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VCE Mathematical Methods ½
Graphs of Circular Function [4.4]
Workbook

Outline:

Learning Objectives:

- ☐ **MM12 [4.4.1] –** Graph Sine, Cosine and Tangent Functions
- ☐ **MM12 [4.4.2] –** Fraction of Periods

Section A: Solving Trigonometric Equations

Sub-Section: Recap of Particular & General Solutions

REMINDER: Particular Solutions

- Solving trigonometric equations for finite solutions.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add and subtract the period to find all other solutions in the domain.

Quad
Ref
Angle (acute)
± Period
sin( ) = ____

REMINDER: General Solutions

- Finding infinitely many solutions to a trigonometric equation.
- **Steps:**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add $\text{period} \cdot n$ where $n \in Z$.

Space for Personal Notes

Question 1 Walkthrough.

Find the solutions to the following equation:

particular

$2\sin\left(2x+\frac{\pi}{6}\right)+1=0 \text{ for } x\in[0,2\pi]$

$\sin\left(2x+\frac{\pi}{6}\right)=-\frac{1}{2}$

S | A
T | C

quad 3 & 4

ref $\frac{\pi}{6}$

(A)

(3) (4)

$2x+\frac{\pi}{6}=\pi+\frac{\pi}{6}, \quad 2\pi-\frac{\pi}{6}$

$2x+\frac{\pi}{6}=\frac{7\pi}{6}, \quad \frac{11\pi}{6}$

$2x=\pi, \quad \frac{5\pi}{3}$

$x=\frac{\pi}{2}, \quad \frac{5\pi}{6}, \quad \frac{3\pi}{2}, \quad \frac{11\pi}{6}$

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(P) π

Question 2

Find the solutions to the following equation:

$$\sqrt{2}\cos\left(2x-\frac{\pi}{2}\right)-1=0 \text{ for } x\in[0,2\pi]$$

$\cos(2x-\frac{\pi}{2}) = \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{2}}{2}$

quad 1 & 4

ref $\frac{\pi}{4}$

per. $\frac{2\pi}{2}=\pi$

$2x-\frac{\pi}{2} = \frac{\pi}{4}$ (1), $2\pi-\frac{\pi}{4}$ (4)

$2x-\frac{\pi}{2} = \frac{\pi}{4}, \frac{7\pi}{4}$

$2x = \frac{3\pi}{4}, \frac{9\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

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Question 3 Walkthrough.

Find the general solutions to the following equation:

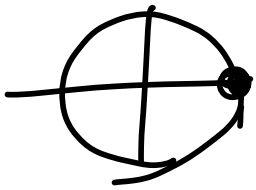
$$2\cos\left(3x-\frac{\pi}{6}\right)=2$$

general solution

$$\cos\left(3x-\frac{\pi}{6}\right)=1$$

quad 1, ~~4~~

ref 0



$$3x-\frac{\pi}{6}=0$$

$$3x=\frac{\pi}{6}$$

$$x=\frac{\pi}{18}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$$

period $\frac{2\pi}{3}$

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actual

quad 1: $\theta=$ ref

quad 2: $\theta=\pi-$ ref

quad 3: $\theta=\pi+$ ref

quad 4: $\theta=2\pi-$ ref

Question 4

Find the general solutions to the following equation:

$$4\sin\left(2x+\frac{\pi}{3}\right)+2=0$$

$$\sin\left(2x+\frac{\pi}{3}\right)=-\frac{1}{2}$$

quad 3 & 4

ref $\frac{\pi}{6}$

ang. $2x+\frac{\pi}{3}=\pi+\frac{\pi}{6}$ (3), $2\pi-\frac{\pi}{6}$ (4)

$2x+\frac{\pi}{3}=\frac{7\pi}{6}$, $\frac{11\pi}{6}$

$2x=\frac{5\pi}{6}$, $\frac{9\pi}{6}$

$2x=\frac{5\pi}{6}$, $\frac{3\pi}{2}$

$x=\frac{5\pi}{12}$, $\frac{3\pi}{4}$

period $=\frac{2\pi}{2}=\pi$

$x=\frac{5\pi}{12}+\pi n,\ n\in\mathbb{Z}$

$x=\frac{3\pi}{4}+\pi n,\ n\in\mathbb{Z}$

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Question 5

Find the general solutions to the following equation:

$$3\tan\left(2x+\frac{\pi}{6}\right)-3\sqrt{3}=0$$

$\tan\left(2x+\frac{\pi}{6}\right)=\sqrt{3}$

quad 1

ref $\frac{\pi}{3}$

$2x+\frac{\pi}{6}=\frac{\pi}{3}$

period: $\frac{\pi}{2}$

$2x=\frac{\pi}{6}$

$x=\frac{\pi}{12}+\frac{\pi}{2}n,\ n\in\mathbb{Z}$

NOTE: The period of tan is $\frac{\pi}{n}$.

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Section B: Graphs of Sine and Cosine

Sub-Section: Understanding the Shape

What does a sine and cosine graph look like?

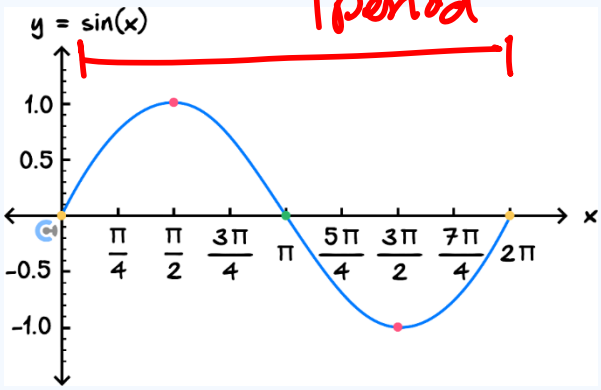
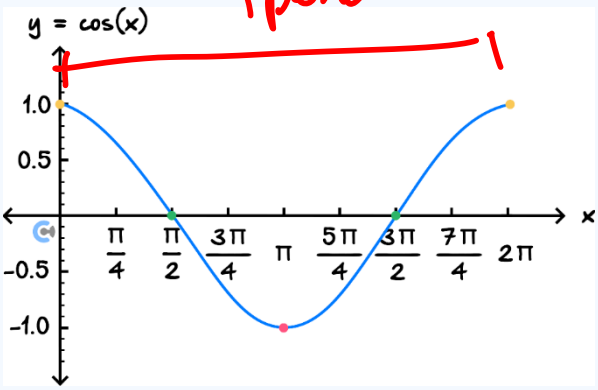
Exploration: Graph of Sine and Cosine

- Scan the following QR code on your device!

Sine	Cosine

Space for Personal Notes

Sine and Cosine Graphs

Sine	Cosine
	

Discussion: Is $\cos(x)$ an even function or an odd function. What about $\sin(x)$?

