $3^x = 9$

$x = 2$

b. $2^{2x+1} = 8$

$x = 1$



Discussion: If a positive power represents the number of bases multiplied, what does a negative power mean?

Just means we are dividing instead of multiplying.

Question 3 Walkthrough.

Evaluate the following.

$$3^{-3}$$

$$\frac{1}{27}$$

Question 4

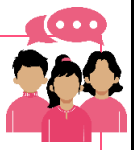
Evaluate the following.

a. 2^{-4}

$$\frac{1}{16}$$

b. 5^{-2}

$$\frac{1}{25}$$



Discussion: How many a's are we multiplying for $a^x \times a^y$?

$x + y$ many a's.

Question 5 Walkthrough.

Evaluate the following.

$$2^{-3} \times 2^5$$

4

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Question 6

Evaluate the following.

a. $3^{-5} \times 3^2$

$\frac{1}{27}$

b. $3^2 \times 3^{-4}$

$\frac{1}{9}$

NOTE: The base must be the same for this to work.

Discussion: How many a's are we multiplying for $(a^x)^y$?

xy many a's.

Question 7 Walkthrough.

Simplify the following.

$$(x^3)^2$$

x^6

Question 8

Simplify the following.

a. $(x^2)^4$

x^8

b. $(x^{a+1})^2$

x^{2a+2}

Discussion: If a^3 triples the number of a's multiplied, what does $a^{\frac{1}{3}}$ do?

It thirds the a multiplied. Hence, it cube roots.

Question 9 Walkthrough.

Simplify the following.

$$8^{\frac{1}{3}}$$

2

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Question 10

Simplify the following.

a. $27^{\frac{1}{3}}$

3

b. $8^{\frac{2}{3}}$

4

Tutors Note: Change it to 8 power of $\frac{1}{3}$ all squared.

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Question 11 Extension.

a. Evaluate:

$$\frac{125^{-\frac{2}{3}} \times 27^{\frac{1}{3}} \times 2^{-2}}{2^{-5}} + 5^{-2}$$

1

b. Solve the following equation for x.

$$2^{3x+2} = 32 \times 4^{x}$$

$x = 3$

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Sub-Section: Index Law

Let's summarise everything!

Index Laws

- All the rules explored above are called ___ index laws ___.

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1$$

$$(a \times b)^x = a^x \times b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{1}{x}} = \sqrt[x]{a}$$

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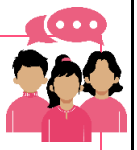


Discussion: Any question with the above rule? We can try proving together.

Tutors Note: Invite the students to ask you any question about the index law above.

Let's mix them all!

Question 12 Walkthrough.

Simplify the following expressions.

a. $\frac{4^x}{8^{3x}}$

$$2^{2x-9x} = 2^{-7x}$$

b. $\frac{a^{2x}}{(a^{3x})^{\frac{1}{2}}}$

$$a^{\frac{x}{2}}$$

Question 13

Simplify the following expressions.

a. $\frac{b^x}{b^{2x-1}}$

$$= \frac{b^x}{b^{2x-1}}$$

$$= b^{1-x}$$

b. $\frac{3^x \times 9^{x-5}}{3^2}$

$$]:= \frac{3^x * 3^{2x-10}}{3^2}$$

$$2]= 3^{-12+3x}$$

Question 14 Extension.

Simplify the following expression.

$$\frac{3^{2x+1} \times 27^{x-2}}{9^x \times 81}$$

$$3^{3x-9}$$

Sub-Section: Inequalities

Question 15

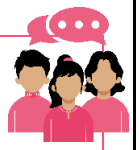
Solve the following inequalities for x.

a. $2^x > 8$

$x > 3$

b. $3^{2x-1} < 27$

$2x - 1 < 3.\ x < 2$



Discussion: If the base is less than 1, does multiplying more of the base increase the number?

No. It decreases it.

Question 16 Walkthrough.

Solve the following inequalities for x.

$$\left(\frac{1}{3}\right)^x > \frac{1}{9}$$

$x < 2$

NOTE: If the base is less than 1, always flip the inequality sign.

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Question 17

Solve the following inequalities for x.

a. $\left(\frac{1}{2}\right)^x \geq \frac{1}{8}$

$$x \leq 3$$

b. $\left(\frac{1}{3}\right)^x < 9$

$$x > -2$$

Question 18 Extension.

