Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Circular Function Exam Skills [4.3]
Workbook

Outline:

Section A: Recap of Particular and General Solutions

If you were here last week, skip to Section B - Warmup Test.

Definition

Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add and subtract the period to find all other solutions in the domain.

Question 1 Walkthrough.

Solve the following equations for x over the domain specified.

$$2\sin(2x) + \sqrt{3} = 0 \text{ for } x \in [0, 2\pi]$$

Question 2

Solve the following equations for x over the domains specified.

a. $\sin(4x) = -1$ for $x \in [-\pi, \pi]$.

b. $2\cos\left(2x - \frac{\pi}{2}\right) + 1 = 0$ for $x \in [0, 2\pi]$.

Question 3

Solve the following equations for x over the domains specified.

$$\sqrt{3}\tan\left(x-\frac{\pi}{3}\right)-1=0 \text{ for } x\in(0,3\pi)$$

Definition

General Solutions

- Finding infinite solutions to a trigonometric equation.
- **Steps**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add $\text{Period}\cdot n$ where $n\in Z$.

Question 4 Walkthrough.

Find the general solutions to the following equations:

$$2\sin\left(2x + \frac{\pi}{2}\right) - 1 = 0$$

Space for Personal Notes

Question 5

Find the general solutions to the following equations:

a. $-2\sin\left(3x+\frac{\pi}{4}\right)=\sqrt{2}$

b. $2\cos\left(2x+\frac{\pi}{6}\right)=1$

Question 6 Walkthrough.

Find the general solutions to the following equations:

$$\tan\left(\frac{1}{2}x - \pi\right) - \frac{1}{\sqrt{3}} = 0$$

NOTE: We only need to find one angle for tangents!

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Question 7

Find the general solutions to the following equations:

a. $2\tan\left(2x-\frac{\pi}{4}\right)=2$

b. $\sqrt{3}\tan\left(3x-\frac{\pi}{6}\right)=1$

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Section B: Warmup Test (12 Marks)

Question 8 (6 marks)

Solve the following equations for x, over the stated domain.

a. $\tan\left(2x - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$, for $x \in (0, 2\pi]$ (3 marks)

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$\therefore 2x - \frac{\pi}{2} = \frac{\pi}{6}$

$2x = \frac{2\pi}{3}$

$\therefore x = \frac{\pi}{3}$

$\therefore P = \frac{\pi}{2}$

$D \in [0, 2\pi]$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{6}, \frac{8\pi}{6}, \frac{11\pi}{6}$

b. $2\sin\left(2x + \frac{\pi}{4}\right) + 1 = 0$, for $x \in [0, 2\pi]$. (3 marks)

$\sin\left(2x + \frac{\pi}{4}\right) = \frac{-1}{2} \Rightarrow Q3, Q4$

$\pi + \theta, -\theta$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

θ

$\therefore 2x + \frac{\pi}{4} = \frac{7\pi}{6}, \frac{-\pi}{6}$

$2x = \frac{11\pi}{12}, \frac{-5\pi}{12}$

$\therefore x = \frac{11\pi}{24}, \frac{-5\pi}{24}$

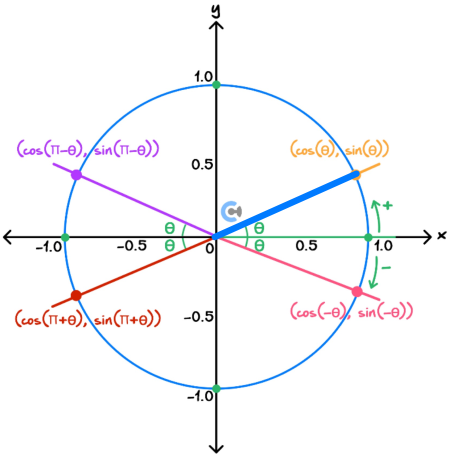
$\therefore P = \pi$

$D \in [0, 2\pi]$

$\therefore x = \frac{11\pi}{24}, \frac{35\pi}{24}, \cancel{\frac{-5\pi}{24}}, \frac{19\pi}{24}, \frac{43\pi}{24}$

Question 9 (6 marks)

Solve the following equations for x:



a. $2\cos\left(2x+\frac{\pi}{3}\right)+1=0$. (3 marks)

$\cos\left(2x+\frac{\pi}{3}\right)=-\frac{1}{2}$

$\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$ R.A

θ

$2x+\frac{\pi}{3}=\frac{2\pi}{3},\frac{4\pi}{3}$

$2x=\frac{\pi}{3},\pi$

$\therefore x=\frac{\pi}{6},\frac{\pi}{2}$

$\therefore P=\pi$

$\therefore x=\frac{\pi}{6}+\pi n, n\in Z$

or

$x=\frac{\pi}{2}+\pi n, n\in Z$

b. $\sqrt{3}\tan\left(4x+\frac{\pi}{6}\right)-3=0$. (3 marks)

$\tan\left(4x+\frac{\pi}{6}\right)=\frac{3}{\sqrt{3}}=\sqrt{3}$

$\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$

$\therefore 4x+\frac{\pi}{6}=\frac{\pi}{3}$

$4x=\frac{\pi}{6}$

$\therefore x=\frac{\pi}{24}$

$P=\frac{\pi}{4}$

$\therefore x=\frac{\pi}{24}+\frac{\pi}{4}n, n\in Z$

Section C: Circular Functions Exam Skills

Sub-Section: Equivalent General Solutions

Let's review some important skills from last week!

