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VCE Mathematical Methods ½
Graphs of Circular Function [4.4]
Workbook

Outline:

Learning Objectives:

- ☐ **MM12 [4.4.1] –** Graph Sine, Cosine and Tangent Functions
- ☐ **MM12 [4.4.2] –** Fraction of Periods

Section A: Solving Trigonometric Equations

Sub-Section: Recap of Particular & General Solutions

REMINDER: Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps:**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add and subtract the period to find all other solutions in the domain.

REMINDER: General Solutions

- Finding infinitely many solutions to a trigonometric equation.
- **Steps:**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add $\text{period} \cdot n$ where $n \in Z$.

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Question 1 Walkthrough.

Find the solutions to the following equation:

$$2\sin\left(2x+\frac{\pi}{6}\right)+1=0 \text{ for } x\in[0,2\pi]$$

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Question 2

Find the solutions to the following equation:

$$\sqrt{2}\cos\left(2x-\frac{\pi}{2}\right)-1=0 \text{ for } x\in[0,2\pi]$$

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Question 3 Walkthrough.

Find the general solutions to the following equation:

$$2\cos\left(3x-\frac{\pi}{6}\right)=2$$

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Question 4

Find the general solutions to the following equation:

$$4\sin\left(2x+\frac{\pi}{3}\right)+2=0$$

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Question 5

Find the general solutions to the following equation:

$$3\tan\left(2x+\frac{\pi}{6}\right)-3\sqrt{3}=0$$

NOTE: The period of tan is $\frac{\pi}{n}$.

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Section B: Graphs of Sine and Cosine

Sub-Section: Understanding the Shape

What does a sine and cosine graph look like?

Exploration: Graph of Sine and Cosine

- Scan the following QR code on your device!

Sine	Cosine

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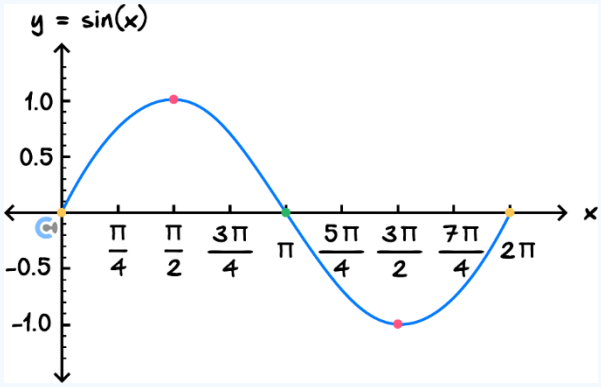
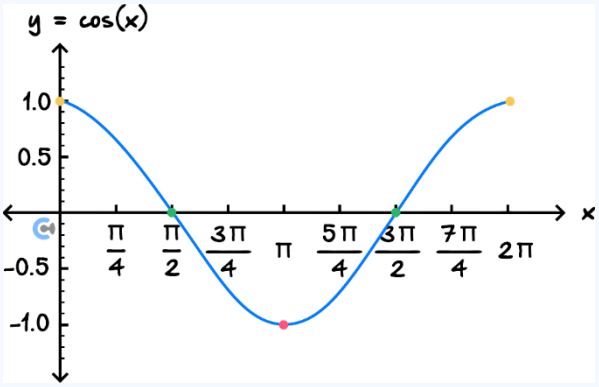
Sine and Cosine Graphs

Sine	Cosine
	

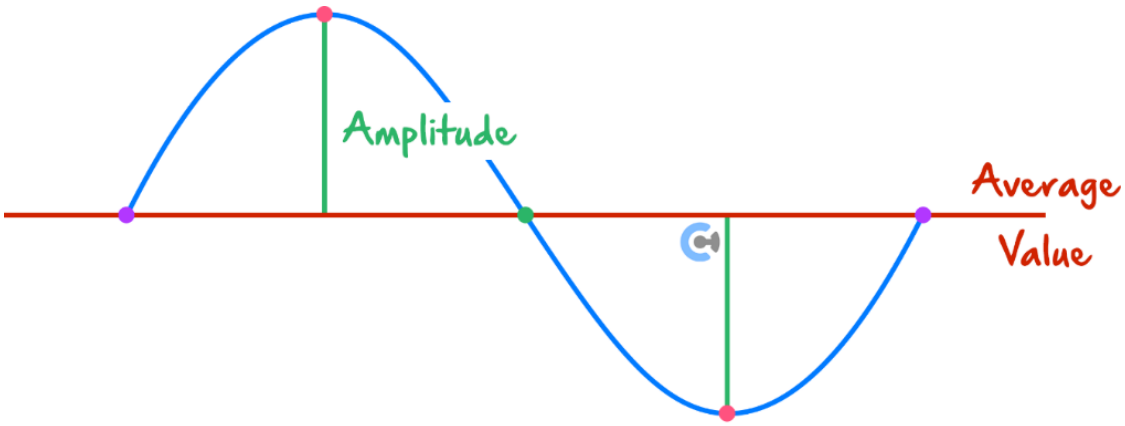
Discussion: Is $\cos(x)$ an even function or an odd function. What about $\sin(x)$?