Solve the following inequality for x.

$$2^{-x^2+3x} > 1$$

$$0 < x < 3$$

$$1 = 2^0 \Rightarrow 2^{-x^2+3x} > 2^0 \Rightarrow -x^2 + 3x > 0 \Rightarrow x^2 - 3x < 0 \Rightarrow x(x-3) < 0 \Rightarrow \boxed{0 < x < 3}$$

In summary!

Inequalities for Exponentials

For $a^x < a^y$

- Flip the inequality sign when base is less than 1.

If $a > 1$ then $x < y$.

If $0 < a < 1$ then $x > y$.

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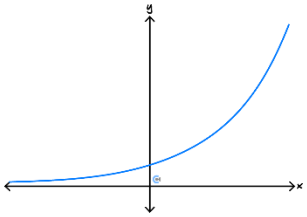
Section B: Graphs of Exponentials

Sub-Section: Graphs of Exponentials

Exponential Functions



a^x where $a > 1$

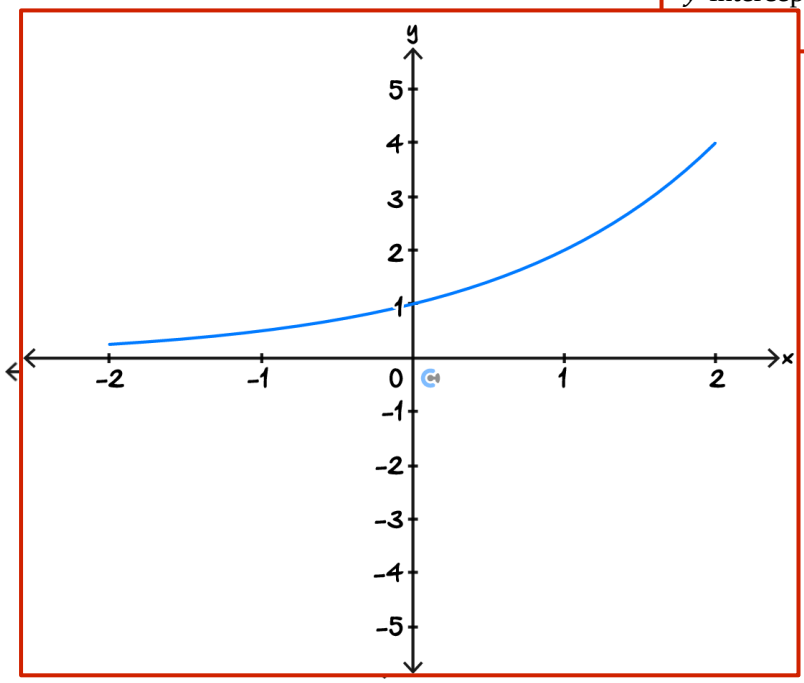
- Domain of the exponential function is R.
- Range of the exponential function is R^+.

Question 19 Walkthrough.

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

$y = 2^x$

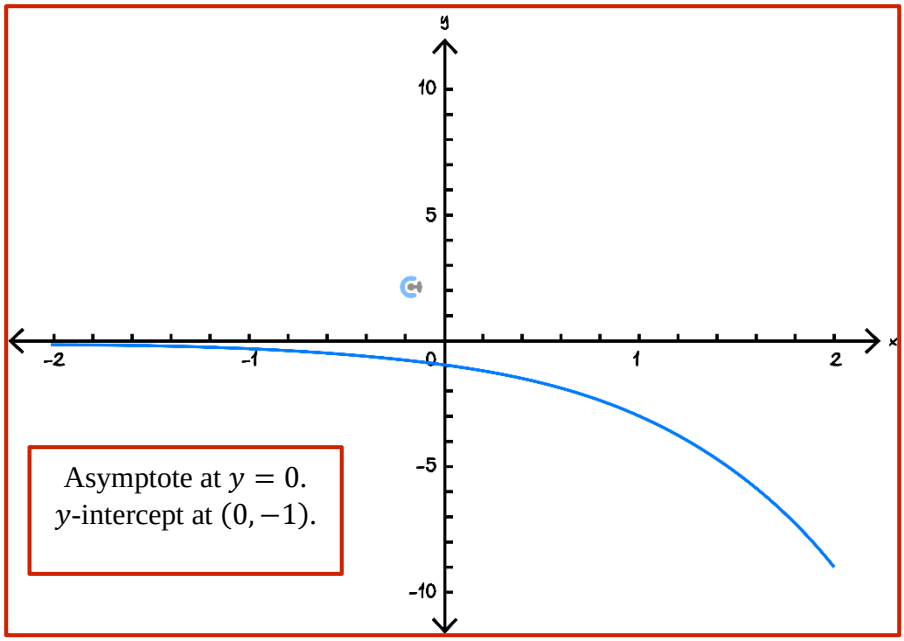
Asymptote at $y = 0$.
y-intercept at $(0,1)$.

