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VCE Mathematical Methods ½
Exponentials [5.1]
Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 20
Supplementary Questions	Pg 21 – Pg 33

Section A: Compulsory Questions

Contour Check

Learning Objective: [5.1.1] – Basics of exponentials

Key Takeaways

Exponentials

$$base \times \cdots \times base = base^{power}$$

- Exponentiation is a _____.
- The power represents the _____.

Index Laws

$$a^x \times a^y = \_\_\_\_\_$$

$$\frac{a^x}{a^y} = \_\_\_\_\_$$

$$(a^x)^y = \_\_\_\_\_$$

$$a^0 = \_\_\_\_\_$$

$$(a \cdot b)^x = \_\_\_\_\_$$

$$\left(\frac{a}{b}\right)^x = \_\_\_\_\_$$

$$a^{-x} = \_\_\_\_\_$$

$$a^{\frac{1}{x}} = \_\_\_\_\_$$

Inequalities for Exponentials

$$\text{For } a^x < a^y$$

- Flip the inequality sign when the base is less than 1.

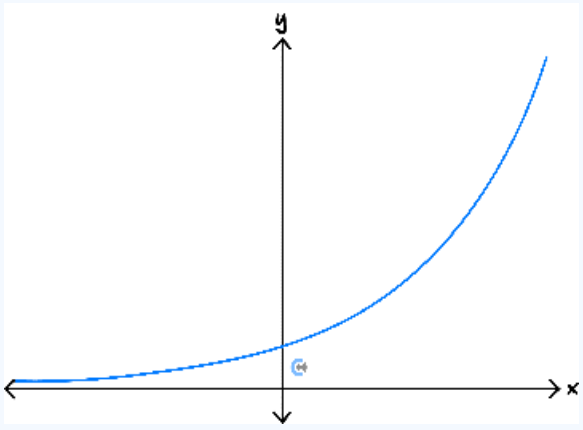
$$\text{If } a > 1 \text{ then } x < y$$

$$\text{If } 0 < a < 1 \text{ then } x > y$$

Learning Objective: [5.1.2] - Graph exponentials

Key Takeaways

Exponential Functions



$$a^x \text{ where } a > 1$$

- Domain of the exponential function is _____.
- Range of the exponential function is _____.

Graphs of Exponential Functions

$$y = a\,base^{b(x-h)} + k$$

- The _____ asymptote is always given by _____.
- **Steps to take when sketching an exponential:**
  1. Find corresponding _____.
  2. Plot x and y-intercepts (if they exist).
  3. Sketch the curve.
  4. Always follow these steps as they minimise potential mistakes.

Learning Objective: [5.1.3] – Solve hidden quadratics of exponentials

Key Takeaways

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

$$Let\ A = \_\_\_\_\_$$

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Sub-Section [5.1.1]: Basics of Exponentials

Question 1

Evaluate $8^{-\frac{2}{3}}$.

Question 2

Solve $49^x = 7$ for x.

Question 3

Solve the equation $2^{3x-3} = 8^{2-x}$ for x.

Question 4

Find the set of values of x for which $9^{x+1} \geq 27^{x-5}$

Question 5

Simplify the following, writing your answer in the positive index form.

a. $\sqrt{25x^5y^4}$

b. $3^n \times 9^{n+1} \times 27^n + 2$

c. $\frac{(-3x^2y^3)^2}{(2xy)^3}$

d. $\frac{x^3yz^{-2} \times (2x^3y^{-2}z)^2}{xyz^{-1}}$

Question 6

Evaluate $3^{-4} \times 27^{-\frac{1}{3}} \times 2^2$

Question 7

Simplify $\frac{12^{-5} \times 2^3 \times 9^{-8}}{6^2}$.

