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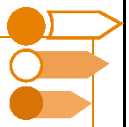
VCE Mathematical Methods ½
Circular Function Exam Skills [4.3]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 14

Section A: Compulsory Questions



Sub-Section [4.3.1]: Equivalent General Solutions

Question 1

Which one of the following solutions is not equivalent to the others?

A. $\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

B. $-\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

C. $\frac{3\pi}{4} + n\pi, n \in \mathbb{Z}$

D. $-\frac{5\pi}{4} + n\pi, n \in \mathbb{Z}$

Question 2

Which of the following is ***not*** a general solution to the equation $\cos(x) = \frac{1}{2}$?

A. $x = \frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$

B. $x = \frac{\pi}{3} + 2n\pi, n \in \mathbb{Z}$

C. $x = \frac{2\pi}{3} + 2n\pi, n \in \mathbb{Z}$

D. $x = -\frac{\pi}{3} + 2n\pi, n \in \mathbb{Z}$

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Question 3

Which one of the following contains all solutions to the equation $\tan(x) = 1$?

A. $x = -\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

B. $x = \frac{7\pi}{4} + n\pi, n \in \mathbb{Z}$

C. $x = -\frac{5\pi}{4} + n\pi, n \in \mathbb{Z}$

D. $x = \frac{\pi}{4} + n\pi, -\frac{3\pi}{4} + n\pi, n \in \mathbb{Z}$

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Sub-Section: Exam 1 Questions

Question 4

Given that $\cos(x) = \frac{4}{5}$ and $x \in \left(0, \frac{\pi}{2}\right)$, find:

a. $\cos(\pi - x)$.

$\cos(\pi - x) = -\cos(x)$

$= -\frac{4}{5}$ 1A

b. $\sin\left(\frac{\pi}{2} - x\right)$.

$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$

$= \frac{4}{5}$ 1A

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Question 5

Solve the following expression for x:

$$\sqrt{2}\sin\left(2x-\frac{\pi}{3}\right)=1$$

$\sin\left(2x-\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$ 1M

$2x-\frac{\pi}{3}=\frac{\pi}{4}+2k\pi, \frac{3\pi}{4}+2k\pi, \quad k\in\mathbb{Z}$ 1M

$2x=\frac{7\pi}{12}+2k\pi, \frac{13\pi}{12}+2k\pi$

$x=\frac{7\pi}{24}+k\pi, \frac{13\pi}{24}+k\pi$ 1A

Question 6

Solve the following expression for $x \in [0,2\pi]$:

$$6\cos\left(3x-\frac{\pi}{6}\right)-3\sqrt{3}=0$$

$\cos\left(3x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ 1M

$3x-\frac{\pi}{6}=\frac{\pi}{6}+2k\pi, \frac{11\pi}{6}+2k\pi, \quad k\in\mathbb{Z}$ 1M

$3x=\frac{\pi}{3}+2k\pi, 2\pi+2k\pi$

$x=\frac{\pi}{9}+\frac{2}{3}k\pi, \frac{2\pi}{3}+\frac{2}{3}k\pi$

$=\frac{\pi+6k\pi}{9}, \frac{2\pi+2k\pi}{3}$

in $x\in[0,2\pi]$: $x=0, \frac{\pi}{9}, \frac{2\pi}{3}, \frac{7\pi}{9}, \frac{4\pi}{3}, \frac{13\pi}{9}, 2\pi$

1A - all solutions except 0 and 2π

2A - all solutions including 0 and 2π

Question 7

Solve the following expression for $x \in [0,2\pi]$:

$$2\sin^2(x) - 3\sin(x) + 1 = 0$$

let $\sin(x) = a \Rightarrow 2a^2 - 3a + 1 = 0$ 1M

$(2a-1)(a-1) = 0 \Rightarrow a = \frac{1}{2}, 1$ 1M

$\Rightarrow \sin(x) = \frac{1}{2}, 1$

$\therefore x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$ 1M

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$ 1A

Question 8

Given that $\sin(x) = \frac{5}{13}$ and $x \in \left(\frac{\pi}{2}, \pi\right)$, find $\cos(x)$.