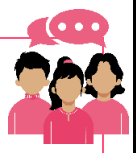


Question 20

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

$$y = -3^x$$





Discussion: What would the graph look like if the base was less than 1? For example: $\left(\frac{1}{2}\right)^x$?

Show them algebraically that $\left(\frac{1}{2}\right)^x$ is same as $(2^{-1})^x = 2^{-x}$.
Hence, it will be flipped around the y-axis.

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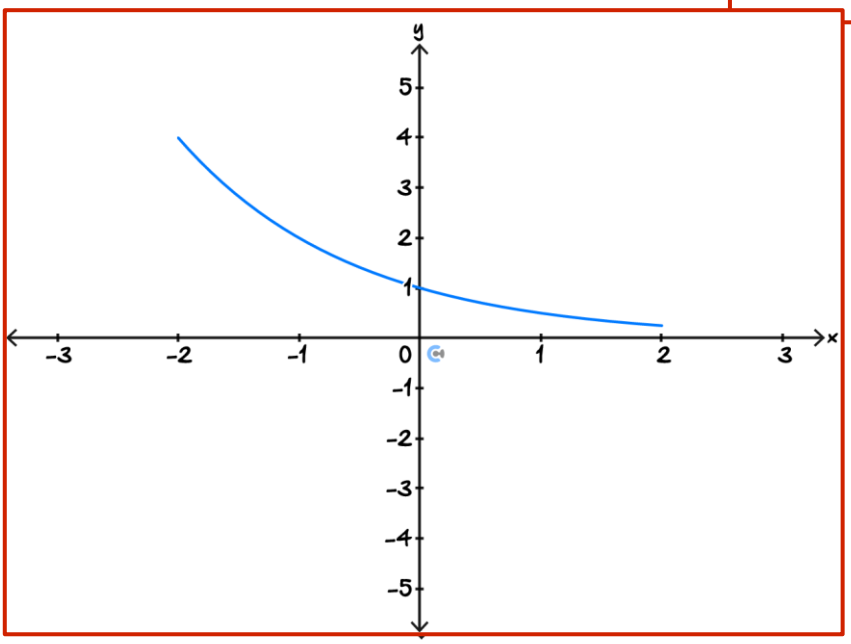
Question 21

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

$$y = \left(\frac{1}{2}\right)^x$$

Asymptote at $y = 0$.
y-intercept at $(0, 1)$.



Let's take a look at more difficult graphs now!

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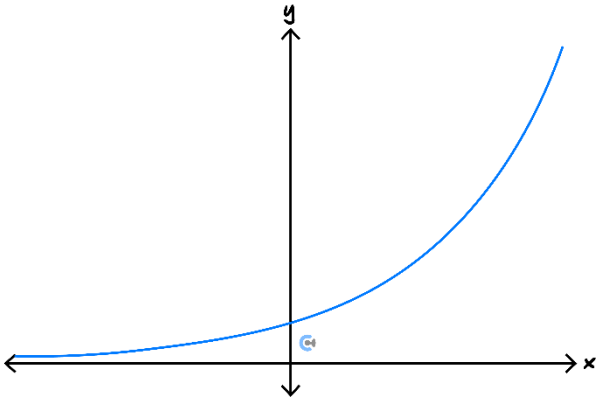
Graphs of Transformed Exponential Functions



$$y = a\,base^{b(x-h)} + k$$

- The horizontal asymptote is always given by ___ $y = k$ ___.
- **Steps to take when sketching an exponential:**

