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VCE Mathematical Methods ½
Graph of Circular Function Exam Skills [4.5]
Homework

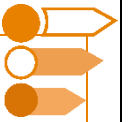### Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02 - Pg 15
Supplementary Questions	Pg 16 – Pg 28

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<u>Section A:</u> Compulsory Questions

<u>Sub-Section:</u> Exam 1

Question 1

Given that $\sin(\alpha) = \frac{12}{13}$ and $0 < \alpha < \frac{\pi}{2}$, find:

a. $\cos\left(-\alpha + \frac{\pi}{2}\right)$

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b. $\sin(\alpha - \pi)$

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c. $\tan\left(\frac{3\pi}{2} - \alpha\right)$

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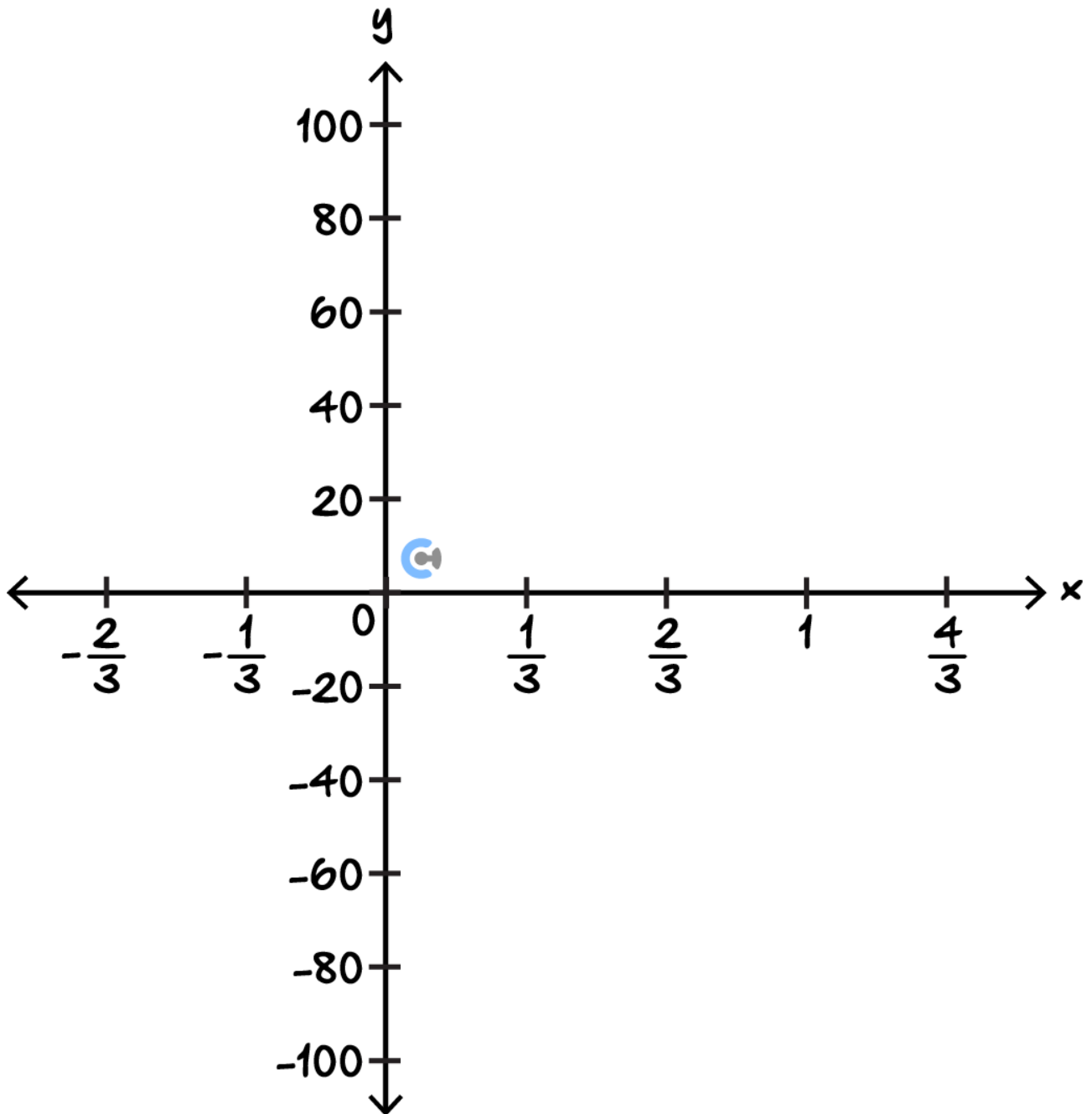**Question 2**

Consider $f(x) = 5\sqrt{3}\tan\left(\pi x - \frac{\pi}{2}\right) - 5, x \in R$.

a. State the transformation from $g(x) = \tan(x)$ to $f(x)$.

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b. Find the general solution for $f(x) = 0$.

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c. Sketch $f(x)$ for $x \in [0, 1]$, label all intercepts and asymptotes.

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Question 3

a. Solve the equation $2\cos^2(\theta) + 3\cos(\theta) = -1$ for $0 \le \theta \le 2\pi$.