Discussion: What does $\sin\left(\frac{\pi}{2}+x\right)$ equal to? So, how can we translate sin function to cosine function?

$= \cos(x)$

↳ Same shape

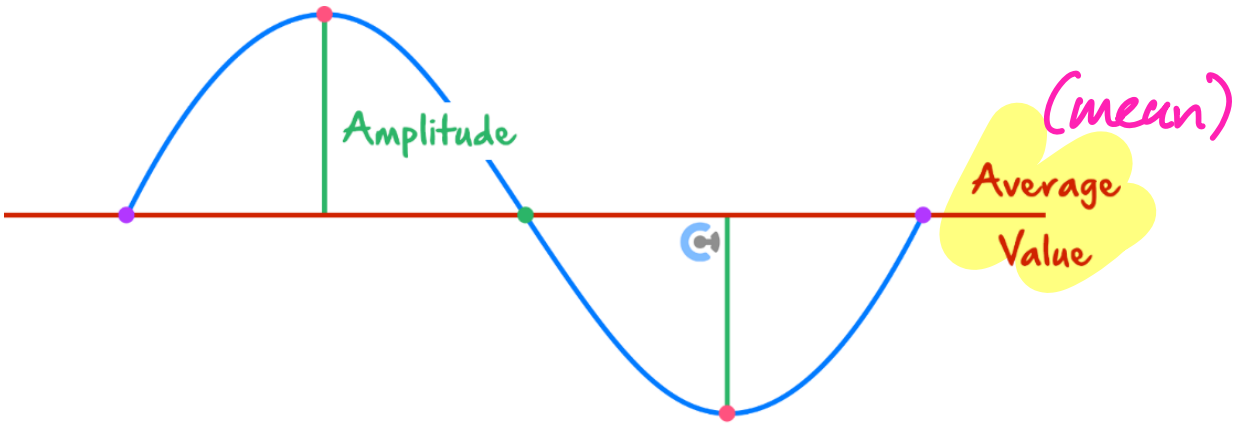
BUT

Sub-Section: Graphing Sine and Cosine Functions

Amplitude, Period and Average Value

MAPSS: mean value, amplitude, shape, starting point, period

For $y = A\sin/\cos(nx + b) + k$



Consider the sign of our graph

Amplitude $= |A|$ ← tve

Period $= \frac{2\pi}{|n|}$

Average Value $= k$

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Question 6

Identify the amplitude, period and average value of the following functions:

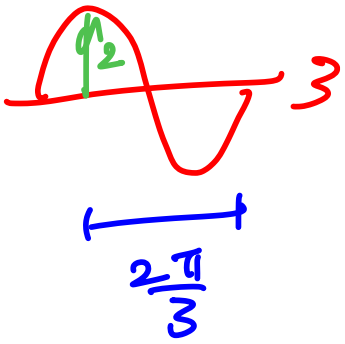
a. $f(x) = 2\sin\left(\frac{\pi}{6} - 3x\right) + 3$

