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VCE Mathematical Methods ½
Graph of Circular Function Exam Skills [4.5]
Workbook

Outline:



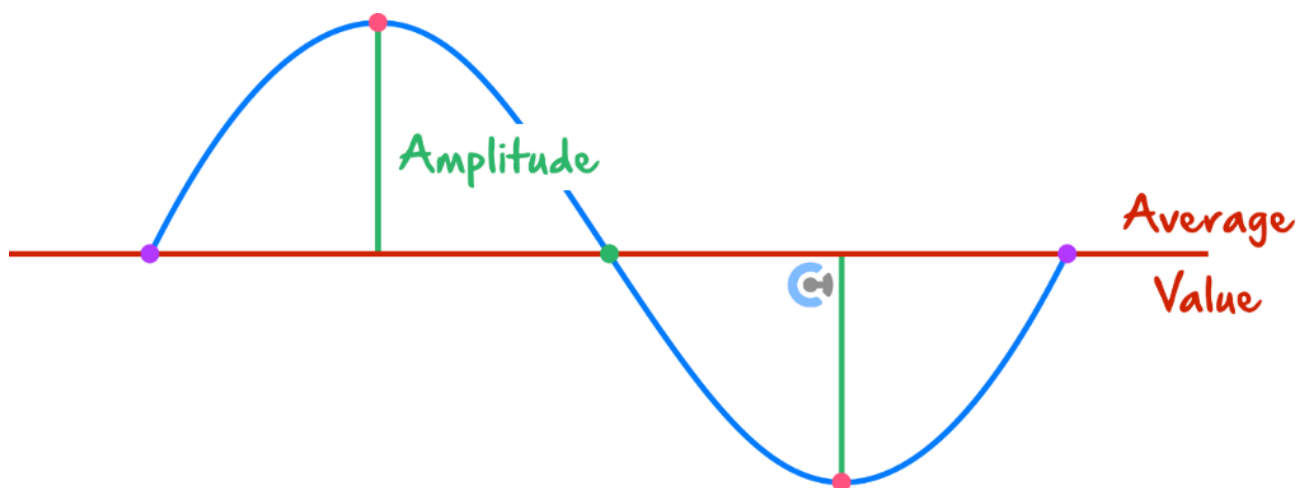
<u>Recap</u>	Pg 2-6		
<u>Warmup Test</u>	Pg 7-10	<u>Technology Exam Skills</u>	Pg 17-19
<u>Exam Skills</u>	Pg 11-16	<u>Exam 2</u>	Pg 20-26
➤ Identifying the Correct Graph			

Section A: Recap



Amplitude, Period and Average Value

For $y = A\sin/\cos(nx + b) + k$



Consider the sign of our graph

$$\text{Amplitude} = |A|$$

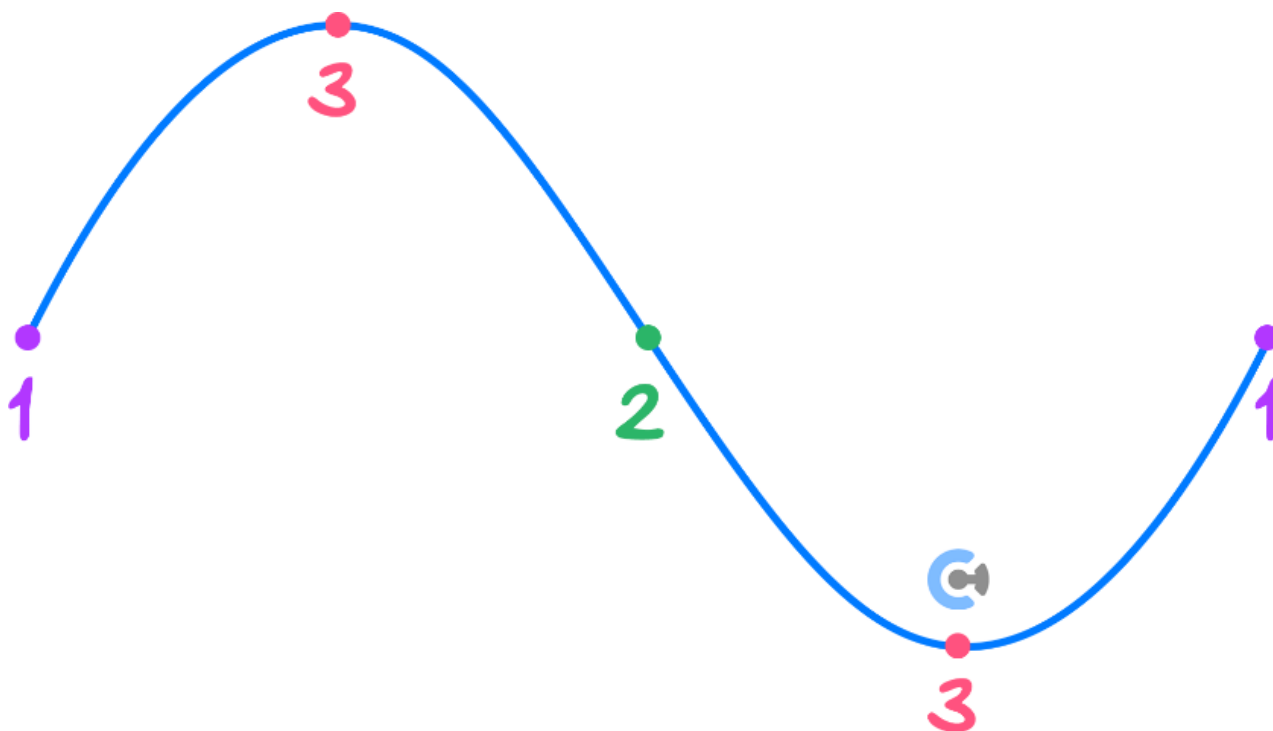
$$\text{Period} = \frac{2\pi}{|n|}$$

$$\text{Average Value} = k$$

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Graphing of sin and cos Functions



► Steps:

1. Identify: _____.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle = _____.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any _____.
6. Join all the points!



