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VCE Mathematical Methods ½
Exponentials [5.1]
Workbook

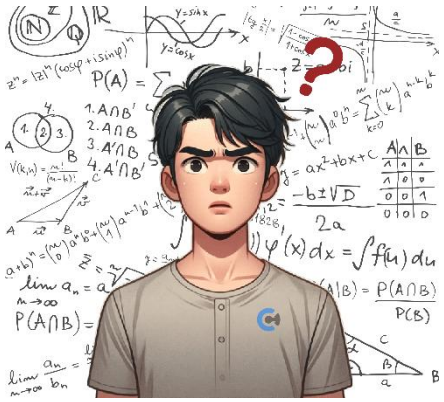
Outline:

Section A: Basics of Exponentials

Sub-Section: Introduction to Exponentials

Context: Sam's Problem

- Sam decides to multiply his weight by 1.1 each time he eats a chocolate bar.



- Sam, over easter, loses control and eats 100 chocolates.
- Now, Sam has a problem other than his weight.
- How does he multiply 1.1 hundred times?
- How does he represent that in a concise way?

Exponentials

Definition

$$base \times \cdots \times base = base^{power}$$

- Exponentiation is a _____.
- The power represents the _____.

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Question 1 Walkthrough.

Solve the following equation for x.

$$4^x = 64$$

NOTE: To solve the power, think about how many bases you need to get 64.

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Question 2

Solve the following equations for x.

a. $3^x = 9$

b. $2^{2x+1} = 8$

Discussion: If a positive power represents the number of bases multiplied, what does a negative power mean?

Question 3 Walkthrough.

Evaluate the following.

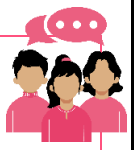
$$3^{-3}$$

Question 4

Evaluate the following.

a. 2^{-4}

b. 5^{-2}



Discussion: How many a's are we multiplying for $a^x \times a^y$?

Question 5 Walkthrough.

Evaluate the following.

$$2^{-3} \times 2^5$$

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Question 6

Evaluate the following.

a. $3^{-5} \times 3^{2}$

b. $3^{2} \times 3^{-4}$

NOTE: The base must be the same for this to work.

Discussion: How many a's are we multiplying for $(a^{x})^{y}$?

Question 7 Walkthrough.

Simplify the following.

$$(x^3)^2$$

Question 8

Simplify the following.

a. $(x^2)^4$

b. $(x^{a+1})^2$

Discussion: If a^3 triples the number of a's multiplied, what does $a^{\frac{1}{3}}$ do?

Question 9 Walkthrough.

Simplify the following.

$$8^{\frac{1}{3}}$$

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Question 10

Simplify the following.

a. $27^{\frac{1}{3}}$

b. $8^{\frac{2}{3}}$

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Question 11 Extension.

a. Evaluate:

$$\frac{125^{-\frac{2}{3}} \times 27^{\frac{1}{3}} \times 2^{-2}}{2^{-5}} + 5^{-2}$$

b. Solve the following equation for x.

$$2^{3x+2} = 32 \times 4^{x}$$

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Sub-Section: Index Law

Let's summarise everything!

Index Laws

- All the rules explored above are called _____.

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1$$

$$(a \times b)^x = a^x \times b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{1}{x}} = \sqrt[x]{a}$$

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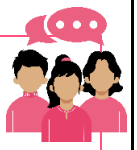

Discussion: Any question with the above rule? We can try proving together.

Let's mix them all!

Question 12 Walkthrough.

Simplify the following expressions.

a. $\frac{4^x}{8^{3x}}$

b. $\frac{a^{2x}}{(a^{3x})^{\frac{1}{2}}}$

Question 13

Simplify the following expressions.

a. $\frac{b^x}{b^{2x-1}}$

b. $\frac{3^x \times 9^{x-5}}{3^2}$

Question 14 Extension.

Simplify the following expression.

$$\frac{3^{2x+1} \times 27^{x-2}}{9^x \times 81}$$

Sub-Section: Inequalities

Question 15

Solve the following inequalities for x.

a. $2^x > 8$

b. $3^{2x-1} < 27$

Discussion: If the base is less than 1, does multiplying more of the base increase the number?

Question 16 Walkthrough.

Solve the following inequalities for x.

$$\left(\frac{1}{3}\right)^x > \frac{1}{9}$$

NOTE: If the base is less than 1, always flip the inequality sign.