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Question 10

Which one of the following is **not** the same as the rest?

A. $\frac{5\pi}{6} + \frac{\pi}{6}n, n \in Z$ $\quad \frac{10\pi}{12}$

B. $\frac{3\pi}{2} + \frac{\pi}{6}n, n \in Z$ $\quad \frac{18\pi}{12}$

C. $-\frac{\pi}{2} + \frac{\pi}{6}n, n \in Z$ $\quad -\frac{6\pi}{12}$

D. $\frac{5\pi}{4} + \frac{\pi}{6}n, n \in Z$ $\quad \frac{15\pi}{12}$

E. $\frac{5\pi}{6} - \frac{\pi}{6}n, n \in Z$ $\quad \frac{10\pi}{12}$

$p = \frac{2\pi}{12}$

$\frac{8\pi}{12} = 4p$

$-\frac{24\pi}{12} = -12p$

$\frac{21\pi}{12} = 10 \cdot 5p$

NOTE: Very important for multiple choice questions in VCAA exams!

Space for Personal Notes

Sub-Section: General Solutions with Domain Restrictions

Misconception

"When there is a domain restriction, we always get particular solutions."

TRUTH: If the domain restriction has either ∞ or $-\infty$, we can still have general solutions.

Question 11 Walkthrough.

Solve for the following trigonometric equation:

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } x \geq 0$$

Q1 Q2

$2x + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$

$2x = -\frac{\pi}{6}, \frac{\pi}{2}$

$\therefore x = -\frac{\pi}{12}, \frac{\pi}{4}$

$\therefore P = \pi$

$x \geq 0$:

$n = 1$ ✓ $\}\ Z^+$

$n = 0$ ✗

$n = -1$ ✗

$$\therefore \boxed{\begin{array}{c} x = -\frac{\pi}{12} + \pi n, n \in Z^+ \\ \text{OR} \\ x = \frac{\pi}{4} + \pi n, n \in Z^+ \cup \{0\} \end{array}}$$

$n = 1$ ✓ $\}\ Z^+$

$n = 0$ ✓

$n = -1$ ✗

General Solution with Domain Restriction

$$\text{E.G trig}\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\text{ for } x \geq 0$$

- We can have infinite solutions for the restricted domain.
- The value of n is also restricted.

Your turn!

Question 12

Solve for the following trigonometric equation:

$$\cos\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}\text{ for } x<0$$

Q1 Q4

$2x+\frac{\pi}{4}=\frac{\pi}{6},-\frac{\pi}{6}$

$2x=-\frac{\pi}{12},-\frac{5\pi}{12}$

$\therefore x=-\frac{\pi}{24},-\frac{5\pi}{24}$ $\quad \therefore P=\pi$

$x<0$:

$n=1$ ✗
$n=0$ ✓
$n=-1$ ✓
$\vdots$

$\therefore x=\frac{-\pi}{24}+\pi n,\ n\in Z^{-}\cup\{0\}$

OR

$x=\frac{-5\pi}{24}+\pi n,\ n\in Z^{-}\cup\{0\}$

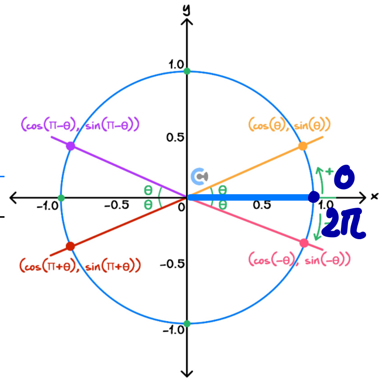
Sub-Section: Hidden Quadratics

Let's have a look at hidden quadratics for circular functions!

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

$$\text{Let } A = f(x)$$



Question 13 Walkthrough.

Solve the following for the values of x:

$$\cos^2\left(x - \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right) = 2, 0 \le x \le 3\pi$$

let $a = \cos\left(x - \frac{\pi}{3}\right)$:

$a^2 + a = 2$

$a^2 + a - 2 = 0$

$(a+2)(a-1) = 0$

$\therefore a = -2$ OR $a = 1$

Reject as $a \in [-1, 1]$

$\therefore \cos\left(x - \frac{\pi}{3}\right) = 1$

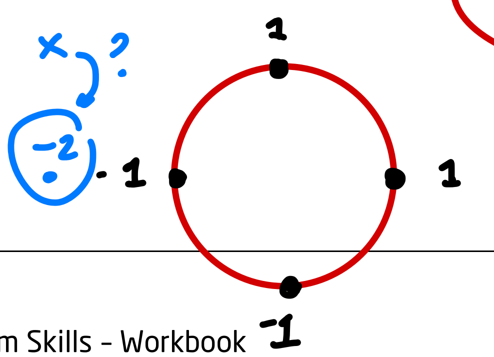
$\therefore x - \frac{\pi}{3} = 0$

$\therefore x = \frac{\pi}{3}$

$\therefore P = 2\pi = \frac{6\pi}{3}$

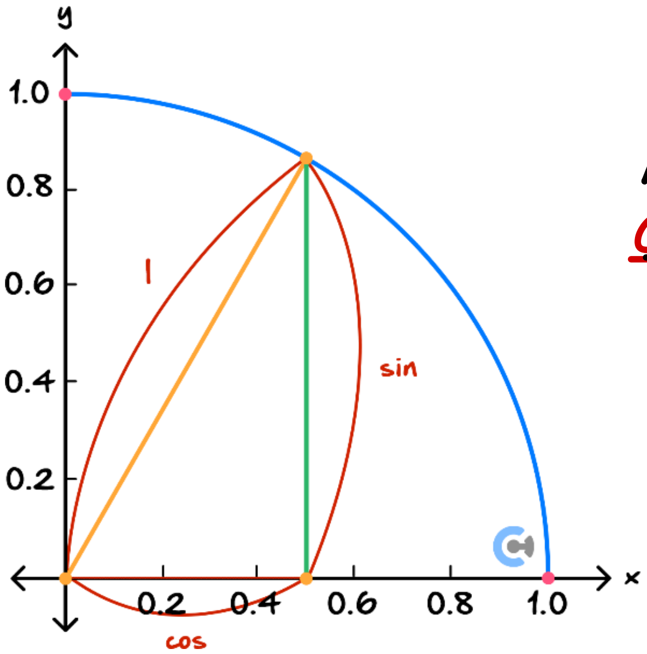
DG $\left[0, \frac{9\pi}{3}\right]$

$\therefore x = \frac{\pi}{3}, \frac{7\pi}{3}$



NOTE: $\sin$ and $\cos$ are between -1 and 1.

<u>REMINDER</u>: Pythagorean Identity



$1-\sin^2(x)$

$\cos^2(x)+\sin(x)+1=0$

Let $a=\sin(x)$:

$$\sin^2(\theta)+\cos^2(\theta)=1 \Rightarrow \cos^2(x)=1-\sin^2(x)$$

- Can be used for finding one trigonometry function by using the other.

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Your turn!

Question 14

Solve the following for the values of x:

$$2\sin^2(2x) + 3\cos(2x) = 2$$

$\sin^2(2x) + \cos^2(2x) = 1$

$\sin^2(2x) = 1 - \cos^2(2x)$

$2(1-\cos^2(2x)) + 3\cos(2x) = 2$

Let $a = \cos(2x)$:

$-2a^2 + 3a + 2 = 2$

$2a^2 - 3a = 0$

$a(2a-3) = 0$

$a = 0$ OR $a = \frac{3}{2}$

reject as $a \in [-1,1]$

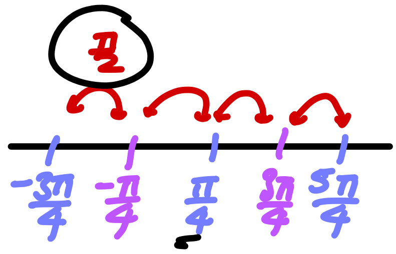
$\therefore \cos(2x) = 0$

$+P = \pi$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow$ $\therefore$ $x = \frac{\pi}{4} + \pi n, n \in Z$ OR $x = \frac{3\pi}{4} + \pi n, n \in Z$



$\Rightarrow \therefore x = \frac{\pi}{4} + \frac{\pi}{2}n, n \in Z$

TIP: $\sin^2(\theta) = 1 - \cos^2(\theta)$.

Section D: Exam 1 (14 Marks)

Question 15 (4 marks)

Solve the equation $2\sin\left(2x+\frac{\pi}{3}\right)+1=0$ for $x \geq 0$.

$\sin\left(2x+\frac{\pi}{3}\right)=\frac{-1}{2}$

$2x+\frac{\pi}{3}=\frac{7\pi}{6}, -\frac{\pi}{6}$

$2x=\frac{5\pi}{6}, -\frac{\pi}{2}$

$\therefore x=\frac{5\pi}{12}, -\frac{\pi}{4}$ $\quad \therefore P=\pi$

$x \geq 0$

$\therefore x=\frac{5\pi}{12}+\pi n, n \in Z^{+}\cup\{0\}$

$x=-\frac{\pi}{4}+\pi n, n \in Z^{+}$

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Question 16 (4 marks)

Consider the function $f(x) = 2\tan(3x) - 2$.

a. Find a general solution to $f(x) = 0$. (3 marks)

$\tan(3x) = 1$

$3x = \frac{\pi}{4}$

$\therefore x = \frac{\pi}{12}$ $\therefore p = \frac{\pi}{3}$

$\therefore x = \frac{\pi}{12} + \frac{\pi}{3}n, \ n \in Z$

b. State an **equivalent** general solution to what you found in **part a.** (1 mark)

$\therefore x = \frac{5\pi}{12} + \pi n, \ n \in Z$

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$k=-4$	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
$-\frac{15\pi}{12}$	$-\frac{11\pi}{12}$	$-\frac{7\pi}{12}$	$-\frac{3\pi}{12}$	$\frac{\pi}{12}$	$\frac{5\pi}{12}$	$\frac{9\pi}{12}$	$\frac{13\pi}{12}$	$\frac{17\pi}{12}$

Question 17 (6 marks)

$1-\cos^2(2x)$

Consider the function $f(x) = 3\sin^2(2x) + 3\cos(2x) - 3\cos^2(2x)$.

a. Show that $f(x) = 3 + 3\cos(2x) - 6\cos^2(2x)$. (1 mark)

$f(x) = 3(1-\cos^2(2x)) + 3\cos(2x) - 3\cos^2(2x)$

$= 3 - 6\cos^2(2x) + 3\cos(2x)$ //

b. Solve the equation $f(x) = 0$ for $x \in [0, \pi]$. (3 marks)