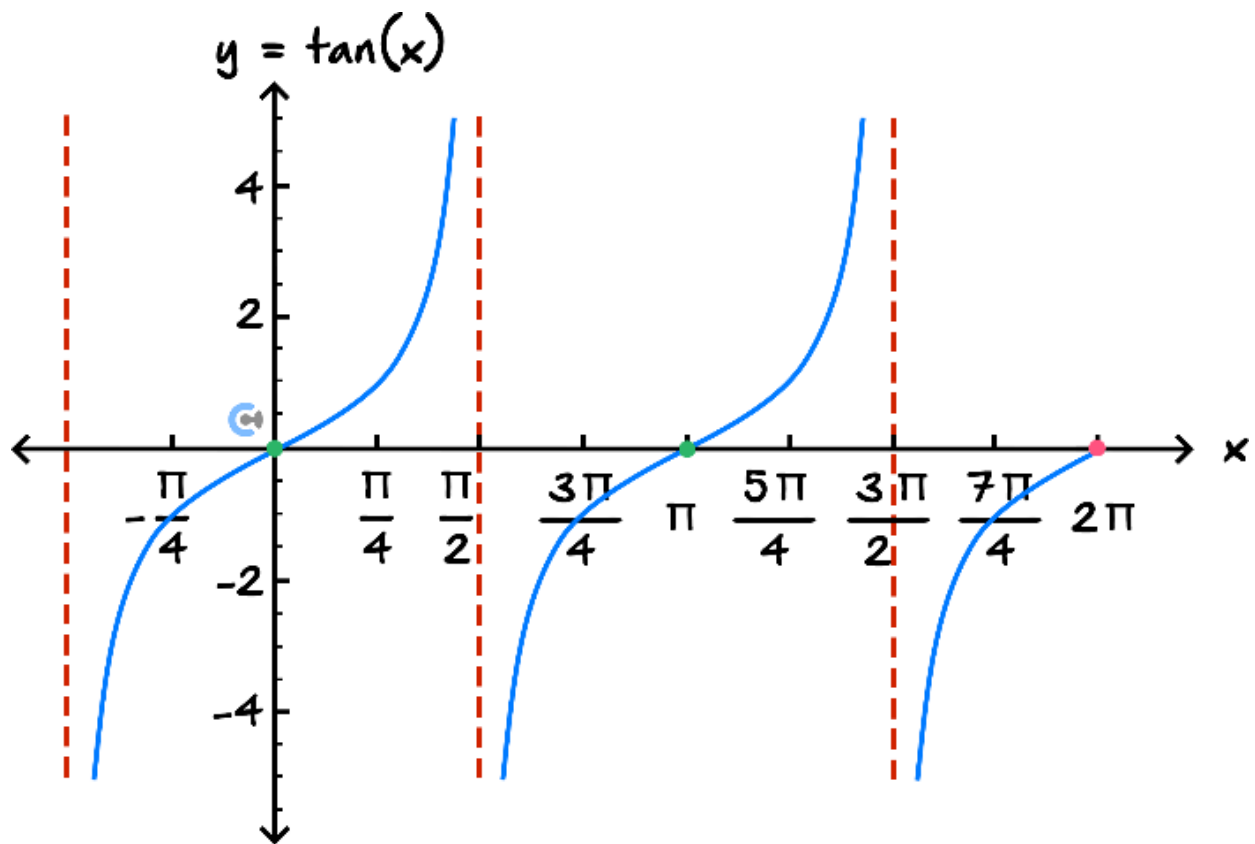
Finding the Rule

$$\text{Amplitude } (A) = \frac{\text{max} - \text{min}}{2}$$

$$\text{Average } (k) = \frac{\text{max} + \text{min}}{2}$$



Graph of Tangent



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Steps for Sketching tan Functions


1. Identify:


 The period = _____.

2. Find the vertical asymptotes by solving for angle = _____.

3. Find other vertical asymptotes within the domain by adding the period to the answer from the previous step.

4. Plot the inflection point (h, k) . (Midpoint of the two _____.)

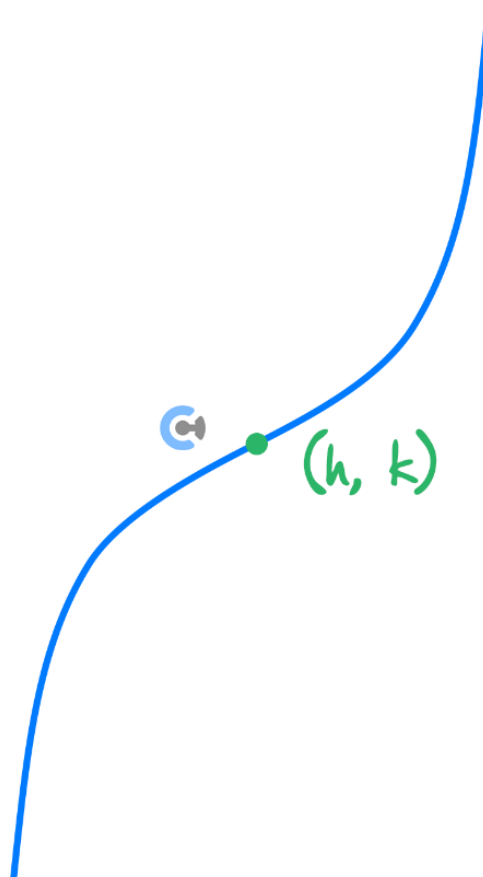
 x -value of the inflection point = x -value which makes angle = 0.

 y -value of the inflection point = vertical translation of the function.

5. Find any _____.

6. Sketch a _____ shape.

eg: $\tan(x-h) + k$





Fraction of Period

$$\text{Fraction of Period} = \frac{\text{Duration}}{\text{Period}}$$

$$\% \text{ of Period} = \frac{\text{Duration}}{\text{Period}} \times 100\%$$

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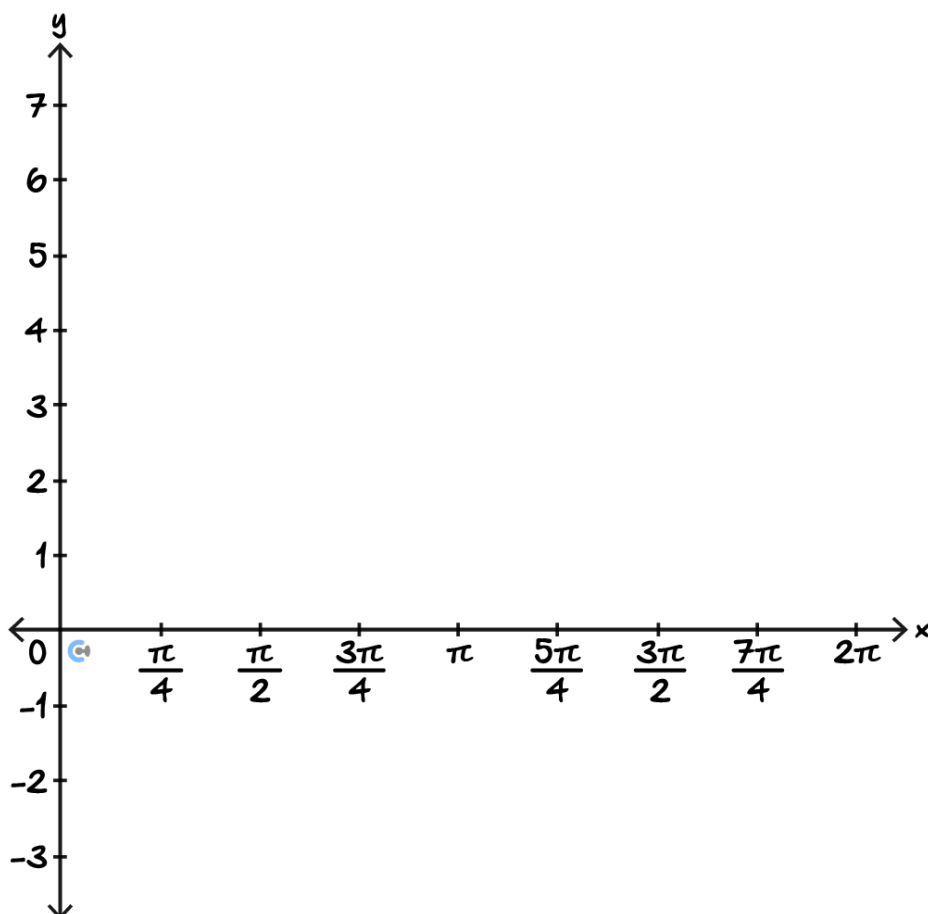
Section B: Warmup Test (16 Marks)

Question 1 (8 marks)

Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2 - 4 \cos(2x)$.

