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VCE Mathematical Methods ½ Circular Function II [4.2]

Workbook

Outline:

Pg 2-14

Particular and General Solutions

- Recap of Particular Solutions
- General Solutions
- Equivalent General Solutions

Advanced Trigonometric Algebra

Pg 15-2

- General Solutions with Domain Restrictions
- Hidden Quadratics

Learning Objectives:

- MM12 [4.2.1] Solve General Solutions for Trigonometric Functions
- MM12 [4.2.2] Solve Hidden Quadratic Equations for Trigonometric Functions



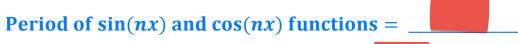


Section A: Particular and General Solutions

Sub-Section: Recap of Particular Solutions



Active Recall: Period of trigonometric function

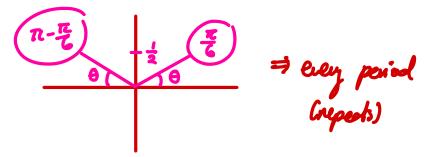




where n = coefficient of x and n > 0



<u>Discussion</u>: How often would the solution to $sin(x) = \frac{1}{2}$ repeat?



Active Recall: Particular Solutions

?

- Solving trigonometric equations for finite solutions.
- Steps
 - 1. Make the trigonometric function the subject.
 - **2.** Find the necessary ___ for one period.
 - **3.** Solve for *x* by equating the necessary angles to the _____ of the trigonometric functions.
 - **4.** Add and subtract the _____ to find all other solutions in the domain.



Question 1 Walkthrough.

Solve the following equation for x over the domain specified:

$$2\cos(2x) + \sqrt{2} = 0 \text{ for } x \in [0, 2\pi]$$

1. Male Trig the Subject:

$$\cos(2x) = -\frac{\sqrt{2}}{2}$$

213. Find angles for 1 penied:

1 Find 0: (exact value)

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$
 (ref angle)

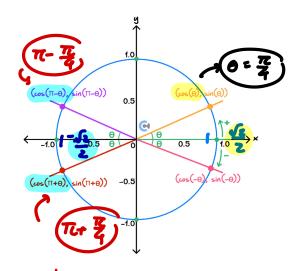
3 Find other angle(s):

$$\frac{\cos(2x) = \frac{\sqrt{2}}{2}}{\pi} \Rightarrow \frac{62}{\pi}, \frac{63}{\pi}$$

 $2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$



Space for Personal Notes



4. ± Period For All Solutions

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Question 2

Solve the following equations for x over the domains specified:

a.
$$\sin(3x) = -1$$
 for $x \in [-\pi, \pi]$.

213. Find angle(s) for 1 penied:

4. I Period For All Solutions

b.
$$2\sin\left(2x - \frac{\pi}{2}\right) - 1 = 0$$
 for $x \in [0, 2\pi]$

1. Male Trig the Subject:

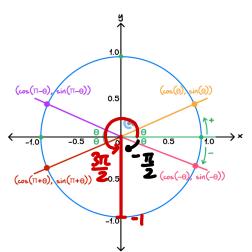


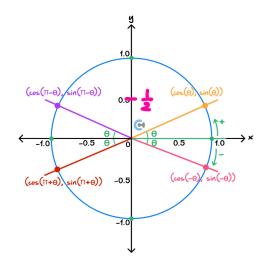
Φ Find θ: (exact value)

$$\operatorname{Sin}\left(\frac{\mathbb{E}}{6}\right) = \frac{1}{2} \qquad R \cdot A$$

② Find other angle(s):



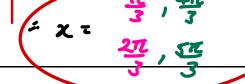




4. I Period For All Solutions

$$\rho = \frac{2\pi}{2} = \frac{3\pi}{3}$$

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Question 3 Walkthrough.

Solve the following equations for x over the domains specified:

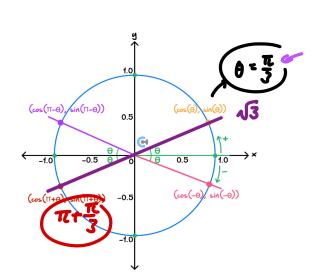
$$3\tan(2x - \pi) - 3\sqrt{3} = 0 \text{ for } x \in [0, 2\pi]$$

1. Make Trig the Subject:
$$tan(2x-\pi) = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

(1) Find θ : (exact value)

$$\tan \left(\frac{\pi}{3}\right) = \sqrt{3} \qquad (R.A)$$

2 Find other angle(s):



$$\dot{2}x - \pi = \frac{\pi}{3}$$

1 ANGLE

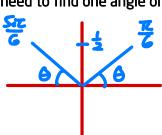
$$2\pi = \frac{2n}{3}$$

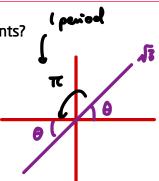


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Discussion: Why do we need to find one angle only for tangents?