1. Find corresponding asymptotes.
2. Plot x and y-intercepts (if they exist).
3. Sketch the curve.

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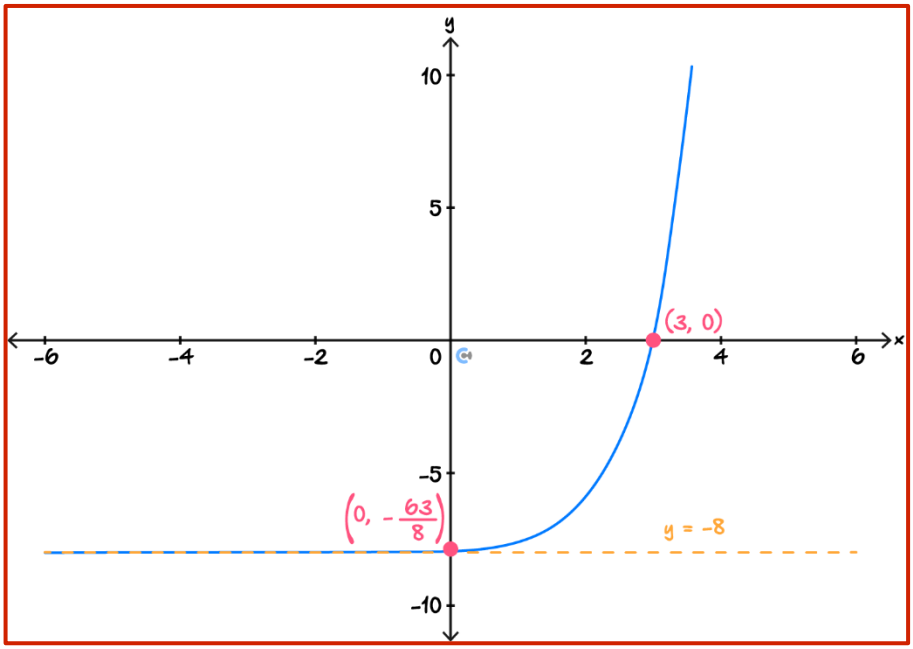
Question 22 Walkthrough.

Sketch the graph of the following function, labelling all key features, including axes intercepts and asymptotes.

$$y = \frac{1}{2} \times 4^{x-1} - 8$$



Asymptote at $y = -8$.
y-intercept at $\left(0, -\frac{63}{8}\right)$, x-intercept at $(3,0)$.

NOTE: Graphing is easy if you strictly follow the steps!

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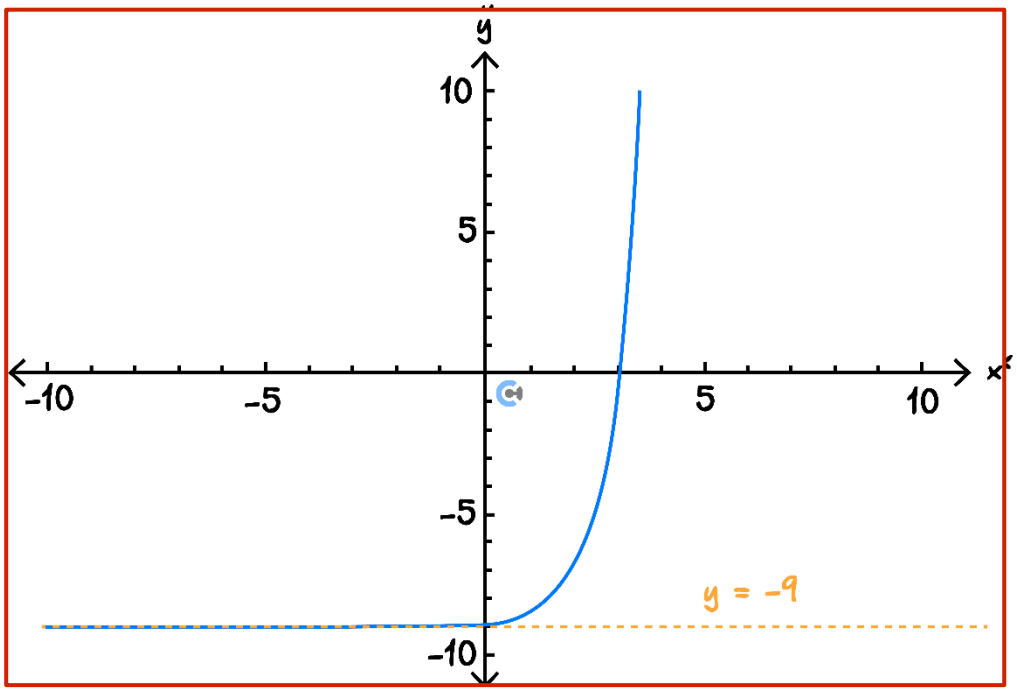
Your turn!