Discussion: What does $\sin\left(\frac{\pi}{2}+x\right)$ equal to? So, how can we translate sin function to cosine function?

Sub-Section: Graphing Sine and Cosine Functions

Amplitude, Period and Average Value

For $y = A\sin/\cos(nx + b) + k$



Consider the sign of our graph

$$\text{Amplitude} = |A|$$

$$\text{Period} = \frac{2\pi}{|n|}$$

$$\text{Average Value} = k$$

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Question 6

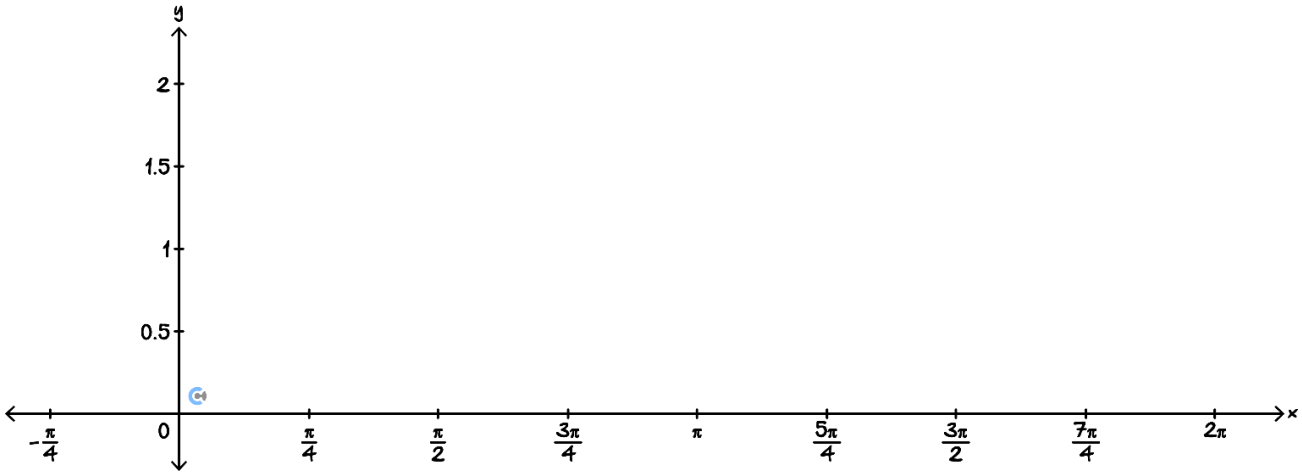
Identify the amplitude, period and average value of the following functions:

a. $f(x) = 2\sin\left(\frac{\pi}{6} - 3x\right) + 3$

b. $g(x) = -5\cos(2x + 7) - 2$

Exploration: Graphing of $\sin$ and $\cos$ Functions

- Let's sketch $\sin(2x + \pi) + 1$ on the axes below!



1. Identify Amplitude, Period, Mean Value and Positive/Negative Shape.

2. Create a "mini-version" of the graph you are about to draw.

3. Start plotting the function from when the angle $= 0$. **Why?**

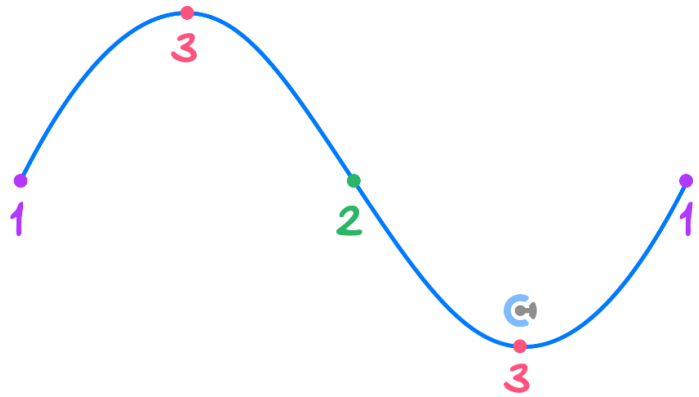
   - **It allows us to always sketch the graph from the _____.**

4. Draw the start and end of the periods, and plot the halves (turning points).

5. Find any x-intercepts.

6. Join all the points!

Graphing of sin and cos Functions



1. Identify Amplitude, Period, Mean Value and Positive/Negative Shape.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle $= 0$.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any x-intercepts.
6. Join all the points!