- a. Find the general solution to $f(x) = 0$. (2 marks)

- b. Sketch the graph of $y = f(x)$ for $x \in [0, 2\pi]$. Label any endpoints, turning points and axes intercepts with coordinates. (3 marks)

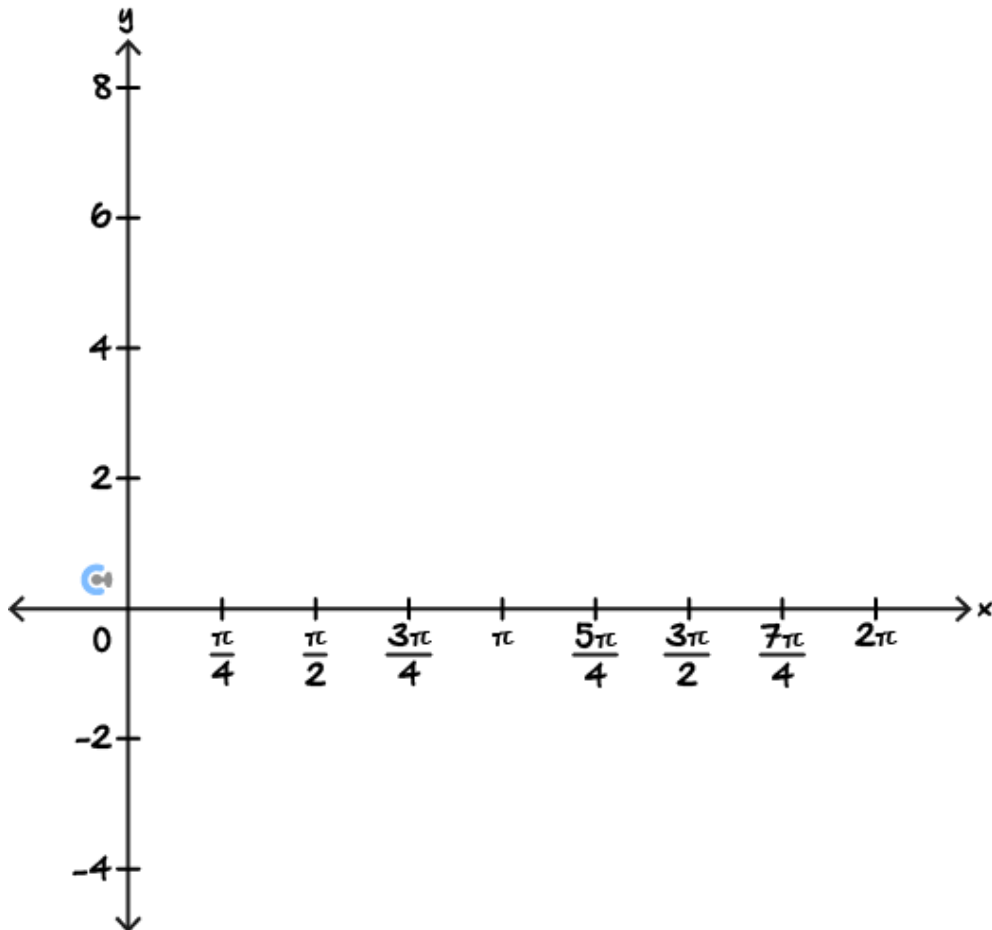


- c. Find the fraction of a period that $f(x)$ is above 4 for. (3 marks)

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Question 2 (4 marks)

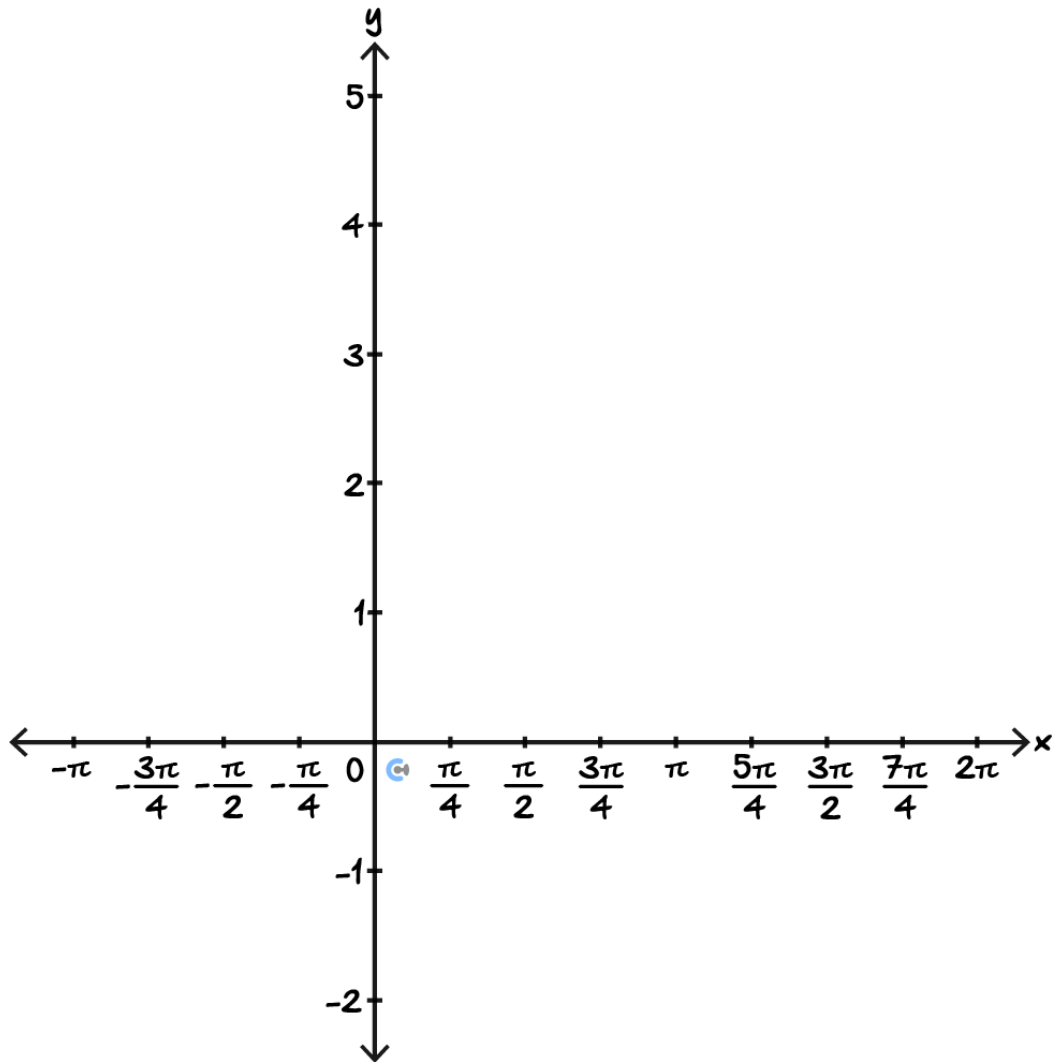
Sketch the graph of $y = -2 \tan\left(x - \frac{\pi}{4}\right) + 2$ for $x \in [0, 2\pi]$. Label any asymptotes with equations and endpoints, axes intercepts, and inflection points with coordinates.



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Question 3 (4 marks)

Sketch the graph of $y = -2 \sin\left(x - \frac{\pi}{4}\right) + 1$ for $x \in [-\pi, 2\pi]$. Label any endpoints, turning points and axes intercepts with coordinates.