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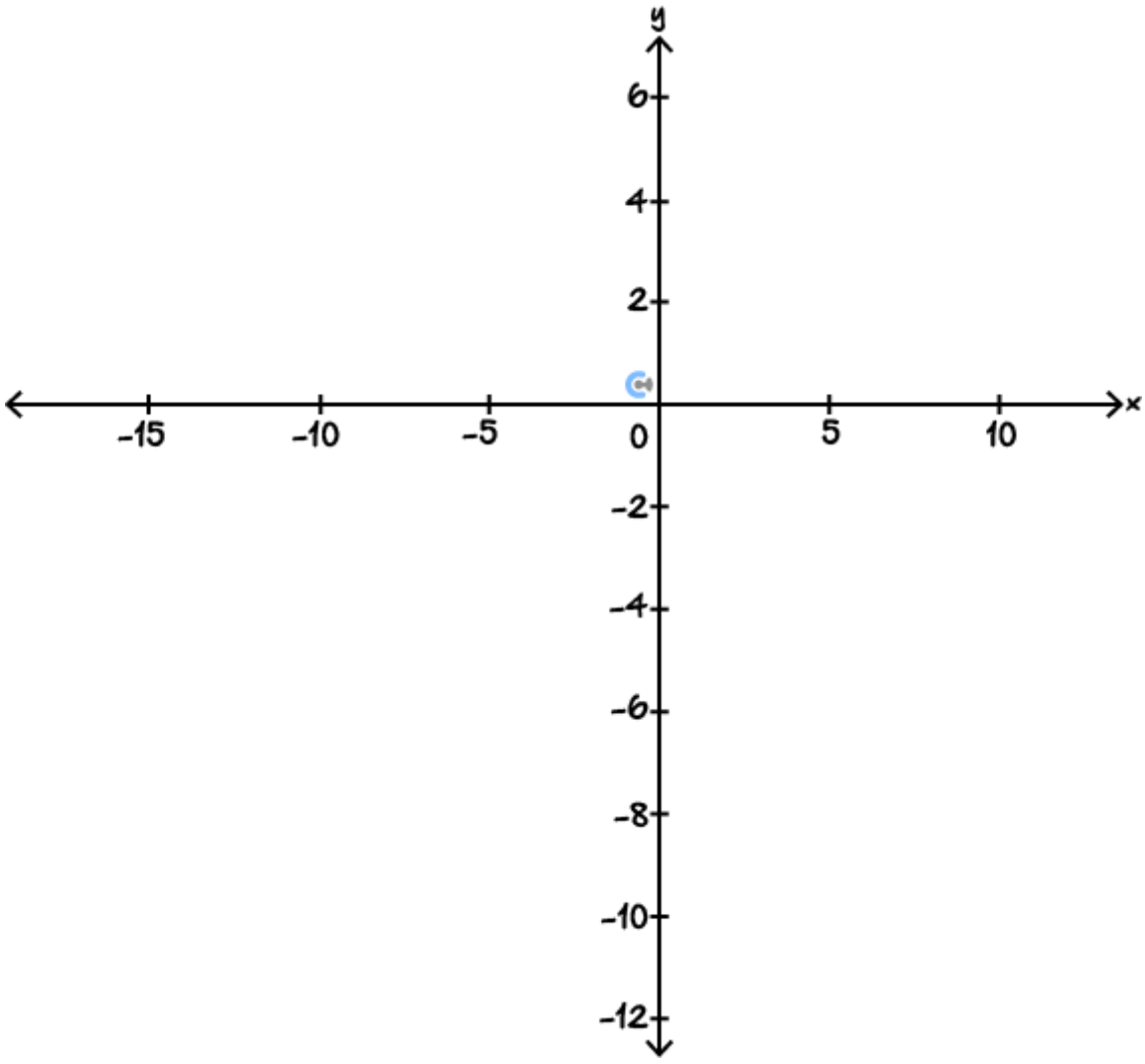
Sub-Section [5.1.2]: Graphs of Exponentials