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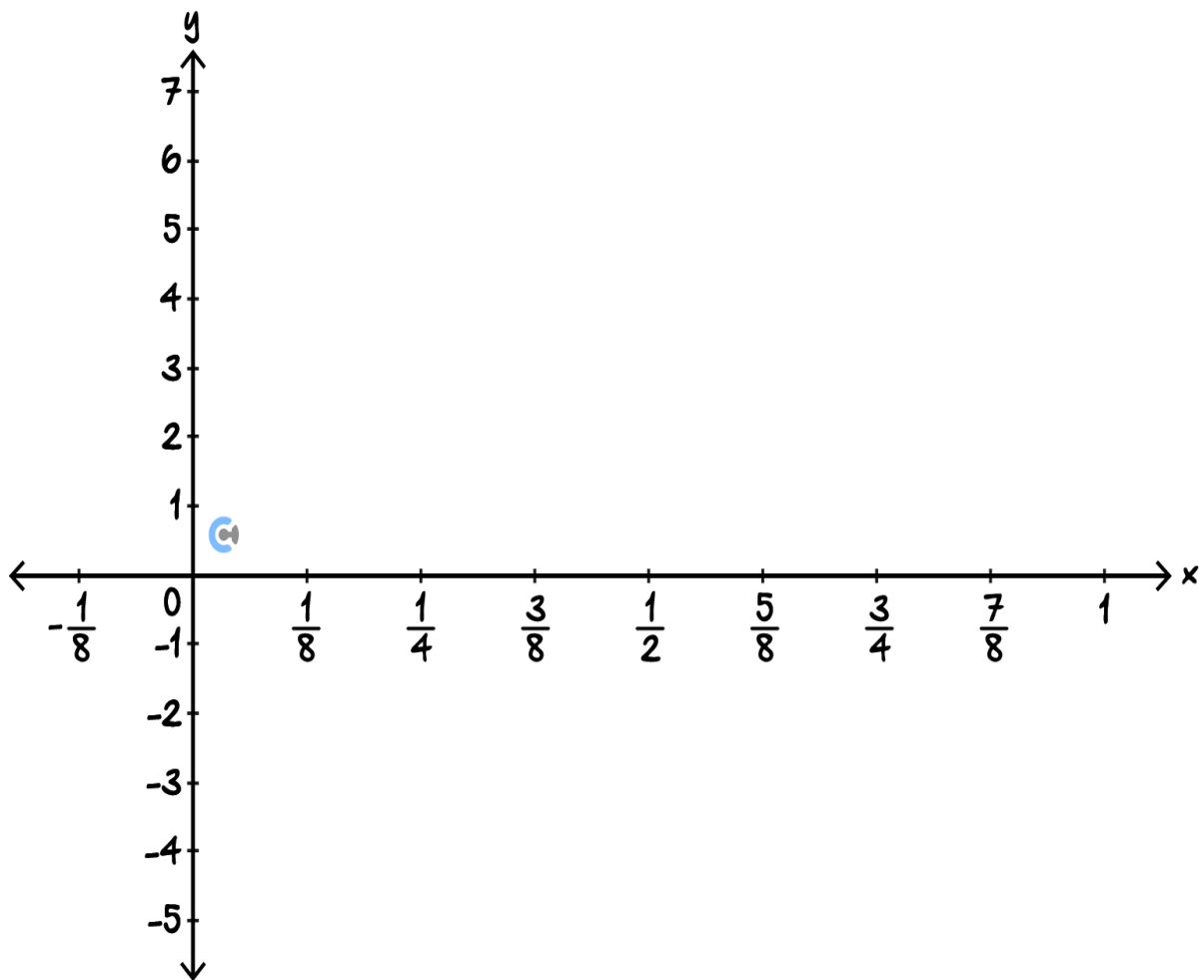**b.** Hence, state the general solution for this equation.

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Question 4

Consider $h(x) = a\cos\left(nx + \frac{\pi}{4}\right) + c$. $h(x)$ has a maximum of 7 when $x = 7$ and has a minimum of -5, and a period of 1.

a. Find $a, n,$ and c.

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b. Find the general solution for $h(x) = 4$.

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c. Graph $h(x)$ for $0 \leq x \leq 1$, label all intercepts.

d. Hence, find the percentage such that $h(x) \in [1, 4]$ when $0 \leq x \leq 1$, correct to 2 decimal places.

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Sub-Section: Exam 2

Question 5

If $5\sin(\theta) = 12\cos(\theta)$ where $\theta \in \left[\pi, \frac{3\pi}{2}\right]$, what is the value of $\cos(\theta)$?

A. $\frac{12}{13}$

B. $-\frac{12}{13}$

C. $\frac{5}{13}$

D. $-\frac{5}{13}$

Question 6

A circular function is given by $f: R \to R$ where $f(x) = -10\cos\left(\frac{\pi x}{5}\right)$. The amplitude and period of f are respectively:

A. $-10, \frac{\pi}{5}$

B. $10, \frac{\pi}{5}$

C. $-10, 10$

D. $10, 10$

Question 7

A circular function is given by $f: R \to R$ where $f(x) = -24\cos\left(\frac{x}{5}\right)$. Which of the following statements is false?

A. The domain is R and the range is $[-24, 24]$.

B. The period is 10π.

C. The function is a many-to-one function.

D. The graph crosses the x-axis at $x = \frac{5k\pi}{2}$ where $k \in Z$.

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Question 8

The graph of $y = \tan\left(\frac{dx}{7}\right)$ where $d \neq 0$, has a vertical asymptote at:

A. $x = 0$

B. $x = \frac{7\pi}{d}$

C. $x = \frac{2\pi}{7d}$

D. $x = \frac{7\pi}{2d}$

Question 9

For the graph $y = 300 \tan\left(\frac{x}{5}\right)$, which of the following options is correct?