$\therefore P = \pi$ $\quad$ $D \in [0, \pi]$

Let $a = \cos(2x)$:

$-6a^2 + 3a + 3 = 0$

$\div(-3)\,(\quad)\div(-3)$

$2a^2 - a - 1 = 0$

$(2a+1)(a-1) = 0$

$\therefore a = -\frac{1}{2}$ OR $a = 1$ $\quad$ $a \in [-1, 1]$

$\therefore \cos(2x) = -\frac{1}{2}$ $\quad$ $\therefore \cos(2x) = 1$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $2x = 0$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$ $\quad$ $\therefore x = 0$

$\therefore x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$

c. Hence, write the general solution to $f(x) = 0$. (2 marks)

$\frac{\pi}{3}$

$-\pi \quad -\frac{2\pi}{3} \quad -\frac{\pi}{3} \quad 0 \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \pi \quad \frac{4\pi}{3} \quad \frac{5\pi}{3}$

$x = 0 + \pi n,\ n \in \mathbb{Z}$

OR

$x = \frac{\pi}{3} + \pi n,\ n \in \mathbb{Z}$

OR

$x = \frac{2\pi}{3} + \pi n,\ n \in \mathbb{Z}$

$\therefore x = \frac{\pi}{3} n,\ n \in \mathbb{Z}$

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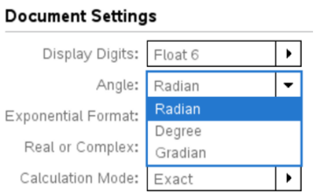
Section E: Technology Exam Skills

Calculator Commands: Degrees and Radians

TI

- Doc → 7 → 2



Casio

- Change at the bottom of the screen.

Mathematica

- In radians by default.
- Write "Degree."

```
In[27]:= Sin[30 Degree]
```

Out[27]= $\frac{1}{2}$

Calculator Commands: Solving trigonometric functions.

TI

- solve$(\text{trig}(..) = a, x)$ | domain restriction
- | is under control equal.

Casio

- solve$(\text{trig}(..) = a, x)$ | domain restriction
- | is under maths 3.

Mathematica

- Solve[trig[] $== a$ && domain restriction, x]

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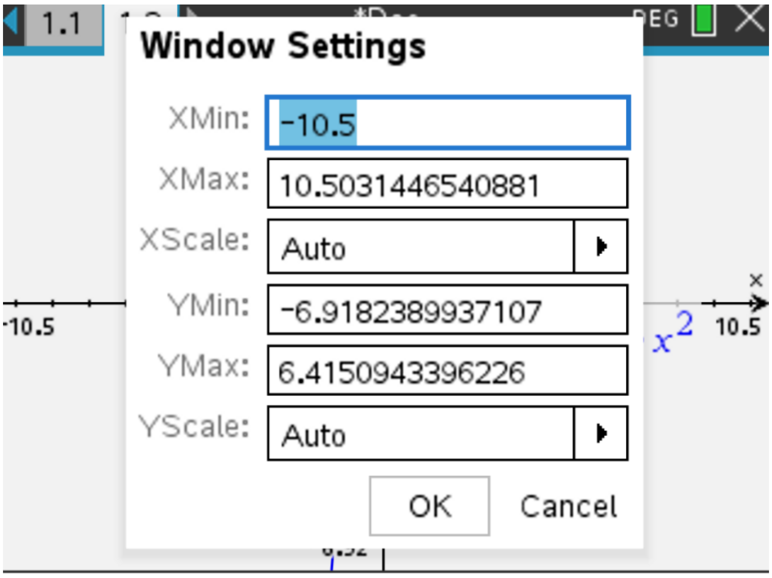
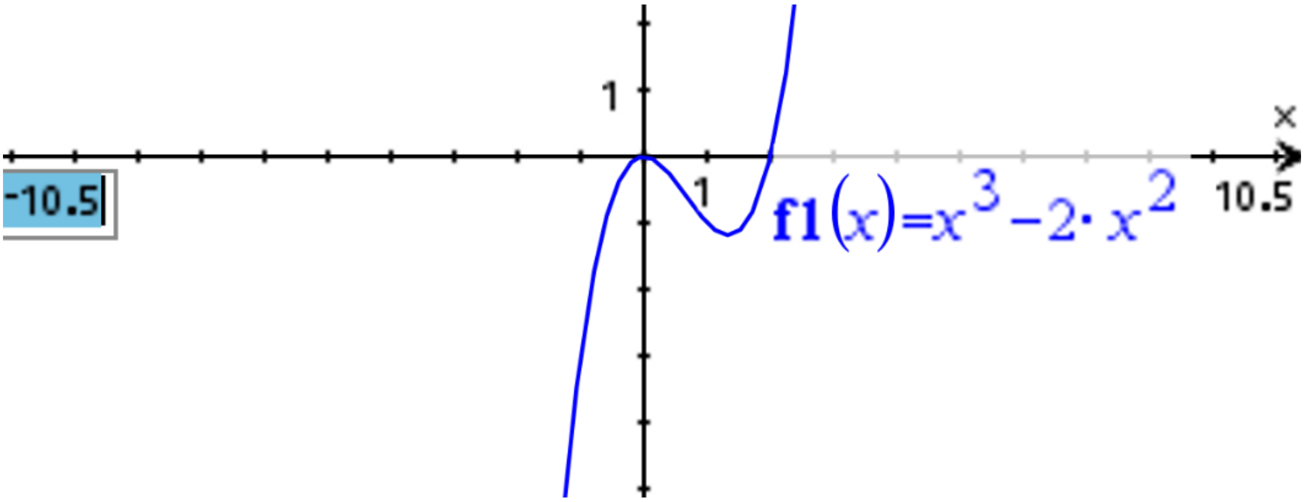
<u>Calculator Commands:</u> Graphing

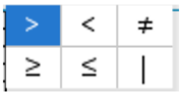
- Open a graph page and plot your function.
- Zoom settings: Menu → 4 (window / zoom) → 1 enter your x and y-ranges.