Question 8

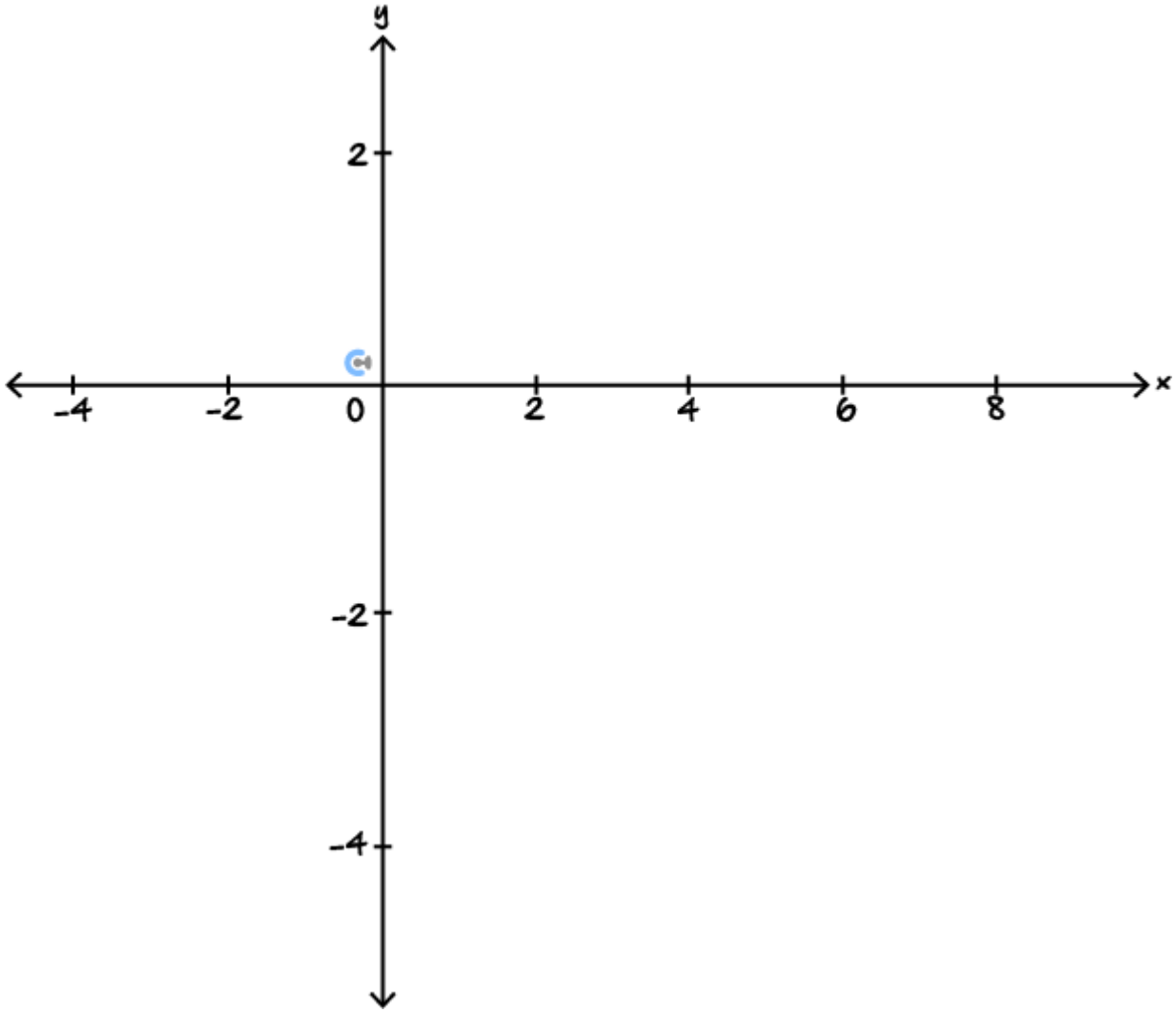
Sketch the graphs of each of the following, labelling all intercept coordinates and asymptotes with equations.

a. $y = 2^x - 8$



b. $y = -3 \times 4^{-x} + 1$

Useful information : $4^{-0.79} = \frac{1}{3}$



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Sub-Section [5.1.3]: Solving Exponential Equations with Hidden Quadratics

Question 9

a. Solve the equation $4^x - 15 \times 2^x = 16$ for x.

b. Solve the equation $2^{2x} + 4 \bullet 2^x - 32 = 0$ for x.

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Sub-Section: Problem Solving

Question 10

Solve $3^x = 6 + 27 \bullet 3^{-x}$ for x.

Question 11

Solve $5 \bullet 3^{-2x} = 11 \bullet 3^{-x} + 12$.

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Question 12

a. If the function $f(x) = 2^x$, describe the transformations that map $f(x)$ onto $-3f\left(\frac{x}{2}\right) + 5$.