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Question 17

Solve the following inequalities for x.

a. $\left(\frac{1}{2}\right)^x \geq \frac{1}{8}$

b. $\left(\frac{1}{3}\right)^x < 9$

Question 18 Extension.

Solve the following inequality for x.

$$2^{-x^2+3x} > 1$$

In summary!

Inequalities for Exponentials

For $a^x < a^y$

- Flip the inequality sign when base is less than 1.

If $a > 1$ then $x < y$.

If $0 < a < 1$ then $x > y$.

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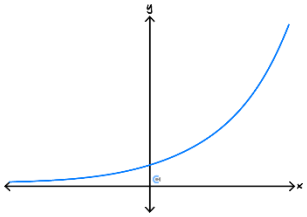
Section B: Graphs of Exponentials

Sub-Section: Graphs of Exponentials

Exponential Functions



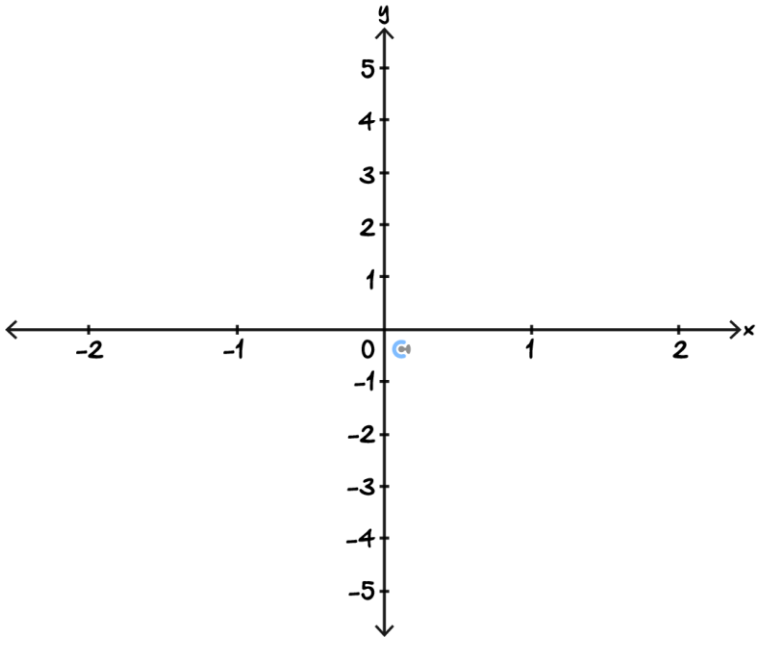
a^x where $a > 1$

- Domain of the exponential function is _____.
- Range of the exponential function is _____.

Question 19 Walkthrough.

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

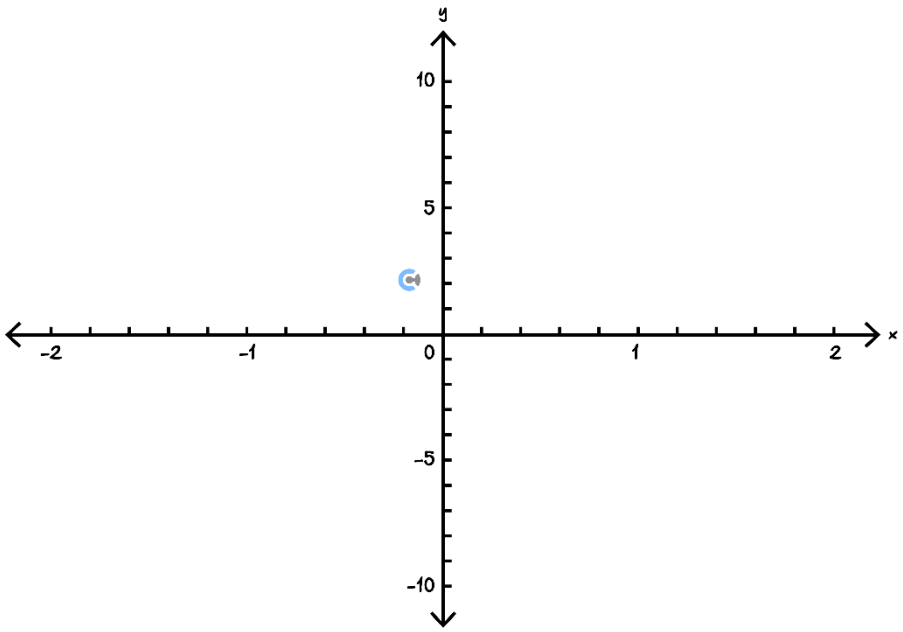
$$y = 2^x$$

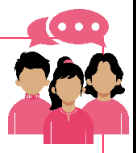


Question 20

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

$$y = -3^x$$





Discussion: What would the graph look like if the base was less than 1? For example: $\left(\frac{1}{2}\right)^x$?

