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VCE Mathematical Methods ½ Graph of Circular Function Exam Skills [4.5]

Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02 - Pg 15
Supplementary Questions	Pg 16 – Pg 28



Section A: Compulsory Questions



Sub-Section: Exam 1

Question 1

Given that $\sin(\alpha) = \frac{12}{13}$ and $0 < \alpha < \frac{\pi}{2}$, find:

a. $\cos\left(-\alpha + \frac{\pi}{2}\right)$

 $\cos\left(-\alpha + \frac{\pi}{2}\right) = \sin(\alpha) = \frac{12}{13}$

b. $\sin(\alpha - \pi)$

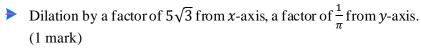
 $\sin(\alpha - \pi) = -\sin(\alpha) = -\frac{12}{13}$

c. $\tan\left(\frac{3\pi}{2} - \alpha\right)$

 $\tan\left(\frac{3\pi}{2} - \alpha\right) = \frac{1}{\tan(\alpha)} = \frac{5}{12}$ Alternatively, use $\tan\left(\frac{3\pi}{2} - \alpha\right) = \frac{\sin\left(\frac{3\pi}{2} - \alpha\right)}{\cos\left(\frac{3\pi}{2} - \alpha\right)}$ to solve.

Consider $f(x) = 5\sqrt{3} \tan \left(\pi x - \frac{\pi}{2}\right) - 5, x \in \mathbb{R}$.

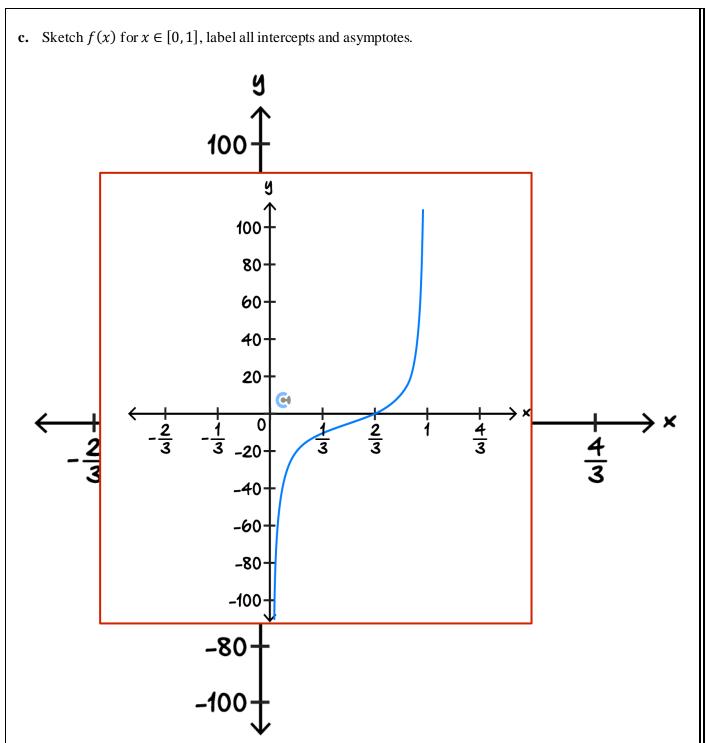
a. State the transformation from $g(x) = \tan(x)$ to f(x).



- Translation $\frac{1}{2}$ unit to the right, 5 units down. (1 mark)
- **b.** Find the general solution for f(x) = 0.

 $5\sqrt{3}\tan\left(\pi x - \frac{\pi}{2}\right) - 5 = 0 \to \tan\left(\pi x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$ We recall: $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \to \pi x - \frac{\pi}{2} = \frac{\pi}{6} + \pi n$ Solve for x: $x = \frac{2}{3} + n, n \in \mathbb{Z}$







a. Solve the equation $2\cos^2(\theta) + 3\cos(\theta) = -1$ for $0 \le \theta \le 2\pi$.