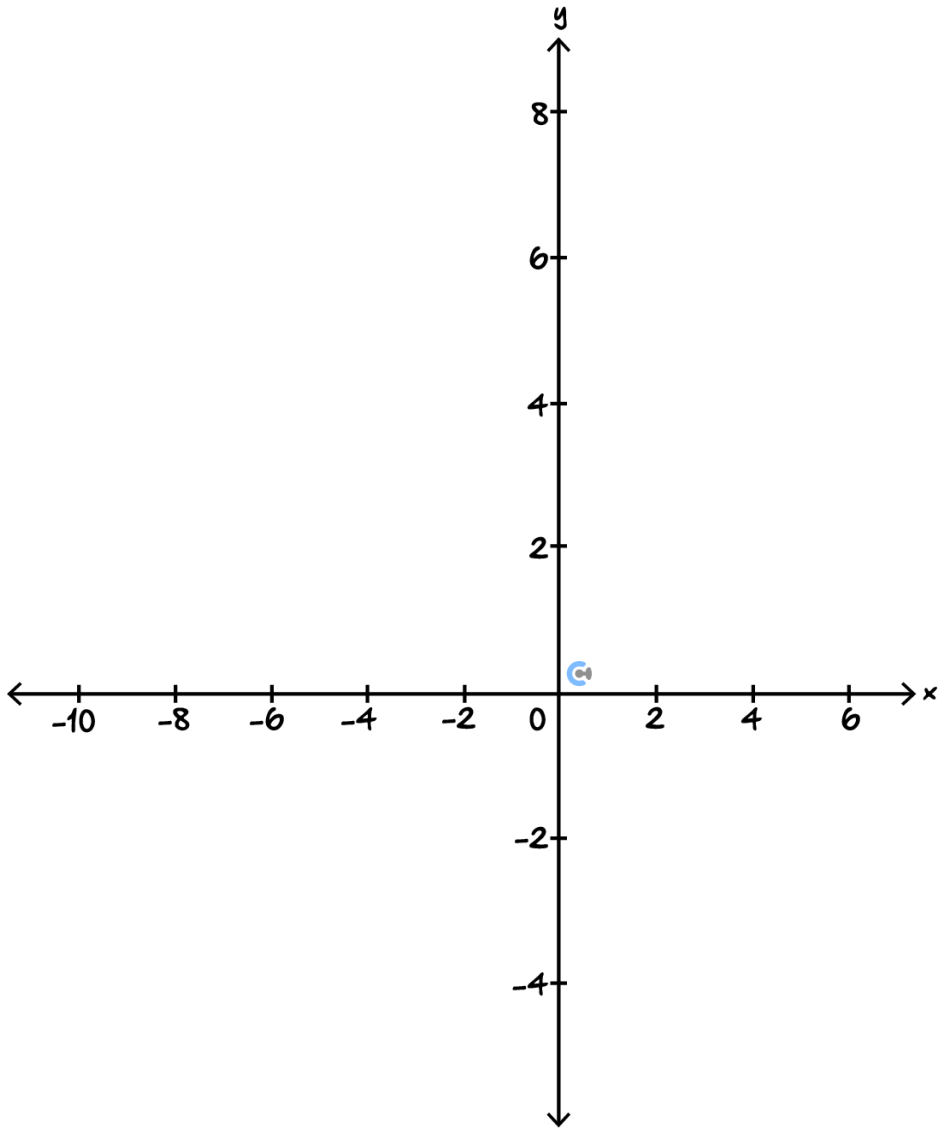
b. State the rule for the function $-3f\left(\frac{x}{2}\right) + 5$. Write down its maximal domain and the corresponding range.

Now consider another function $g(x)$.

c. If $g(x)$ in the form of $a \bullet b^x + c$ and passes through points $(1, 7)$ and $(2, 17)$, and has an asymptote at $y = -3$, find the value(s) of a, b and c.

d. Find the **point of intersection** between $-3f\left(\frac{x}{2}\right)+5$ and $g(x)$.

e. Sketch the graph of $-3f\left(\frac{x}{2}\right)+5$ and $g(x)$. Label any **asymptotes** with their equations and the ***y*-intercepts** with their coordinates.



f. If you do not want any intersections between $-3f\left(\frac{x}{2}\right)+5$ and $g(x)$, what translation should you apply to $g(x)$?

Question 13 Tech-Active.

The value of a piece of equipment initially valued at \$40000 depreciates at a rate of 8% per annum.

a. Find the value of the equipment after 1 year.

b. Find the value of the equipment after n years.

c. After how many years will the value of the equipment be less than \$20000? Will the value of the equipment ever reach \$0?

Question 14 Tech-Active.

There are approximately ten times as many red kangaroos as grey kangaroos in a certain area. If the population of grey kangaroos increases at a rate of 13% per annum while that of the red kangaroos decreases at 7% per annum, find how many years must elapse before the proportions are reversed, assuming the same rates continue to apply.

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Sub-Section: The Tech-Free 'Final Boss'

Question 15

Consider the following functions:

$$h(x) = -2 \bullet 3^{x-1} + \frac{2}{3}, p(x) = \frac{1}{2} \bullet 3^{2x} - \frac{1}{2}$$

a. Describe the transformation(s) applied to the base function $y = 3^x$ to obtain:

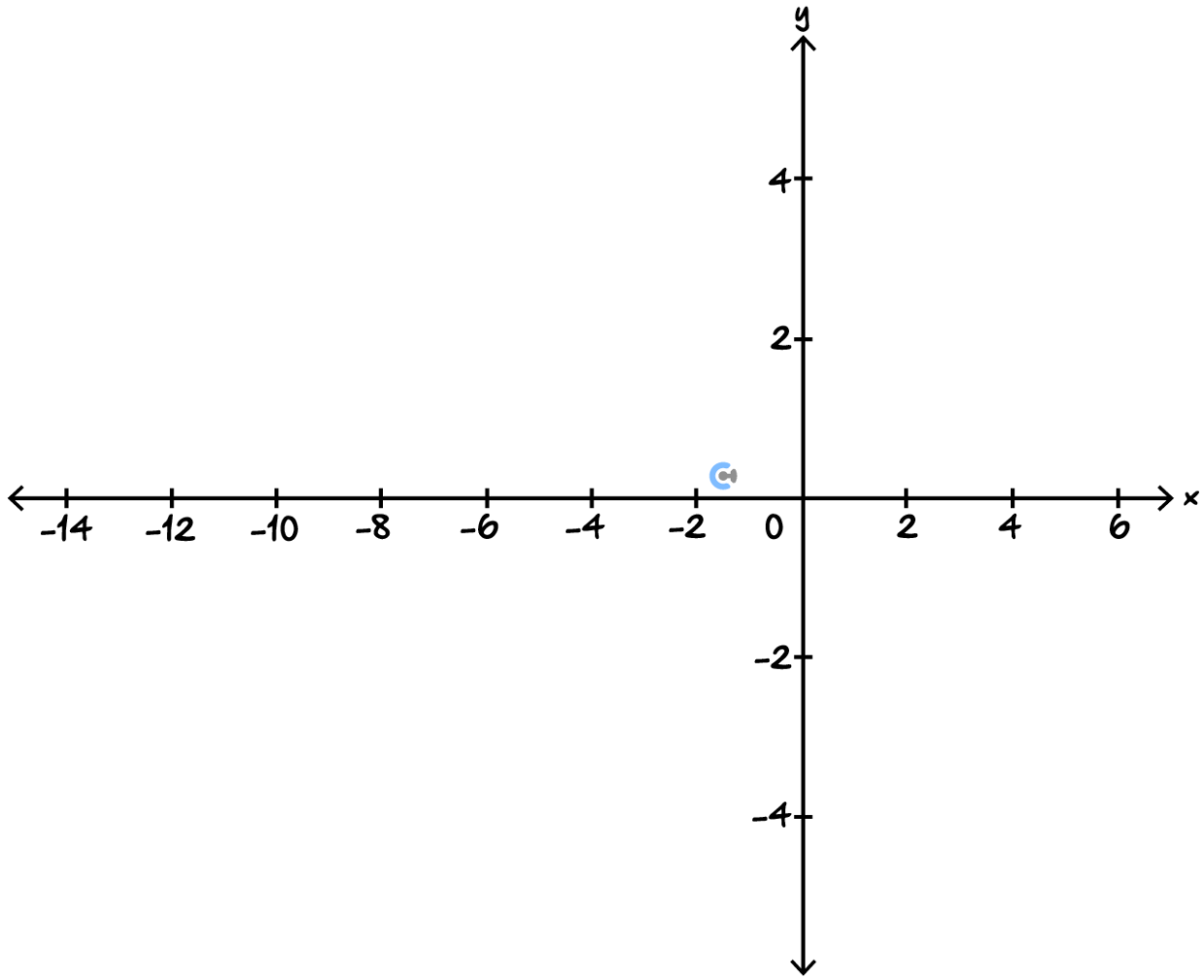
i. 3^{x-2}, using dilation only.

ii. $h(x)$.

iii. $p(x)$.

b. Solve $h(x) > p(x)$ for x.

c. On the same axes, sketch the graphs of $h(x)$ and $p(x)$. Label all **asymptotes** and ***y*-intercepts**.



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Sub-Section: The Tech-Active 'Final Boss'

Question 16

Liam recently started a new job at a hospital in the eastern suburbs of Melbourne. As more residents have moved into the area over time, peak-hour traffic has worsened, and the time it takes Liam to commute has increased significantly. The travel time is found to increase exponentially with the number of cars on the road.

A model of the amount of time, in minutes, it takes Liam to get to work is $T(n) = a \bullet 2^{(kn)}$ where T is the time (in minutes), and the number of drivers is n (in hundreds of thousands); a and k are constants.

When Liam first started commuting, it used to take him 39 minutes, and there were 120,000 drivers on the road. Recent transport data shows that 10 years later, the same trip now takes 2 hours and 36 minutes, with 360,000 drivers on the road.

a. Show that $k = \frac{5}{6}$ and $a = \frac{39}{2}$.

b. At this rate, if the number of drivers is expected to reach 450,000 in the future, how long, to the nearest minute, will the same trip take?

c. How many cars are predicted to be on the road if a trip is expected to take 3 hours? Give your answer to the nearest 5000 drivers.

d. How much longer (in minutes) is the trip expected to take if the number of drivers increases from 760,000 to 900,000?

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Section B: Supplementary Questions

Sub-Section: Exam 1 (Tech-Free)

Question 17

Find the set of values of t for which $4 \times 2^{0.2t} > 2$.

