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VCE Mathematical Methods ½
Graphs of Circular Function [4.4]
Homework Solutions

Admin Info & Homework Outline:

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Questions You Need Help For	
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Section A: Basics

Question 1

Consider the function $f(x) = 3\sin(x)$.

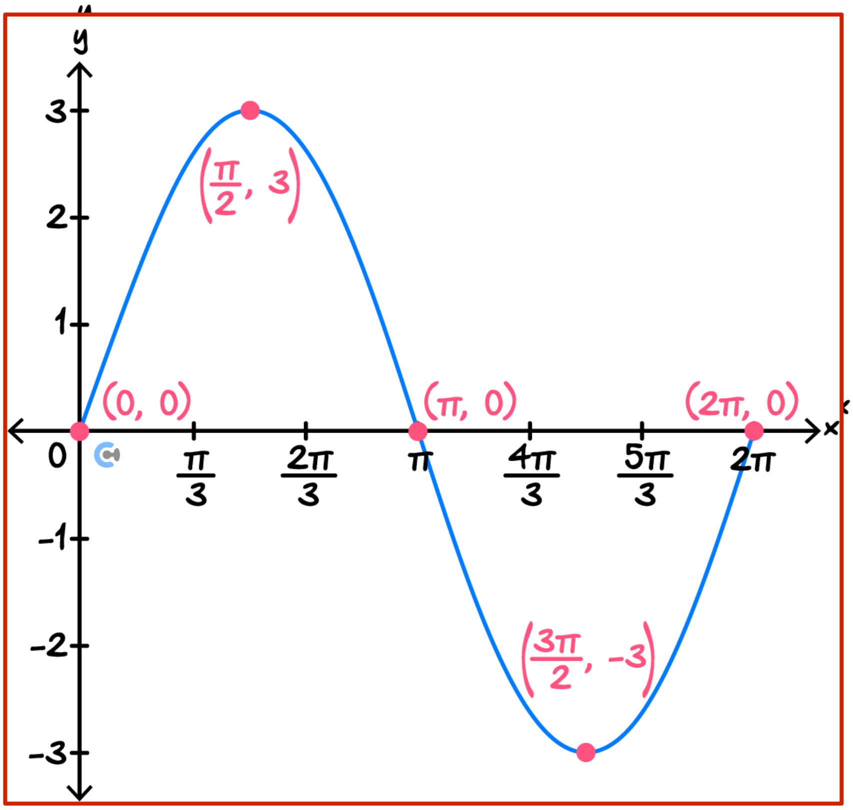
a. State the amplitude of f.

$\text{Amplitude} = |coefficient\ of\ \sin| = 3$

b. State the range of f.

$[-3,3]$

c. Sketch the graph of $y = f(x)$ for $x \in [0, 2\pi]$ on the axes below, labelling axes intercepts and turning points with their coordinates.



Question 2

Consider the function $f(x) = 2\cos(2x) - 1$.

a. State the amplitude of f.

2

b. State the period of f.

$\text{Period} = \frac{2\pi}{coefficient\ of\ x} = \frac{2\pi}{2} = \pi$

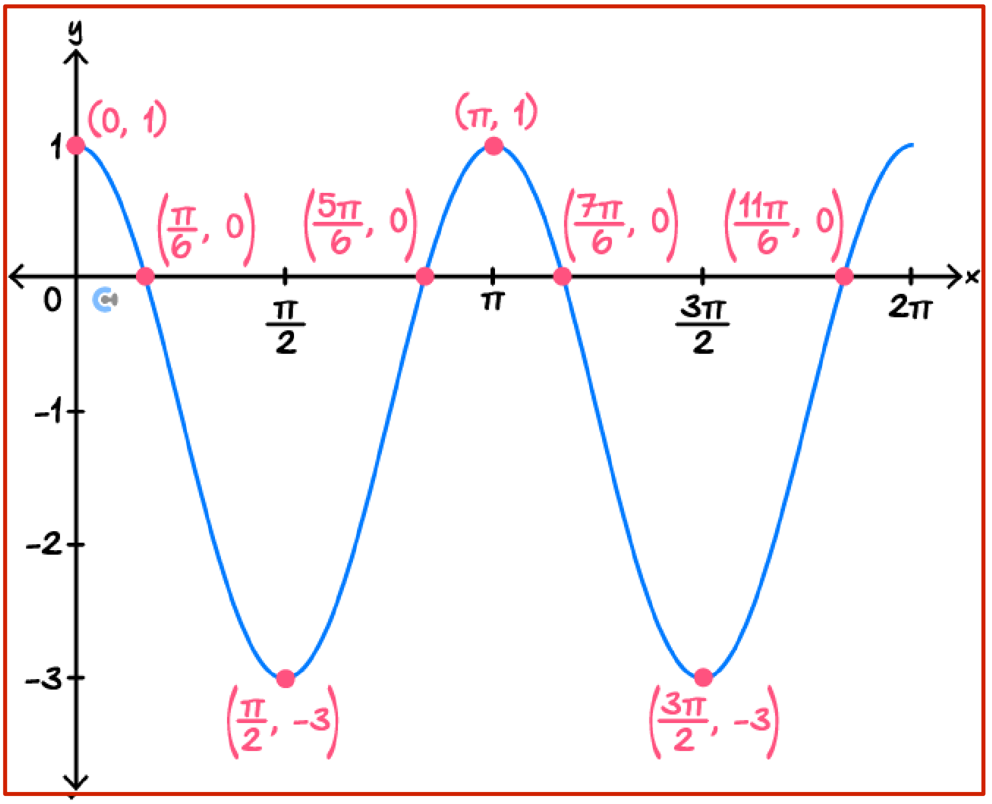
c. State the average value of f.