Let $\cos(\theta) = u$, then rewrite the equation as: $2u^2 + 3u + 1 = 0, u = -1 \text{ and } -\frac{1}{2}$ Then, $\cos(\theta) = -1 \rightarrow \theta = \pi$; $\cos(\theta) = -\frac{1}{2} \rightarrow \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

Final Answer: $\theta = \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

b. Hence, state the general solution for this equation.

 $\cos(\theta) = -1 \to \theta = \pi + 2n\pi;$ $\cos(\theta) = -\frac{1}{2} \to \theta = \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n$

Question 4

Consider $h(x) = a \cos\left(nx + \frac{\pi}{4}\right) + c$. h(x) has a maximum of 7 when x = 7 and has a minimum of -5, and a period of 1.

a. Find a, n, and c.

$$a = \frac{7 - (-5)}{2} = 6, \quad c = \frac{7 + (-5)}{2} = 1$$

$$period = \frac{2\pi}{n} = 1 \rightarrow n = 2\pi$$



b. Find the general solution for h(x) = 4.

$$h(x) = 6\cos\left(2\pi x + \frac{\pi}{4}\right) + 1 = 4 \to \cos\left(2\pi x + \frac{\pi}{4}\right) = \frac{1}{2}$$

$$- 2\pi x + \frac{\pi}{4} = \pm \frac{\pi}{3} + 2n\pi$$

$$1. 2\pi x + \frac{\pi}{4} = \frac{\pi}{3} + 2n\pi$$

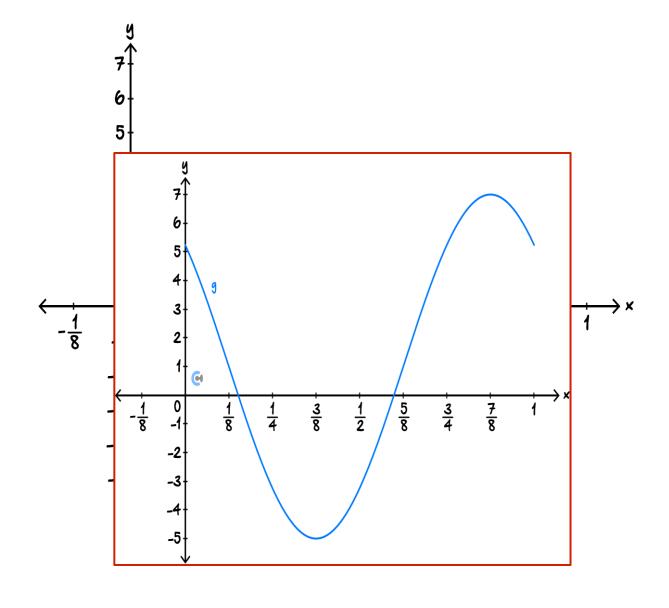
$$- 2\pi x = \frac{\pi}{3} - \frac{\pi}{4} + 2n\pi = \frac{4\pi - 3\pi}{12} + 2n\pi = \frac{\pi}{12} + 2n\pi$$

$$\Rightarrow x = \frac{1}{24} + n$$

2. $2\pi x + \frac{\pi}{4} = -\frac{\pi}{3} + 2n\pi$ $2\pi x = -\frac{\pi}{3} - \frac{\pi}{4} + 2n\pi = \frac{-7\pi}{12} + 2n\pi$ $\Rightarrow x = -\frac{7}{24} + n$

Final answer: $x=rac{1}{24}+n, \quad x=-rac{7}{24}+n, \quad n\in\mathbb{Z}$

c. Graph h(x) for $0 \le x \le 1$, label all intercepts.



d. Hence, find the percentage such that $h(x) \in [1, 4]$ when $0 \le x \le 1$, correct to 2 decimal places.