MAPSS

mean value: 3

amplitude : 2

period : $\frac{2\pi}{n} = \frac{2\pi}{3}$



b. $g(x) = -5\cos(2x + 7) - 2$

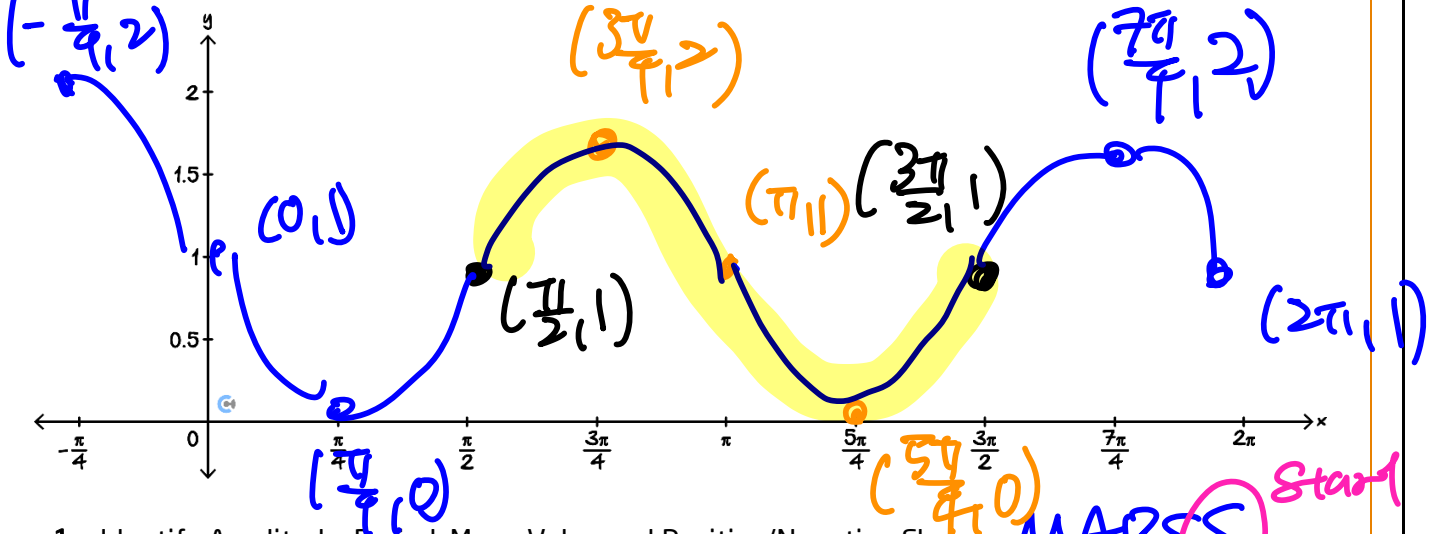
Mean : -2

Amp : 5

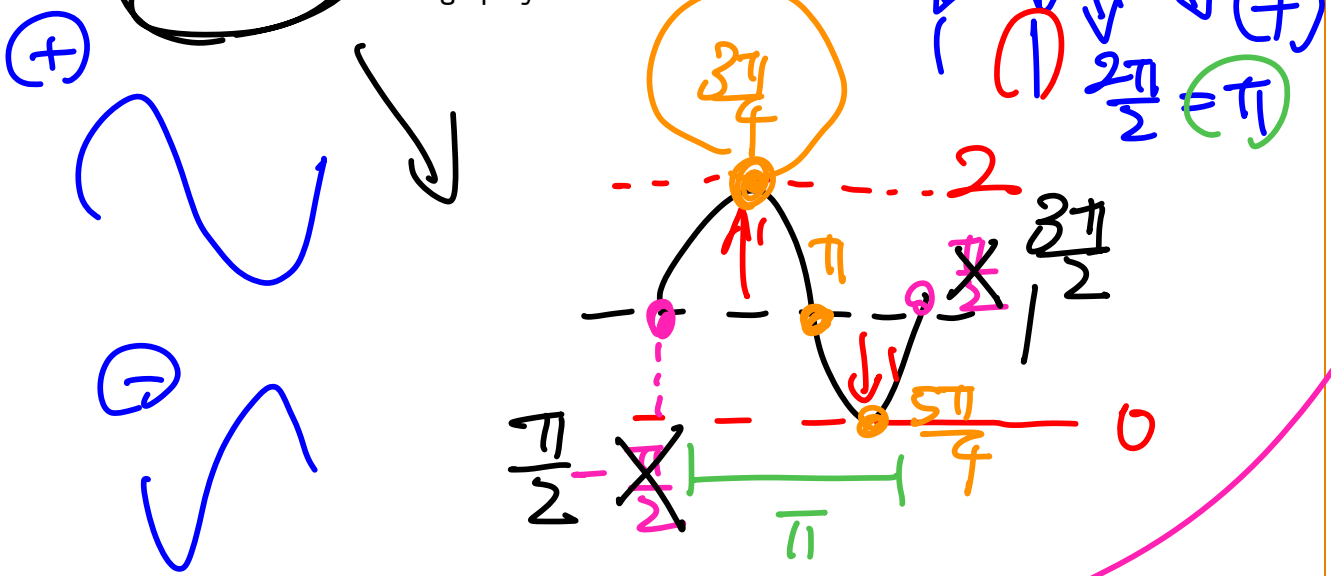
Period : $\frac{2\pi}{n} = \frac{2\pi}{2} = \pi$

Exploration: Graphing of sin and cos Functions

- Let's sketch $\sin(2x+\pi)+1$ on the axes below!

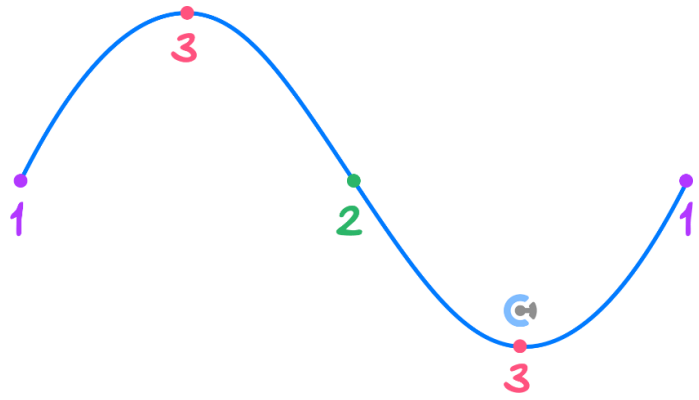


1. Identify ~~Amplitude~~, Period, Mean Value and Positive/Negative Shape.
2. Create a "mini-version" of the graph you are about to draw.



3. Start plotting the function from when the angle $= 0$. **Why?**
   - **It allows us to always sketch the graph from the** Start.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

Graphing of sin and cos Functions



1. Identify Amplitude, Period, Mean Value and Positive/Negative Shape.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle $= 0$.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

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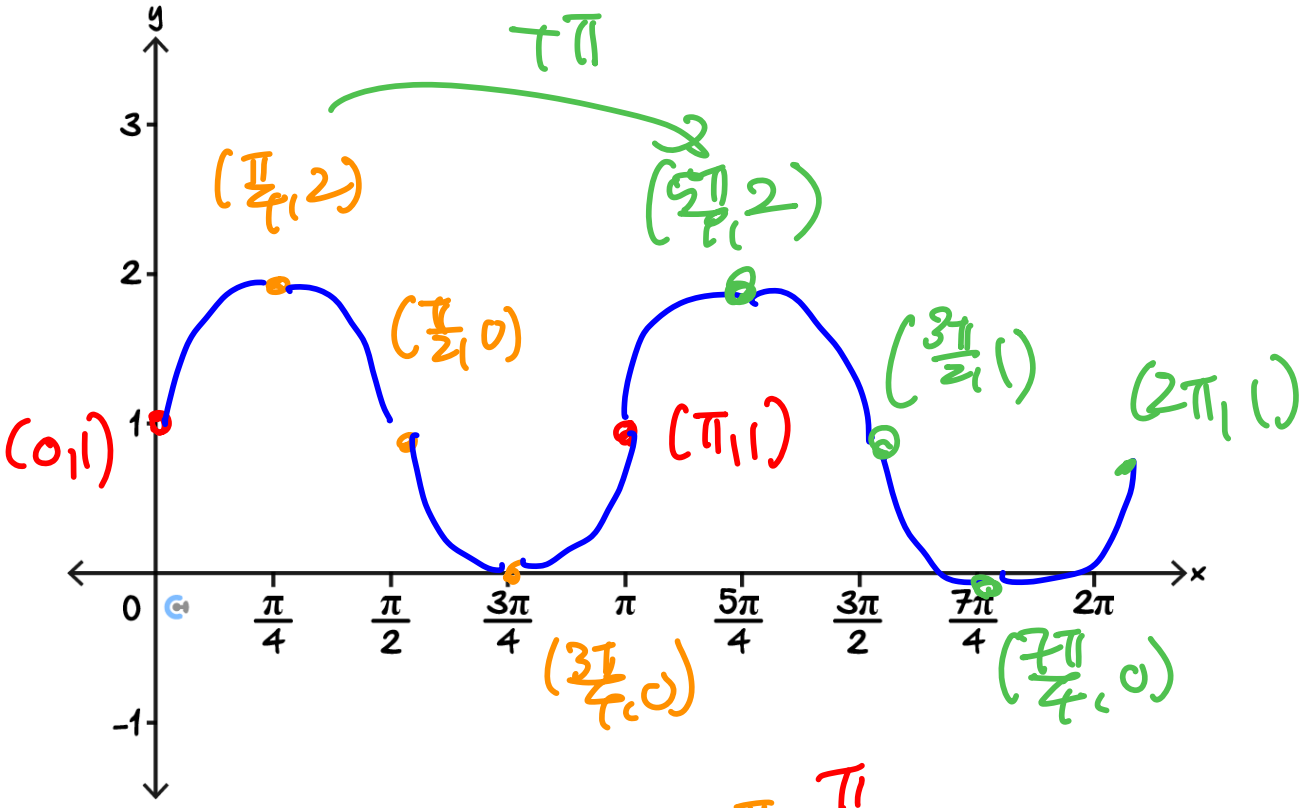
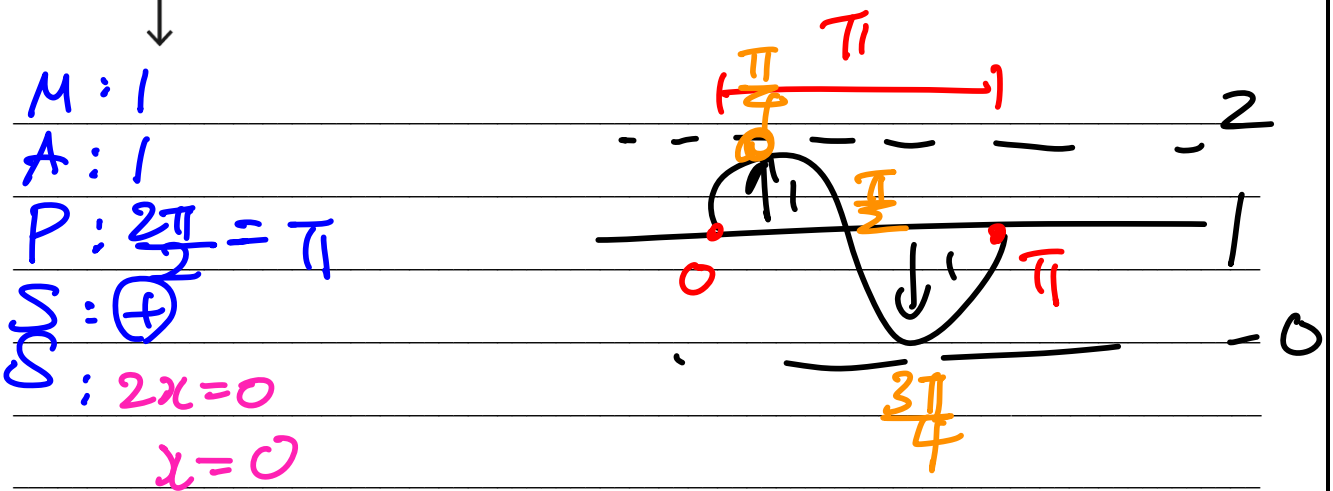
Question 7 Walkthrough.