Question 23

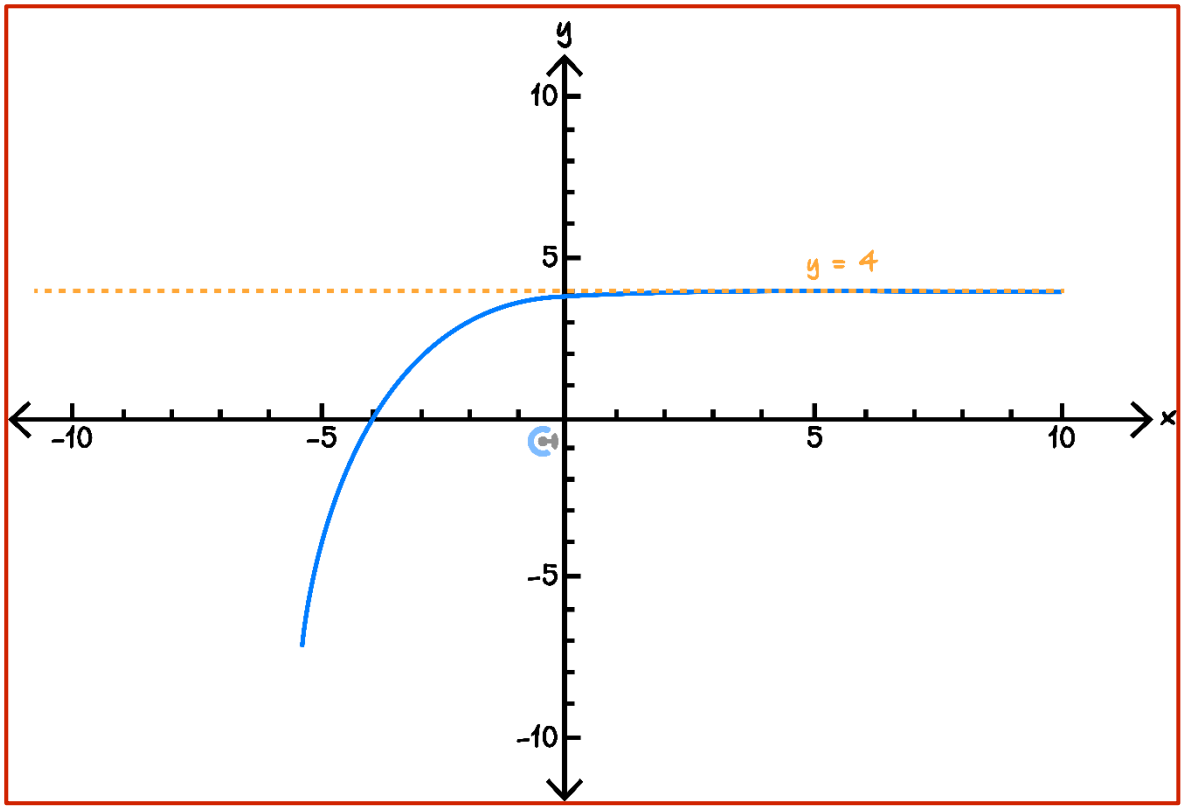
Sketch the graphs of the following functions, labelling all key features including axes intercepts and asymptotes.

a. $y = 3^{x-1} - 9$



Asymptote at $y = -9$
y-intercept at $\left(0, -\frac{26}{3}\right)$, x-intercept at $(3,0)$.