A. The domain is $[-500, 500]$ and the period is 10π.

B. The domain is $[-500, 500]$ and the period is 5π.

C. The domain is R and the period is 5π.

D. The range is R and the period is 5π.

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Question 10

The population of bats in a particular location varies according to the rule $B(t) = 840 + 70\cos\left(\frac{\pi t}{3}\right)$, where B is the number of bats and t is the number of months after 1 April 2025.

a. Find the period and amplitude of the function B.

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b. Find the maximum and minimum populations of bats in this location.

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c. Find the number of bats on 1 Aug 2025.

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d. Over the 12 months from 1 April 2025, find the fraction of time when the population of bats in this location was less than the number of bats on 1 Aug 2025.

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Question 11

The power output, $P(t)$, in kilowatts (kW) of a solar panel array at a solar farm at t hours after midnight on a particular day is given by:

$$P(t) = 10 + 3\sin\left(\frac{\pi t}{6}\right), 0 \leq t \leq 24$$

To operate certain machinery, the system requires at least w kilowatts of power for a continuous period of 1 hour.

Find, correct to one decimal place, the largest value of w that satisfies this condition.

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Question 12

A young boy and girl are lifted onto a seesaw in the park. At the moment, the seesaw is horizontal with respect to the ground. Initially, the boy's end of the seesaw rises. His height above the ground, h metres, t seconds after the seesaw starts to move is modelled by $h(t) = a\sin(nt) + k$.

The greatest height above the ground that the boy reaches is 2.4 metres, and the least distance above the ground that he reaches is 0.8 metres. It takes 3 seconds for him to seesaw between these heights.

a. Find the values of a, n, and k.

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b. Draw the graph showing the height of the boy above the ground for $0 \leq t \leq 6$, and label all key points.
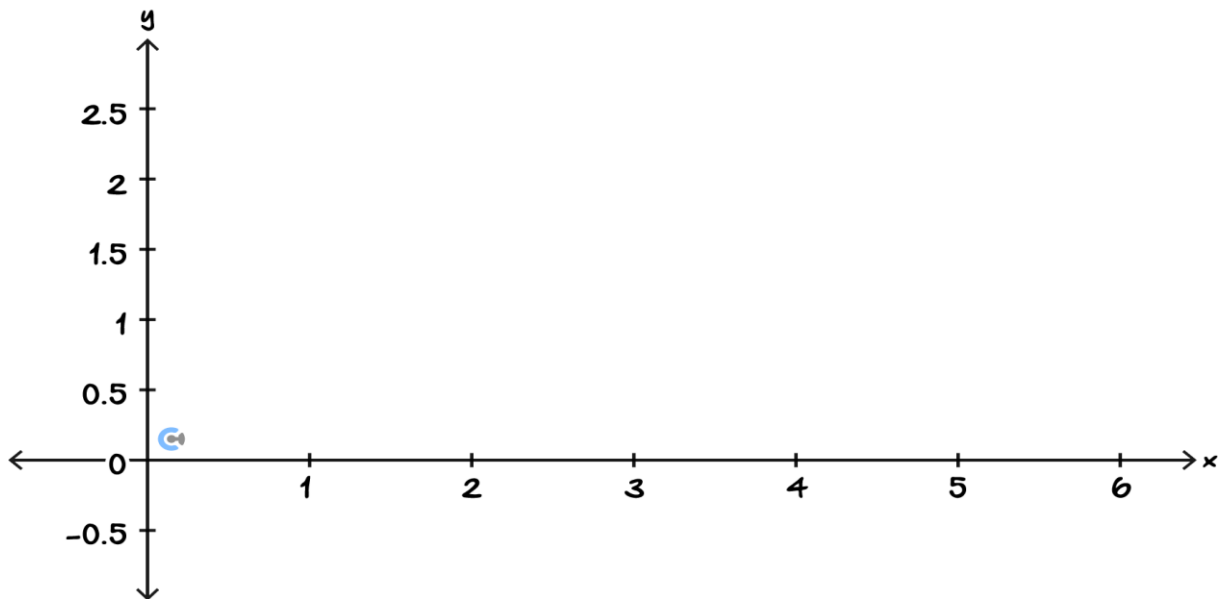
c. For what length of time during the first 6 seconds of the motion of the seesaw is the boy's height above the ground 2.0 metres or higher?

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d. Sketch the graph showing the height of the girl above the ground during the first 6 seconds and state its equation.