Average value $=$ *vertical translation* $= -1$

d. State the maximum value of f.

Maximum value $=$ *vertical translation* $+$ *amplitude* $= -1 + 2 = 1$

e. Sketch the graph of $y = f(x)$ for $x \in [0, 2\pi]$ on the axes below, labelling axes intercepts and turning points with their coordinates.



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Question 3

Consider the function $f(x) = 3\tan\left(\frac{x}{2}\right)$.

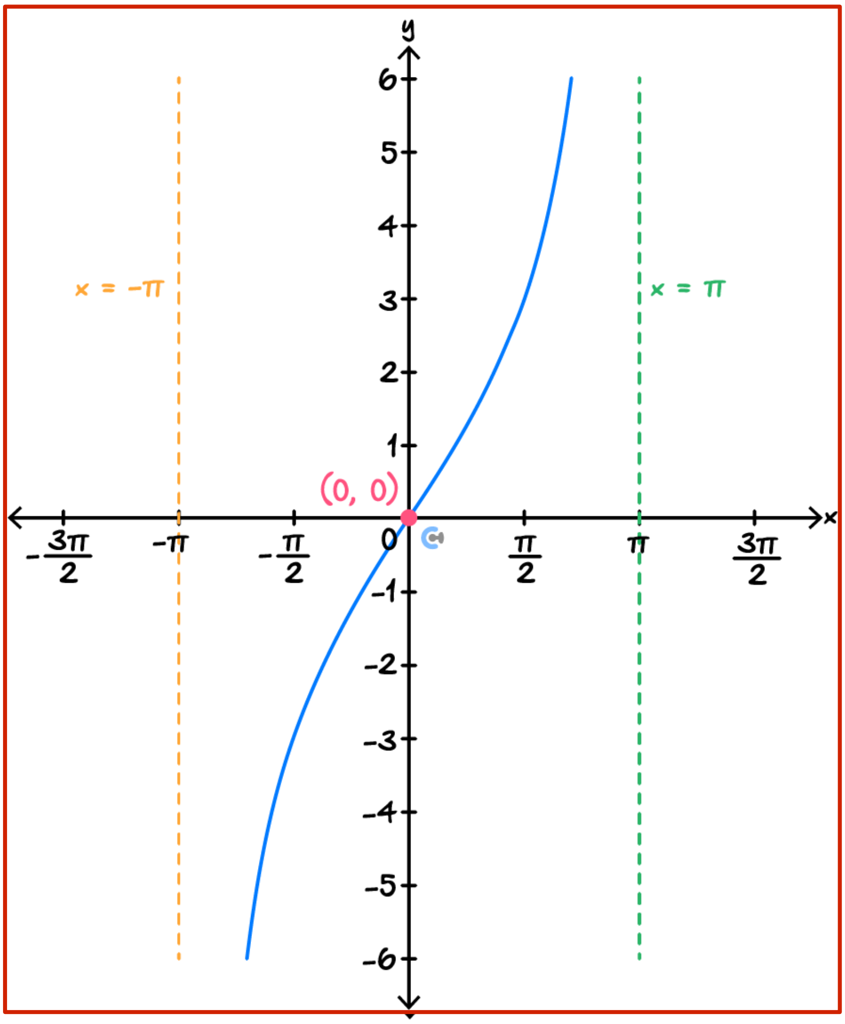
a. State the range of f.

$(-\infty, \infty)$ which can also be written as $\mathbb{R}$.

b. State the period of f.

$$\text{Period} = \frac{\pi}{coefficient\ of\ x} = \frac{\pi}{\frac{1}{2}} = 2\pi$$

c. Sketch the graph of $y = f(x)$ for $x \in (-\pi, \pi)$ on the axes below, labelling axis intercepts with their coordinates and asymptotes with their equations.



Question 4

Consider the function $f(x) = -4\sin(\pi x) + 3$.

a. State the amplitude of f.

4 – remember that amplitude is always positive.

b. State the period of f.