Question 18

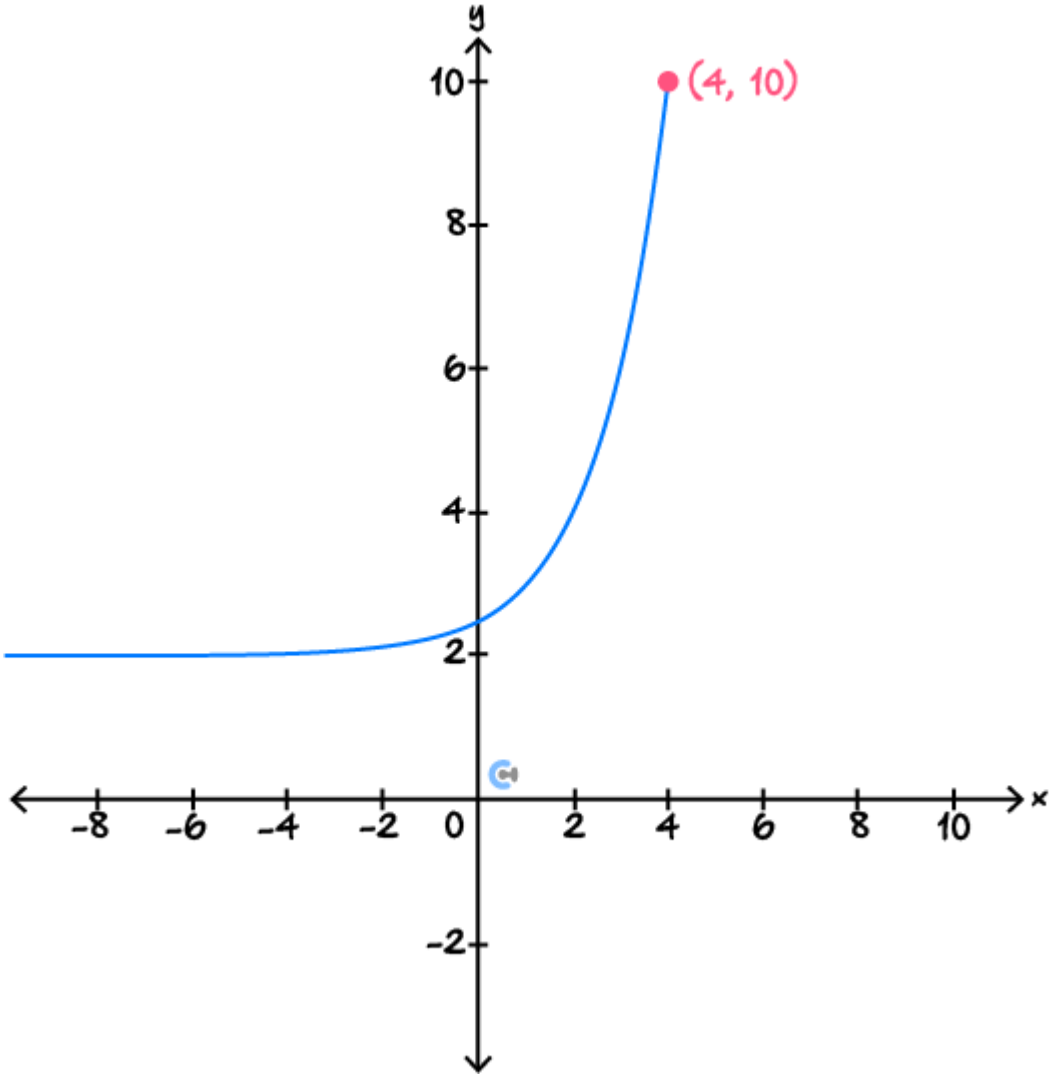
Solve for x where $256^{-x} = \frac{4^x}{16^{3-x}}$.

Question 19

Simplify $\frac{x}{1+x^{-1}} + \frac{x}{x^{-1}-1}$.

Question 20

The graph of the function defined by $f: (-\infty, 4] \to R, f(x) = 2^{x+b} + c$ is shown below.



a. Use algebra to show that $b = -1$ and $c = 2$.

b. Determine the range of $f(x)$.

c. Does the inverse function $f^{-1}(x)$ exist? Explain why.

Now consider $g(x) = -16 \bullet 2^{-x-2} + \frac{13}{2}$.

d. Find the set of values for x such that $g(x) > f(x)$.

e. On the same set of axes above, sketch $g(x)$. Label asymptote, intercepts and point of intersection(s).

Useful information: $2^{0.70} = \frac{13}{8}$

f. Describe the sequence of transformations applied to $g(x)$ to obtain 2^x.

Sub-Section: Exam 2 (Tech-Active)

Question 21

If $3^{\frac{1}{x}} = \frac{1}{9}$ **then:**

A. $x = -2$

B. $x = 2$

C. $x = -\frac{1}{2}$

D. $x = \frac{1}{2}$

Question 22

Let $g(x) = 2^x, x \in R$. Which one of the following equations is true for all positive real values of x?