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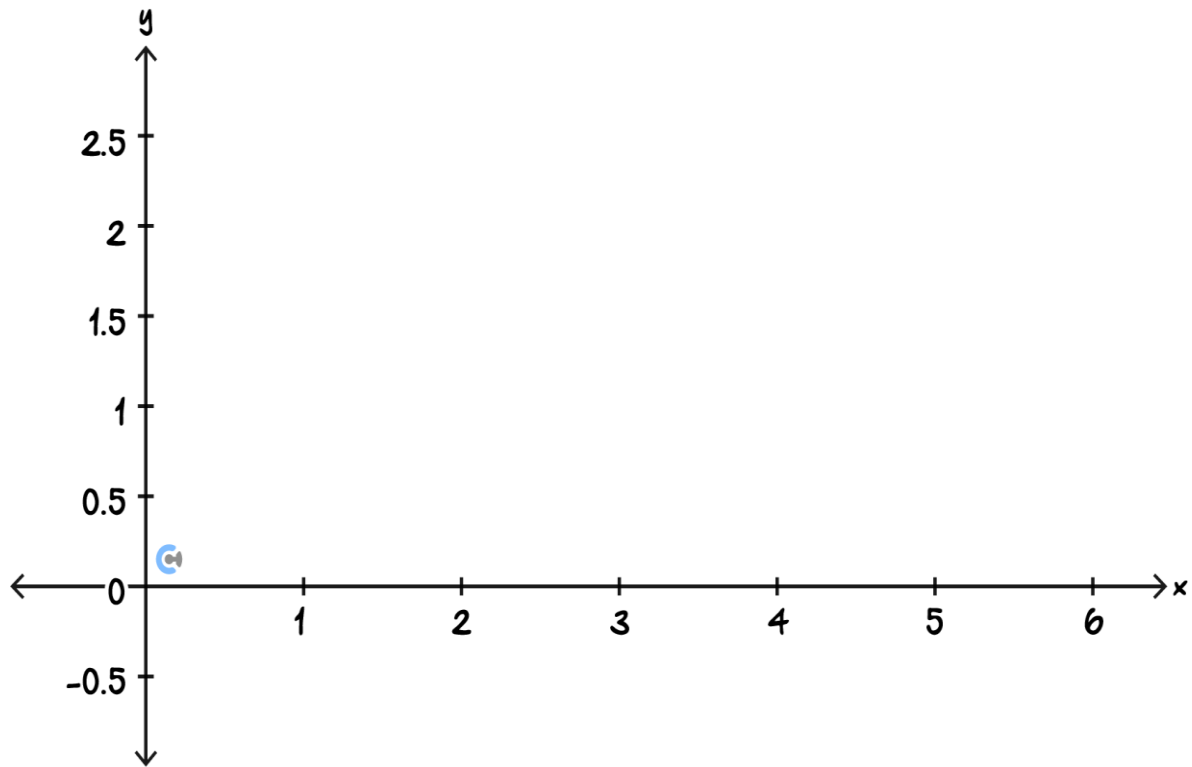
Question 13

The **oxygen concentration** in a hydroponic tank is modelled by the function:

$$O(t) = 6 + 4\sin\left(\frac{\pi t}{12} - \frac{\pi}{3}\right), 0 \le t \le 24$$

Where $O(t)$ is the concentration of dissolved oxygen (in mg/L) and t is the number of hours after midnight.

a. Sketch the graph of $O(t)$ over the interval $0 \le t \le 24$, and clearly label:

- Maximum and minimum values.

- Time(s) when the oxygen level is exactly $6\ mg/L$.
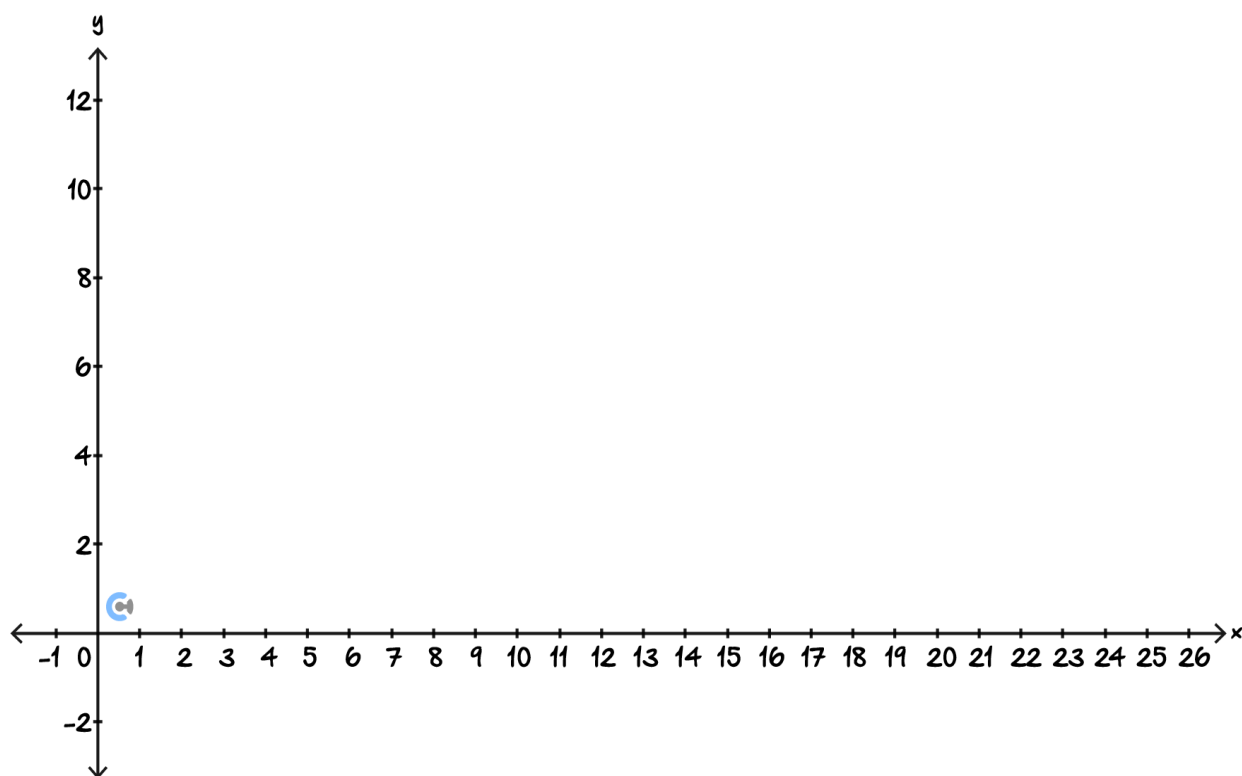
A particular type of plant in the tank requires the oxygen concentration to be **above 7.5 mg/L for at least 60% of the day** in order to remain healthy. A pump is added to **increase the oxygen level uniformly by a constant amount c** throughout the day.