Sketch the graph of $f(x) = \sin(2x) + 1$ for $x \in [0, 2\pi]$ on the axes below, labelling all intercepts and endpoints with their coordinates.





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Active Recall: Graphing of $\sin$ and $\cos$ Functions

- **Steps:**

1. Identify: MAPSS.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle = 0.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-ints (y-ints) y = 0.
6. Join all the points!

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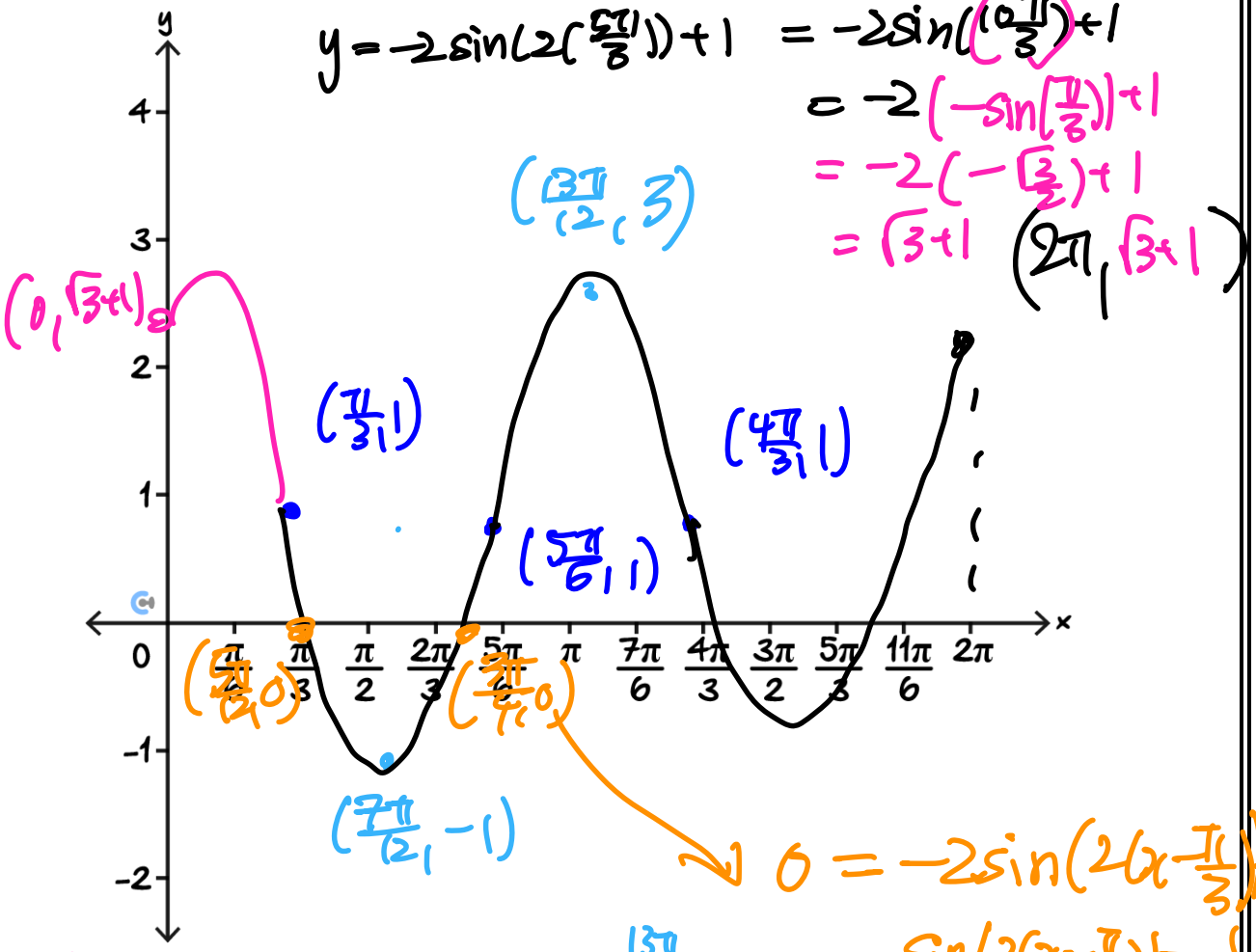
Question 8 *For walkthrough, watch "Mathematica" recording!