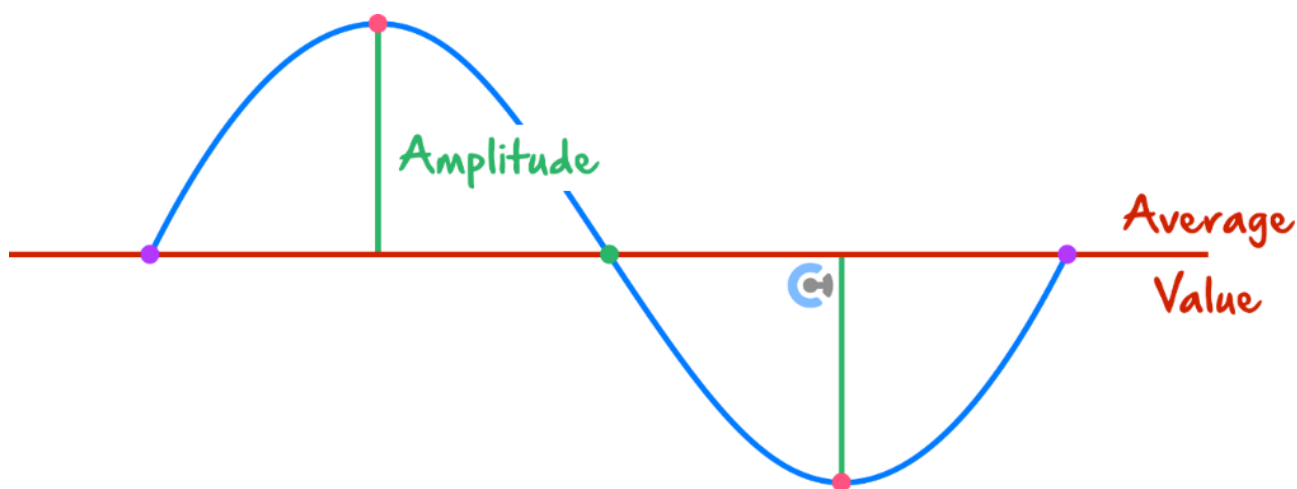
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Section C: Exam Skills

Sub-Section: Identifying the Correct Graph

REMINDER: Amplitude, Period and Average Value

For $y = A \sin/\cos (nx + b) + k$



Consider the sign of our graph

$$\text{Amplitude} = |A|$$

$$\text{Period} = \frac{2\pi}{|n|}$$

$$\text{Average Value} = k$$

REMINDER: Finding the Rule

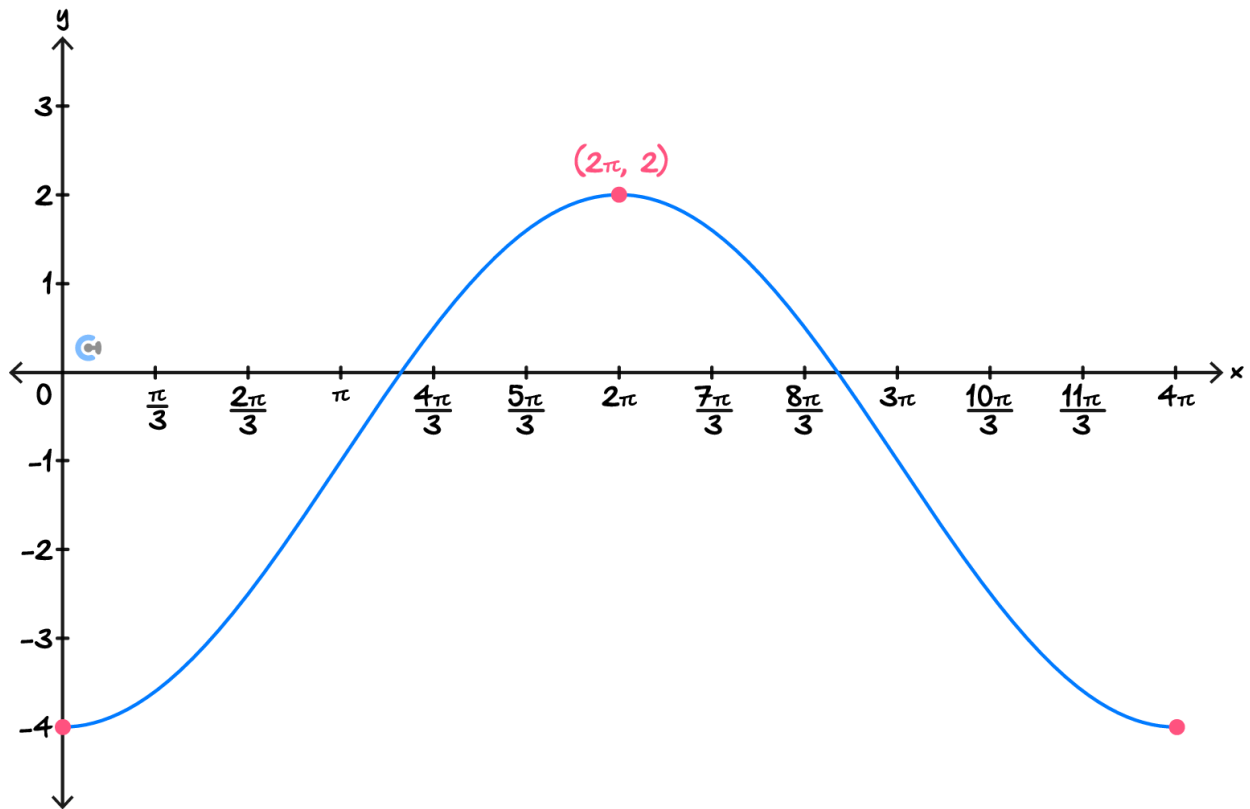
$$\text{Amplitude } (A) = \frac{\text{max} - \text{min}}{2}$$

$$\text{Average } (k) = \frac{\text{max} + \text{min}}{2}$$

Question 4 Walkthrough.

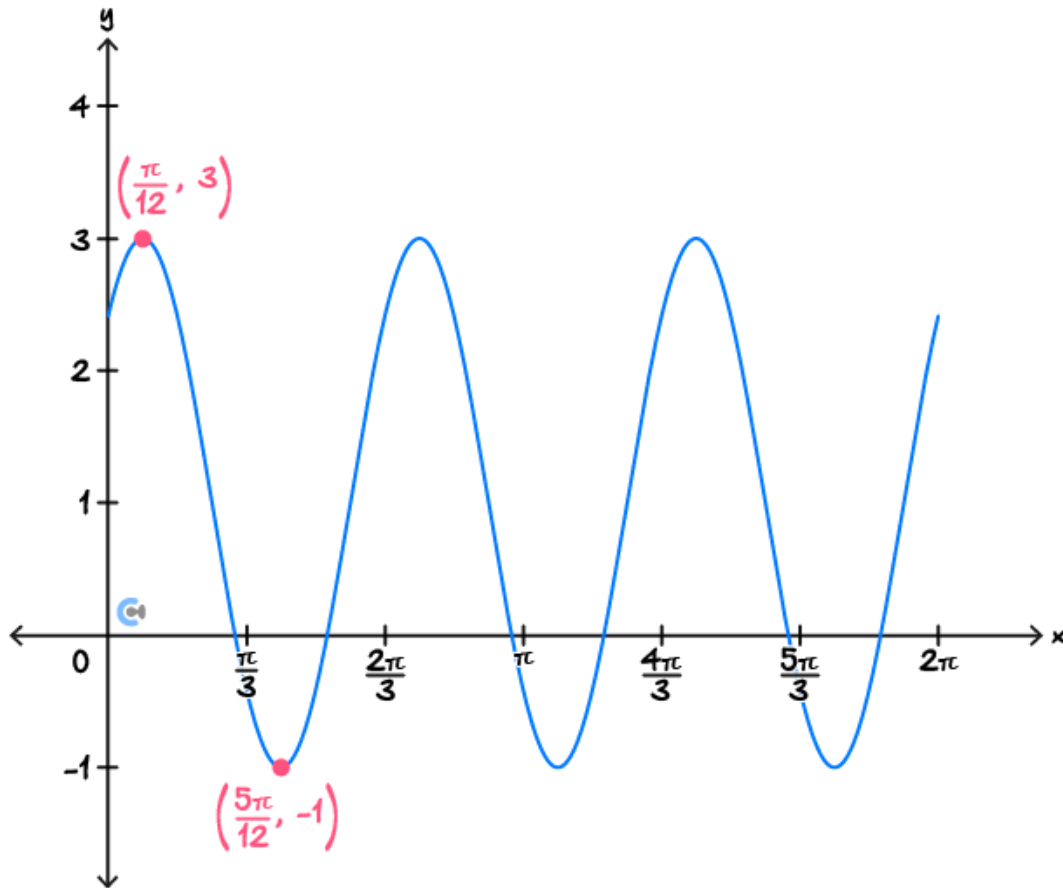
The graph shown below has a rule of the form $y = a \cos(nx) + k$.