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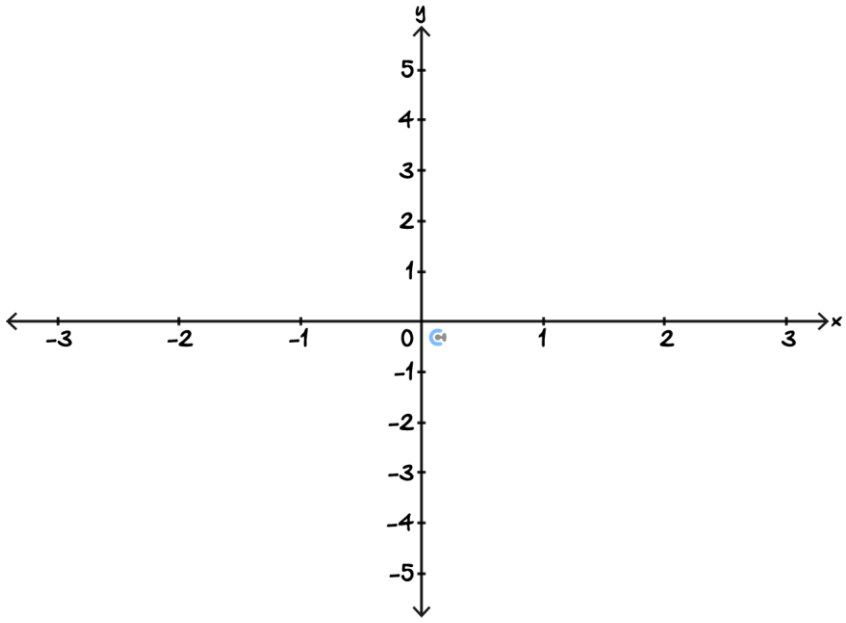
Question 21

Sketch the graph of the following function, labelling all key features including axes intercepts and asymptotes.

$$y = \left(\frac{1}{2}\right)^x$$



Let's take a look at more difficult graphs now!

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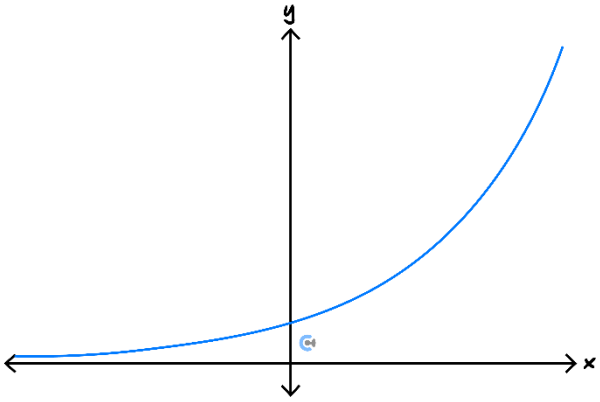
Graphs of Transformed Exponential Functions



$$y = a\,base^{b(x-h)} + k$$

- The horizontal asymptote is always given by _____.
- **Steps to take when sketching an exponential:**