Solve $h(x) = 1 \rightarrow$ $h(x) = 6\cos\left(2\pi x + \frac{\pi}{4}\right) + 1 = 1 \rightarrow \cos\left(2\pi x + \frac{\pi}{4}\right) = 0$ $2\pi x + \frac{\pi}{4} = \frac{\pi}{2} + n\pi$ $2\pi x = \frac{\pi}{4} + n\pi$ $x = \frac{1}{8} + \frac{n}{2} \rightarrow x = \frac{1}{8} \text{ or } \frac{5}{8}$ From previous question: $h(x) = 4 \rightarrow x = \frac{1}{24} \text{ or } \frac{17}{24}$

Percentage = $\frac{\left(\frac{1}{8} - \frac{1}{24}\right) + \left(\frac{17}{24} - \frac{5}{8}\right)}{1} = \frac{1}{6} = 16.67\%$



Sub-Section: Exam 2

Question 5

If $5\sin(\theta) = 12\cos(\theta)$ where $\theta \in \left[\pi, \frac{3\pi}{2}\right]$, what is the value of $\cos(\theta)$?

- **A.** $\frac{12}{13}$
- **B.** $-\frac{12}{13}$
- C. $\frac{5}{13}$
- **D.** $-\frac{5}{13}$

Ouestion 6

A circular function is given by $f: R \to R$ where $f(x) = -10\cos\left(\frac{\pi x}{5}\right)$. The amplitude and period of f are respectively:

- **A.** $-10, \frac{\pi}{5}$
- **B.** 10, $\frac{\pi}{5}$
- $\mathbf{C.} -10, 10$
- **D.** 10, 10

Question 7

A circular function is given by $f: R \to R$ where $f(x) = -24 \cos(\frac{x}{5})$. Which of the following statements is false?

- **A.** The domain is R and the range is [-24, 24].
- **B.** The period is 10π .
- **C.** The function is a many-to-one function.
- **D.** The graph crosses the x-axis at $x = \frac{5k\pi}{2}$ where $k \in \mathbb{Z}$.



The graph of $y = \tan\left(\frac{dx}{7}\right)$ where $d \neq 0$, has a vertical asymptote at:

- **A.** x = 0
- **B.** $x = \frac{7\pi}{d}$
- $\mathbf{C.} \ \ x = \frac{2\pi}{7d}$
- **D.** $x = \frac{7\pi}{2d}$

Question 9

For the graph $y = 300 \tan \left(\frac{x}{5}\right)$, which of the following options is correct?

- **A.** The domain is [-500, 500] and the period is 10π .
- **B.** The domain is [-500, 500] and the period is 5π .
- C. The domain is R and the period is 5π .
- **D.** The range is R and the period is 5π .



The population of bats in a particular location varies according to the rule $B(t) = 840 + 70\cos\left(\frac{\pi t}{3}\right)$, where *B* is the number of bats and *t* is the number of months after 1 April 2025.

a. Find the period and amplitude of the function B.

Period = 6, amplitude = 70

b. Find the maximum and minimum populations of bats in this location.

Maximum = 910, minimum = 770

c. Find the number of bats on 1 Aug 2025.

 $B(4) = 840+70 \cdot \cos\left(\frac{\pi \cdot x}{3}\right) |x=4|$

d. Over the 12 months from 1 April 2025, find the fraction of time when the population of bats in this location was less than the number of bats on 1 Aug 2025.

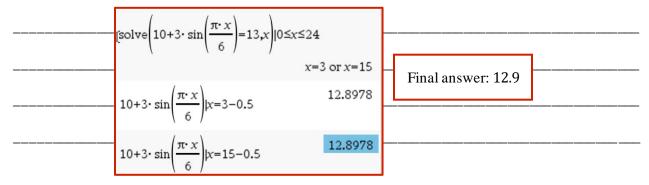


The power output, P(t), in kilowatts (kW) of a solar panel array at a solar farm at t hours after midnight on a particular day is given by:

$$P(t) = 10 + 3\sin\left(\frac{\pi t}{6}\right), 0 \le t \le 24$$

To operate certain machinery, the system requires at least w kilowatts of power for a continuous period of 1 hour.

Find, correct to one decimal place, the largest value of w that satisfies this condition.