Question 4

Solve the following equation for x over the domain specified:

$$\sqrt{3}\tan\left(x - \frac{\pi}{4}\right) + 1 = 0 \text{ fo} \quad x \in (0, 3\pi)$$



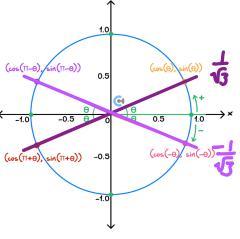
$$\tan\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{3}} \quad (e.A)$$



$$\lambda = \frac{\pi}{4} = \pi - \frac{\pi}{6}$$

$$\lambda = \frac{5\pi}{6} + \frac{\pi}{4} = \frac{26\pi}{24}$$

$$\frac{7}{12} = \frac{13E}{12}$$



I Period For All Solutions

$$D \in (0, \frac{34\pi}{12})$$

$$2 = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{25\pi}{12}$$



Sub-Section: General Solutions



<u>Discussion:</u> How many solutions yould there be for $x \in R$?

infinite soltions = General form par base angle + Brisd . n

General Solutions



- Finding ___in___ solutions to a trigonometric equation.
- Steps
 - 1. Make the trigonometric function the subject.
 - **2.** Find the necessary angle for one period.
 - **3.** Solve for *x* by equating the necessary angles to the inside of the trigonometric functions.
 - **4.** Add $Period \cdot n$ where $n \in \mathbb{Z}$.





Question 5 Walkthrough.

Find the general solutions to the following equations:

$$2\sin\left(2x + \frac{\pi}{2}\right) - 1 = 0$$

$$2\sin\left(2x + \frac{\pi}{2}\right) - \frac{\pi}{2}$$

$$2x + \frac{\pi}{2} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$2x = -\frac{2\pi}{6}, \frac{2\pi}{6}$$

$$2x = -\frac{\pi}{6}, \frac{\pi}{6}$$

$$2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

Active Recall: General Solutions



Steps

- 1. Make the trigonometric function the subject.
- 2. Find the necessary ___ for one period.
- **3.** Solve for x by equating the necessary angles to the inside of the trigonometric functions.

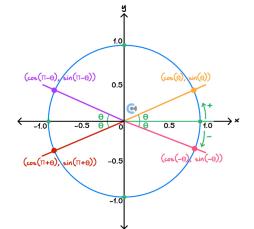
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Question 6

Find the general solutions to the following equations:

a.
$$-2\sin\left(3\right) + \frac{\pi}{4} = \sqrt{2}$$

$$\frac{3}{\sin(3x+\frac{\pi}{4})} = \frac{-\sqrt{2}}{2} \Rightarrow \frac{0.3,04}{(2+0)-0}$$



$$Sin(\frac{2}{4}) = \frac{42}{2}$$

$$= 32 + \frac{\pi}{4} = \pi + \frac{\pi}{4}, -\frac{\pi}{4}$$

$$3x = \pi, -\frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{3}, -\frac{\pi}{6}$$

$$($$
 $\pi)$

$$\frac{3}{3}$$
 Revival = $\frac{2\pi}{3}$

$$2 \times 1 = \frac{\pi}{3} + \frac{2\pi}{3}n, n \in 2$$

b.
$$2\cos\left(2x + \frac{\pi}{6}\right) = 1$$

$$\frac{\cos(2\pi + \frac{\pi}{6})}{\cos(2\pi + \frac{\pi}{6})} = \frac{1}{3}$$

$$\frac{\cos(2\pi + \frac{\pi}{6})}{\cos(2\pi + \frac{\pi}{6})} = \frac{1}{3}$$

$$\frac{\cos(2\pi + \frac{\pi}{6})}{\cos(2\pi + \frac{\pi}{6})} = \frac{1}{3}$$

$$2x + \frac{\pi}{6} = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$2x = \frac{\pi}{6}, -\frac{\pi}{2}$$

$$\frac{1}{2} \left(\frac{2\pi}{2} - \pi \right)$$

$$2 = \frac{\pi}{12} + \pi n_1 n \epsilon \epsilon$$

$$\frac{08}{2}$$

$$2 \times 2 = \frac{\pi}{12} + \pi n_1 n \epsilon \epsilon$$

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c.
$$4\sin(3) - \frac{\pi}{6} = 2$$

$$\sin(3) - \frac{\pi}{6} = 2$$

$$\sin(3) - \frac{\pi}{6} = 2$$

$$3x - \frac{\pi}{6} = \frac{1}{2}$$

$$3x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{\pi}{6}$$

$$3x = \frac{\pi}{3}, \pi$$

$$2z = \frac{\pi}{6}, \frac{\pi}{6}$$



Question 7 Walkthrough.