$\sin^2(x) + \cos^2(x) = 1$

$\Rightarrow \cos(x) = \pm\sqrt{1 - \sin^2(x)}$

$= \pm\sqrt{1 - \frac{25}{169}}$

$= \pm\sqrt{\frac{144}{169}}$

$= \pm\frac{12}{13}$ 1M

$\cos(x)$ -ve in Q2 $\therefore \cos(x) = -\frac{12}{13}$ 1A

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Question 9

Solve the following expression for x:

$$\tan\left(\frac{\pi}{4} - x\right) = \sqrt{3}$$

$\frac{\pi}{4} - x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$ 1M

$x = -\frac{\pi}{12} - k\pi$ 1A

Question 10

Solve the following expression for $x \in [-\pi, \pi]$:

$$2\sin\left(x + \frac{\pi}{3}\right) = \sqrt{3}$$

$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ 1M

$x + \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$x = 2k\pi, \frac{\pi}{3} + 2k\pi$ 1M

$\therefore x = 0, \frac{\pi}{3}$ 1A

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Sub-Section: Exam 2 Questions

Question 11

If $\tan(\theta) = k$ and $k \neq 0$, then $\tan\left(\frac{\pi}{2} - \theta\right)$ is equal to:

A. k

B. $-k$

C. $\frac{1}{k}$

D. $-\frac{1}{k}$

Question 12

Which one of the following represents the correct general solution for the equation $\cos(x) = -\frac{1}{2}$?

A. $x = \frac{2\pi}{3} + 2n\pi, x = \frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$

B. $x = \frac{2\pi}{3} + 2n\pi, x = -\frac{2\pi}{3} + 2n\pi, n \in \mathbb{Z}$

C. $x = \frac{\pi}{3} + 2n\pi, x = \frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$

D. $x = -\frac{2\pi}{3} + 2n\pi, x = \frac{\pi}{3} + 2n\pi, n \in \mathbb{Z}$

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Question 13

If $\sin(\theta) = \frac{8}{17}$ and θ is in the first quadrant, then $\cos(\theta) =$

A. $-\frac{8}{17}$.

B. $\frac{8}{17}$.

C. $-\frac{15}{17}$.

D. $\frac{15}{17}$.

Question 14

Which of the following is equivalent to $\sin\left(\frac{\pi}{2} + \theta\right)$?

A. $\cos(\theta)$

B. $-\cos(\theta)$

C. $\sin(\theta)$

D. $-\sin(\theta)$

Question 15

Which of the following is NOT a root of the function $f(x) = 2\cos^3(\theta) - \cos^2(\theta) - 2\cos(\theta) + 1$?

A. $\cos(\theta) = -\frac{1}{2}$

B. $\cos(\theta) = \frac{1}{2}$

C. $\cos(\theta) = 1$

D. $\cos(\theta) = -1$

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Question 16

On a certain trip to Woolworths, Sam's distance x, measured in metres, from the chocolate aisle at time t, measured in minutes since he entered the store, is modelled by the function:

$$x(t) = 10\cos\left(\frac{\pi t}{6}\right) + 10$$

a. State the maximum distance that Sam strays from the chocolate aisle

Plot graph and inspect the maximum or alternatively, maximum of sin and cos is equal to amplitude + vertical translation, in this case amplitude $= 10$ and vertical translation $= 10$.
$20m$ – 1A (units needed)

b. How long does it take for Sam to visit the chocolate aisle for the first time after entering Woolies?

6 minutes (units needed)
$1M$ – equation equal to 0, 1A – correct answer

Solve[10 Cos[$\frac{\pi}{6}$ t] + 10 == 0, t] // Expand

Out[18]= {{t → -6 + 12 c_1 if $c_1 \in \mathbb{Z}$}, {t → 6 + 12 c_1 if $c_1 \in \mathbb{Z}$}}

c. After his first visit to the chocolate aisle, how much time passes before Sam visits the chocolate aisle again?

Find the period of $x(t)$
12 minutes (units needed) - 1A

d. Hence, state a general solution that includes all of the times Sam visits the chocolate aisle.

$t = 6 + 12k, k \in \mathbb{Z}^+$
1A - $6 + 12k$
1A - $k \in \mathbb{Z}^+$ since $t > 0$ due to the implied restriction in the question context.

Sam leaves the store after 36 minutes.

e. State the number of times Sam visited the chocolate aisle.

Plot graph for $x \in [0,36]$ and inspect the number of x-intercepts
3 times – 1A

f. How much time did Sam spend within 5 metres of the chocolate aisle? Give your answer in minutes.

$$\text{Reduce}\left[10\cos\left[\frac{\pi}{6}t\right] + 10 \leq 5 \;\&\&\; 0 \leq t \leq 36,\; t\right]$$

$$4 \leq t \leq 8 \;||\; 16 \leq t \leq 20 \;||\; 28 \leq t \leq 32$$

$(8-4)+(20-16)+(32-28) = 12$ minutes
1M – correct inequality or equation
1M – correct interpretation of solution $(8-4)+(20-16)+(32-28)$
1A – correct answer

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Question 17

During a late night exam cram, a student tracks their energy level while drinking an energy drink. The energy level follows a predictable cycle as the caffeine levels rise and fall with every sip, modelled by the function:

$$E(t) = 5\sin\left(\pi\left(\frac{t}{2} - \frac{1}{3}\right)\right)$$

Where, E is their energy level in study productivity points (SP) at time t hours after they take their first sip of the energy drink.

a. How much energy in SP does the student have as they take their first sip of the energy drink? Give your answer correct to 2 decimal places.

```
e[t_] := 5 Sin[π (t/2 - 1/3)]

e[0] // N

-4.33013
```

1A – correct answer

The student has $-4.33\ SP$ as they take their first sip of the energy drink

b. What is the maximum energy level the student can reach before they start losing study productivity points?