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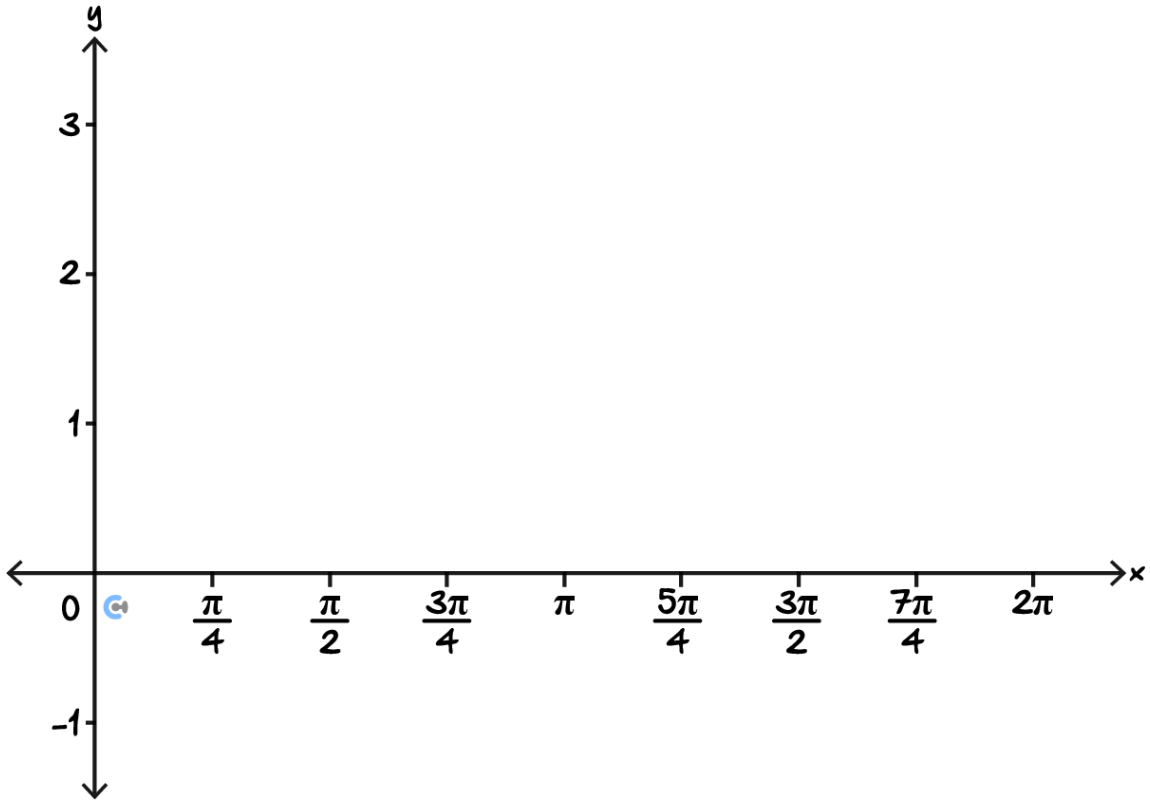
Question 7 Walkthrough.

Sketch the graph of $f(x) = \sin(2x) + 1$ for $x \in [0, 2\pi]$ on the axes below, labelling all intercepts and endpoints with their coordinates.



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Active Recall: Graphing of $\sin$ and $\cos$ Functions

- **Steps:**

1. Identify: _____.
2. Create a "mini-version" of the graph you are about to draw.
3. Start plotting the function from when the angle = _____.
4. Draw the start and end of the periods, and plot the halves (turning points).
5. Find any _____.
6. Join all the points!