1. Find corresponding asymptotes.
2. Plot x and y-intercepts (if they exist).
3. Sketch the curve.

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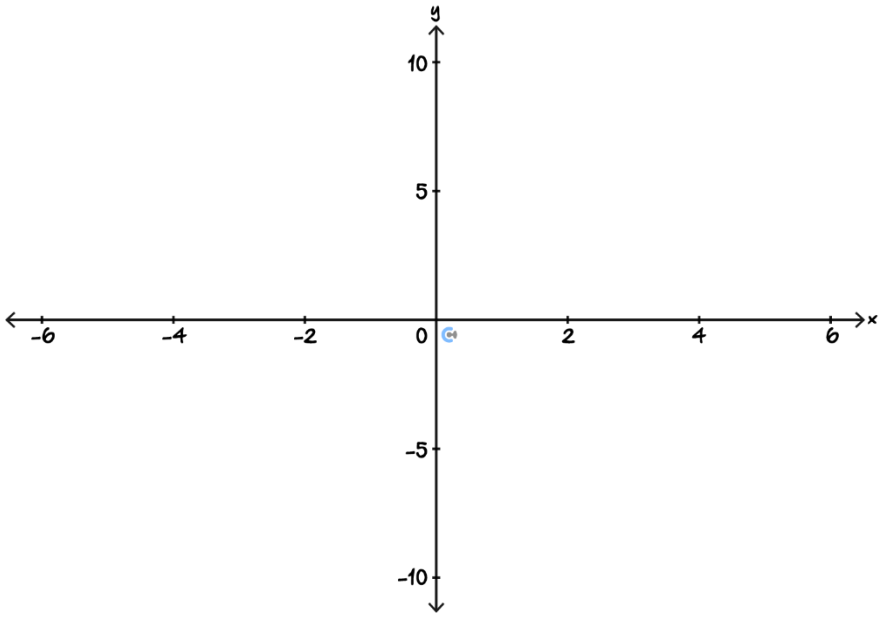
Question 22 Walkthrough.

Sketch the graph of the following function, labelling all key features, including axes intercepts and asymptotes.

$$y = \frac{1}{2} \times 4^{x-1} - 8$$



NOTE: Graphing is easy if you strictly follow the steps!

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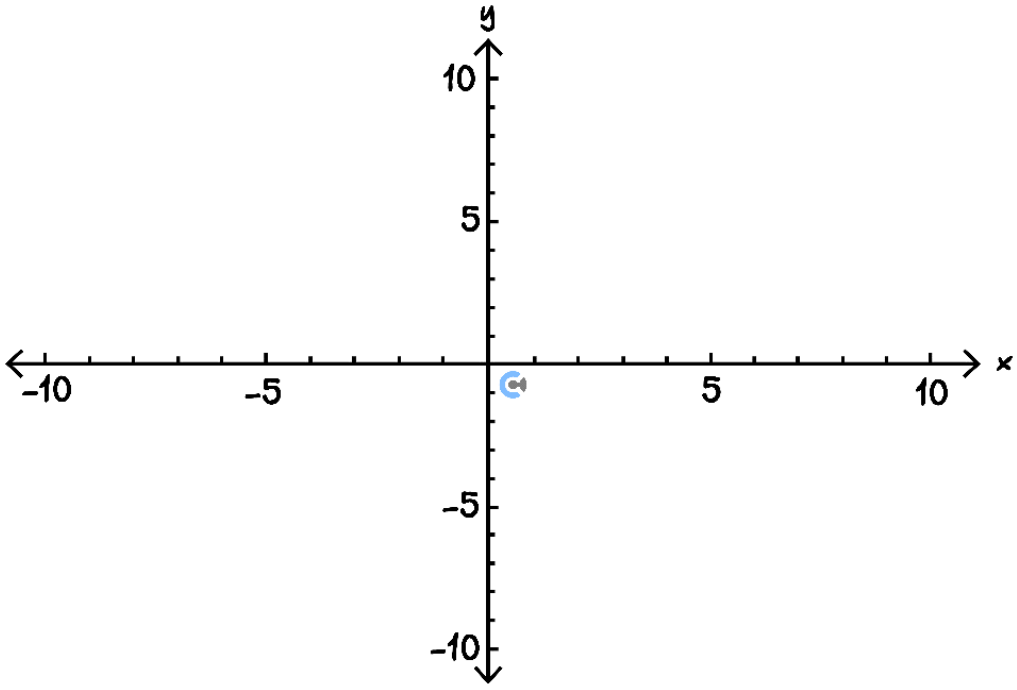
Your turn!