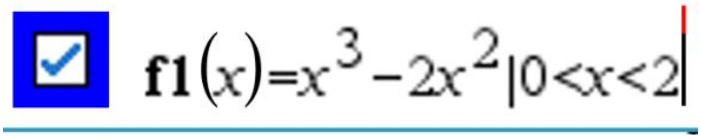


- Can also click the axis numbers on the graph and alter them directly.

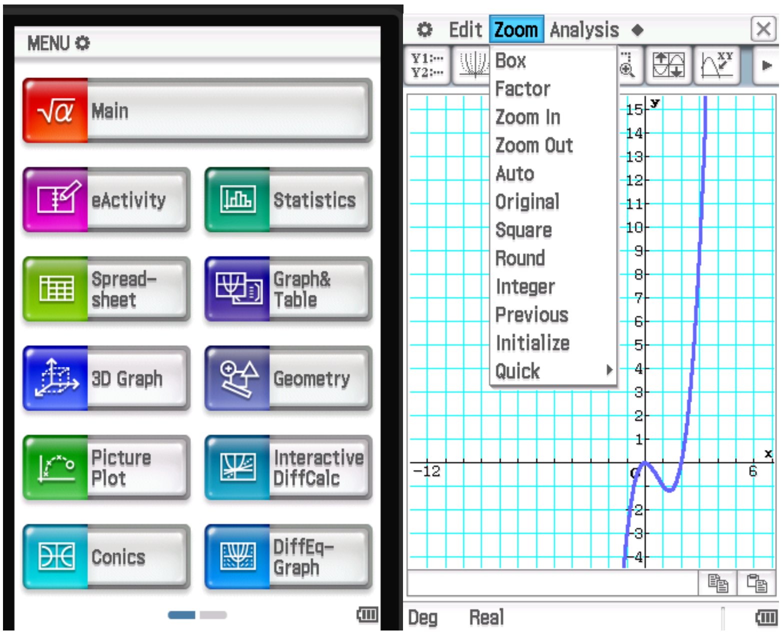


- Menu → 6 (Analyse) to find min / max x and y-intercepts.
- Restrict domain to $0 < x < 2$, use the bar to get it from $\text{ctrl}+=$

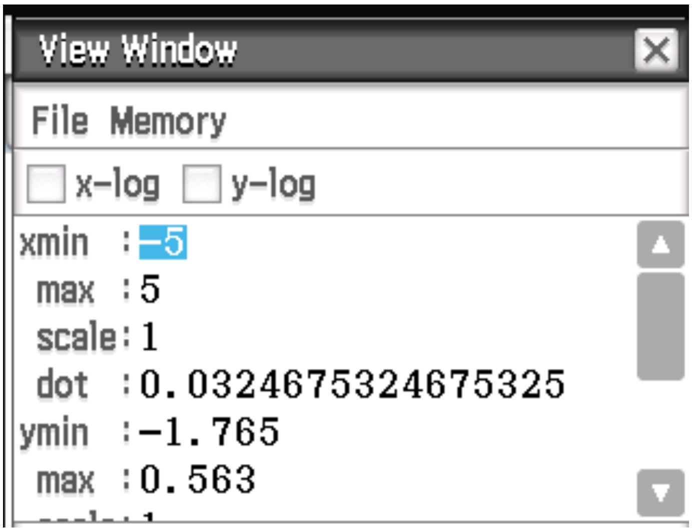




- **Casio:** Click graph & table, and enter the function.



- Analysis → G-Solve to find intercepts.
- Use this button  to set the view window.



- Use | to restrict domain → find it in Math 3.

$$y1=x^3-2\cdot x^2\,|\,0<x<2$$

- **Mathematica:** Plot[function, $\{x, xmin, xmax\}$, PlotRange → $\{ymin, ymax\}$]
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Section F: Exam 2 (32 Marks)

Question 18 (1 mark)

In a right-angled triangle, the two shorter side lengths are 5 *cm* and 12 *cm*. To the nearest degree, the smallest angle is:

A. 22°

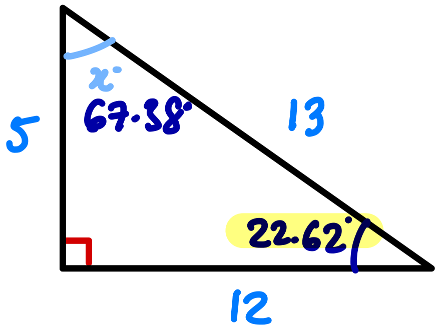
B. 23°

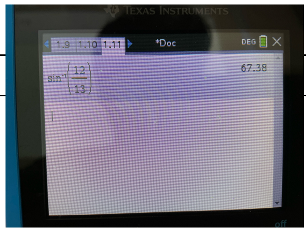
C. 24°

D. 67°

$\sin(x) = \frac{12}{13}$

$x = \sin^{-1}\left(\frac{12}{13}\right)$





Question 19 (1 mark)