Let the new oxygen function be:

$$O_c(t) = 6 + 4\sin\left(\frac{\pi t}{12} - \frac{\pi}{3}\right) + c$$

b. Determine, **correct to one decimal place**, the **minimum value of c** such that the oxygen level stays **above 7.5 mg/L for at least 60% of the 24-hour period**.

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Section B: Supplementary Questions

Sub-Section: Exam 1

Question 14

Let $(\tan(\theta) + 1)(\sqrt{3}\sin(\theta) - \cos(\theta))(\sqrt{3}\sin(\theta) + \cos(\theta)) = 0$.

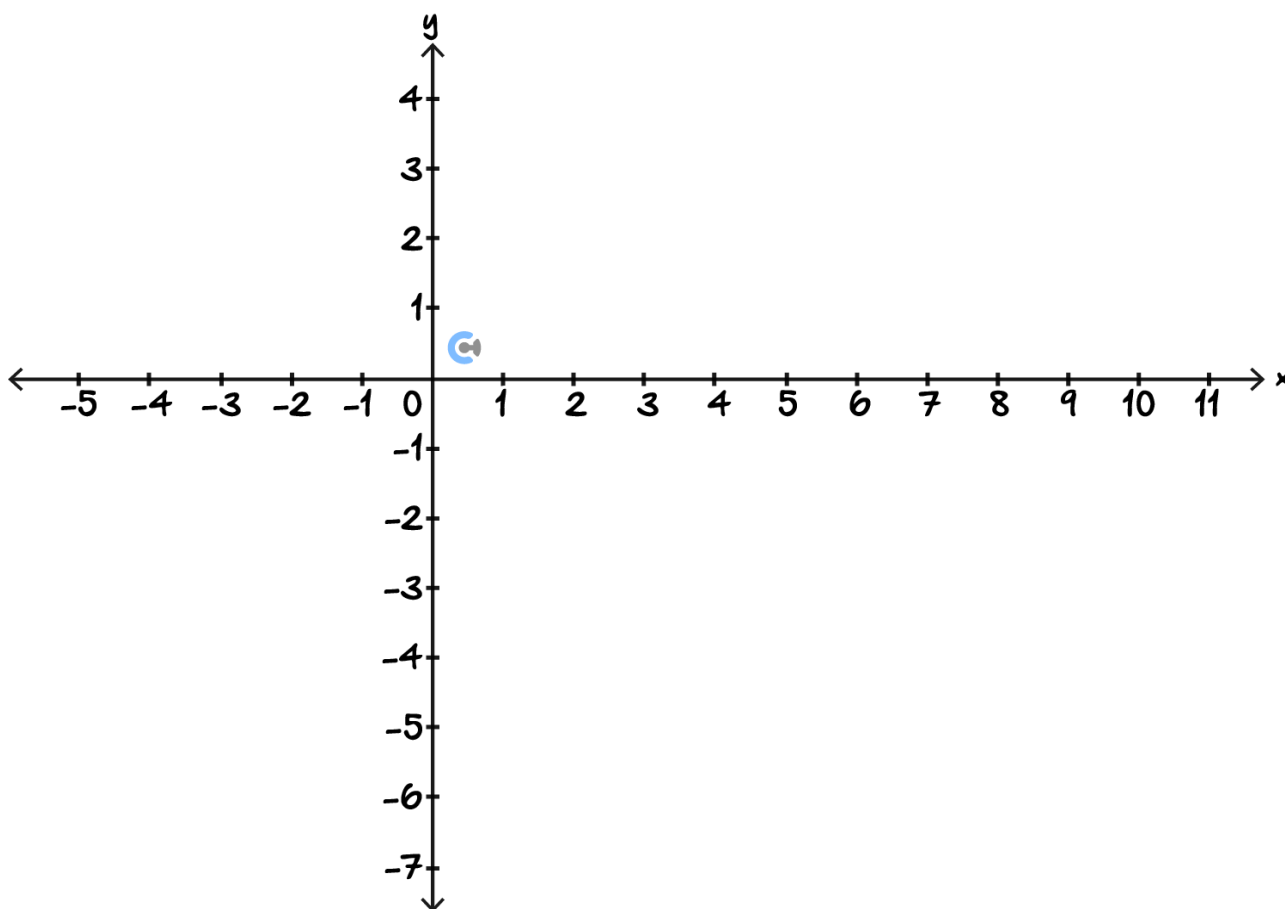**a.** State all possible values of $\tan(\theta)$.

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b. Hence, find all possible solutions for $(\tan(\theta) + 1)(3\sin^2(\theta) - \cos^2(\theta)) = 0$, where $0 \leq \theta \leq \pi$.