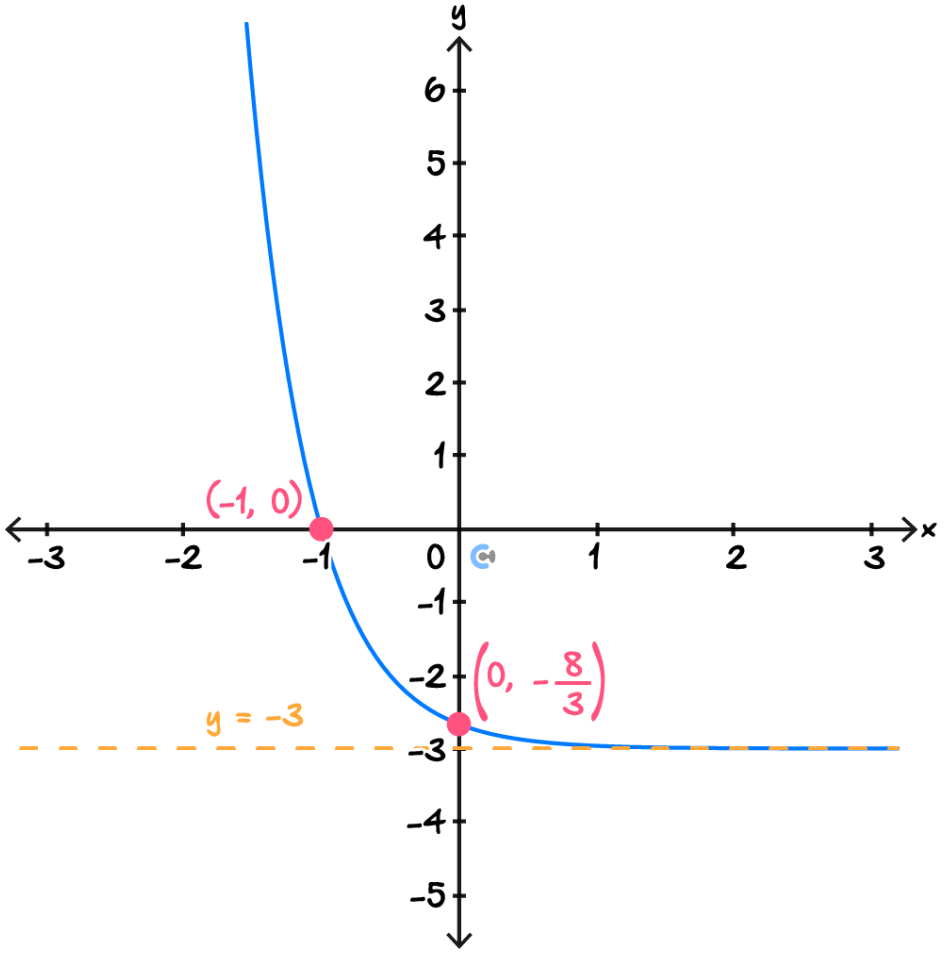
b. $y = -2^{-x-2} + 4$



Asymptote at $y = 4$.
y-intercept at $\left(0, \frac{15}{4}\right)$, x-intercept at $(-4,0)$.

Question 24 Extension.

Find a rule of the form $y = a^{bx+1} - d$, where $a, b, d > 0$, for the graph shown below.



Immediately, we see $d = 3$.

Then when $x = 0$ we get the equation:

$$a^1 = \frac{1}{3} \Rightarrow a = \frac{1}{3}$$

When $x = -1$ we get the equation:

$$a^{1-b} = 3$$
$$1 - b = -1$$
$$b = 2$$

So,

$$y = \left(\frac{1}{3}\right)^{2x+1} - 3$$

Section C: Hidden Quadratics

Sub-Section: Understanding Hidden Quadratics

What is a hidden quadratic?

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

Question 25 Walkthrough.

Consider the following equation:

$$(x^2)^2 + 2x^2 + 1 = 0$$

Convert the equation to be a quadratic of A by appropriate substitution of A.

$A^2 + 2A + 1 = 0$, where $A = x^2$.

NOTE: Look for something and it squared!

Your turn!

Question 26

Convert the following equation to be a quadratic of A by appropriate substitution of A.

$$x - 5\sqrt{x} + 3 = 0$$

$A^2 - 5A + 3 = 0$ where $A = \sqrt{x}$.

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Sub-Section: Hidden Quadratics for Exponentials

Hidden Quadratics for Exponentials

$$a \times exp^2 + b \times exp + c = 0$$

Let $A = exp$ where $A > 0$

- Look for "**same base** and **double power**" pattern.

Discussion: Why does A have to be bigger than 0? HINT: Recall the graph of exponentials!

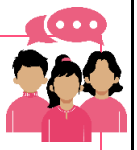
Because exponentials cannot be 0 or negative. (Tutors give them examples here.)