$\text{Period} = \frac{2\pi}{\pi} = 2$

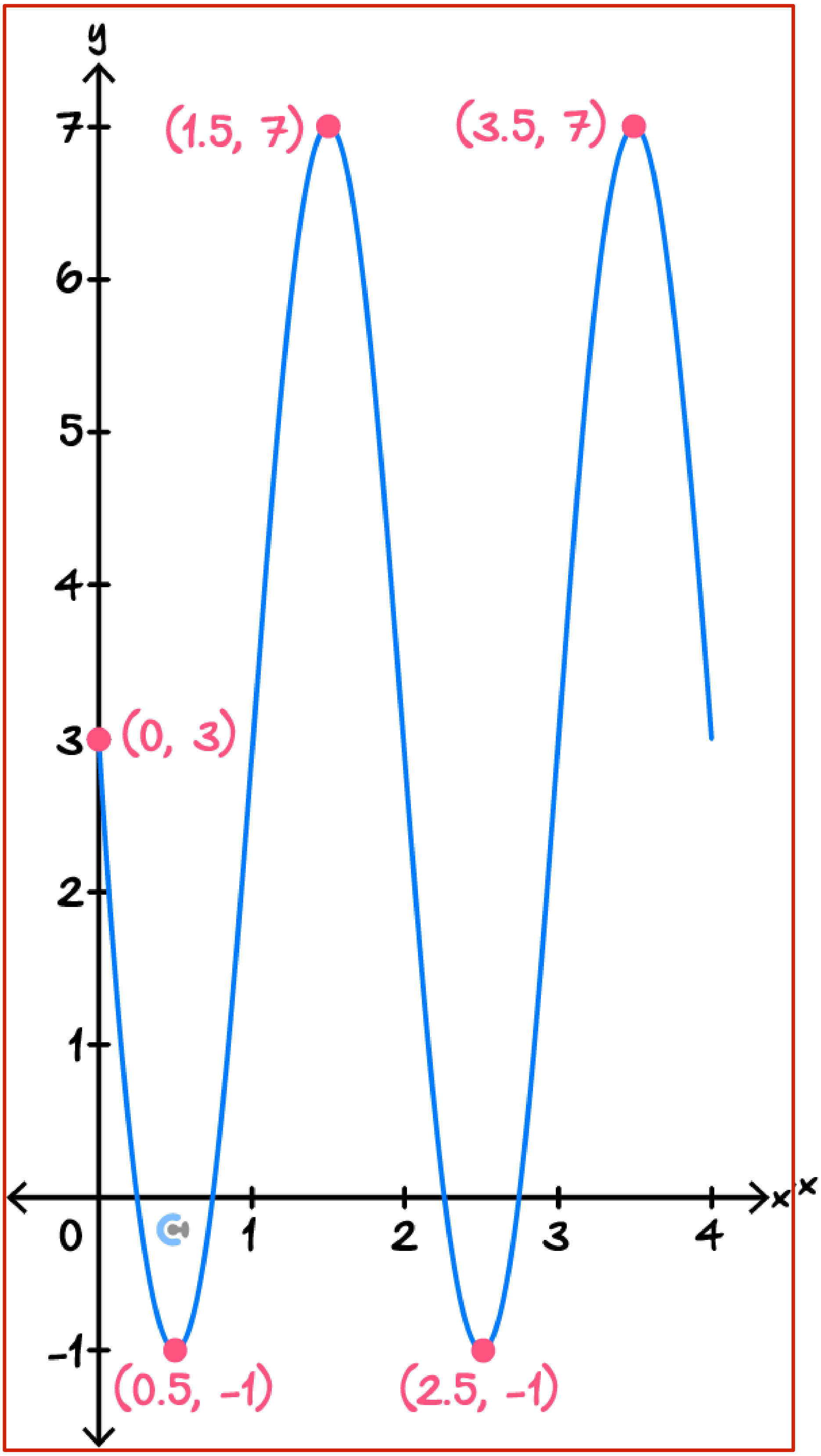
c. State the minimum value of f.

-1

d. State the average value of f.

3

e. Sketch the graph of $y = f(x)$ for $x \in [0,4]$ on the axes below, labelling y-intercepts and turning points with their coordinates.



Question 5

The function $f(x) = a\cos(bx) + c$, where $a, b \in \mathbb{R}^+$ and $c \in \mathbb{R}$, has the following properties:

- Maximum value $= 5$
- Minimum value $= -1$
- Period $= \pi$

Find a, b and c.

b:

Period $= \frac{2\pi}{b} = \pi$

$\therefore b = 2$

c:

average value $= \frac{Max + Min}{2} = c$

$\therefore c = \frac{5 + (-1)}{2} = 2$

a: a is +ve $\therefore$ amplitude $= a$

amplitude $= \frac{Max - Min}{2} = a$

$\therefore a = \frac{5 - (-1)}{2} = 3$

$\therefore a = 3,\ b = 2,\ c = 2$

Question 6

Which of the following is true for $f(x) = \sin(x)$?

A. Amplitude of 2 and period π.

B. $Range = [0,1]$.

C. $f(x)$ has no turning points.

D. Average value $= 0$.

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Section B: Problem Solving

Question 7

Consider the functions $f(x) = \sin(x)$ and $g(x) = 2\sin\left(x - \frac{\pi}{3}\right) + 1$.

a. Describe a series of transformations that map the function of $f(x)$ onto the function $g(x)$.