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Question 15

a. Determine the period of the function $f(x) = -\tan\left(\frac{\pi}{4}x\right) + 3$.

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b. Sketch the graph of the function with rule $f:[0,2] \to R, f(x) = -\tan\left(\frac{\pi}{4}x\right) + 3$. Label all intercept(s) and asymptote(s), correct to two decimal places.

Question 16

Determine the vertical asymptotes for the function $f : (-\pi, \pi) \to R, f(x) = -\tan\left(x - \frac{\pi}{3}\right)$.

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Question 17

a. If $\cos(\alpha) = 0.7$, find the value of $\sin\left(\frac{3\pi}{2} + \alpha\right)$.

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b. If $\tan(\theta) = 0.6$, find the value of $\tan\left(\frac{\pi}{2} - \theta\right)$.

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Question 18

A pendulum's height above the ground (in cm) is modelled by the function:

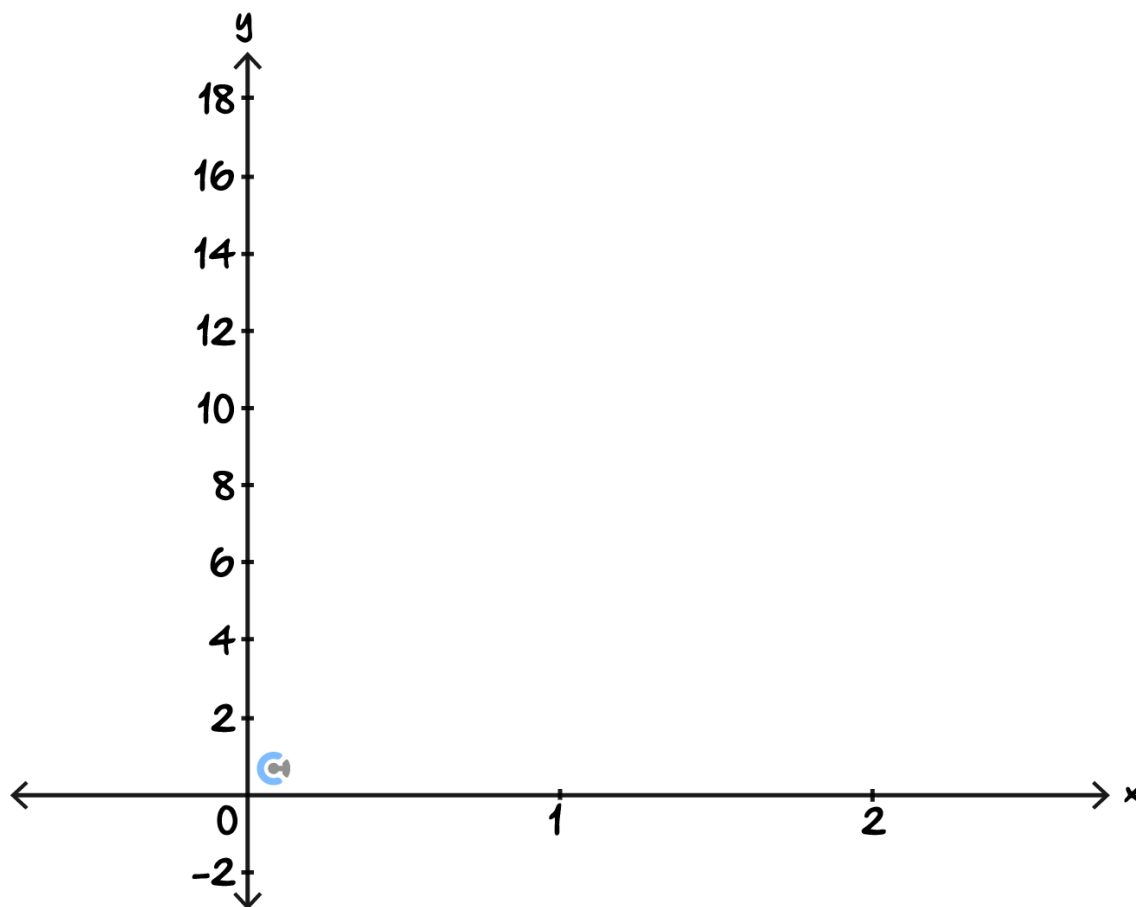$$h(t) = a\sin\left(kt + \frac{\pi}{3}\right) + d$$

where t is the time in seconds.

The pendulum reaches a maximum height of $18\ cm$ at $t = 0$, and a minimum height of $2\ cm$ after 0.6 seconds.

a. Find the values of a, k, and d.

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b. Find the general solution for when the height is **exactly 10 cm**.

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c. Sketch the graph of $h(t)$ for $0 \leq t \leq 1.8$, labelling intercepts, max, and min.

d. Determine the fraction of time over a full cycle that the pendulum is higher than $6\ cm$ but lower than $14\ cm$.

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Sub-Section: Exam 2

Question 19

Select the false statement from the following:

A. $\sin(\pi + \theta) + \sin(\pi - \theta) = 0$

B. $\cos(\pi + \theta) + \cos(\pi - \theta) = 0$

C. $\sin(\pi - \theta) - \sin(2\pi + \theta) = 0$

D. $\tan(\pi + \theta) + \tan(2\pi - \theta) = 0$

Question 20

The number of wombats, n, in a particular area is given by the function $n(t) = 100 \sin\left(\frac{\pi(t+12)}{12}\right) + 200$, where t is the time in months, and $t \geq 0$.