Find a possible rule for the graph.



Question 5


The graph shown below has a rule of the form $y = a \sin(nx + b) + k$.
Find a possible rule for the graph, given that a, n, b, k are all greater than zero.






REMINDER: Steps for Sketching \tan Functions

1. Identify:


 The period = $\frac{\pi}{n}$.


2. Find the vertical asymptotes by solving for angle = $\frac{\pi}{2}$.

3. Find other vertical asymptotes within the domain by adding the period to answer from the previous step.

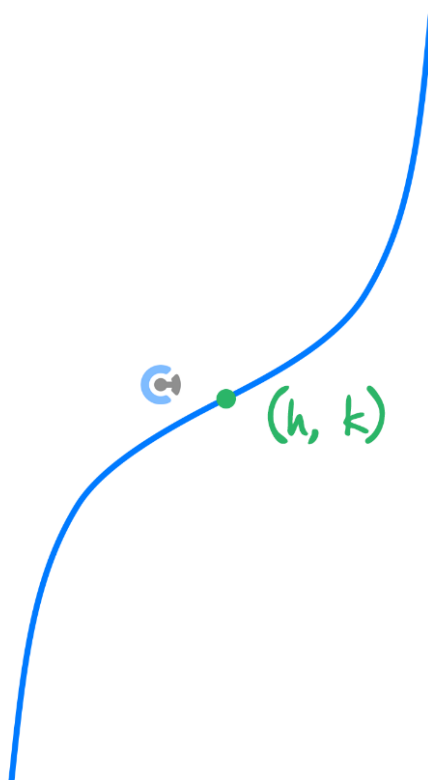
 For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x .

4. Plot the inflection point (h, k) . (Midpoint of the two vertical asymptotes.)

 x -value of inflection point = x -value which makes angle = 0.

 y -value of inflection point = vertical translation of the function.

eg: $\tan(x-h) + k$

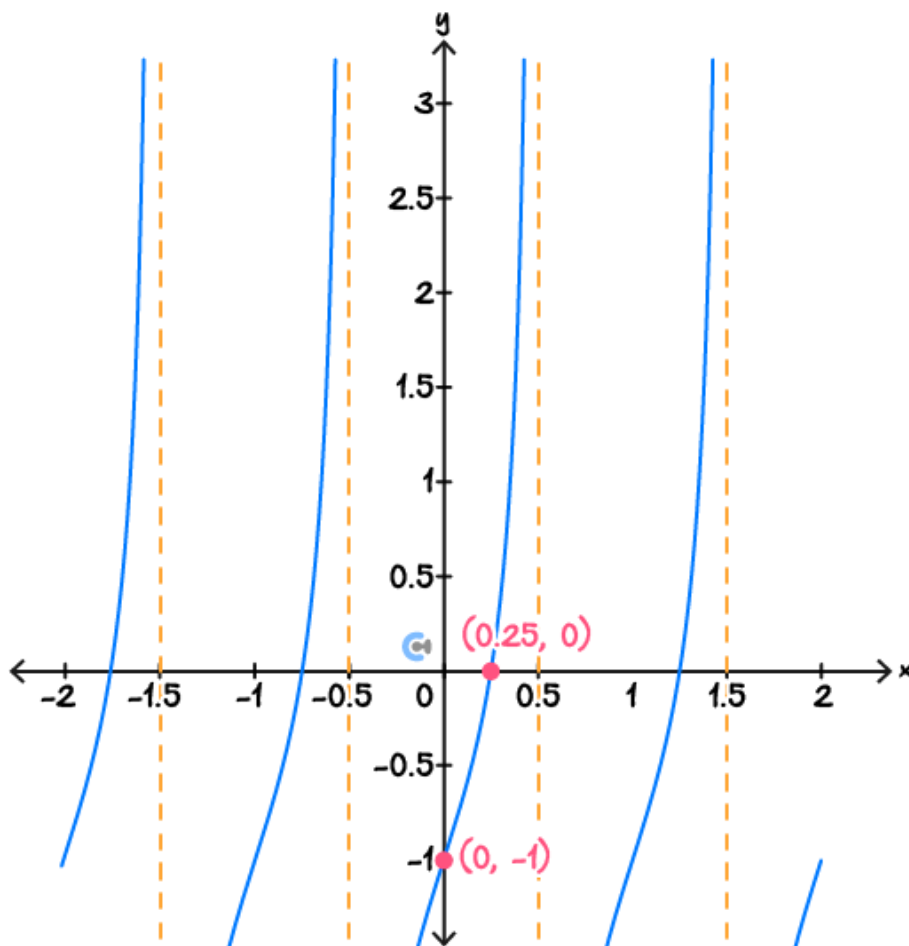


5. Find any x -intercepts.

6. Sketch a "cubic-like" shape.

Question 6

The graph shown below has a rule of the form $y = \tan(nx) + k$, where $n, k \in \mathbb{R}$. Find a possible rule for the graph.