5 SP

1A – correct answer

c. State the period of the study productivity cycle.

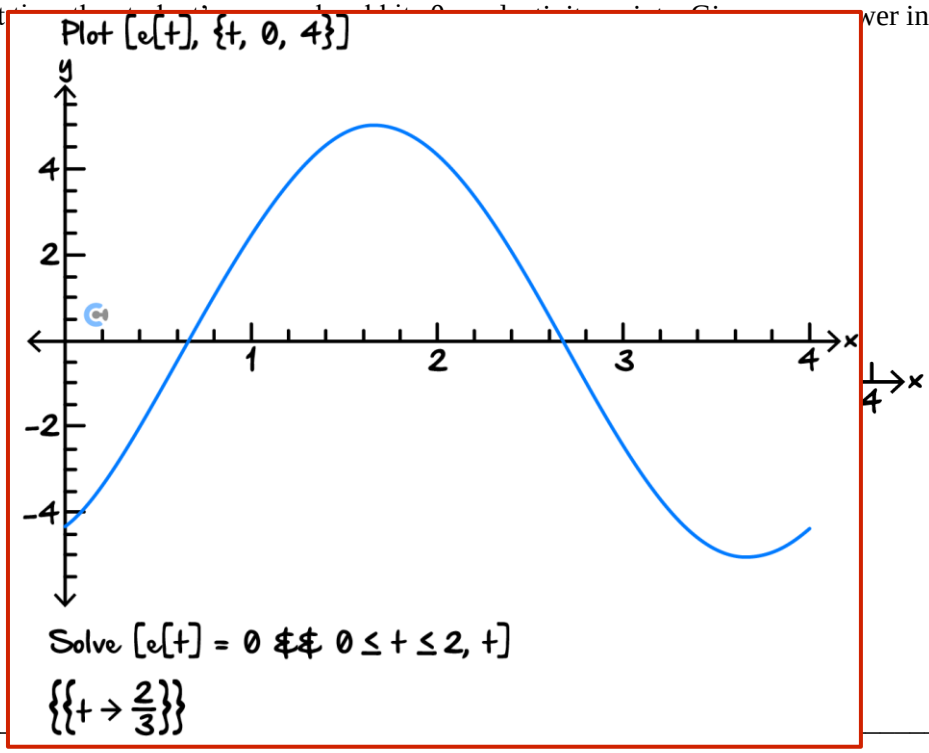
```
FunctionPeriod[e[t], t]

4
```

$$\text{Period} = \frac{2\pi}{coefficient\ of\ t} = \frac{2\pi}{\left(\pi * \frac{1}{2}\right)} = 4$$

4 hours (units required)

1A – correct answer

d. Find the first [illegible] wer in minutes



Plot graph and solve with a rough domain restriction
40 minutes
1M - $E(t) = 0$, 1A – correct answer

e. Find a general solution for when the student's energy level is 2.5 SP.

$t = 1 + 4k, \frac{7}{3} + 4k, k \in \mathbb{Z}^+$
1A – correct expressions ($t = 1 + 4k, \frac{7}{3} + 4k$)
1A – correct k restriction ($k \in \mathbb{Z}^+$)

Solve[e[t] == 5/2, t] // Expand

{{t → 1 + 4 c1 if c1 ∈ ℤ}, {t → 7/3 + 4 c1 if c1 ∈ ℤ}}

The student studies for 4 hours after first sipping their energy drink.

f. State the times at which the student's energy level is 2.5 SP.

Energy level is 2.5 SP when $t = 1, \frac{7}{3}$.

g. When the student's energy level is below 0 SP, they are scrolling on Instagram Reels. Find the amount of time spent scrolling reels in the 4 hour study session. Give your answer in hours and minutes,

```
Reduce[e[t] < 0 && 0 ≤ t ≤ 4, t]
```

$0 \leq t < \frac{2}{3} \;||\; \frac{8}{3} < t \leq 4$

$\left(\frac{2}{3} - 0\right) + \left(4 - \frac{8}{3}\right) = 2$ hours spent scrolling reels

1M – correct inequality

1A – correct answer

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