Dilation by a factor of 2 from the x-axis, followed by,
Translation of $\frac{\pi}{3}$ units to the right, followed by
Translation of 1 unit upwards.
NOTE: Translations can be in any order, but dilations must come before the translation up.

b. Hence, or otherwise, state the amplitude, period, range and average value of g.

Amplitude: 2, Period: 2π, Range: $[-1,3]$, Average value: 1

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Question 8

A circular function $f(x)$ has the following characteristics:

- Amplitude: 3
- Average value: 2
- Period: π
- Has a maximum occurring at $x = 0$

a. Write a possible rule for $f(x)$ in the form $f(x) = a\cos(bx) + c$, where $a, b \in \mathbb{R}\backslash\{0\}$ and $c \in \mathbb{R}$.

$$f(x) = 3\cos(2x) + 2$$

b. Convert the rule from **part a.** into an equivalent form in terms of sine.

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

Let $\theta = 2x$

$f(x) = 3\cos(\theta) + 2 = 3\sin\left(\frac{\pi}{2} - \theta\right) + 2 = 3\sin\left(\frac{\pi}{2} - 2x\right) + 2$

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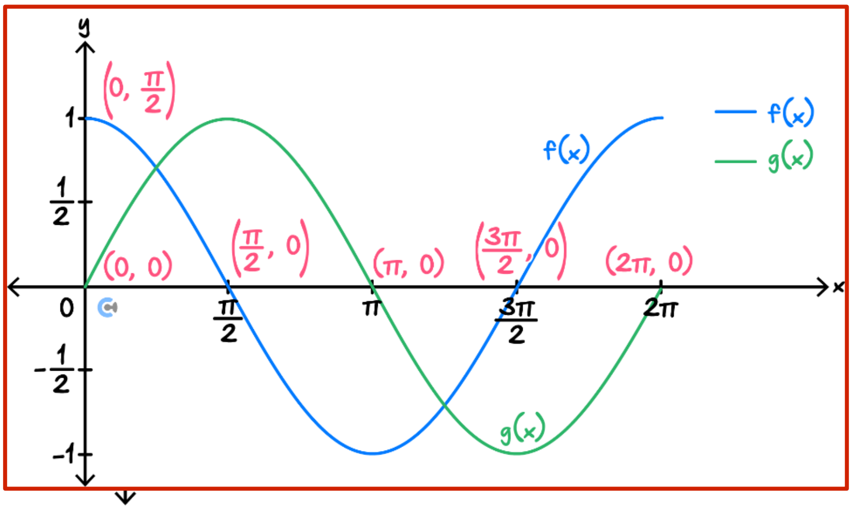
Question 9

Let $f(x) = \cos(x)$ and $g(x) = \sin(x)$.

a. Sketch the graphs of $f(x)$ and $g(x)$ for $x \in [0, 2\pi]$ on the axes below, labelling axes intercepts with their coordinates.



b. Hence, state a single transformation that maps the graph of $f(x)$ onto the graph of $g(x)$.

Translate $\frac{\pi}{2}$ units to the right.

c. Hence, state the equation of the relationship shown in **part b.**

$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$

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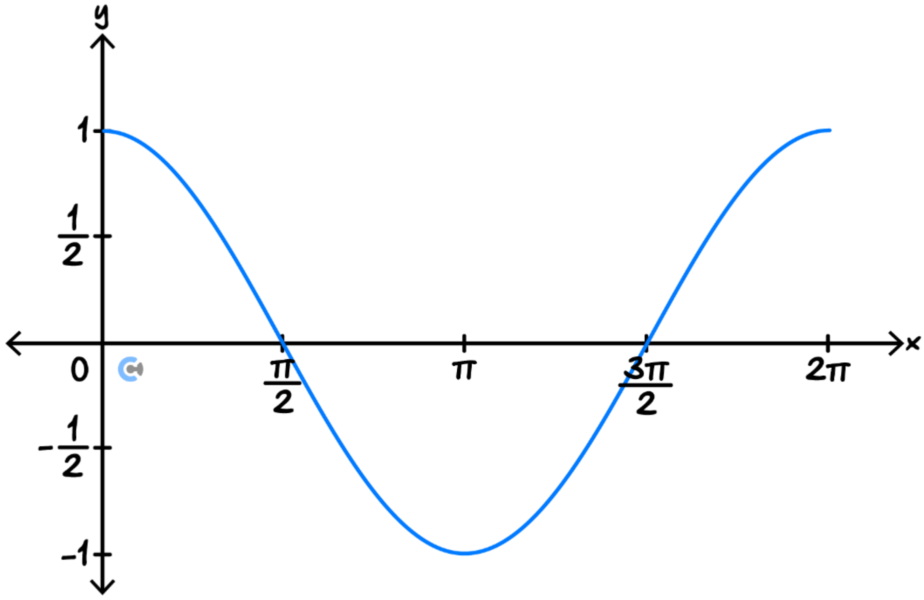
Question 10