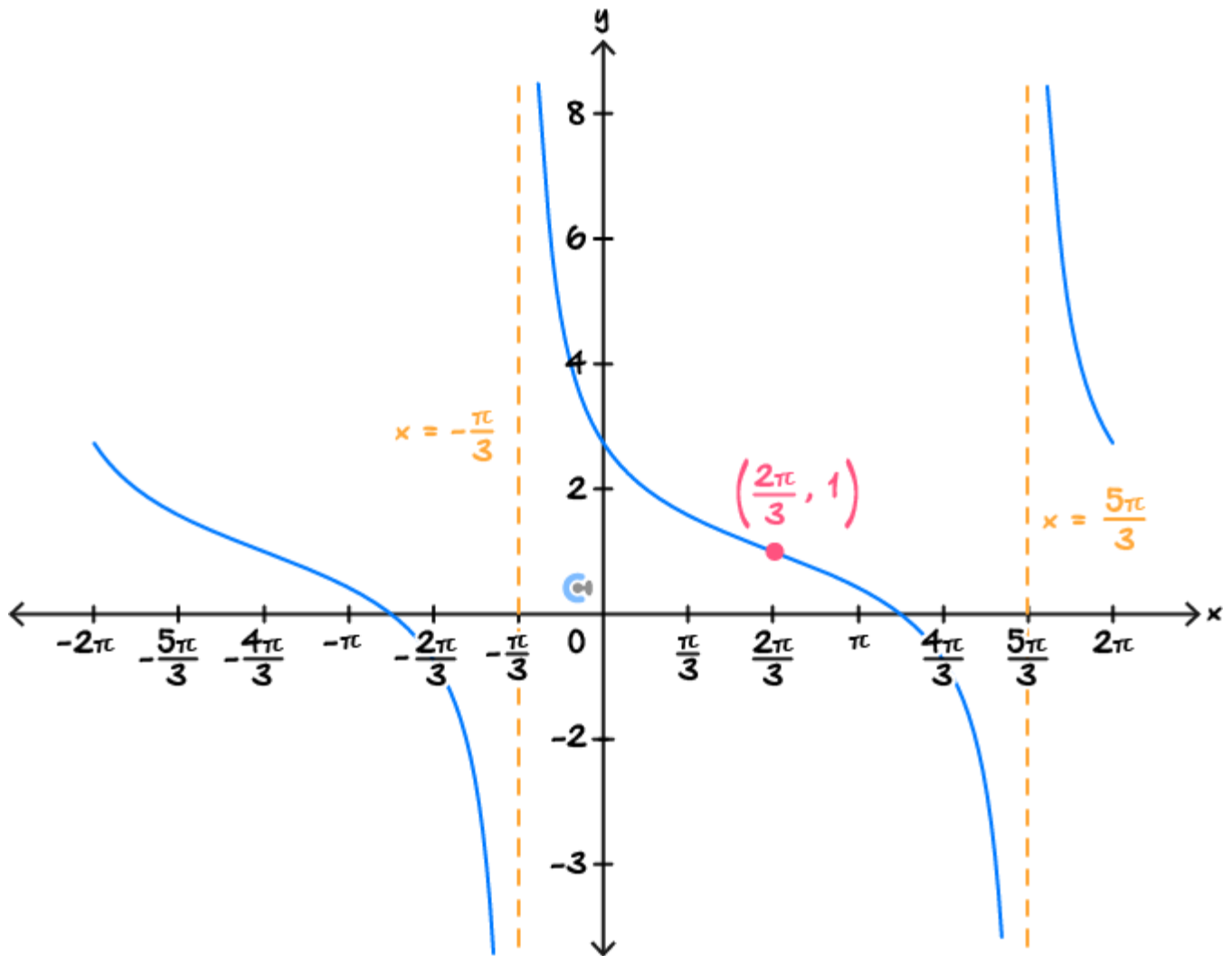
TIP: Simply check if the asymptotic x -value makes inside equal to $90 + 180n$ degrees.



Question 7

The graph shown below has a rule of the form $y = -\tan(nx - b) + k$, where $b, n, k > 0$.

Find a possible rule for the graph.



Section D: Technology Exam Skills

Calculator Commands: Degrees and Radians



TI

Doc → 7 → 2

Document Settings

Display Digits:	Float 6
Angle:	Radian
Exponential Format:	Radian
Real or Complex:	Degree
Calculation Mode:	Exact

Casio

Change at the bottom of the screen.

Alg	Decimal	Real	Rad
-----	---------	------	-----

Mathematica

In radians by default.

Write "Degree".

In[27]:= Sin[30 Degree]

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving Trigonometric Functions.



TI

solve(trig(..) = a, x) | domain restriction.

| is under control equal.

Casio

solve(trig(..) = a, x) | domain restriction.

| is under maths 3.

Mathematica

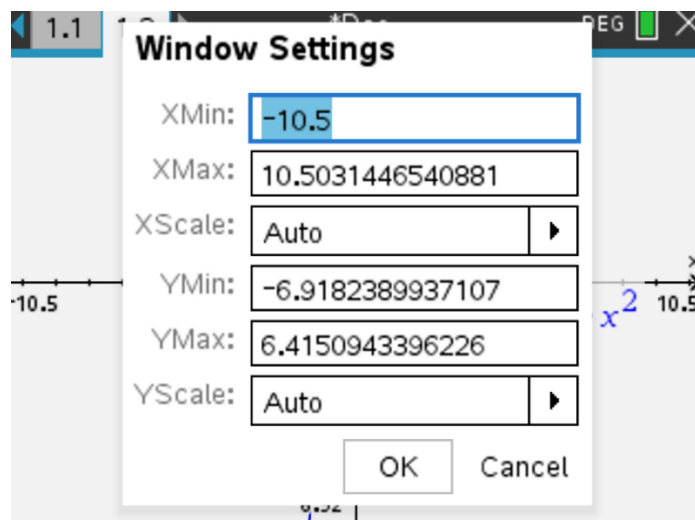
Solve[trig[] == a && domain restriction, x].

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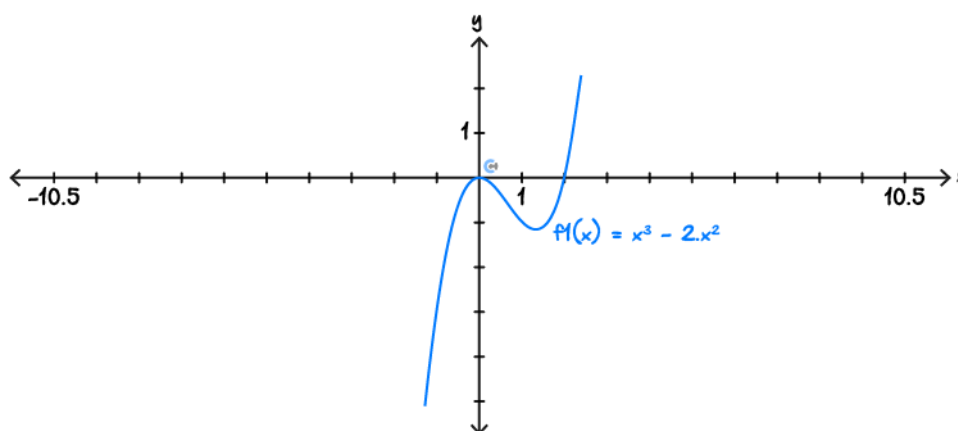


Calculator Commands: Graphing

- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window/zoom) → 1 enter your x and y -ranges.



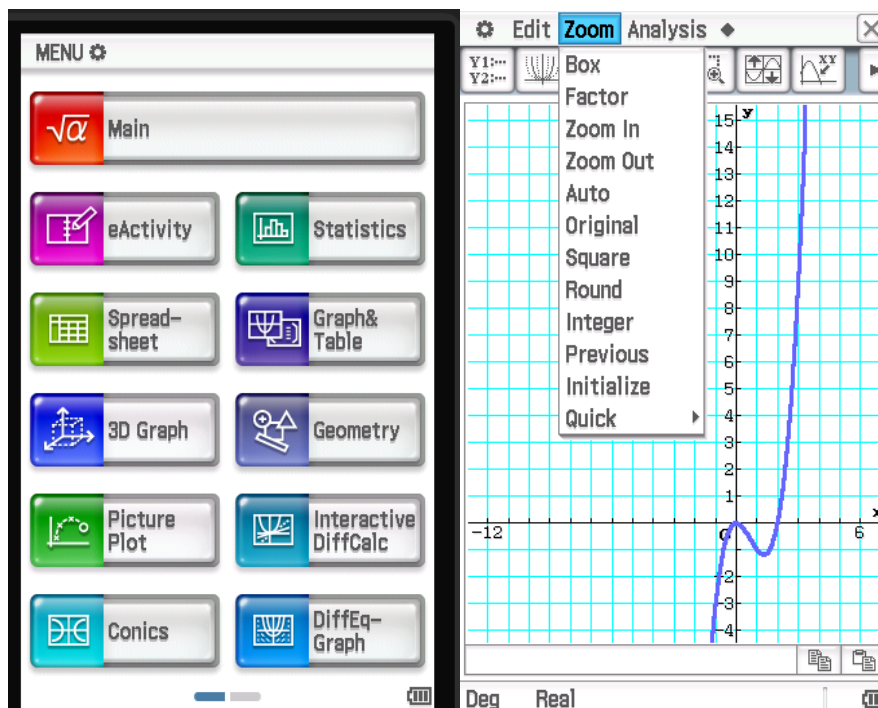
- Can also click the axis numbers on the graph and alter them directly.