Question 27

Convert the following into a quadratic equation of A. You do not need to solve.

$$3^{2x} - 3 \times 3^x + 5 = 0$$

$A^2 - 3A + 5 = 0$ where $A = 3^x$.



Discussion: What pattern does the exponentials need to have?

Same base, but double the power.

3^{2x} and 3^{x}.

It is important to adjust if needed to the pattern of same base and double power before we come up with A

Your turn!

Question 28

Convert the following into a quadratic equation of a. You do **not** need to solve.

a. $4^{2x} - 7 \times 4^{x} + 8 = 0$

$a^2 - 7a + 8 = 0$

b. $4^{x} + 2^{x+1} - 3 = 0$

$a^2 + 2a - 3 = 0$ where $a = 2^{x}$.

NOTE: Look for the same base, double power!

Let's now solve them!

Question 29 Walkthrough.

Solve the following equation for x.

$$2^{2x+1} - 12 \times 2^x + 16 = 0$$

$(2^x)^2 = 2^{2x}$

$2^{2x} \cdot 2^1 - 12 \cdot (2^x) + 16 = 0$

let $2^x = a > 0$

$a^2 \cdot 2 - 12 \cdot a + 16 = 0$

$a^2 - 6a + 8 = 0$

$(a-2)(a-4) = 0$

$a = 2, 4$

$a \neq -$

$2^x = 2, \quad 2^x = 4$

$x = 1, 2$

Discussion: In the above question, both our A values were positive. What happens if one is negative or zero?

Since, exponential must be positive, we simply reject the value of A.

Your turn!

Question 30

Solve the following equations.

a. $3^{2x} - 4 \times 3^x + 3 = 0$

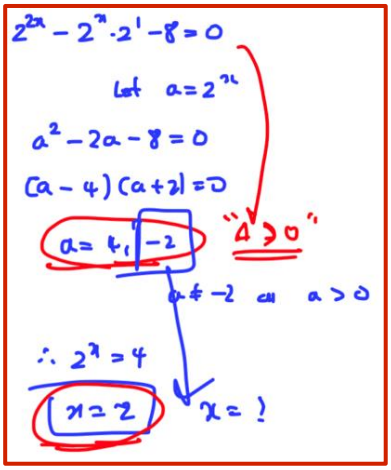
$A = 1,3$. Hence, $x = 0,1$.

b. $2^{2x} - 3 \times 2^{x+1} - 16 = 0$

$A = -2,8$, reject $A = -2$ as $A > 0$.

Hence, $x = 3$.

c. $4^x - 2^{x+1} - 8 = 0$



NOTE: You must show the process of rejecting your A value for the marking scheme!

Question 31 Extension.

Solve the following equations.

a. $3^{2x} + 2 \times 3^{x} - 15 = 0$

$x = 1$

b. $2^{2x} - 5 \times 2^{x+1} + 16$

$x = 1{,}3$

Cheat Sheet

[5.1.1] – Basics of exponentials

Exponentials

$$base \times \cdots \times base = base^{power}$$

- Exponentiation is a stacked multiplication.
- The power represents the number of bases we are multiplying.

Index Laws

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1$$

$$(a \times b)^x = a^x \times b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{1}{x}} = \frac{1}{a^x}$$

Inequalities for Exponentials

For $a^x < a^y$

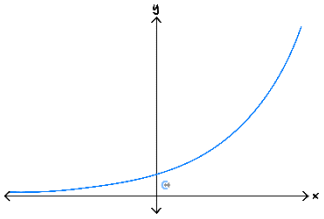
- Flip the inequality sign when base is less than 1.

If $a > 1$ then $x < y$.

If $0 < a < 1$ then $x > y$.

[5.1.2] – Graph exponentials

Exponential Functions



a^x where $a > 1$

- Domain of the exponential function is R.
- Range of the exponential function is R^+.

Graphs of Exponential Functions

$$y = a\, base^{b(x-h)} + k$$

- The horizontal asymptote is always given by $y = k$
- **Steps to take when sketching an exponential:**
  1. Find corresponding asymptotes.
  2. Plot x and y-intercepts (if they exist).
  3. Sketch the curve.
  4. Always follow these steps as they minimise potential mistakes.

[5.1.3] – Solve hidden quadratics of exponentials

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

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