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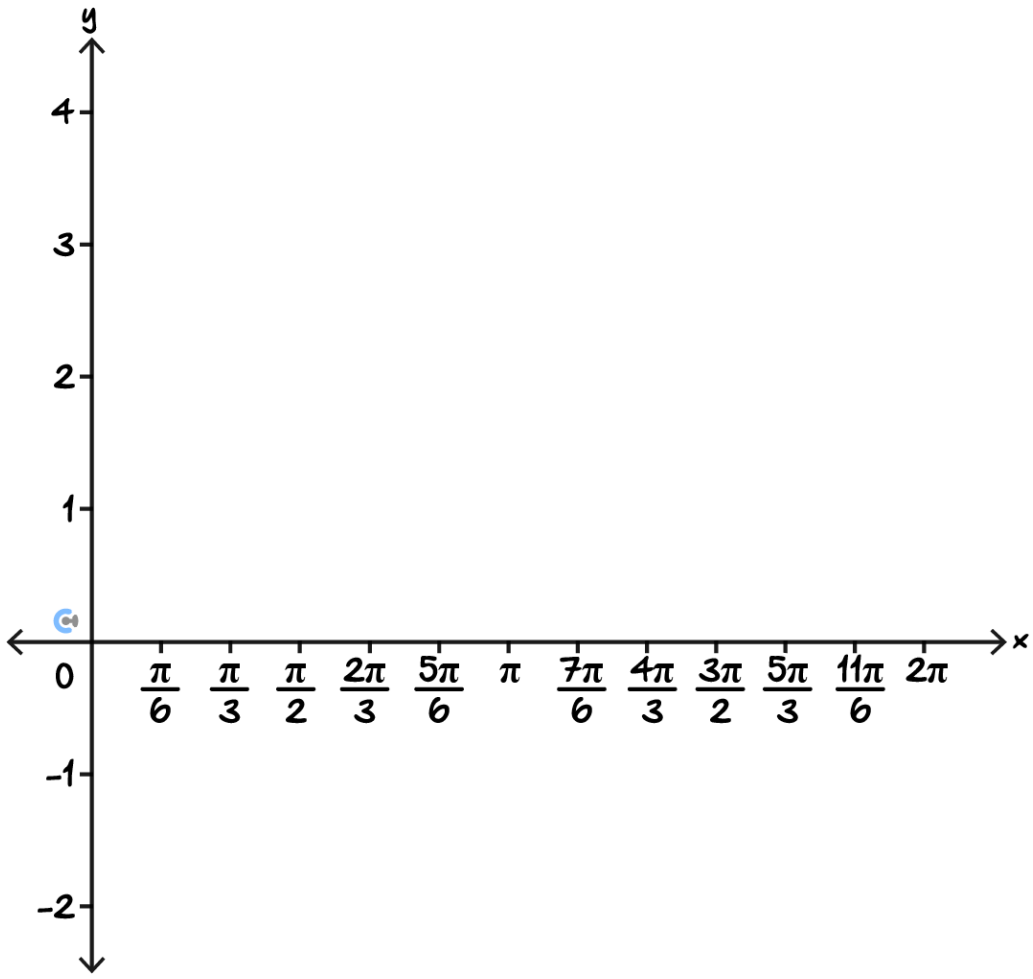
Question 8

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

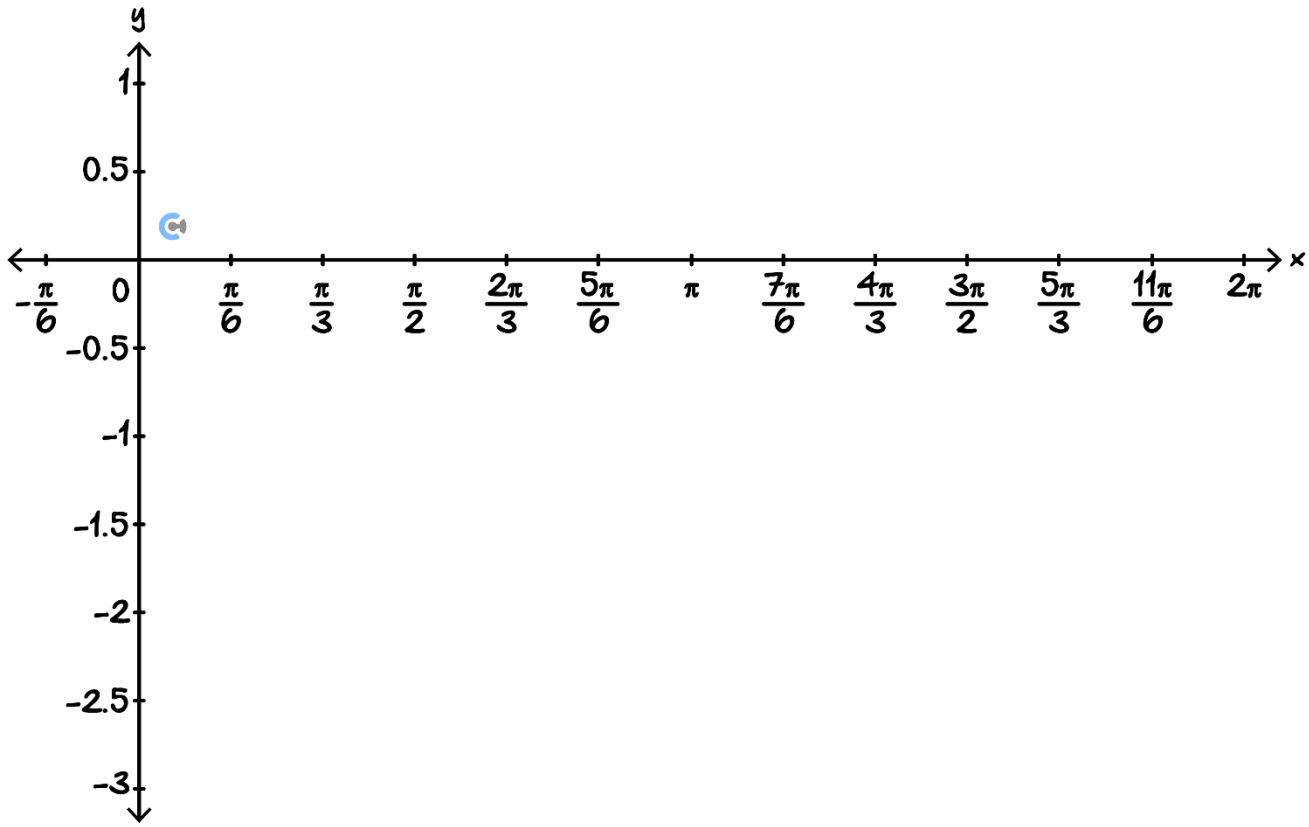
$$y = -2\sin\left(2\left(x - \frac{\pi}{3}\right)\right) + 1 \text{ for } x \in [0, 2\pi]$$