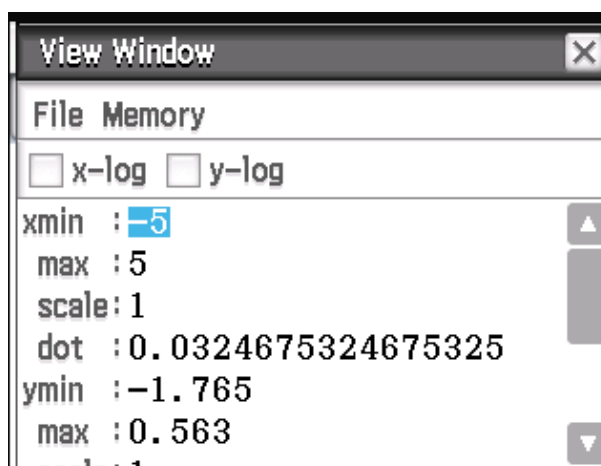
- Menu → 6 (Analyse) to find *min/max* x and y -intercepts.
- Restrict domain to $0 < x < 2$ use the bar can get it from ctrl+ =

☒ $f1(x) = x^3 - 2x^2 | 0 < x < 2 |$

- **Casio:** Click Graph & Table, and enter the function.




- Analysis→ G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict domain → find it in Math 3.

$$\checkmark y1=x^3-2\cdot x^2 \mid 0<x<2$$

- **Mathematica:** Plot[function, {x, xmin, xmax}, PlotRange → {ymin, ymax}]

 PlotRange is optional but can be used to make the scale appropriate for the question.

Section E: Exam 2 (30 Marks)

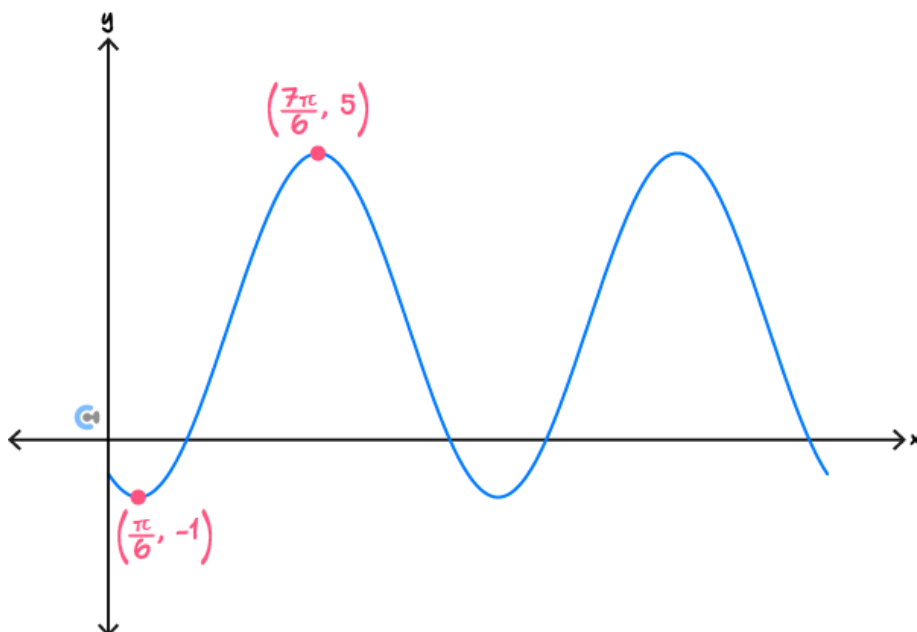
Question 8 (1 mark)

The x -intercepts for $y = 3 \sin \frac{x}{3} + 3$, where $x \in [-3\pi, 5\pi]$, are equal to:

- A. $-2\pi, 0, 2\pi$
- B. $-\frac{3\pi}{2}, \frac{9\pi}{2}$
- C. $-\pi, 0, \pi$
- D. $-\frac{3\pi}{2}, \frac{3\pi}{2}$

Question 9 (1 mark)

The graph shown below could have the rule:



- A. $-3 \cos \left(x + \frac{\pi}{3} \right) + 2$
- B. $3 \sin \left(x + \frac{\pi}{3} \right) + 1$
- C. $-3 \sin \left(x + \frac{\pi}{3} \right) + 2$
- D. $3 \cos \left(x + \frac{\pi}{6} \right) + 2$

Question 10 (1 mark)

Let $f : \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \rightarrow \mathbb{R}$, where $f(x) = 3 \cos(2x)$.

The range of f is:

- A. $[-3, 3]$
- B. $\left[-3, \frac{3}{2}\right)$
- C. $\left[-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right]$
- D. $\left(-3, \frac{3}{2}\right)$

Question 11 (1 mark)

Let $f : [0, a] \rightarrow \mathbb{R}$, where $f(x) = \cos(3x - \pi)$. If the inverse function f^{-1} exists, then the largest value that a can take is:

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{3}$
- C. $\frac{\pi}{2}$
- D. π

Question 12 (1 mark)

Let $g : \left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \rightarrow \mathbb{R}$, with $g(x) = \tan(3x)$. The graph of $y = g(x)$ is transformed by a dilation by a factor of 4 from the x -axis, followed by a reflection in the x -axis. The resulting function h is given by:

- A. $h : \left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \rightarrow \mathbb{R}, h(x) = -4 \tan(3x)$
- B. $h : \left(-\frac{2\pi}{3}, \frac{2\pi}{3}\right) \rightarrow \mathbb{R}, h(x) = -4 \tan(3x)$
- C. $h : \left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \rightarrow \mathbb{R}, h(x) = \frac{1}{4} \tan(3x)$
- D. $h : \left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \rightarrow \mathbb{R}, h(x) = -\frac{1}{4} \tan(3x)$

Question 13 (1 mark)

Given the function $f : \left[0, \frac{4\pi}{k}\right] \rightarrow \mathbb{R}$, where $f(x) = a + b \sin(kx)$, and $a, b, k \in \mathbb{R}$, which of the following statements is **FALSE**?

- A. f has a maximum at $y = a + b$ and occurs when $x = \frac{\pi}{2k}$.
- B. f has a minimum at $y = a - b$ and occurs when $x = \frac{3\pi}{2k}$.
- C. The range of f is $[a - b, a + b]$.
- D. The graph of $y = f(x)$ consists of three full cycles.