Question 12

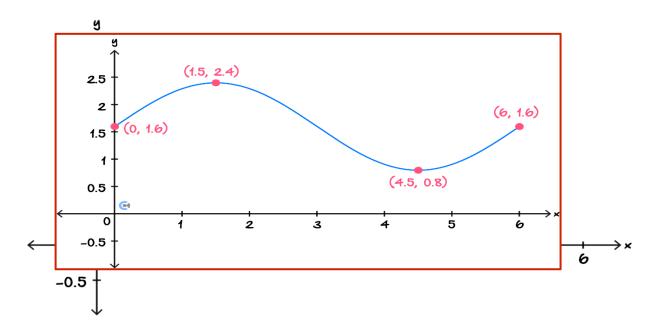
A young boy and girl are lifted onto a seesaw in the park. At the moment, the seesaw is horizontal with respect to the ground. Initially, the boy's end of the seesaw rises. His height above the ground, h metres, t seconds after the seesaw starts to move is modelled by $h(t) = a \sin(nt) + k$.

The greatest height above the ground that the boy reaches is 2.4 metres, and the least distance above the ground that he reaches is 0.8 metres. It takes 3 seconds for him to seesaw between these heights.

a. Find the values of a, n, and k.

 $a = \frac{2.4 - 0.8}{2} = 0.8$	
$k = \frac{2.4 + 0.8}{2} = 1.6$	
 $period T = 6 = \frac{2\pi}{n} \to n = \frac{\pi}{3}$	

b. Draw the graph showing the height of the boy above the ground for $0 \le t \le 6$, and label all key points.

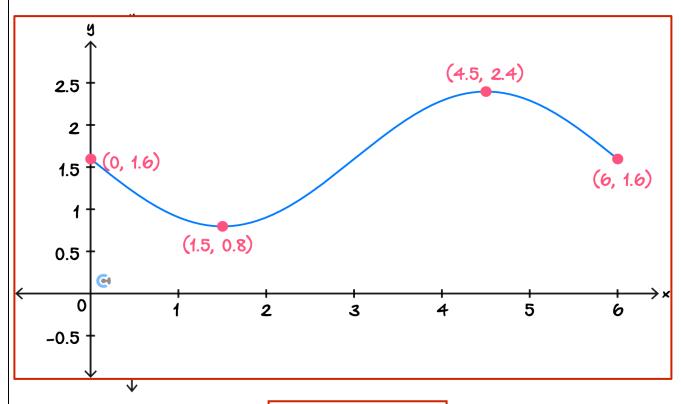


c. For what length of time during the first 6 seconds of the motion of the seesaw is the boy's height above the ground 2.0 metres or higher?

 $h(t) = 0.8 \sin\left(\frac{\pi}{3}t\right) + 1.6 = 2$	
 solve $\left(0.8 \cdot \sin\left(\frac{\pi \cdot x}{3}\right) + 1.6 = 2, x\right) 0 \le x \le 6$	
x=0.5 or x=2.	
 2.5 - 0.5 = 2 seconds	

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d. Sketch the graph showing the height of the girl above the ground during the first 6 seconds and state its equation.



$$y = -0.8\sin\left(\frac{\pi}{3}t\right) + 1.6$$

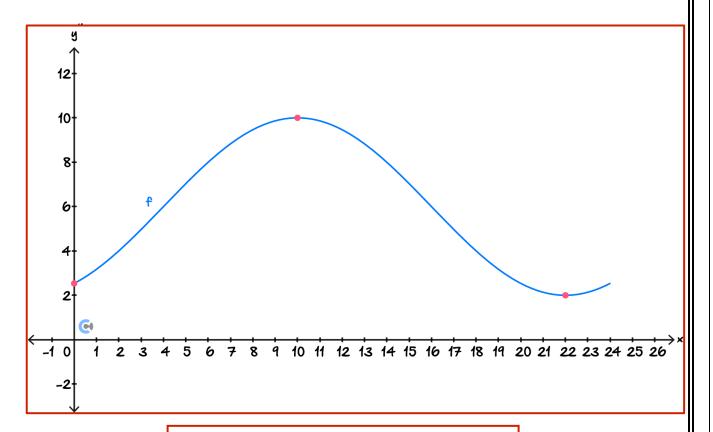


The **oxygen concentration** in a hydroponic tank is modelled by the function:

$$O(t) = 6 + 4\sin\left(\frac{\pi t}{12} - \frac{\pi}{3}\right), 0 \le t \le 24$$

Where O(t) is the concentration of dissolved oxygen (in mg/L) and t is the number of hours after midnight.