Question 9

Sketch the following on the axes below, labelling all intercepts, endpoints, and turning points with their coordinates.

$$y = 2\cos\left(2x + \frac{\pi}{3}\right) - 1 \text{ for } x \in [0, 2\pi]$$



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Sub-Section: Finding the Rule

Finding the Rule

$$\text{Amplitude } (A) = \frac{max - min}{2}$$

$$\text{Average } (k) = \frac{max + min}{2}$$

Question 10 Walkthrough.

A function with rule $y = A\sin(nt) + b$ where $A > 0$ has a range $[-5,3]$ and period 6. Find A, n and b.

TIP: Graphing helps!

Active Recall: Finding the Rule

$$\text{Amplitude } (A) = \_\_\_\_\_$$

$$\text{Average } (k) = \_\_\_\_\_$$

Your turn!

Question 11

A function with rule $y = A\cos(nt + \pi) + b$ where $A < 0$ has a range $[-6,8]$ and period 3. Find A, n and b.

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Section C: Graphs of Tangent

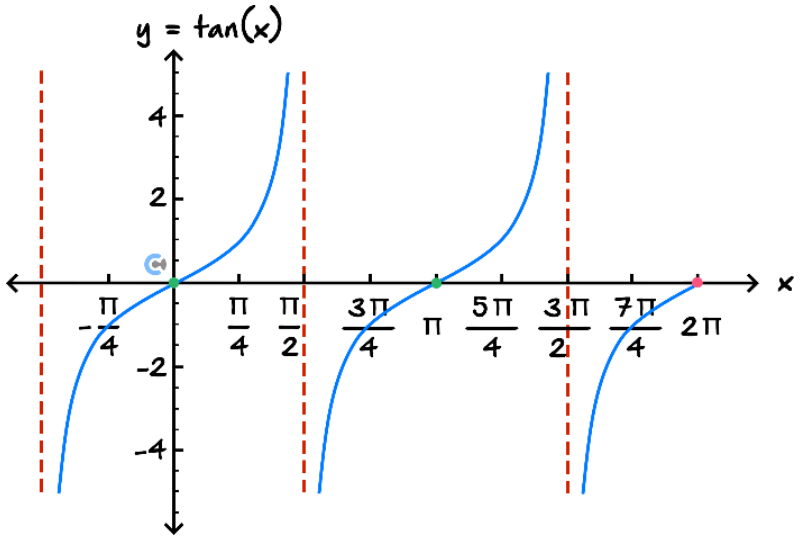
Sub-Section: Understanding Tangent Graphs

What does the tangent graph look like?

Exploration: Graph of Tangents

- Scan the QR code below on your device!