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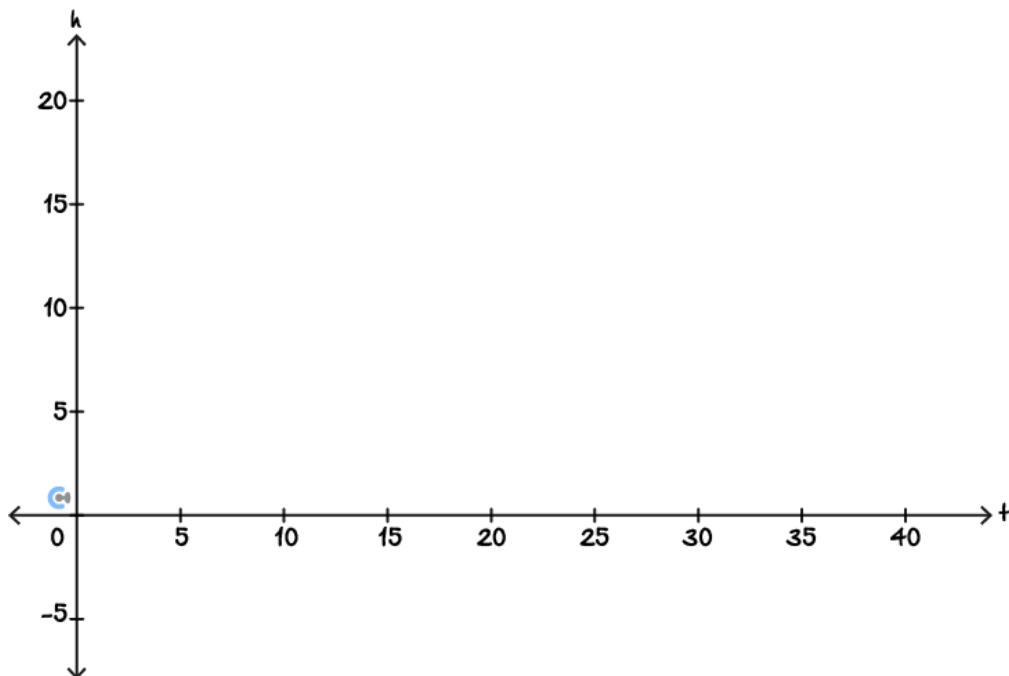
Question 14 (12 marks)

A theme park ride features a pirate ship that swings back and forth in a circular motion. The lowest point of the ride is 2 metres above the ground, and the highest point is 14 metres above the ground. The height of the top of the ship follows a sinusoidal pattern with time. The ship completes one full swing every 20 seconds, starting at its lowest point when $t = 0$.

- a. Show that the height $h(t)$, in metres, of the top of the ship above the ground after t seconds can be modelled by: (2 marks)

$$h(t) = 8 - 6 \cos\left(\frac{\pi t}{10}\right)$$

- b. Sketch the graph of $h(t)$ for $t \in [0, 40]$ on the axes below. Label any maxima and minima with coordinates. (3 marks).



- c. Determine the height of the top of the ship 5 seconds after the ride begins. (1 mark)

- d. Find the first time after the ride begins when the top of the ship reaches a height of 10 metres. Give your answer in seconds correct to two decimal places. (2 marks)

- e. For how many seconds in the first 60 seconds is the top of the ship higher than 12 metres? (3 marks)

- f. The operator adjusts the ride so it starts at its maximum height instead. Write a new rule for $h(t)$ under this condition. (1 mark)

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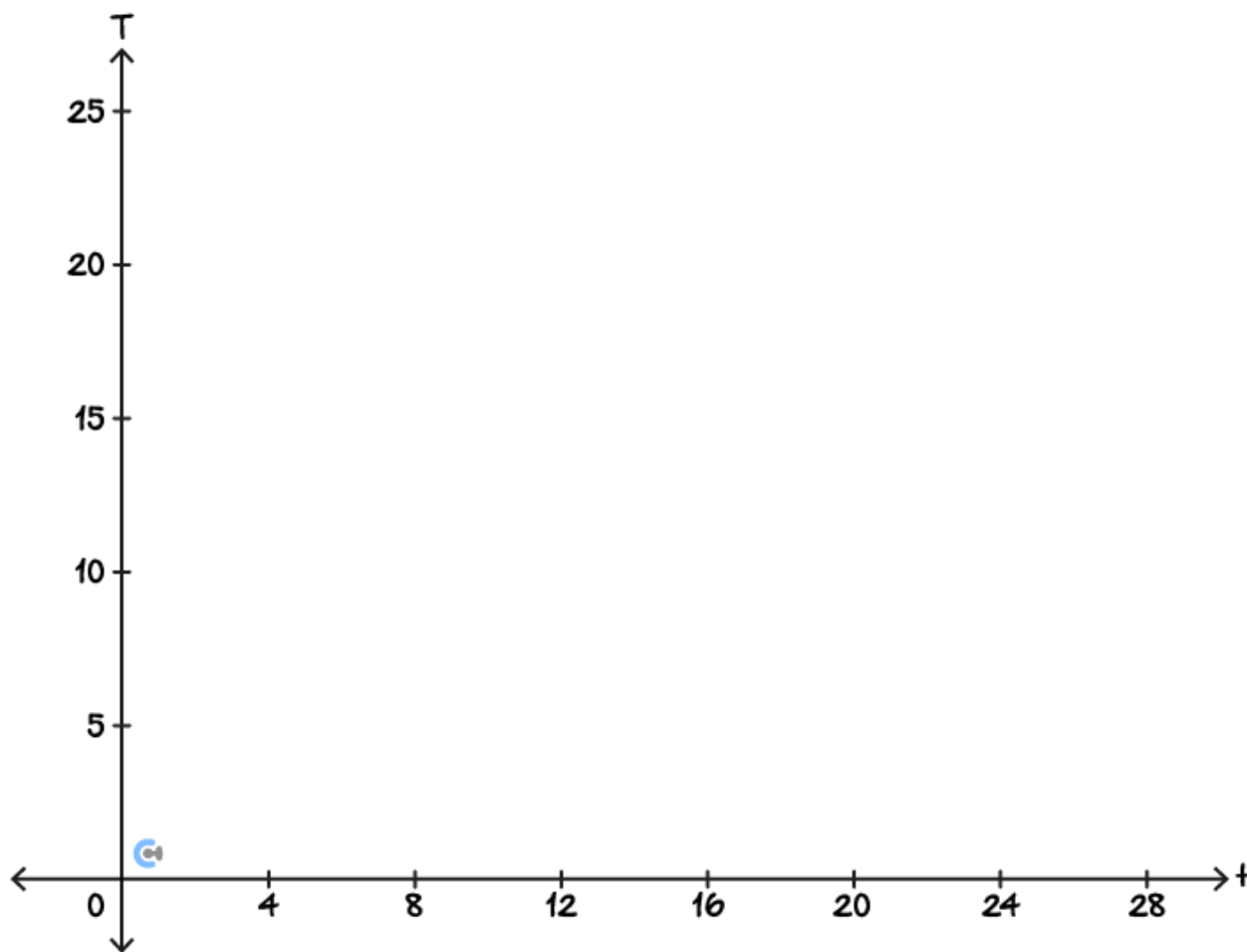
Question 15 (12 marks)

In an underground mine, the temperature varies periodically over a 24-hour cycle and follows a sinusoidal pattern.

The temperature $T(t)$, in degrees Celsius, at t hours after midnight is given by the rule:

$$T(t) = 16 + 6 \sin\left(\frac{\pi}{12}(t - 10)\right)$$

- a. Sketch a graph of $T(t)$ for $0 \leq t \leq 24$. Label any endpoints and minima and maxima with coordinates. (3 marks)



- b. Find an equivalent rule for $T(t)$ in the form $a - b \cos\left(\frac{\pi}{12}(t + c)\right)$, where $a, b, c \in [0, 24]$. (2 marks)

- c. Determine the times during the day when the temperature is exactly 16°C . (2 marks)

- d. Calculate the amount of time in the 24-hour period that the temperature is above 18°C . Give your answer in hours correct to the nearest minute. (2 marks)

- e. The average temperature in the mine is too warm for the workers to work at maximum efficiency. The ventilation system is adjusted so that now the temperature in the mine is given by $T_1(t) = T(t) - k$, where $k \in \mathbb{R}$. Find the value of k if the temperature in the mine is below 15°C for 80% of the time. (3 marks)



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VCE Mathematical Methods ½

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