A. $g(x) + g(y) = 2^{x+y}$

B. $g(2x) = 2g(x)$

C. $2g(2x) = 4g(x)$

D. $g(x)g(y) = 2^{x+y}$

E. $g(x) - g(y) = 2^{2x+y}$

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Question 23

The table below shows the mass m, in grams, of a substance at a time t hours.

t	1	2	3	4	5	6
m	8.2	6.0	4.3	3.1	2.3	1.6

The mass and time would be best modelled using:

A. A linear function.

B. An exponential function.

C. A power function.

D. A circular function.

Question 24

The size of a population of rabbits is determined by the rule $P = 7200 \times 3^{0.3t} - 500$, where P is the size of the population t years after January 2016.

a. Find the size of the population when:

i. $t = 0$

_____

_____

ii. $t = 20$

_____

_____

b. After how many years does the size of the population exceed 1000000?

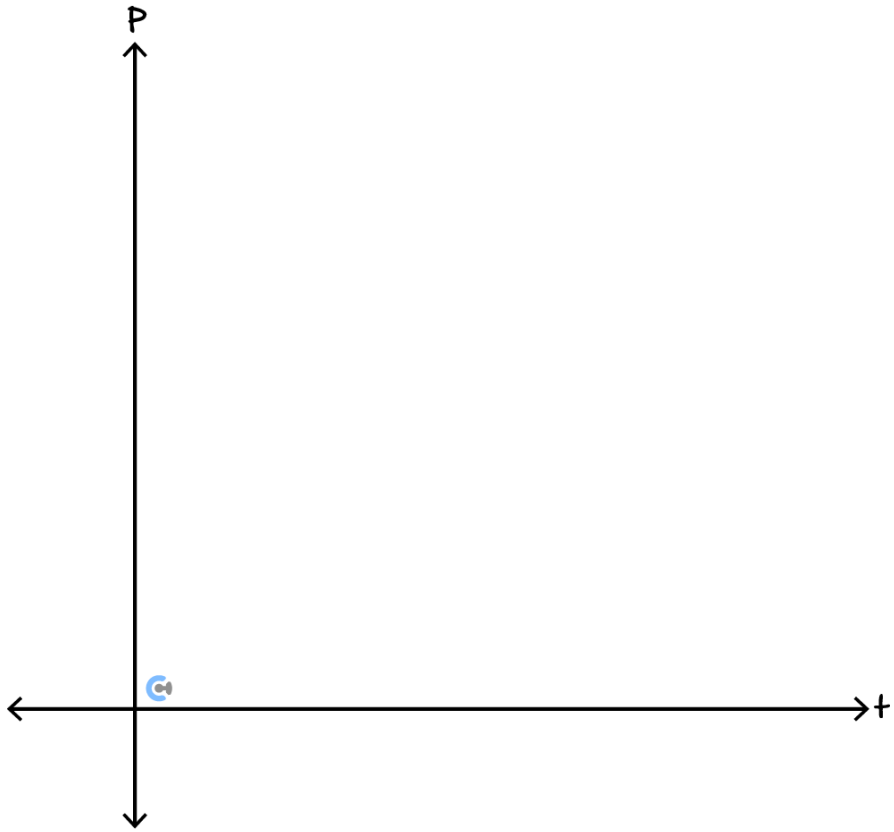
_____

_____

_____

_____

c. Sketch the graph of P against t. Label y-intercept of the graph.



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Question 25

The half-life of plutonium-239 is 24,000 years. A geologist discovers a rock sample containing 10 grams of plutonium-239. How long will it take until only 10% of the original plutonium remains in the rock? Give your answer to the nearest year.

Hint: The equation that describes how the mass of plutonium changes over time looks like: $A = A_0 \times 2^{kt}$, where A_0 and k are constants.

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Question 26

A wooden sculpture discovered in an ancient shipwreck contains 40% of the original carbon-14 it once had. Carbon-14 is a radioactive isotope with a half-life of 5730 years.

How old is the sculpture? Give your answer to the nearest year.

Hint: Similar to the previous question, the equation used to model this problem looks like: $A = A_0 \times 2^{kt}$, where A_0 and k are constants.

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Question 27

The population of a town increases by 6% every year. In January 2016, the population was 5500.

a. Find the population of the town:

i. In January 2017.

ii. After n years from 2016.

b. Find the year in which the population will reach 11000.

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Question 28

After a conservation program was introduced in a wildlife sanctuary on **1st January 2015**, the population of **birds** was modelled by:

$$B(t) = 50 \times 2^{0.4t}$$

Where t is the number of months after the start of the program.