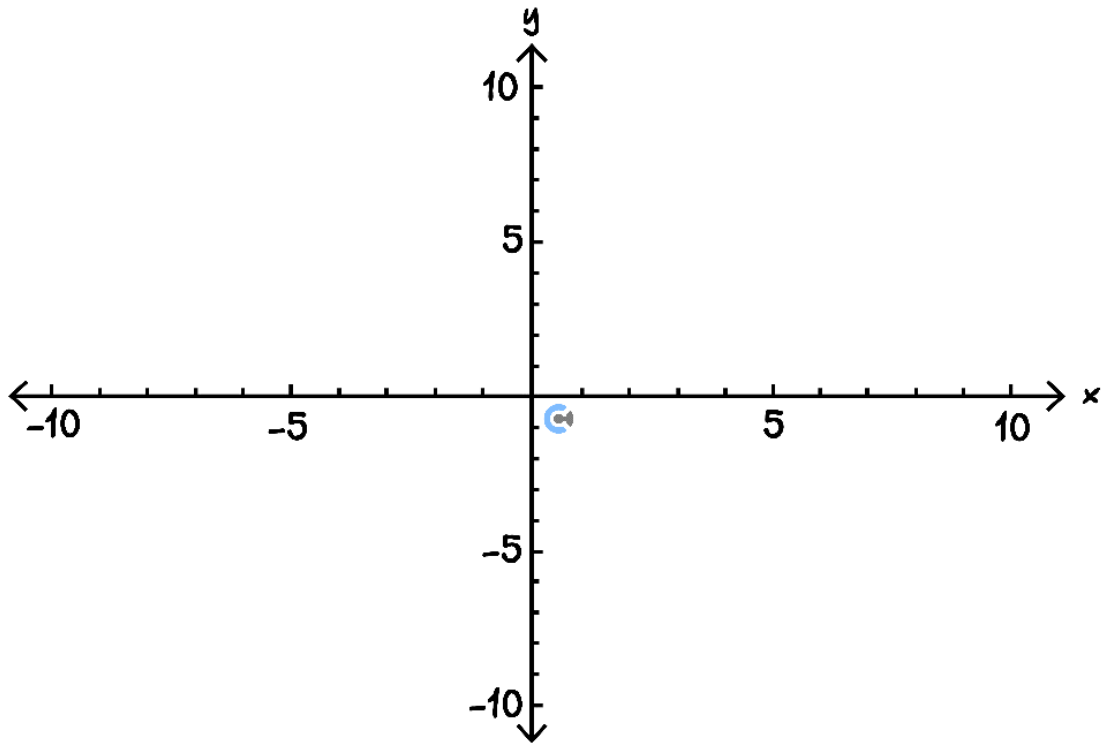
Question 23

Sketch the graphs of the following functions, labelling all key features including axes intercepts and asymptotes.