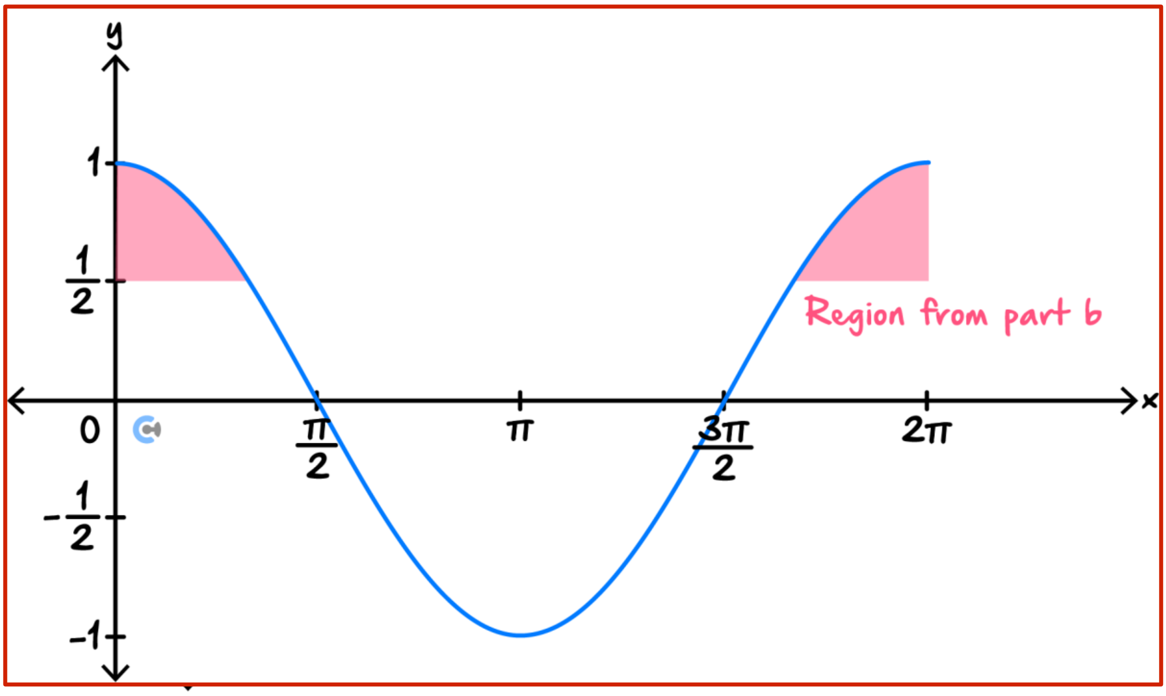
Let $f : [0,2\pi] \to \mathbb{R}, f(x) = \cos(x)$. The graph of $f(x)$ is shown below.



a. Solve the equation $f(x) = \frac{1}{2}$ for x.

$$\cos(x) = \frac{1}{2}, x \in [0,2\pi]$$
$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

b. Shade the part of the graph where $f(x) \geq \frac{1}{2}$ on the graph below.



c. Hence, find the values of x for which $f(x) \geq \frac{1}{2}$ and find the fraction of the period for which this occurs.

$$x \in \left[0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$$

$$\text{Fraction of period} = \frac{\text{size of intervals that meet condition}}{\text{period}}$$

$$= \frac{\left(\frac{\pi}{3} - 0\right) + \left(2\pi - \frac{5\pi}{3}\right)}{2\pi}$$

$= \frac{1}{3}$ of the period

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Question 11 Tech-Active.

A population of endangered butterflies on an island follows a predictable pattern and is modelled by the function:

$$b: [0, \infty) \to \mathbb{R}, b(t) = 0.8 \sin\left(\frac{\pi t}{3}\right) + 1.2$$

where b is the population of the butterflies measured in thousands and t is the number of months since the butterflies were brought to the island.

a. Find the maximum and minimum population of butterflies on the island.

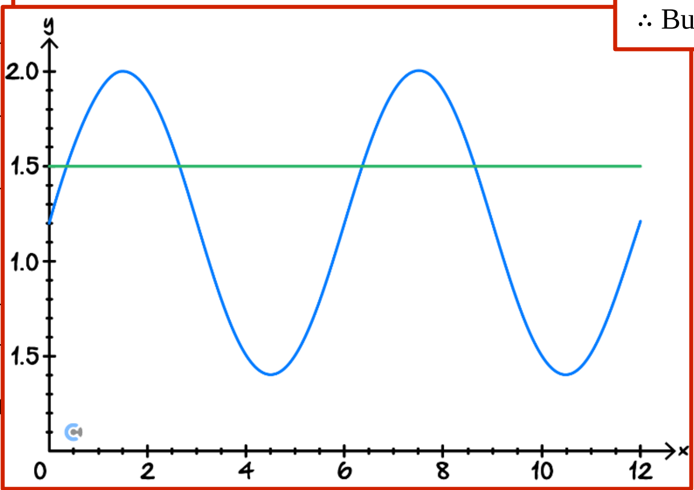
Maximum = $(1.2 + 0.8) \times 1000 = 2000$ butterflies
Minimum = $(1.2 - 0.8) \times 1000 = 400$ butterflies
NOTE: Units are necessary.