Graph of Tangent

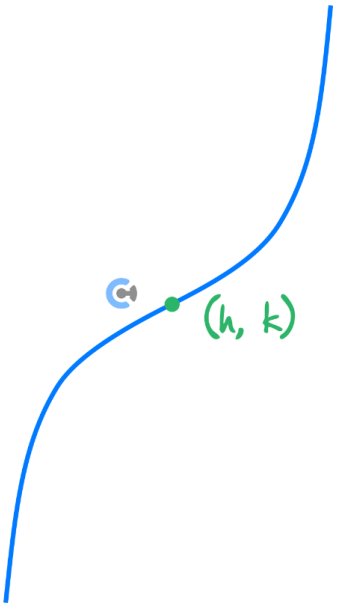


Sub-Section: Graphing Tangent Functions

Steps for Sketching tan Functions

1. Identify:
   - The period $= \frac{\pi}{n}$.
2. Find the vertical asymptotes by solving for angle $= \frac{\pi}{2}$.
3. Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
   - For instance, for $\tan\left(2x - \frac{\pi}{3}\right)$, solve $2x - \frac{\pi}{3} = \frac{\pi}{2}$ for x.
4. Plot the inflection point (h, k). (Midpoint of the two vertical asymptotes.)
   - x-value of inflection point $= x$-value which makes angle $= 0$.
   - y-value of inflection point = vertical translation of the function.

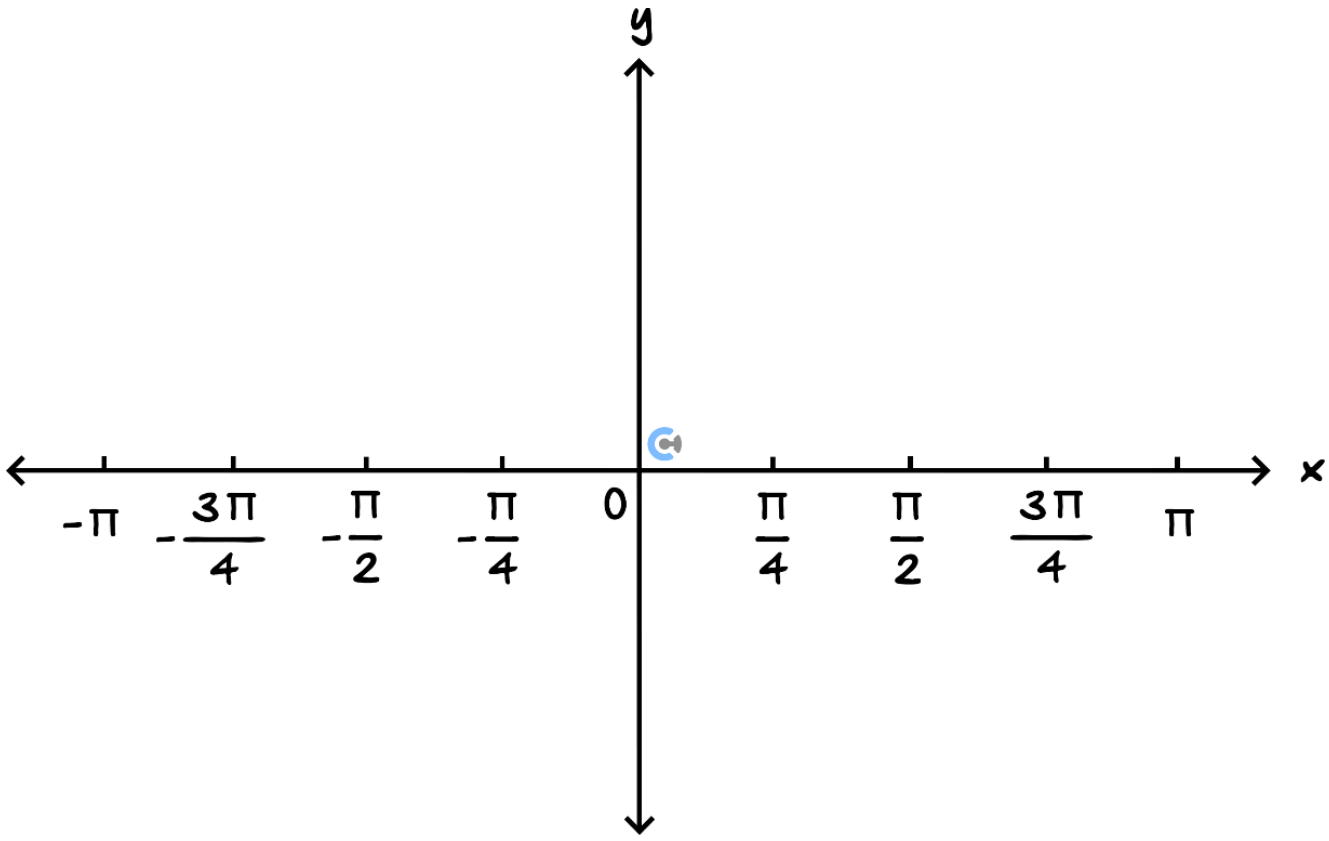
eg: $\tan(x-h)+k$



5. Find any x-intercepts.
6. Sketch a "cubic-like" shape.

Question 12 Walkthrough.