Which of the following statements is false?

A. The maximum number of wombats is 300.

B. The minimum number of wombats is 100.

C. The initial number of wombats is 100.

D. After one year, the number of wombats is 200.

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Question 21

The diagram shows two cycles of a circular function. The period of the function is:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. π
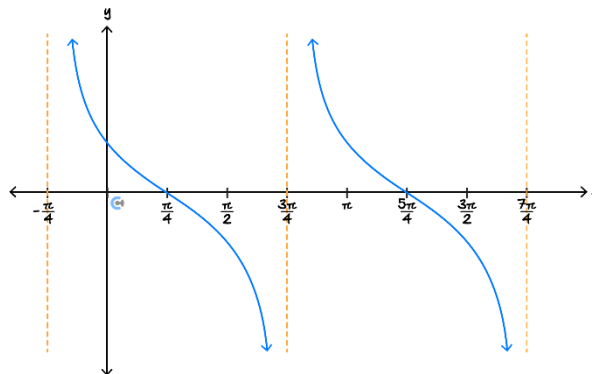
Question 22

The graph of $y = \tan(ax)$, where $a > 0$, has a vertical asymptote at $x = \frac{\pi}{6}$ and exactly one intercept in the interval $\left(0, \frac{\pi}{3}\right)$. The value of a is:

A. 1

B. 3

C. 6

D. 2

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Question 23

The graph of $y = \tan\left(\frac{bx}{5}\right)$, where $b \neq 0$, has a vertical asymptote at:

A. $x = 0$

B. $x = \frac{2\pi}{5b}$

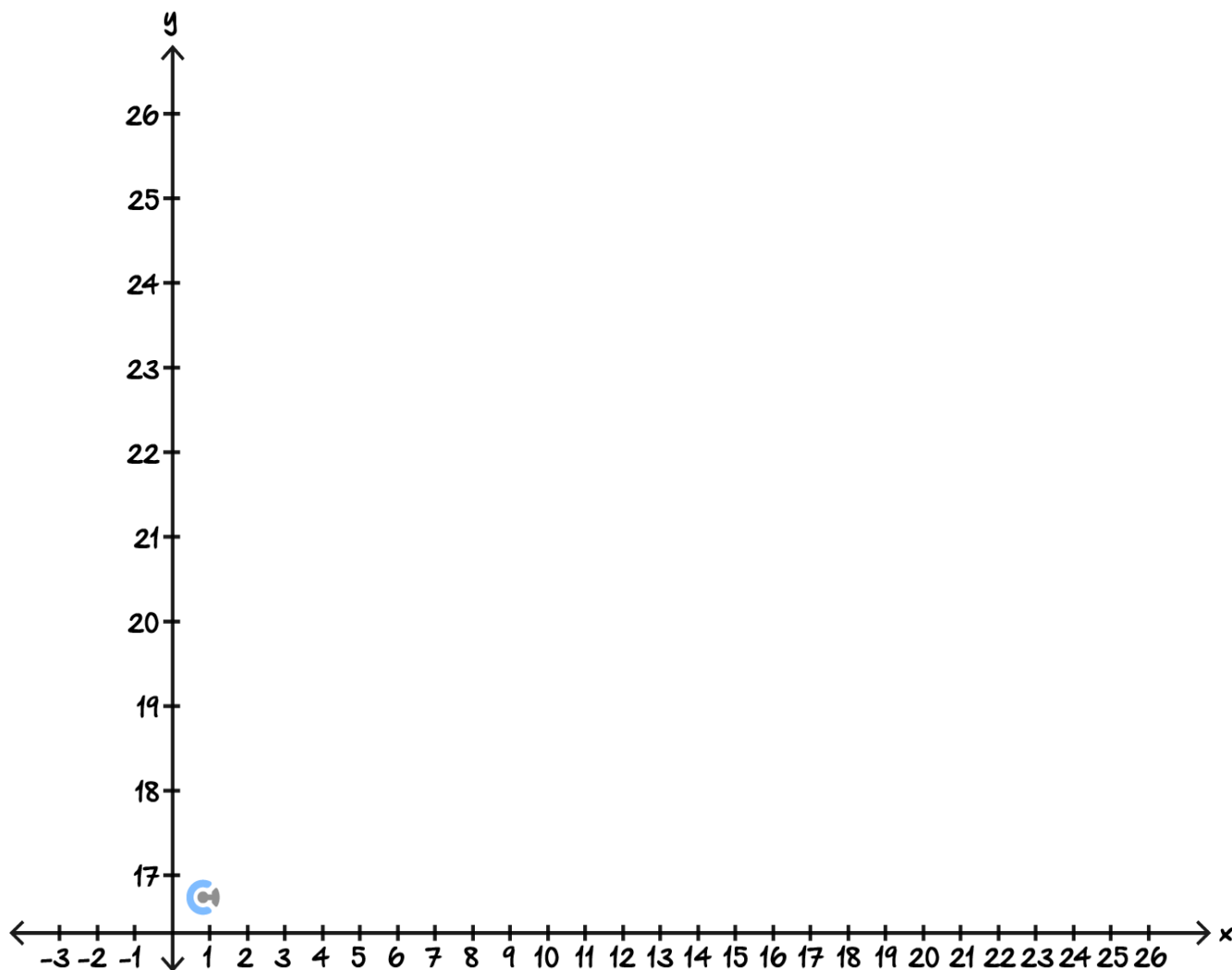C. $x = \frac{\pi}{5b}$

D. $x = \frac{5\pi}{2b}$

Question 24

The temperature in a greenhouse at time t hours after midnight is modelled by the function:

$$T(t) = 22 + 4\cos\left(\frac{\pi t}{12}\right), 0 \leq t \leq 24$$

where $T(t)$ is in degrees Celsius.

a. State the amplitude and period for T.