At the same time, the number of **squirrels** was modelled by

$$S(t) = 400 \times 4^{-0.1t}$$

Find

a. The number of **birds** and **squirrels** in the sanctuary on **1st January 2015**.

b. Which of the two animals will have the highest numbers on **1st April 2016**?

c. The **date** when the bird population will equal the squirrel population.

If the number of squirrels falls below 25, they are at risk of extinction in the sanctuary.

d. According to the model, will this happen? If so, **when** will it happen?

Question 29

Alex takes a bottle of juice out of the fridge at **9:00 AM**. The juice starts at a cold temperature of **4**℃ and begins to warm up in a room. The room temperature is constant, and the juice's temperature increase is modelled by the equation:

$$T(t) = a \bullet \left(\frac{9}{4}\right)^{-kt}$$

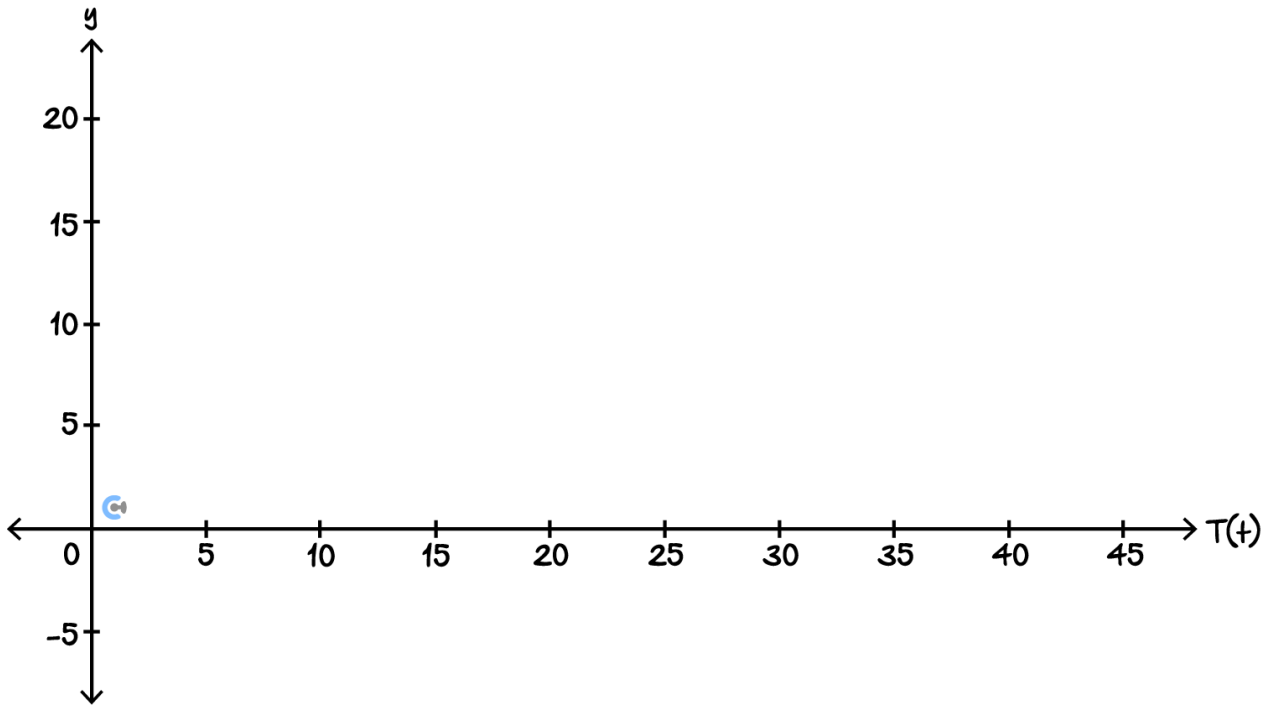
Where, $T(t)$ is the **difference** in temperature (in ℃) between the juice and the **ambient room temperature of 22**℃, t is the number of minutes since **9:00 AM**, and a and k are constants.

a. Determine the value of the constant a.

b. If the juice reaches **14**℃ at **9:20 AM**, determine the value of k.

c. By the time Alex remembers the juice, it is **9: 45 AM**. Using your values of a and k, calculate the juice's temperature at that time, to the **nearest degree**.

d. Sketch the graph of $T(t)$ over 45 minutes, clearly labelling start and endpoints.



At 9: 45 AM, Alex puts the juice in the fridge again. The **cooling** of the juice is now modelled by:

$$C(t) = 4 + A \times 2^{-\frac{t-45}{10}}$$

Where $C(t)$ is the juice temperature at time t, and t is the number of minutes since **9: 00 AM**.

e. Find the value of A, correct to 1 decimal place.

f. What is the appropriate **domain** for $C(t)$? Justify your answer.

g. For how long (to the nearest minute) is the juice **above 14**°C?

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