Sketch the graph of $y = 3\tan(2x)$ for $x \in [-\pi, \pi]$.



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Active Recall: Steps for Sketching $\tan$ Functions

1. Identify:
   - The period = _____.
2. Find the vertical asymptotes by solving for angle = _____.
3. Find other vertical asymptotes within the domain by adding the period to answer from the previous step.
4. Plot the inflection point (h, k). (Midpoint of the two _____.)
   - x-value of inflection point = x-value which makes angle = 0.
   - y-value of inflection point = vertical translation of the function.
5. Find any _____.
6. Sketch a _____ shape.

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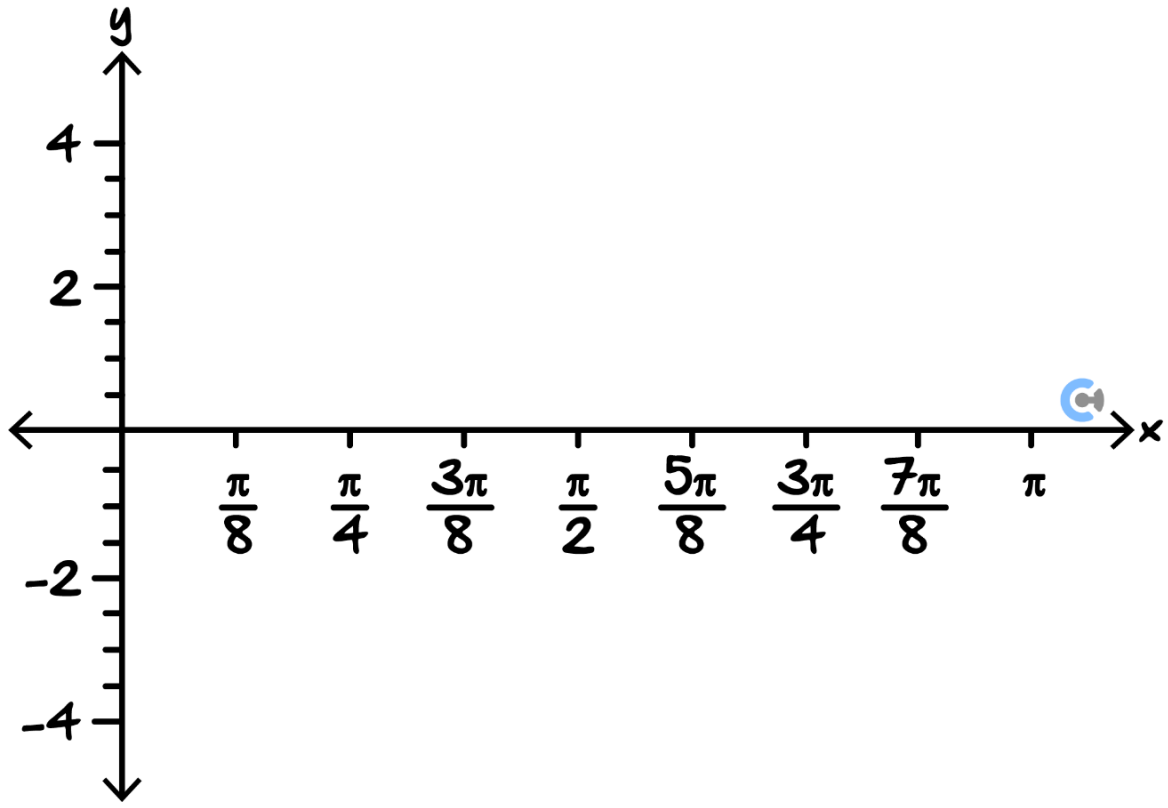
Your turn!

Question 13

Sketch the following on the axes below, labelling all intercepts and points of inflection with coordinates and all asymptotes with their equations.