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b. Sketch the graph of $T(t)$, label minimum, maximum and intercept(s).

c. To ensure optimal plant growth, the temperature must stay **below $k°C$** for **at least 5 consecutive hours**.

Find, correct to one decimal place, the **smallest value of k** that satisfies this condition.

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Question 25

The sunlight intensity $I(t)$, in lumens, shining on a solar sensor at a research station is modelled by the function:

$$I(t) = 500 \sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) + 600$$

Where t is the number of hours after midnight on a particular day. You may assume $0 \leq t \leq 24$.

The sensor must remain in low-power mode whenever the intensity falls below 400 lumens to avoid overheating. Scientists are trying to conduct a sensitive experiment that can only happen when the sensor is **not** in low-power mode.

a. State the amplitude, period, and average sunlight intensity of this function.

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b. Find the times in the day when the sunlight intensity is exactly 400 lumens, correct to the nearest minute.

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c. Determine the total amount of time in a day the sensor can operate above 400 lumens, correct to the nearest minute.

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d. An optimal time to begin the experiment is the first moment in the day when the sunlight intensity reaches 400 lumens and is increasing.

i. State this time, correct to the nearest minute.

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ii. The experiment takes 3 hours and 10 minutes to complete.

Determine whether the scientists have enough time for the experiment before the sensor must return to low-power mode. If not, calculate how much extra time they would need, correct to the nearest second.

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Question 26

A drone is flying over a mountainous region to record environmental data. Its altitude (in kilometres above sea level) is modelled by the function:

$$A(t) = 2.4 \cos\left(\frac{\pi}{3}(t - k)\right) + 2.5$$

Where $A(t)$ is the drone's altitude in km, t is the time in seconds after takeoff, $k \in R$ is a horizontal shift caused by delayed GPS calibration.

a. State the amplitude, period, and average cruising altitude of the drone.

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b. Find the maximum and minimum altitudes reached by the drone.

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c. The drone begins its flight at an altitude of 2.5 km and must initially ascend. Determine all possible value(s) of $k \in (0, 3)$ that satisfy these conditions. Express your final answer(s) correct to 2 decimal places, showing working.

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d. Using your value of k, determine all values of $t \in [0, 6]$, where the drone is flying higher than 2 km. Given answers to 2 decimal places.

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e. Sketch the graph of $A(t)$ for two full cycles, labelling all intercepts, minimum and maximum points, and the midline. Coordinates must be accurate to 2 decimal places.
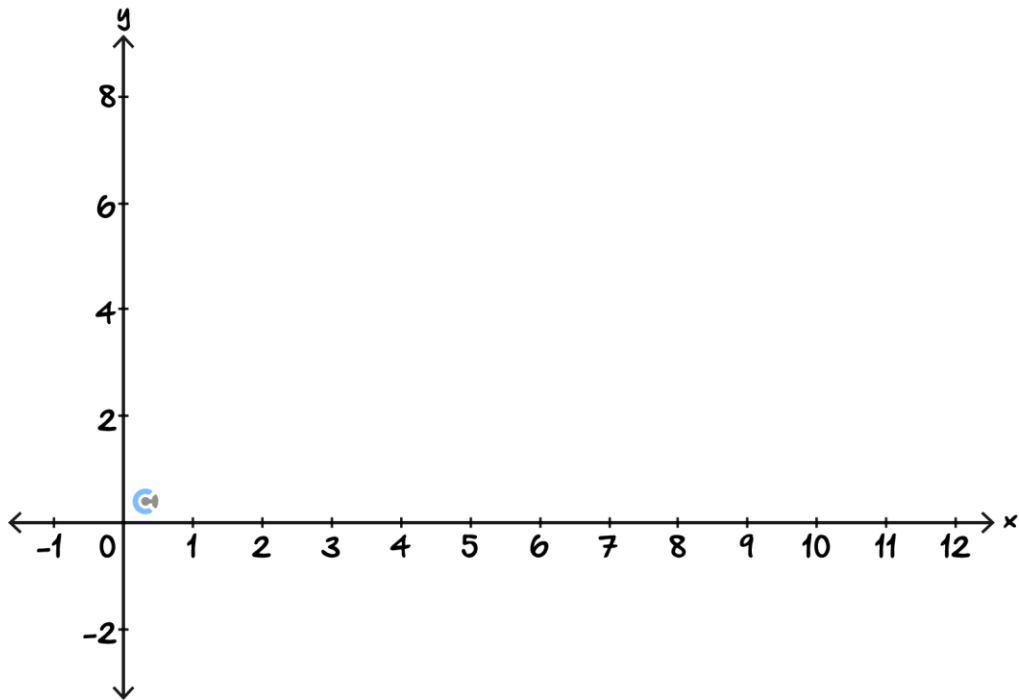
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