- **a.** Sketch the graph of O(t) over the interval $0 \le t \le 24$, and clearly label:
 - Maximum and minimum values.
 - \bigcirc Time(s) when the oxygen level is exactly 6 mg/L.



Label: (10, 10), (22, 2), $(0, 6 - 2\sqrt{3})$, (4, 6), (16, 6)



A particular type of plant in the tank requires the oxygen concentration to be above 7.5 mg/L for at least 60% of the day in order to remain healthy. A pump is added to increase the oxygen level uniformly by a constant amount c throughout the day.

Let the new oxygen function be:

$$O_c(t) = 6 + 4 \sin\left(\frac{\pi t}{12} - \frac{\pi}{3}\right) + c$$

b. Determine, **correct to one decimal place**, the **minimum value of** c such that the oxygen level stays **above** 7.5 mg/L for at least 60% of the 24-hour period.

$$60\% \times 24 = 14.4$$

$$\frac{14.4}{2} = 7.2$$

$$0(10 - 7.2) = 4.76393$$

$$c = 7.5 - 4.76393 = 2.73607 = 2.7$$



Section B: Supplementary Questions



Sub-Section: Exam 1

Question	14
Question	14

Let $(\tan(\theta) + 1)(\sqrt{3}\sin(\theta) - \cos(\theta))(\sqrt{3}\sin(\theta) + \cos(\theta)) = 0$.

a. State all possible values of $tan(\theta)$.

 $\tan(\theta) = -1, \pm \frac{\sqrt{3}}{3}$

b. Hence, find all possible solutions for $(\tan(\theta) + 1)(3\sin^2(\theta) - \cos^2(\theta)) = 0$, where $0 \le \theta \le \pi$.

1 mark for factorise $3\sin^2(\theta) - \cos^2(\theta)$ 1 mark for solving $\tan(\theta) = -1, \pm \frac{\sqrt{3}}{2}$

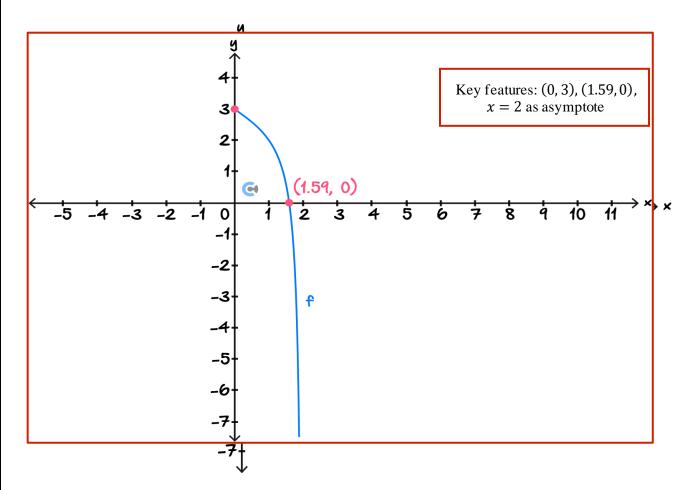
1 mark for answer $heta=rac{\pi}{6},rac{5\pi}{6},rac{3\pi}{4}$



a. Determine the period of the function $f(x) = -\tan\left(\frac{\pi}{4}x\right) + 3$.

 $period = \frac{\pi}{\frac{\pi}{4}} = 4$

b. Sketch the graph of the function with rule $f:[0,2] \to R$, $f(x) = -\tan(\frac{\pi}{4}x) + 3$. Label all intercept(s) and asymptote(s), correct to two decimal places.