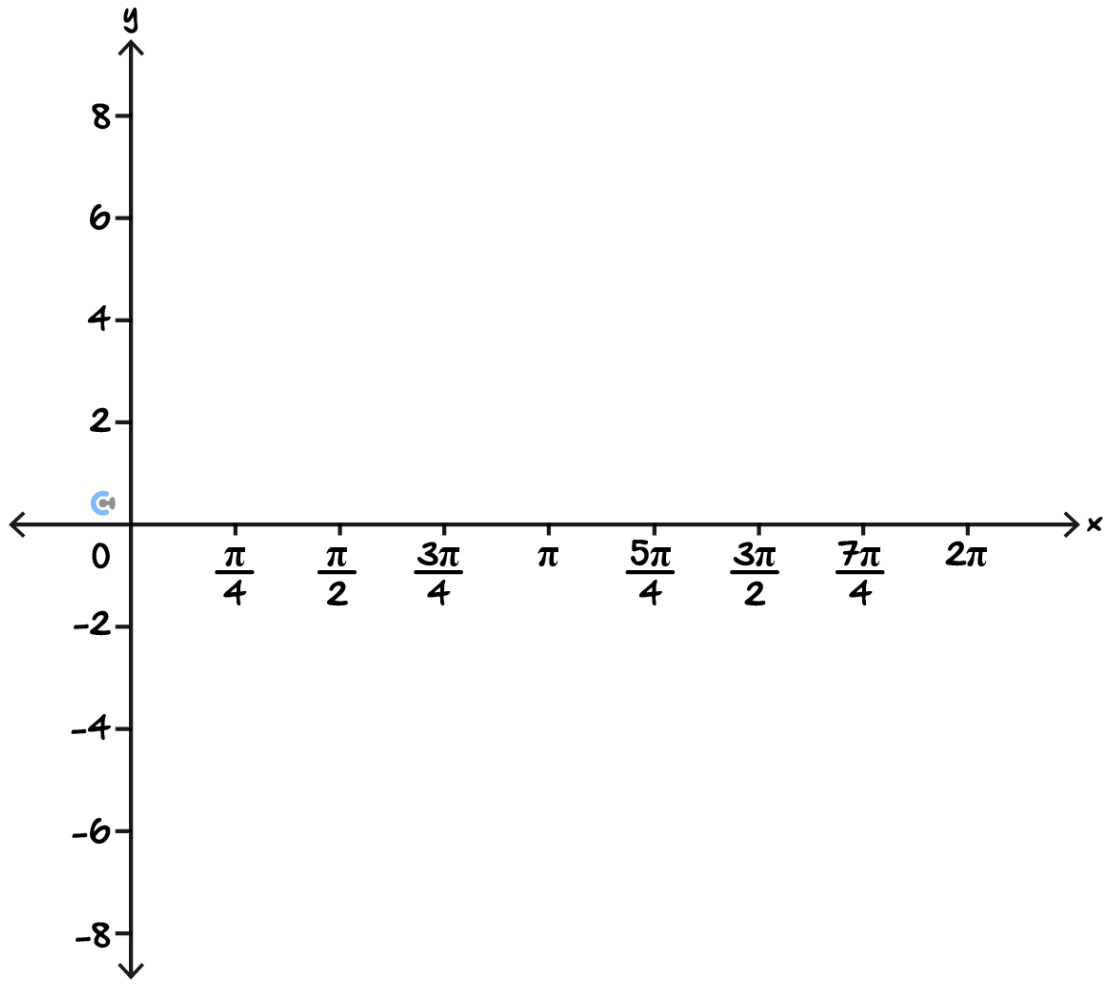
$$y = \tan\left(2x + \frac{\pi}{2}\right) + 1 \text{ for } x \in (0, \pi)$$



Question 14

Sketch the following on the axes below, labelling all intercepts, points of inflection, and endpoints with their coordinates, and all asymptotes with their equations.

$$f: [0,2\pi] \to \mathbb{R}, f(x) = -2\tan(\pi + 2x) + 2$$



Section D: Fraction of Period

Sub-Section: Fraction of Period

Definition: Fraction of Period

$$\textit{Fraction of Period} = \frac{\textit{Duration}}{\textit{Period}}$$

$$\%\ \textit{of Period} = \frac{\textit{Duration}}{\textit{Period}} \times 100\%$$

Question 15 Walkthrough.

The population of dogs in a certain household is modelled by $P(t)$.

$$P(t) = 4 - 2\cos\left(\frac{\pi}{4}t\right)$$

Where $P(t)$ is the number of dogs t years since 2024. Find the fraction of time where the population is above 5.

NOTE: Always sketch the function to find the duration!

Active Recall: Fraction of Period

$$\textbf{\textit{Fraction of Period}} = \_\_\_\_\_$$

$$\%\ \textbf{\textit{of Period}} = \_\_\_\_\_ \times 100\%$$

Question 16

The population of cats in a certain household is modelled by $P(t)$.

$$P(t) = 10 - 4\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right)$$

Where $P(t)$ is the number of cats t years since 2024.

Find the fraction of time where the population is above 8.

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