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

$$y = -2\sin\left(2\left(x - \frac{\pi}{3}\right)\right) + 1 \text{ for } x \in [0, 2\pi]$$



$y = -2\sin(2(\frac{5\pi}{6})) + 1 = -2\sin(\frac{10\pi}{3}) + 1$

$= -2(-\sin(\frac{\pi}{3})) + 1$

$= -2(-\frac{\sqrt{3}}{2}) + 1$

$= \sqrt{3} + 1$ $(2\pi, \sqrt{3}+1)$

CQRS

$(\frac{\pi}{3}, 1)$ $(\frac{4\pi}{3}, 1)$ $(\frac{5\pi}{6}, 1)$ $(\frac{3\pi}{2}, 3)$ $(0, \sqrt{3}+1)$ $(\frac{\pi}{6}, 0)$ $(\frac{7\pi}{12}, -1)$

$0 = -2\sin(2(x - \frac{\pi}{3})) + 1$

$\sin(2(x - \frac{\pi}{3})) = \frac{1}{2}$

$2(x - \frac{\pi}{3}) = \frac{\pi}{6}, \frac{5\pi}{6}$

$x - \frac{\pi}{3} = \frac{\pi}{12}, \frac{5\pi}{12}$

$x = \frac{5\pi}{12}, \frac{9\pi}{12}$

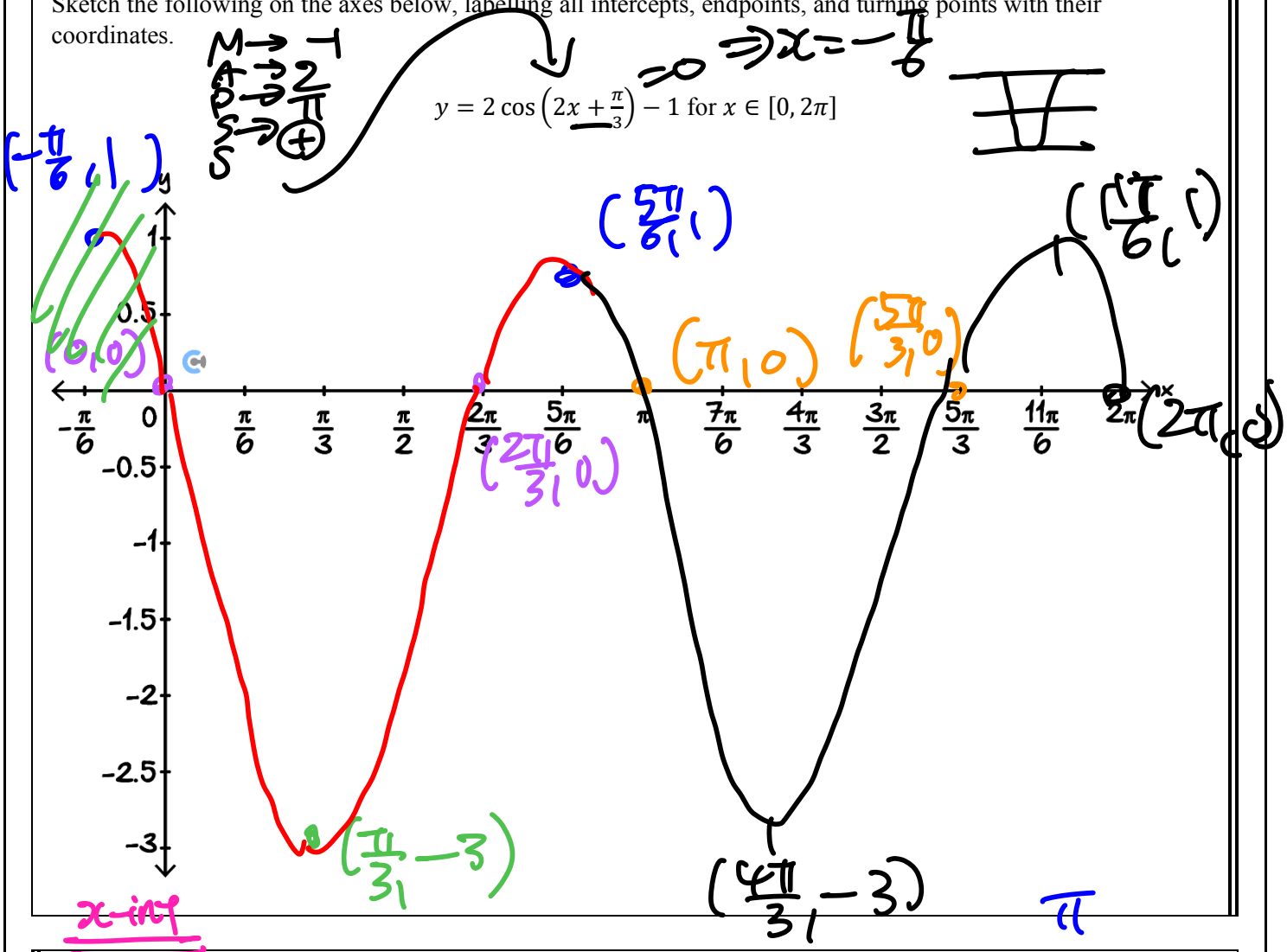
M 1
A 2
P π
S ⊖
S $2(x - \frac{\pi}{3}) = 0$
$\Rightarrow x = \frac{\pi}{3}$

$\frac{\pi}{3} + \frac{\pi}{2} = \frac{2\pi}{6} + \frac{3\pi}{6} = \frac{5\pi}{6}$

Question 9

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

$$y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1 \text{ for } x \in [0, 2\pi]$$



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$0 = 2\cos(2x + \frac{\pi}{3}) - 1$

$\cos(2x + \frac{\pi}{3}) = \frac{1}{2}$

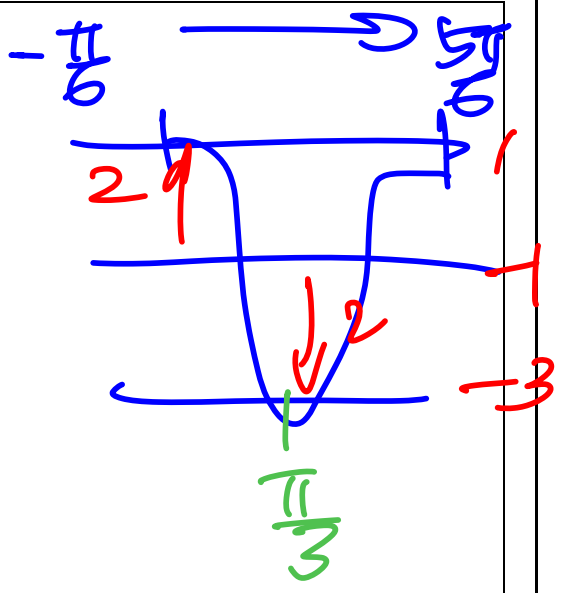
Quad 1 & 4

Ref Angle $\frac{\pi}{3}$

$2x + \frac{\pi}{3} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$2x = 0, \frac{4\pi}{3}$

$x = 0, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}$



Sub-Section: Finding the Rule

Finding the Rule

$$\text{Amplitude } (A) = \frac{max - min}{2}$$

$$\text{Average } (k) = \frac{max + min}{2}$$

Question 10 Walkthrough.

A function with rule $y = A\sin(nt) + b$ where $A > 0$ has a range $[-5,3]$ and period 6. Find A, n and b.