b. Find the total amount of time in the first year where the butterfly population is above 1500. Give your answer in months correct to 2 decimal places.

```
b[t_] := 0.8 Sin[π t / 3] + 1.2

(*THE FIRST YEAR MEANS THE FIRST 12 MONTHS*)

Plot[{b[t], 1.5}, {t, 0, 12}, PlotRange → {0, 2}]
```



∴ Butterfly population above 1500 for 4.53 months.

```
Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a correspo
0.367072 < t < 2.63293 || 6.36707 < t < 8.63293

(2.6329281193826306` - 0.36707188061736945`) + (8.632928119382631` - 6.367071880617369`)
4.53171
```

Spa

Question 12 Tech-Active.

The height of a Ferris wheel carriage during one ride is modelled by the function:

$$h: [0,10] \to \mathbb{R}, h(t) = -12\cos\left(\frac{\pi t}{5}\right) + 17$$

where h is the height of the carriage in metres above the ground and t is the time in minutes from when the carriage departs the boarding platform.

a. Find the height of the boarding platform above the ground.

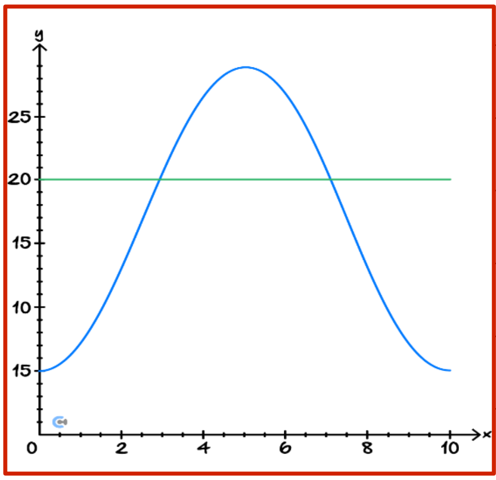
$17 - 12 = 5\ m$ – units necessary

b. Find the height of the Ferris wheel, given that the bottom of the Ferris wheel lies on the ground.

$17 + 12 = 29\ m$ – units necessary

It is possible to get an aesthetic view of the nearby harbourfront if you are more than 20 metres above the ground on the Ferris wheel.

c. Find the percentage of total ride time during a ride where riders have an aesthetic view of the harbourfront. Give your answer correct to 2 decimal places.




```
FunctionPeriod[h[t], t]
10
Reduce[h[t] > 20 && 0 ≤ t ≤ 10, t] // N
2.90215 < t < 7.09785
(7.097846883724168` - 2.902153116275832`)/10 * 100
41.9569
```

$Percentage = Fraction\ of\ period \times 100\% = 41.96\%$

Section C: Exam 1 (18 Marks)

Question 13 (4 marks)

Consider the function $f(x) = 2\cos(x) + 1$.

a. State the amplitude, and average value of the function. (1 mark)

Amplitude $= 2$, Average value $= 1$
1A – both correct answers

b. Find the maximum and minimum values of f. (2 marks)

Minimum $= 1 - 2 = -1$ (1A)
Maximum $= 1 + 2 = 3$ (1A)

c. Hence, state the range of f. (1 mark)

$[-1,3]$ (1A)

Question 14 (5 marks)

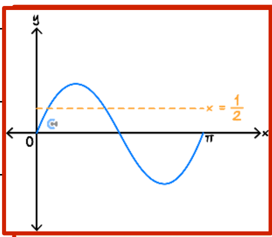
Let $f(x) = \sin(2x)$.

a. Find the values of x where $f(x) = \frac{1}{2}$ in the interval $x \in [0, \pi]$. (2 marks)

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

1M – reference angles, 1A – correct answers

b. Find the maximum and minimum values of f. (3 marks)



$\therefore f(x) > \frac{1}{2}$ for $x \in \left(\frac{\pi}{12}, \frac{5\pi}{12}\right)$ from part a. 1M

fraction of period $= \dfrac{\frac{5\pi}{12} - \frac{\pi}{12}}{\pi}$ 1M

$= \frac{1}{3}$ 1A

Question 15 (2 marks)

The function $f(x) = \tan(kx)$, where $k \in \mathbb{R}$, completes one full period over the interval $(0, 4\pi)$.