The value of $\sin\left(\frac{5\pi}{6}\right)$ is equal to:

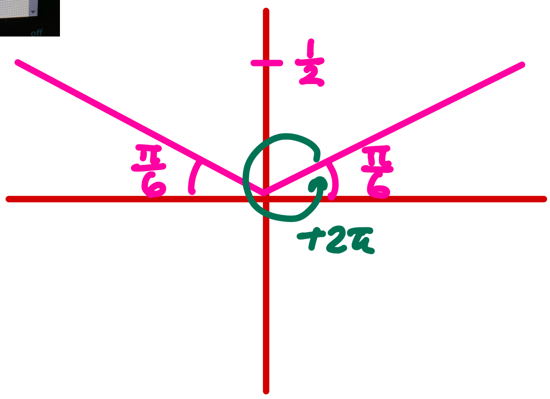
$= \frac{1}{2}$

A. $-\sin\left(\frac{\pi}{6}\right)$

B. $\sin\left(\frac{\pi}{3}\right)$

C. $\sin\left(\frac{13\pi}{6}\right)$ $= \sin\left(\frac{12\pi}{6} + \frac{\pi}{6}\right)$

D. $-\sin\left(\frac{\pi}{3}\right)$ $= \sin\left(2\pi + \frac{\pi}{6}\right)$



Question 20 (1 mark)

The value of $\cos\left(\frac{7\pi}{2} - \theta\right)$ is equal to:

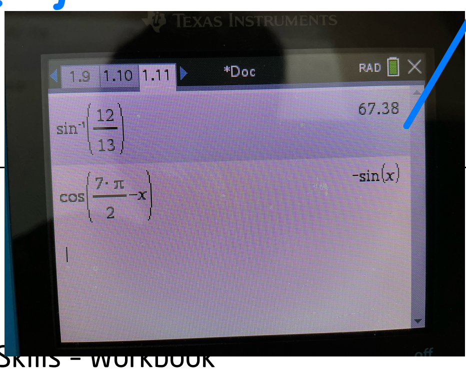
A. $\cos(\theta)$ $\Rightarrow \cos\left(\frac{4\pi}{2} + \frac{3\pi}{2} - \theta\right)$

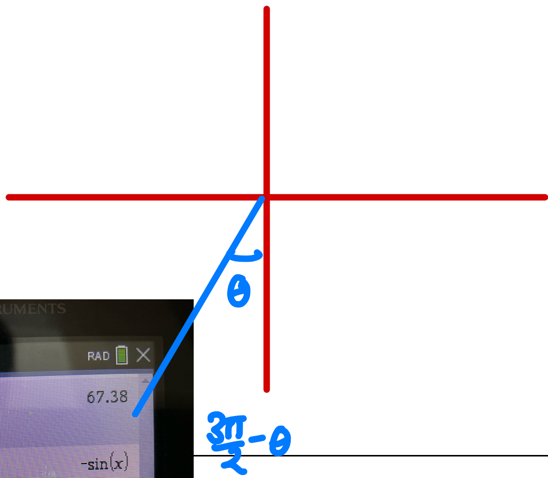
B. $-\cos(\theta)$ $= \cos\left(\cancel{2\pi} + \frac{3\pi}{2} - \theta\right)$

C. $\sin(\theta)$

$= \cos\left(\frac{3\pi}{2} - \theta\right)$

D. $-\sin(\theta)$ $\ominus$ Q3





Question 21 (1 mark)

The minimum value of $7 - 9\cos(3x)$ is:

(1)

A. 2

B. -5

C. -2

D. -16

$\text{Min} = 7 - 9 = -2$

Question 22 (1 mark)

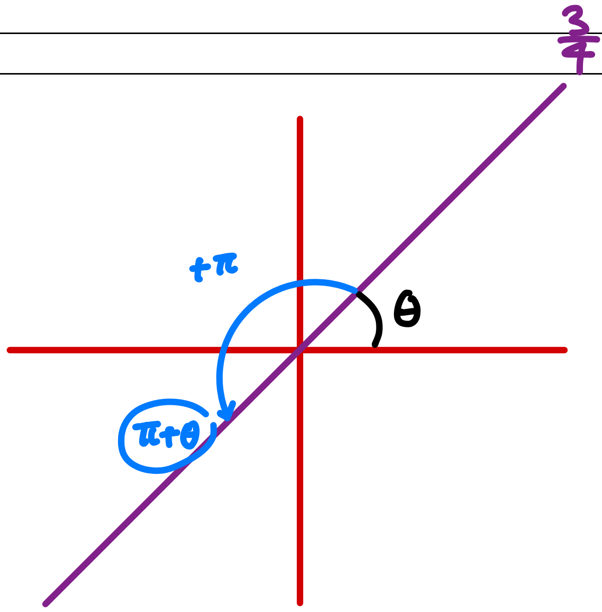
If $\tan(\theta) = \frac{3}{4}$, then $\tan(\pi + \theta)$ is:

A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. $-\frac{4}{3}$



Question 23 (1 mark)