$A = \pm 4$

$b = -1$

$A = 4$

$\frac{2\pi}{n} = 6$

$n = \frac{2\pi}{6} = \frac{\pi}{3}$

4

3

−1

−5

TIP: Graphing helps!

Active Recall: Finding the Rule

$$\text{Amplitude } (A) = \frac{max - min}{2}$$

$$\text{Average } (k) = \frac{max + min}{2}$$

Your turn!

Question 11

A function with rule $y = A\cos(nt + \pi) + b$ where $A < 0$ has a range $[-6,8]$ and period 3. Find A, n and b.

$A < 0$

$A = -7$

$b = 1$

$8 - 1 = 7$

8

1

-6

$\frac{2\pi}{n} = 3$

$n = \frac{2\pi}{3}$

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Section C: Graphs of Tangent

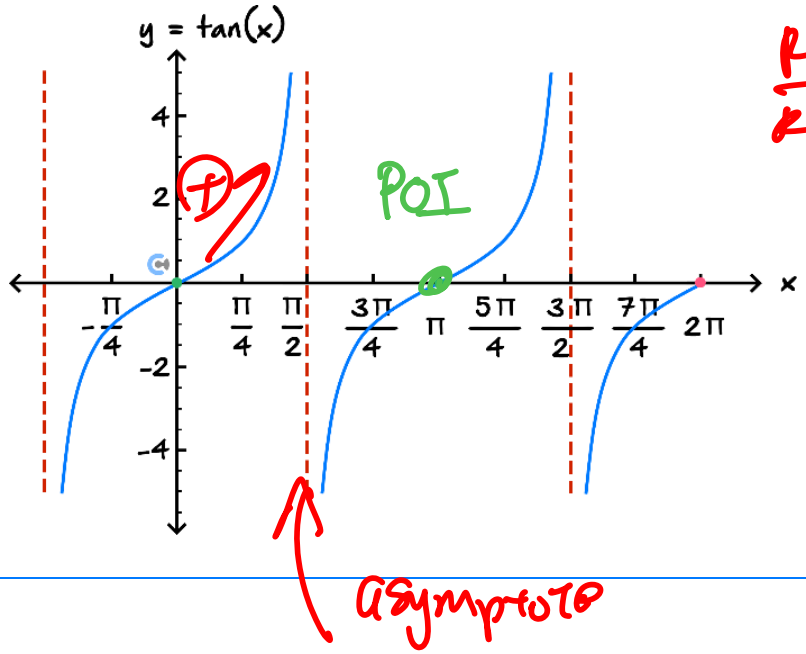
Sub-Section: Understanding Tangent Graphs

What does the tangent graph look like?

Exploration: Graph of Tangents

- Scan the QR code below on your device!

Graph of Tangent

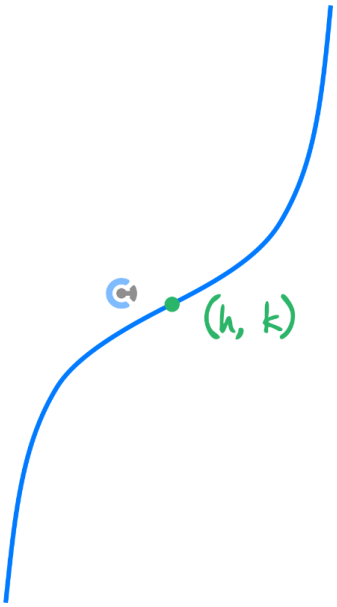


Sub-Section: Graphing Tangent Functions

Steps for Sketching tan Functions

1. Identify:
   - The period $= \frac{\pi}{n}$.
2. Find the vertical asymptotes by solving for angle $= \frac{\pi}{2}$.
3. Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
   - For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x.
4. Plot the inflection point (h, k). (Midpoint of the two vertical asymptotes.)
   - x-value of inflection point $= x$-value which makes angle $= 0$.
   - y-value of inflection point = vertical translation of the function.