Find the value of k.

period $= 4\pi - 0 = \dfrac{\pi}{k}$ 1M

$\therefore 4\pi = \dfrac{\pi}{k}$ $\therefore k = \dfrac{1}{4}$ 1A

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Question 16 (4 marks)

The graph of a circular function $f(x)$ has the following features:

- Maximum value: 1
- Minimum value: -5
- Period: 3
- Minimum at $x = 0$

Find a rule for $f(x)$. Express your answer in the form $f(x) = a\cos(bx) + c$, where $a, b \in \mathbb{R}\backslash\{0\}$ and $c \in \mathbb{R}$.

average value $= \frac{\text{Max} + \text{Min}}{2} = c$

$\therefore c = \frac{1 + (-5)}{2} = -2$ 1M

Minimum value at $x = 0$ $\therefore f(0) = -5$:

$f(0) = a\cos(0) + c = a + c = -5$, $c = -2$

$\therefore a - 2 = -5$ $\therefore a = -3$ 1M

period $= \frac{2\pi}{b} = 3$ $\therefore b = \frac{2\pi}{3}$ 1M

$\therefore a = -3$, $b = \frac{2\pi}{3}$, $c = -2$

$\therefore f(x) = -3\cos\left(\frac{2\pi}{3}x\right) - 2$ 1A

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Question 17 (3 marks)

The function $T(t) = 2\sin\left(\frac{\pi t}{6}\right) + 4$ models the temperature of a fridge in °C over time, measured in minutes.

a. Find the period of T in minutes. (1 mark)

$Period = \frac{2\pi}{\frac{\pi}{6}} = 12$ minutes – 1A (units not required)

b. Find the maximum temperature reached in the fridge in °C. (1 mark)

$Max\ temp = 4 + 2 = 6$°C – 1A (units not required)

c. Find the percentage of time that the fridge spends above its average temperature. (1 mark)

By definition of an average, 50% of values will be above average and 50% will be below average.
50% - 1A

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Section D: Exam 2 (24 Marks)

Question 18 (1 mark)

The function $f(x) = 23\sin(2x) - \pi$ has a period of:

A. $-\pi$

B. π

C. 23

D. $\frac{\pi}{2}$

Question 19 (1 mark)

The graph of a circular function $f(x)$ has the following features:

- Minimum value: 2
- Maximum value: 10
- Period: 1

A possible rule for $f(x)$ is:

A. $4\sin(x) + 6$

B. $4\cos\left(\frac{x}{2\pi}\right) + 6$

C. $4\sin(\pi x) + 6$

D. $4\cos(2\pi x) + 6$

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Question 20 (1 mark)

Let $f(x) = 6\sin\left(\frac{x}{2}\right) - 3$. The fraction of the period where $f(x) < -1$ is closest to:

A. 0.608

B. 0.764

C. 0.108

D. 0.392

Question 21 (1 mark)

The function $f(x) = a\sin(x) + b$ is greater than 2 for exactly π units per period. Which of the following are possible values of a and b?

A. $a = 2, b = 0$

B. $a = 2, b = 2$

C. $a = 3, b = 1$

D. $a = 1, b = 1$

Question 22 (9 marks)

Sam is bouncing on a trampoline at Bounce and Subu is trying to model his height above the ground. Subu models Sam's height above the ground using the function:

$$h(t) = \frac{5}{4}\sin\left(\frac{\pi}{2}(t + k)\right) + 1$$

where h is Sam's height above the ground measured in metres, t is the time in seconds since Sam started bouncing and $k \in \mathbb{R}$.

a. State the maximum height reached by Sam in metres. (1 mark)

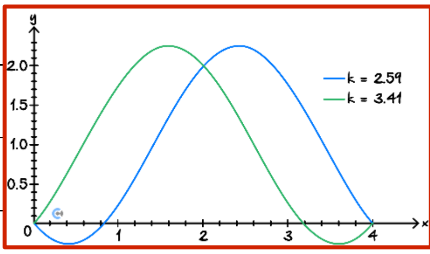
$1 + \frac{5}{4} = \frac{9}{4}\ m$ – 1A (correct answer, units not required)

b. State Sam's average height above the ground when he is bouncing. (1 mark)

1 m – 1A (correct answer, units needed)

c. Given that Sam starts on ground level and has to press down on the trampoline to start his bounce, meaning that the graph of $y = h(t)$ must fall below the t-axis immediately after $t = 0$, find a possible value of k if $0 < k < 4$. Give your answer correct to 2 decimal places. (3 marks)

```
h[t_] := 5/4 Sin[π/2 (t + k)] + 1

Solve[{h[0] == 0, 0 ≤ k ≤ 4}, k] // N

{{k → 3.40967}, {k → 2.59033}}
```