If $\cos(x) = \frac{2}{3}$ and x is in the fourth quadrant, then $\sin(x)$ is equal to:

A. $\frac{\sqrt{5}}{3}$

B. $-\frac{\sqrt{5}}{3}$

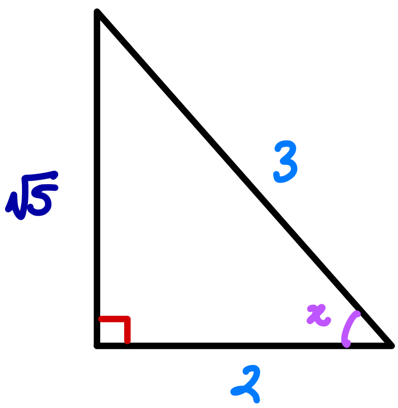
C. $\frac{\sqrt{5}}{2}$

D. $-\frac{\sqrt{5}}{2}$

$\sin(x) = \pm\frac{\sqrt{5}}{3}$

y Q4

$-$



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Question 24 (1 mark)

The number of solutions to the equation $\sin(x) = \frac{1}{2}$ in the interval $[0, 8\pi]$ is:

$p = 2\pi$

8

$\times 2$

4 Periods

A. 2

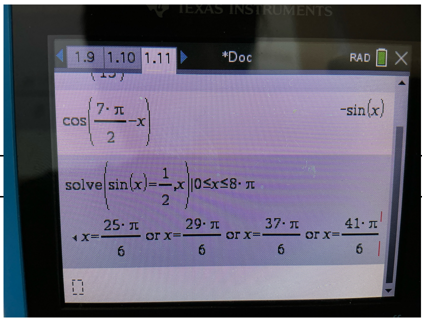
B. 4

C. 6

D. 8

$\text{solve}\left(\sin(x) = \frac{1}{2}, x\right) \mid 0 \le x \le 8\pi$

2 soln/period



Question 25 (1 mark)

$p = \frac{\pi}{3}$

A general solution to the equation $\tan\left(3x - \frac{\pi}{3}\right) = \sqrt{3}$ for $x \ge 0$ is:

A. $x = \frac{\pi}{9} + \frac{n\pi}{3}$, where $n \in \mathbb{Z}$ and $n \ge 0$.

B. $x = \frac{\pi}{6} + \frac{n\pi}{3}$, where $n \in \mathbb{Z}$ and $n \ge 0$.

C. $x = \frac{\pi}{3} + \frac{2n\pi}{3}$, where $n \in \mathbb{Z}$ and $n \ge 0$.

D. $x = \frac{2\pi}{9} + \frac{n\pi}{3}$, where $n \in \mathbb{Z}$ and $n \ge 0$.

$\text{solve}\left(\tan\left(3x - \frac{\pi}{3}\right) = \sqrt{3}, x\right) \mid x \ge 0$

$x = \frac{2\pi}{9} + \frac{\pi}{3}n, \quad n \ge 0$

$-\frac{\pi}{9} \quad \frac{\pi}{3} \quad \frac{2\pi}{9}$

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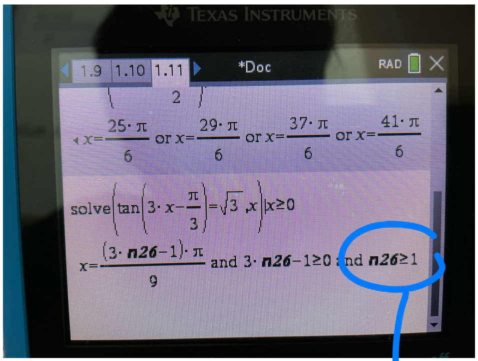


$\Rightarrow \therefore x = \frac{3n\pi}{9} - \frac{\pi}{9}$

$\therefore x = -\frac{\pi}{9} + \frac{\pi}{3}n$

$n \ge 1$

Question 26 (7 marks)

The Surf Life Saving HQ receives automatic tide alerts from a coastal sensor. The height of the tide (in metres), is modelled by the equation:

$$H(t) = 1.5 + \cos\left(\frac{\pi t}{6}\right)$$

where t is the time in hours after midnight, and $H(t)$ is the height in metres above mean sea level.

A **red warning alert** is triggered **when the tide drops below 1 metre**, as this exposes shallow sandbars near the shoreline and makes it dangerous for vessels to operate.

a. State the period of the tide function. (1 mark)

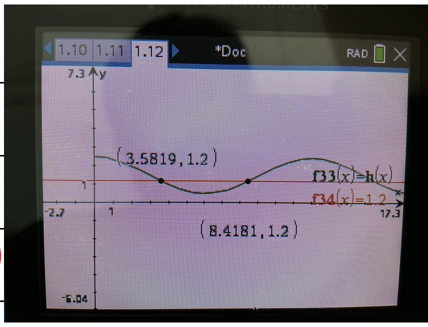
$\therefore P = \frac{2\pi}{\left(\frac{\pi}{6}\right)} = 12$ hours

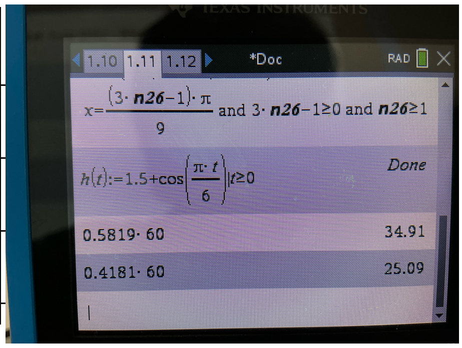
b. Find the first two times after midnight when the tide height is exactly 1.2 metres. Give your answers correct to the nearest minute. (2 marks)