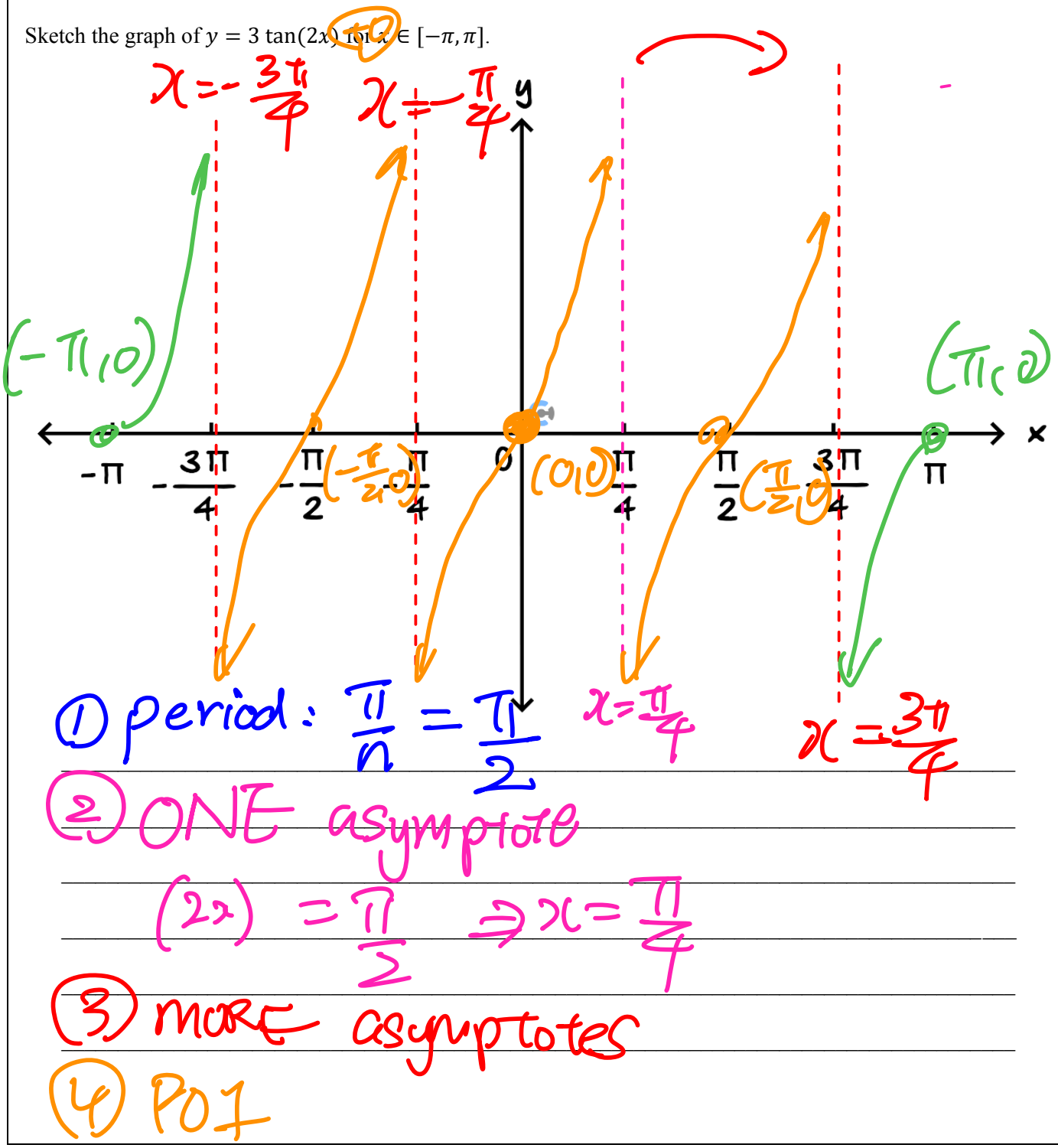
eg: $\tan(x-h)+k$



5. Find any x-intercepts.
6. Sketch a "cubic-like" shape.

Question 12 Walkthrough.

Sketch the graph of $y = 3\tan(2x)$ for $x \in [-\pi, \pi]$.



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Active Recall: Steps for Sketching tan Functions

1. Identify:
   - The period = $\frac{\pi}{b}$.
2. Find the vertical asymptotes by solving for angle = $\frac{\pi}{2}$
3. Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
4. Plot the inflection point (h, k). (Midpoint of the two asymptotes.)
   - x-value of inflection point = x-value which makes angle = 0.
   - y-value of inflection point = vertical translation of the function.
5. Find any x-int.
6. Sketch a cubic-like shape.

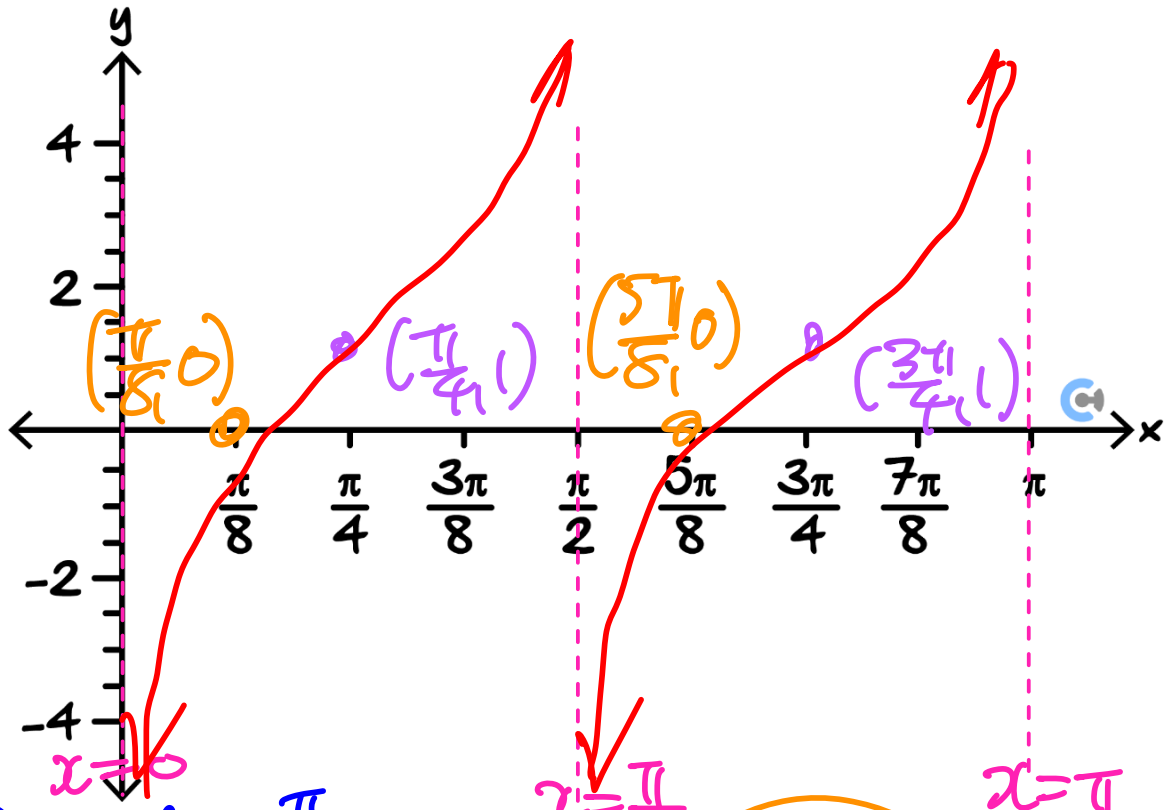
Space for Personal Notes

Your turn!

Question 13

Sketch the following on the axes below, labelling all intercepts and points of inflection with coordinates and all asymptotes with their equations.