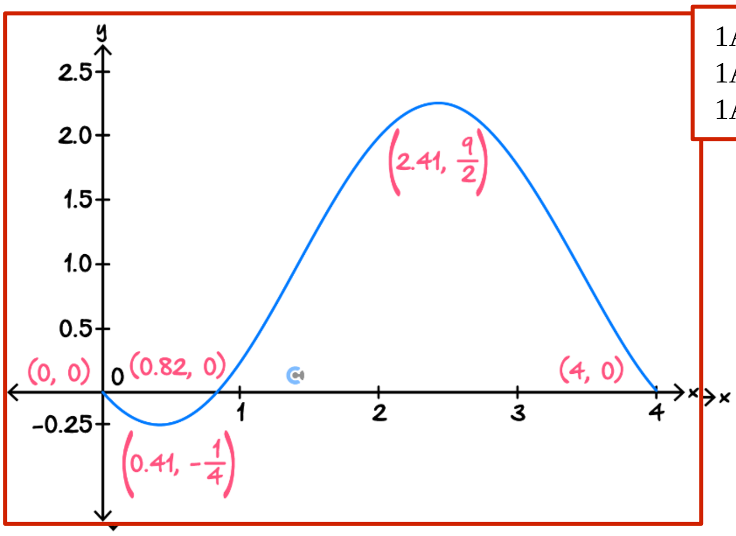
Solving for k gives 2 potential values, but Sam needs to press down on the trampoline, meaning that h must fall below 0 immediately after $t = 0$. Plotting $h(t)$ for both values of k shows that only $k \approx 2.59$ satisfies this second condition as shown below.

1M – Statement saying "solve for k when $h(0) = 0$."
1M – Both potential values of k.
1A – Rejection of $k = 3.41$ to end up with $k = 2.59$.

d. Hence, state an expression that gives all possible values of k. Give non-exact values correct to 2 decimal places. (1 mark)

$k = 2.59 + 4n, n \in \mathbb{Z}$ – 1A (general solution)

e. Sketch one of Sam's bounces on the axes below, labelling axes intercepts and turning points with their coordinates. Give all non-exact values to 2 decimal places. (3 marks)



1A – Shape
1A – Turning points
1A – Intercepts

Question 23 (11 marks)

The water level on a beach rises and falls according to the function:

$$w(t) = 3\sin\left(\frac{\pi t}{6}\right) + 4$$

where w is the water level in metres and t is the time in hours after midnight.

The ocean conceals the entrance to a secret underwater cave which contains the secret to getting a raw 50 in Methods. The entrance to the cave is only accessible when the water level is below 1.5 m.

a. State the amplitude, period and average tide height. (2 marks)

Amplitude: 3 m, Period: 12 hours, Average height: 4 m
1A – amplitude and average
1A – period

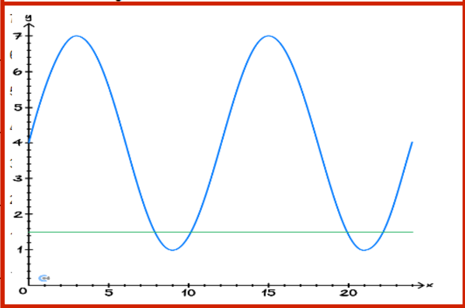
b. Find the times in the day when the water level is exactly 1.5 m, correct to the nearest minute. (3 marks)

```
Solve[3 Sin[π t / 6] + 4 == 1.5 && 0 ≤ t ≤ 24, t]

Solve: Solve was unable to solve the system with inexact coefficients. The answe

{{t → 7.88142}, {t → 10.1186}, {t → 19.8814}, {t → 22.1186}}

{0.881423007935976`, 0.118576992064025`} * 60

{52.8854, 7.11462}
```

Water level is 1.5 m at 7: 53 AM, 10: 07 AM, 7: 53 PM, 10: 07 PM
1M – Statement saying "solve for t when $h(t) = 1.5$"
1M – 4 solutions of t in decimals
1A – converting to hours and minutes

c. Hence, determine the total amount of time in a day that the cave is accessible, correct to the nearest minute. (2 marks)

```
Plot[{3 Sin[πt/6] + 4, 1.5}, {t, 0, 24}, PlotRange → {0, 7}]
```




```
2 * (10.118576992064025` - 7.881423007935976`)
4.47431

0.474307968256097` * 60
28.4585
```

The cave is accessible for 4 hours and 28 minutes per day.
1M – Correct interpretation of accessible time (correct expression)
1A – Correct answer in hours and minutes

d. An optimal time to enter the cave is when the water level hits 1.5 m and continues falling.

i. State the first optimal time in a day, correct to the nearest minute. (1 mark)

7: 53 AM– 1A (found in **part b.**)

ii. A daring student knows that they need 130 minutes to be able to retrieve the secret to getting a 50 from the cave. Given that they enter at an optimal time, determine whether they have enough time, and state how much time they have to spare/how much more time they would require, correct to the nearest second. (3 marks)

```
(*time window for 1 opening*)
(10.118576992064025` - 7.881423007935976`)
2.23715

(*130 minutes = 13/6 hours*)
2.2371539841280486` - 13/6
0.0704873

(*converting hours to minutes*)
0.07048731746138204` * 60
4.22924

(*converting decimals to seconds*)
0.22923904768292 2` * 60
13.7543
```

There is enough time with 4 minutes and 14 seconds to spare.
1M – Finding the time window for 1 opening
1M – Finding the amount of time leftover
1A – Final statement including the time in minutes and seconds

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