Find the general solutions to the following equations:

$$\tan\left(\frac{1}{2}x - \pi\right) - \frac{1}{\sqrt{3}} = 0$$

$$\tan\left(\frac{1}{2}x-\pi\right) = \frac{1}{\sqrt{3}}$$

$$\frac{1}{2}x-\pi = \frac{\pi}{6}$$

$$\frac{1}{2}x-\pi=\frac{\pi}{6}$$

$$\Rightarrow x = \frac{2\pi}{3}$$

$$\frac{1}{2} \int_{-\infty}^{\infty} \frac{\pi}{2\pi} = 2\pi$$

NOTE: We only need to find one angle for tangents!



Question 8

Find the general solutions to the following equations:

$$\mathbf{a.} \quad \sqrt{3} - \tan\left(2\left(x + \frac{\pi}{3}\right)\right) = 0$$



$$\tan\left(2x+\frac{2\pi}{3}\right)=\sqrt{3}$$

$$2n^{2} - \frac{\pi}{3} \Rightarrow \left(x^{2} - \frac{\pi}{6} \right)$$

b.
$$2 \tan \left(2x - \frac{\pi}{4}\right) = 2$$

$$\left(\frac{\pi}{2} \right)$$

$$\varepsilon \approx \frac{\pi}{4}$$

$$\mathbf{c.} \quad \sqrt{3}\tan\left(3x - \frac{\pi}{6}\right) = 1$$

$$\tan\left(3\pi-\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}$$

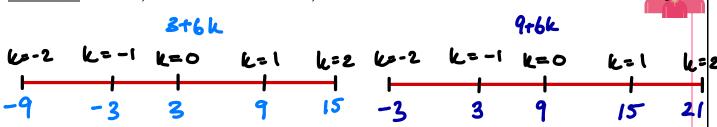
$$3x = \frac{\pi}{3} \Rightarrow \left(2x = \frac{\pi}{9}\right)$$



Sub-Section: Equivalent General Solutions



Discussion: Is 3 + 6k, $k \in Z$ the same as 9 + 6k, $k \in Z$?



=) If base angle is a periode away from the other



Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Question 9 Walkthrough.

Which one of the following is **not** the same as the rest?



NOTE: Very important for multiple choice questions in VCAA exams!



Question 10

Which one of the following is **not** the same as the rest?

$$\frac{2\pi}{3} + \frac{\pi}{4}n, n \in Z$$

$$\mathbf{B.} \int_{8}^{5\pi} + \frac{\pi}{4} n, n \in \mathbb{Z}$$

B.
$$\frac{6\pi}{8} + \frac{\pi}{4}n, n \in \mathbb{Z}$$

D.
$$\frac{7\pi}{6} + \frac{\pi}{4}n, n \in \mathbb{Z}$$



Section B: Advanced Trigonometric Algebra

Sub-Section: General Solutions with Domain Restrictions

- Don Restriction



Discussion: What is the main difference between the general and particular solution questions?



Question 11 Walkthrough.

Solve the following trigonometric equation:

sin
$$(2x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}$$
 for $x \ge 0$

$$2n + \frac{\pi}{4} = \frac{\pi}{4}, \pi - \frac{\pi}{4}$$

$$\frac{1}{2} \rho = \frac{2\pi}{2} = \pi$$

n=0 / 1=1

$$x = 0 + \pi n, n \in 2^{\dagger} U_{20}^{*}$$

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MM12 [4.2] - Circular Function II - Workbook





General Solution with Domain Restriction



$$E.G \operatorname{trig}\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \operatorname{for} x \geq 0$$

- We can have infinite solutions for a restricted domain.
- \blacktriangleright The value of n is also restricted.