$\therefore H(t) = 1.2$

$\therefore t \approx 3.582, 8.418$

$\therefore$ 3:35AM, 8:25AM



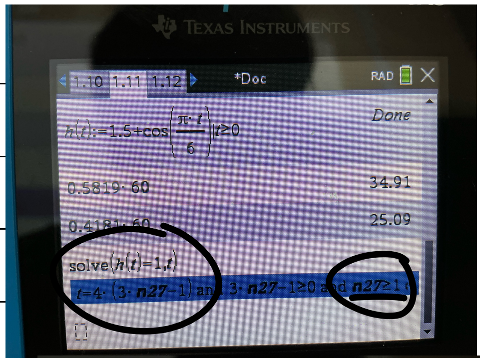


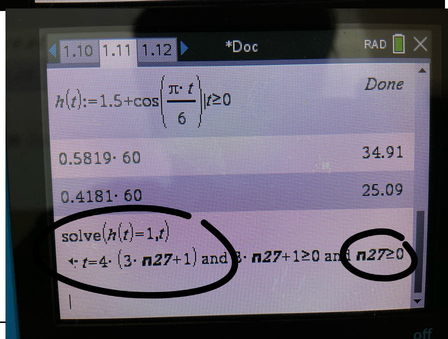
c. Write the general solution for t when $H(t) = 1$. (2 marks)

$\therefore t = 4(3n-1), n \geq 1$

OR

$t = 4(3n+1), n \geq 0$





d. For how many hours in a 24-hour period is the red alert active? (2 marks)

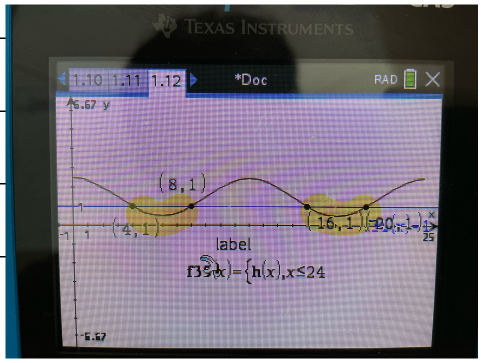
$H(t) < 1$

$\therefore H(t) = 1:$

$\therefore t = 4, 8, 16, 20$

$\Rightarrow \therefore$ No. of hours $= (8-4)+(20-16)$

$= 8$ hours



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Question 27 (7 marks)

A child builds a sandcastle at a position C on a beach. The waves wash up and down the beach in such a way that, after t minutes, the distance p metres from C to the edge of the water is given by:

$$p(t) = 2.5\sin(n\pi t) + 4$$

where n is a real constant.

a. Calculate the closest distance the water gets to the sandcastle. (1 mark)

Min distance $(\sin(n\pi t) = -1)$:

$\Rightarrow 1.5\text{m}$

b. Over a period of 60 minutes, the child counts 48 complete wave cycles. Find the value of n. (2 marks)

$P = \frac{2\pi}{n\pi} = \frac{2}{n}$ mins/wave cycle

$\frac{60}{48}$ mins/wave cycle

$\Rightarrow \frac{2}{n} = \frac{60}{48}$

$\frac{2}{n} = \frac{5}{4}$

$\therefore n = \frac{8}{5}$

c. Later in the day, the distance from the water's edge to the sandcastle is modelled by:

$$p_2(t) = a\sin(3\pi t) + 4$$

$\in [-1, 1]$

If the water just reaches the sandcastle, find the value of a. Hence, find how many times in 20 minutes the water reaches the sandcastle. (3 marks)

$\therefore a = 4$ OR $a = -4$

$P = \frac{2\pi}{3\pi} = \frac{2}{3}$ mins/wave cycle

$\Rightarrow \frac{20\text{min}}{\left(\frac{2}{3}\right)} = 30$ wave cycles

$\therefore$ reach sandcastle 30 times

d. In which of the two models is the wave frequency greater? Justify your answer. (1 mark)

#1 : 60min ⇒ 48 wave cycles

#2 : 20min ⇒ 30 wave cycles → 60mins ⇒ 90 wave cycles

∴ The second model is greater.

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Question 28 (10 marks)

A lost hiker walks in a circular pattern, trying to find a phone signal. Her position east of the rescue base, in kilometres, is modelled by:

$$x(t) = 3\sin\left(\frac{\pi t}{4}\right)$$

where $x(t)$ is the displacement east of the base, and t is the time in hours after she started walking.

a. What is the maximum distance east she travels, and how long does it take to reach this point for the first time? (2 marks)

b. Find all values of $t \in [0, 8]$ for which her eastward position is exactly $1.5\ km$. (2 marks)

c. The function $x(t)$ can also be expressed in the form $x(t) = 3\cos\left(\frac{\pi}{4}(t-a)\right)$, where $a \in [0, 8]$.

Find the value of a. (2 marks)

d. Write the general solution for when she is 1.5 km east of the base. (2 marks)

e. The rescue helicopter can only pick her up when she is between 2 km and 3 km east of the base. For how long, during each 8-hour cycle, is she in the pickup zone? Give your answer correct to two decimal places. (2 marks)

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