a. $y = 3^{x-1} - 9$

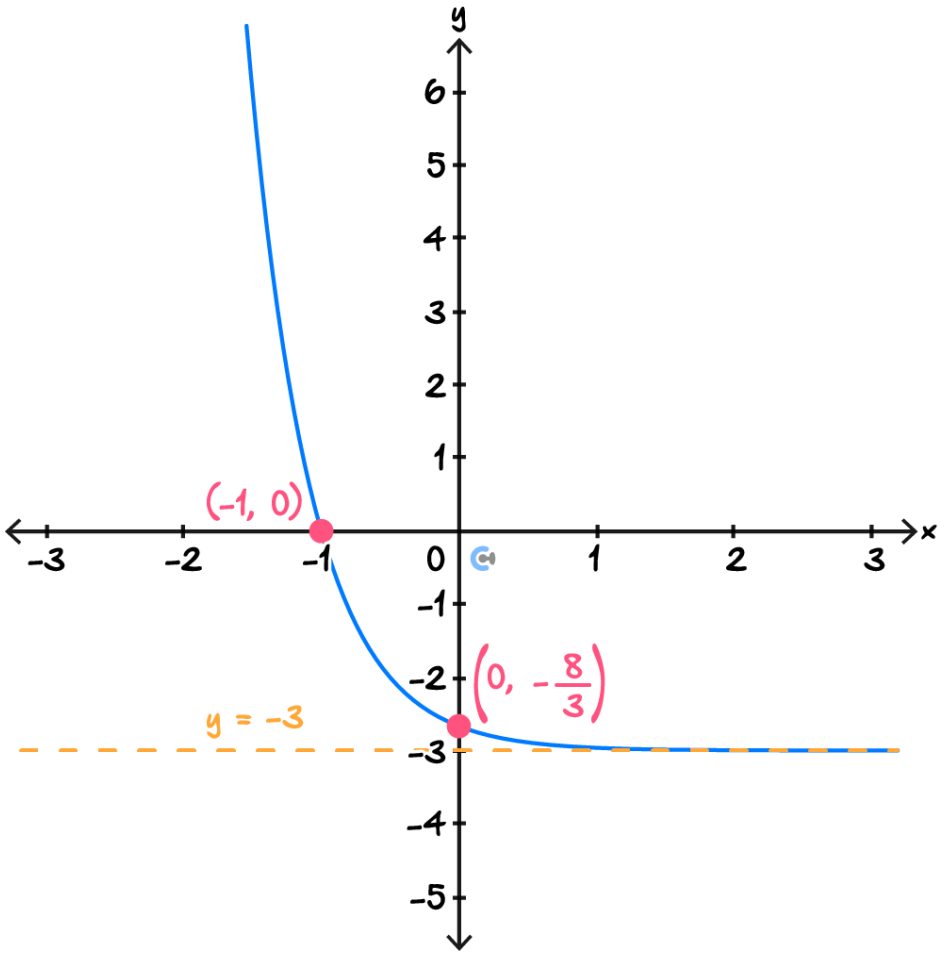


b. $y = -2^{-x-2} + 4$



Question 24 Extension.

Find a rule of the form $y = a^{bx+1} - d$, where $a, b, d > 0$, for the graph shown below.



Section C: Hidden Quadratics

Sub-Section: Understanding Hidden Quadratics

What is a hidden quadratic?

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

Question 25 Walkthrough.

Consider the following equation:

$$(x^2)^2 + 2x^2 + 1 = 0$$

Convert the equation to be a quadratic of A by appropriate substitution of A.

NOTE: Look for something and it squared!

Your turn!

Question 26

Convert the following equation to be a quadratic of A by appropriate substitution of A.

$$x - 5\sqrt{x} + 3 = 0$$

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Sub-Section: Hidden Quadratics for Exponentials

Hidden Quadratics for Exponentials

$$a \times exp^2 + b \times exp + c = 0$$

Let $A = exp$ where $A > 0$

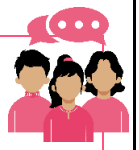
- Look for "**same base** and **double power**" pattern.

Discussion: Why does A have to be bigger than 0? HINT: Recall the graph of exponentials!

Question 27

Convert the following into a quadratic equation of A. You do not need to solve.

$$3^{2x} - 3 \times 3^x + 5 = 0$$



Discussion: What pattern does the exponentials need to have?

Your turn!

Question 28

Convert the following into a quadratic equation of a. You do **not** need to solve.

a. $4^{2x} - 7 \times 4^{x} + 8 = 0$

b. $4^{x} + 2^{x+1} - 3 = 0$

NOTE: Look for the same base, double power!

Let's now solve them!

Question 29 Walkthrough.

Solve the following equation for x.

$$2^{2x+1} - 12 \times 2^x + 16 = 0$$

Discussion: In the above question, both our A values were positive. What happens if one is negative or zero?

Your turn!

Question 30

Solve the following equations.

a. $3^{2x} - 4 \times 3^x + 3 = 0$

b. $2^{2x} - 3 \times 2^{x+1} - 16 = 0$

c. $4^x - 2^{x+1} - 8 = 0$

NOTE: You must show the process of rejecting your A value for the marking scheme!

Question 31 Extension.

Solve the following equations.

a. $3^{2x} + 2 \times 3^{x} - 15 = 0$

b. $2^{2x} - 5 \times 2^{x+1} + 16 = 0$

Cheat Sheet

[5.1.1] – Basics of exponentials

Exponentials

$$base \times \cdots \times base = base^{power}$$

- Exponentiation is a stacked multiplication.
- The power represents the number of bases we are multiplying.

Index Laws

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1$$

$$(a \times b)^x = a^x \times b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{1}{x}} = \sqrt[x]{a}$$

Inequalities for Exponentials

For $a^x < a^y$

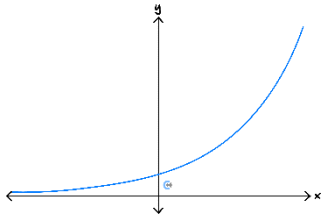
- Flip the inequality sign when base is less than 1.

If $a > 1$ then $x < y$.

If $0 < a < 1$ then $x > y$.

[5.1.2] – Graph exponentials

Exponential Functions



a^x where $a > 1$

- Domain of the exponential function is R.
- Range of the exponential function is R^+.

Graphs of Exponential Functions

$$y = a\, base^{b(x-h)} + k$$

- The horizontal asymptote is always given by $y = k$
- **Steps to take when sketching an exponential:**
  1. Find corresponding asymptotes.
  2. Plot x and y-intercepts (if they exist).
  3. Sketch the curve.
  4. Always follow these steps as they minimise potential mistakes.

[5.1.3] – Solve hidden quadratics of exponentials

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

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