Determine the vertical asymptotes for the function $f:(-\pi,\pi)\to R$, $f(x)=-\tan\left(x-\frac{\pi}{3}\right)$.

$$x-rac{\pi}{3}=rac{\pi}{2}+n\pi\Rightarrow x=rac{\pi}{2}+rac{\pi}{3}+n\pi=rac{5\pi}{6}+n\pi$$

Consider n = -1 and n = 0 to fit in the domain restriction. Therefore, the final answer is:

$$x = -\frac{\pi}{6} \text{ and } x = \frac{5\pi}{6}$$

Question 17

a. If $\cos(\alpha) = 0.7$, find the value of $\sin(\frac{3\pi}{2} + \alpha)$.

 $\sin\left(\frac{3\pi}{2}\right)$

 $\sin\left(\frac{3\pi}{2} + \alpha\right) = -\cos(\alpha) = -0.7$

b. If $tan(\theta) = 0.6$, find the value of $tan(\frac{\pi}{2} - \theta)$.

 $\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta) = \frac{1}{0.6} = \frac{5}{3}$



A pendulum's height above the ground (in cm) is modelled by the function:

$$h(t) = a\sin\left(kt + \frac{\pi}{3}\right) + d$$

where *t* is the time in seconds.

The pendulum reaches a maximum height of 18 cm at t = 0, and a minimum height of 2 cm after 0.6 seconds.

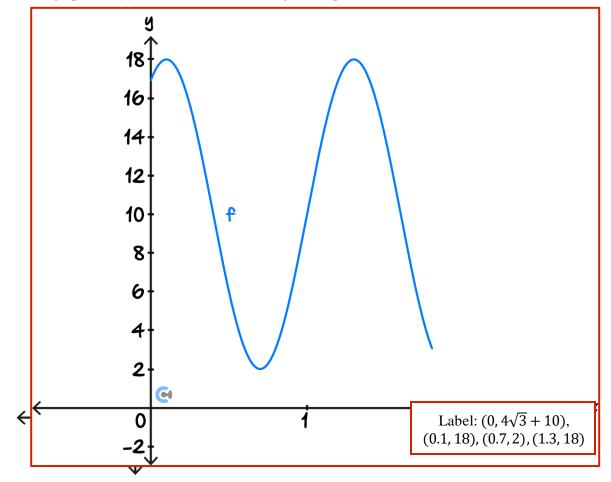
a. Find the values of a, k, and d.

 $a=\frac{\max-\min}{2}=\frac{18-2}{2}=8$	
$d=\frac{\max+\min}{2}=\frac{18+2}{2}=10$	
 $\mathrm{Period} = \frac{2\pi}{k} = 1.2 \Rightarrow k = \frac{2\pi}{1.2} = \frac{10\pi}{6} = \frac{5\pi}{3}$	

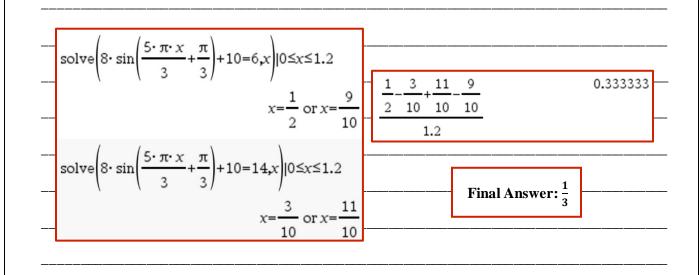
b. Find the general solution for when the height is **exactly 10** *cm*.

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c. Sketch the graph of h(t) for $0 \le t \le 1.8$, labelling intercepts, max, and min.



d. Determine the fraction of time over a full cycle that the pendulum is higher than 6 cm but lower than 14 cm.