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Question 12

Solve the following trigonometric equations:

a.
$$\cos(2x - \frac{\pi}{6}) = \frac{1}{2} \text{ for } x < 0.$$

a.
$$\cos(2x - \frac{\pi}{6}) = \frac{1}{2}$$
 for $x < 0$.

$$2x - \frac{\pi}{6} = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$2n = \frac{\pi}{2}, -\frac{\pi}{6}$$

$$\frac{1}{2} = \frac{\pi}{4}, \frac{-\pi}{4}$$

Period =
$$\frac{2\pi}{4} = \pi$$

$$= \frac{\pi}{4} + \pi \cdot \kappa, ne \frac{\pi}{2}$$

b.
$$2\sin\left(3\right) + \frac{\pi}{3} = \sqrt{3} \text{ fo } x > 0.$$

$$\sin\left(3c+\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$$

$$3\kappa + \frac{\pi}{3} = \frac{\pi}{3}, \frac{2\pi}{3}$$

Ferrod =
$$\frac{2\pi}{3}$$

$$x = 0 + \frac{2\pi}{3} \cdot n, n \in 2^+$$

$$x = \frac{\pi}{9} + \frac{2\pi}{3} \cdot \eta, n \in 2^{\dagger} U205$$



c.
$$\tan(2x - \frac{\pi}{4}) + \sqrt{3} = 0 \text{ for } x \le 0.$$

$$\tan \left(\frac{2\lambda - \frac{\pi}{4}}{2} \right) = -\sqrt{3}$$

$$2x - \frac{\pi}{4} = -\frac{\pi}{3}$$

$$2n - \frac{\pi}{4} = -\frac{\pi}{3}$$

$$2x = \frac{-\pi}{12}$$

$$2 = \frac{\pi}{24}$$

$$\eta = 1 \times n = 0$$

$$= \frac{-\pi}{24} + \frac{\pi}{2} \cdot n , n \in \overline{2} U \wr o J$$

NOTE: This was assessed in a VCAA exam!





Sub-Section: Hidden Quadratics



Let's have a look at hidden quadratics for circular functions!



Hidden Quadratics



$$af(x)^{2} + \underbrace{bf(x)}_{l} + c = 0$$
Let $A = f(x)$

Question 13 Walkthrough.

Solve the following for the values of x:

$$\sin^2\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{\pi}{3}\right) = 2\left(0 \le x \le 3\pi\right)$$

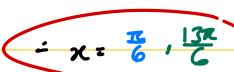
Let
$$a = sin(x + \frac{\pi}{3})$$
: $\frac{\pi}{3}$

$$a^2+a=2$$

$$(a+2)(a-1)=0$$

$$2 \sin(0) + \frac{\pi}{3} = 1$$

NOTE: \sin and \cos are between -1 and 1.







Question 14

Solve the following for the values of x:

a. $2\cos^2(2x) + 5\cos(2x) = 3, 0 \le x \le 2\pi$

$$2a^2 + 5a - 3 = 0$$

$$(2a-1)(a+3)=0$$

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$$\cos(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$2x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$\rho = \frac{2\pi}{2} = \pi$$

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b.
$$4 \tan^2 \left(x - \frac{\pi}{4} \right) - 3 \tan^2 \left(x - \frac{\pi}{4} \right) = 1, -\pi \le x \le \pi$$



$$\rho = \pi$$

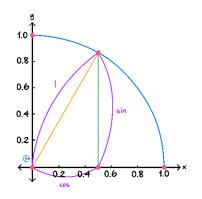


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REMINDER: Pythagorean Identity





$$\sin^2(\theta) + \cos^2(\theta) = 1$$

Can be used for finding one trigonometry function by using the other.

Question 15 Extension.

Find the general solution to the following equation:

$$-4\sin^2(3x) + 6\cos(3x) = 0$$

$$\sin^2(3x) = 1 - \cos^2(3x)$$

$$-4(1-\cos^2(3z))+6\cos(3z)=6$$

$$4a^2 + 6a - 4 = 0$$

$$(2a-1)(a+2)=0$$

$$3x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$\therefore x = \frac{n}{9}, \frac{-n}{9}$$

+ neget on 16[-1,1] = x = + 1 + 1 1,168

TIP: $\sin^2(\theta) = 1 - \cos^2(\theta)$







Contour Check

□ <u>Learning Objective</u>: [4.2.1] – Solve general solutions for trigonometric functions

Key Takeaways

- General Solutions
 - Finding ____ solutions to a trigonometric equation.
 - Steps
 - 1. Make the trigonometric function the subject.
 - **2.** Find the necessary _ for one period.
 - **3.** Solve for *x* by equating the necessary angles to the _____ of the trigonometric functions.
 - **4.** Add _____ where $n \in Z$.
 - If there is a domain restriction, only step 4 changes, and we need to be more careful in specifying what values n can take.
- ☐ Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the _____ of a and b is a multiple of period.



□ <u>Learning Objective</u>: [4.2.2] – Solve hidden quadratic equations for trigonometric functions

Key Takeaways

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

• May need to use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) =$



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VCE Mathematical Methods ½

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