$$y = \tan\left(2x + \frac{\pi}{2}\right) + 1 \text{ for } x \in (0, \pi)$$



① period = $\frac{\pi}{2}$

② ONE asymptote

$2x + \frac{\pi}{2} = \frac{\pi}{2}$

$x = 0$

③ OTHER asymptotes

x-int $y = 0$

$0 = \tan\left(2x + \frac{\pi}{2}\right) + 1$

$\tan\left(2x + \frac{\pi}{2}\right) = -1$

$2x + \frac{\pi}{2} = \frac{3\pi}{4}$

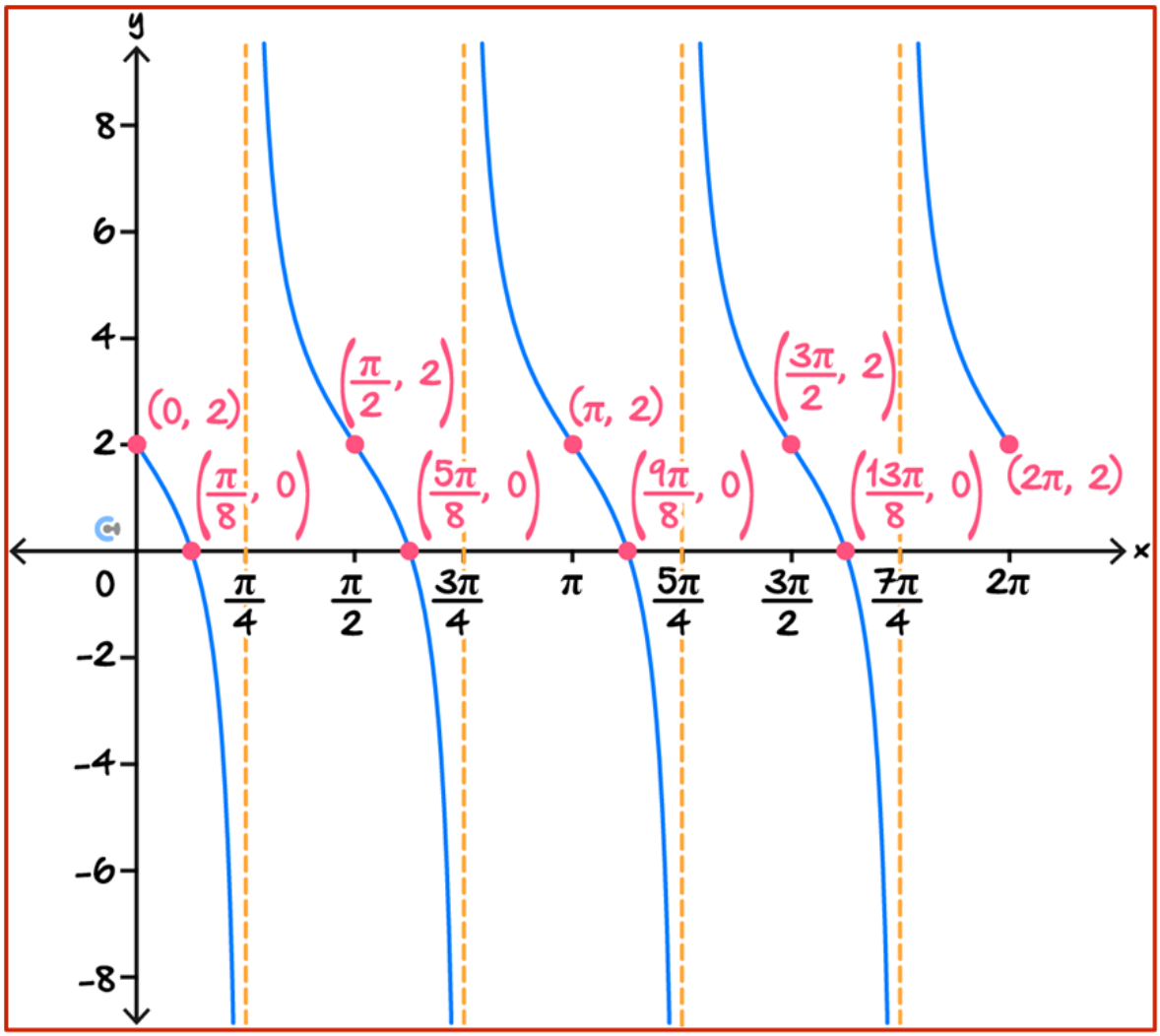
$2x = -\frac{\pi}{4}$

$x = -\frac{\pi}{8}$

Question 14

Sketch the following on the axes below, labelling all intercepts, points of inflection, and endpoints with their coordinates, and all asymptotes with their equations.

$$f: [0,2\pi] \to \mathbb{R}, f(x) = -2\tan(\pi + 2x) + 2$$



Section D: Fraction of Period

Sub-Section: Fraction of Period

1 sleep 8 hrs a day

8/24

Definition: Fraction of Period

$$\text{Fraction of Period} = \frac{\text{Duration}}{\text{Period}}$$

$$\%\ \text{of Period} = \frac{\text{Duration}}{\text{Period}} \times 100\%$$

Question 15 Walkthrough.

The ~~population of dogs~~ number of lollies in a certain household is modelled by $P(t)$.

$$P(t) = 4 - 2\cos\left(\frac{\pi}{4}t\right)$$

Where $P(t)$ is the number of ~~dogs~~ lollies t years since 2024. Find the fraction of time where the population is above 5.

$P(t) > 5$

$P(t) = 5$

$4 - 2\cos\left(\frac{\pi}{4}t\right) = 5$

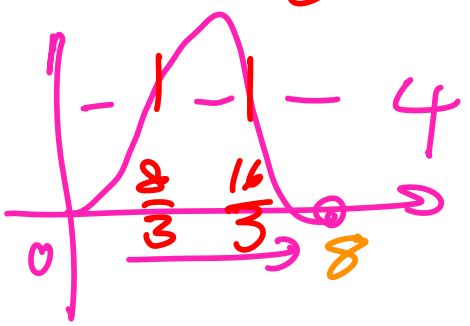
$-2\cos\left(\frac{\pi}{4}t\right) = 1$

$\cos\left(\frac{\pi}{4}t\right) = -\frac{1}{2}$

② $\frac{\pi}{4}t = \pi - \frac{\pi}{3}$ ③ $\pi + \frac{\pi}{3}$

$\frac{\pi}{4}t = \frac{2\pi}{3}, \frac{4\pi}{3}$

$t = \frac{8}{3}, \frac{16}{3}$



$2\pi \div \frac{\pi}{4} = 8$

duration: $\frac{16}{3} - \frac{8}{3} = \frac{8}{3}$

fraction: $\frac{8/3}{8} = \frac{1}{3}$

NOTE: Always sketch the function to find the duration!

Active Recall: Fraction of Period

$$\textit{Fraction of Period} = \frac{\text{duration}}{\text{period}}$$

$$\%\ \textit{of Period} = \frac{\quad}{\quad} \times 100\%$$

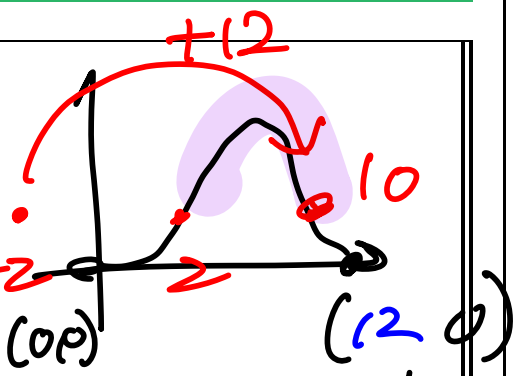
Question 16

The population of cats in a certain household is modelled by $P(t)$.

$$P(t) = 10 - 4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right)$$

Where $P(t)$ is the number of cats t years since 2024.

Find the fraction of time where the population is above 8.



period $= 2\pi \div \frac{\pi}{6}$

$= 2\pi \times \frac{6}{\pi} = 12$

$P(t) > 8$

$P(t) = 8$

$10 - 4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) = 8$

$-4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) = -2$

$\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) = \frac{1}{2}$

quad 1 & 2

ref $\frac{\pi}{6}$

① $\frac{\pi}{6}t + \frac{\pi}{2} = \frac{\pi}{6}$ ② $\frac{5\pi}{6}$

$\frac{\pi}{6}t = -\frac{\pi}{3}$, $\frac{\pi}{3}$

$t = -2$, 2

duration $= 10 - 2$

$= 8$

fraction: $\frac{8}{12} = \frac{2}{3}$

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