Sub-Section: Exam 2

Question 19

Select the false statement from the following:

A.
$$\sin(\pi + \theta) + \sin(\pi - \theta) = 0$$

$$\mathbf{B.} \ \cos(\pi + \theta) + \cos(\pi - \theta) = 0$$

C.
$$\sin(\pi - \theta) - \sin(2\pi + \theta) = 0$$

D.
$$tan(\pi + \theta) + tan(2\pi - \theta) = 0$$

Question 20

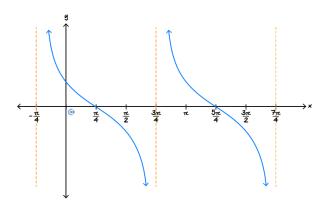
The number of wombats, n, in a particular area is given by the function $n(t) = 100 \sin\left(\frac{\pi(t+12)}{12}\right) + 200$, where t is the time in months, and $t \ge 0$.

Which of the following statements is false?

- **A.** The maximum number of wombats is 300.
- **B.** The minimum number of wombats is 100.
- C. The initial number of wombats is 100.
- **D.** After one year, the number of wombats is 200.



The diagram shows two cycles of a circular function. The period of the function is:



- A. $\frac{\pi}{2}$
- $\mathbf{B.} \ \frac{\pi}{4}$
- C. $\frac{3\pi}{4}$
- **D.** π

Question 22

The graph of $y = \tan(ax)$, where a > 0, has a vertical asymptote at $x = \frac{\pi}{6}$ and exactly one intercept in the interval $\left(0, \frac{\pi}{3}\right)$. The value of a is:

- **A.** 1
- **B.** 3
- **C.** 6
- **D.** 2



The graph of $y = \tan\left(\frac{bx}{5}\right)$, where $b \neq 0$, has a vertical asymptote at:

- **A.** x = 0
- **B.** $x = \frac{2\pi}{5b}$
- **C.** $x = \frac{\pi}{5b}$
- **D.** $x = \frac{5\pi}{2b}$

Question 24

The temperature in a greenhouse at time t hours after midnight is modelled by the function:

$$T(t) = 22 + 4\cos\left(\frac{\pi t}{12}\right), 0 \le t \le 24$$

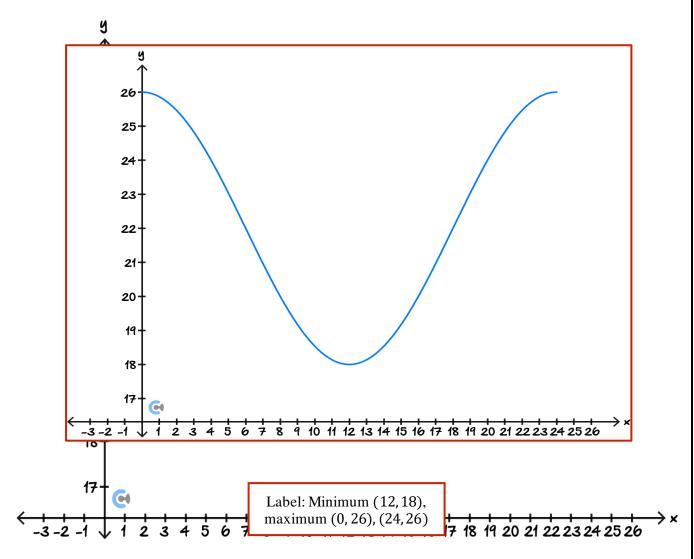
where T(t) is in degrees Celsius.

a. State the amplitude and period for T.

Amplitude = 4, period = 24

CONTOUREDUCATION

b. Sketch the graph of T(t), label minimum, maximum and intercept(s).



c. To ensure optimal plant growth, the temperature must stay below k° C for at least 5 consecutive hours.

Find, correct to one decimal place, the **smallest value of** k that satisfies this condition.

k = T(12 - 2.5) = T(9.5) = 18.8



The sunlight intensity I(t), in lumens, shining on a solar sensor at a research station is modelled by the function:

$$I(t) = 500 \sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) + 600$$

Where t is the number of hours after midnight on a particular day. You may assume $0 \le t \le 24$.

The sensor must remain in low-power mode whenever the intensity falls below 400 lumens to avoid overheating. Scientists are trying to conduct a sensitive experiment that can only happen when the sensor is **not** in low-power mode.

a. State the amplitude, period, and average sunlight intensity of this function.

Amplitude = 500, period = 24, average = 600

b. Find the times in the day when the sunlight intensity is exactly 400 lumens, correct to the nearest minute.

solve $\left(500 \cdot \sin\left(\pi \cdot \frac{x}{12} - \frac{\pi}{2}\right) + 600 = 400, x\right) | 0 \le x \le 2$ Answer: 4 : 26 AM and 19 : 34 PM

c. Determine the total amount of time in a day the sensor can operate above 400 lumens, correct to the nearest minute.

19.5719-4.42812 15.1438 15.14378· 60 908.627

_____ 909 minutes

- **d.** An optimal time to begin the experiment is the first moment in the day when the sunlight intensity reaches 400 lumens and is increasing.
 - i. State this time, correct to the nearest minute.

4: 26 AM

ii. The experiment takes 3 hours and 10 minutes to complete.

Determine whether the scientists have enough time for the experiment before the sensor must return to low-power mode. If not, calculate how much extra time they would need, correct to the nearest second.

3 hours and 10 minutes = 190 minutes < 909 minutes, so enough time

Question 26

A drone is flying over a mountainous region to record environmental data. Its altitude (in kilometres above sea level) is modelled by the function:

$$A(t) = 2.4 \cos\left(\frac{\pi}{3}(t - k)\right) + 2.5$$

Where A(t) is the drone's altitude in km, t is the time in seconds after takeoff, $k \in R$ is a horizontal shift caused by delayed GPS calibration.

a. State the amplitude, period, and average cruising altitude of the drone.

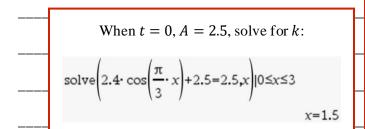
Amplitude = 2.4, period = 6, average cruising altitude = 2.5

b. Find the maximum and minimum altitudes reached by the drone.

Max = 4.9, Min = 0.1

c. The drone begins its flight at an altitude of 2.5 km and must initially ascend. Determine all possible value(s) of $k \in (0,3)$ that satisfy these conditions. Express your final answer(s) correct to 2 decimal places, showing

working.



k=1.5, check if the graph is ascending: k=1.5, check if the graph is ascending:Yes, it is! So k=1.5 is the final answer

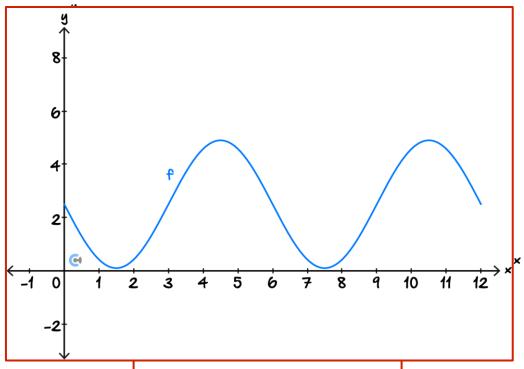
d. Using your value of k, determine all values of $t \in [0,6]$, where the drone is flying higher than $2 \, km$. Given answers to 2 decimal places.

solve $\left(2.4 \cdot \cos\left(\frac{\pi}{3} \cdot (x-1.5)\right) + 2.5 = 2, x\right) | 0 \le x \le 6$ x = 3.20041 or x = 5.79959

Hence, $t \in [0,3.20] \cap [5.80,6]$



e. Sketch the graph of A(t) for two full cycles, labelling all intercepts, minimum and maximum points, and the midline. Coordinates must be accurate to 2 decimal places.



Label: (0, 2.67), (1.57, 0.1), (4.57, 4.9), (7.57, 0.1), (10.57, 4.9), (12, 2.67)



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