Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Circular Function II [4.2]
Workbook

Outline:

Learning Objectives:

- ☐ **MM12 [4.2.1]** - Solve General Solutions for Trigonometric Functions
- ☐ **MM12 [4.2.2]** - Solve Hidden Quadratic Equations for Trigonometric Functions

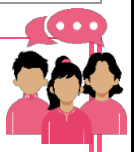
CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

Mock Exams Term 1 Info Sheet

Purpose & Rationale for Mock Exams

- We'll be running mock exams in the Term 1 School Holidays for **free** for Contour students, and they are **compulsory to attend,** as it offers you an opportunity to experience an entire exam-length exam under timed and invigilated conditions.
- It is an **extremely high-yield session** which is an excellent reality check for your progress and also helps inform your tutors about your learning.



Subjects Running Mock Exams

- **MM12:** AOS 1, 2, 3
- **MM34:** AOS 1, 2
- **SM12:** AOS 1, 2, 3
- **SM34:** AOS 1, 2, 3
- **BI12:** AOS 1, 2 (Unit 1)
- **BI34:** AOS 1, 2 (Unit 3)
- **CH12:** AOS 1 & Half of AOS 2 (2.1-2.3)
- **CH34 (Normal):** AOS 1 & 2.1-2.4
- **CH34 (Flipped):** AOS 1 & 2.6-2.9
- **PH12:** AOS 1, 2, 3
- **PH34:** AOS 1, & 2.1
- **EN34:** Section A & Section B (No Section C)

Subjects Not Running Mock Exams

- **MA07:** (Already run CATs)
- **MA08:** (Already run CATs)
- **MA09:** (Already run CATs)
- **MA10:** (Already run CATs)
- **EL34:** (New Subject)

How Will Booking Work?

Students will receive a mock exam email with a booking link for each Contour subject they're enrolled in.

There will be one email per subject, sent by Friday 4th April. Be sure to book ASAP to get your preferred session ☺

Frequently asked questions and their answers will also be emailed–e.g., what to bring, how exams will run, what to do if you cannot attend, marking and reviewing exams on the LMS.

Subject-Specific Notes

- **Chemistry:** Running two sets of exams
  - **Normal Exam:** Covers AOS 1 & 2.1-2.4
  - **Flipped Exam:** Covers AOS 1 & 2.6-2.9
  - **Booking:** Students select either *Normal* or *Flipped* on the booking form. Both exams run at the same time and location. Students sit in the same room but receive different exams.
- **English:**
  - **Mock Exam Duration:** 2 hours writing + 10 minutes reading (instead of 3 hours writing + 15 minutes reading)
  - **Content Note:** Section C (Argument Analysis) is excluded as most students haven't covered it yet. Reading time is reduced since there's no Section C passage.
- **Additional Things For Students to Bring:**
  - **MM12:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
  - **MM34:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
  - **SM12:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
  - **SM34:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
  - **BI12:** N/A
  - **BI34:** N/A
  - **CH12:** Scientific Calculator (only VCAA Data Book Provided)
  - **CH34:** Scientific Calculator (only VCAA Data Book Provided)
  - **PH12:** Scientific Calculator, Cheat Sheet (only VCAA Formula Sheet Provided)
  - **PH34:** Scientific Calculator, Cheat Sheet (only VCAA Formula Sheet Provided)
  - **EN34:** Dictionary

Mock Exams Term Info Sheet

Purpose & Rationale for Mock Exams

We run the mock exams for **free** for students and are **compulsory to attend**, as it offers students an opportunity to experience an entire exam-length exam under timed and invigilated conditions. It is an **extremely high-yield** session which is an excellent reality check for their progress, and also helps inform tutors about their learning.

Campuses & Days Run, Subjects

Days

Normal Classes will still be run during mock exams (no rescheduled classes).

Exam Dates: Term 1 Holidays

- **Week 1:** Mon 7/04 – Fri 11/04
- **Week 2:** Mon 14/04 only

Exceptions: No CBD exams on Wednesday & limited spots at Glen due to regular 1:45 PM - 3:45 PM classes. Less availabilities on Thursday due to regular 1:45 PM - 3:45 PM

Point Cook Tuesday Only, **Narre Warren** Thursday Only

Campuses

Glen Waverley: 07/04 - 11/04 (Mon - Fri) & 14/04 (Mon)

CBD: 07/04 - 08/04 & 10 - 11/04 (Mon, Tue, Thu, Fri) & 14/04 (Mon)

Box Hill: 07/04 - 11/04 (Mon - Fri) & 14/04 (Mon)

Point Cook: 08/04 (Tue) – MM, SM12, CH, BI, PH34

Narre Warren: 10/04 (Thu) - MM, SM, CH, BI, PH

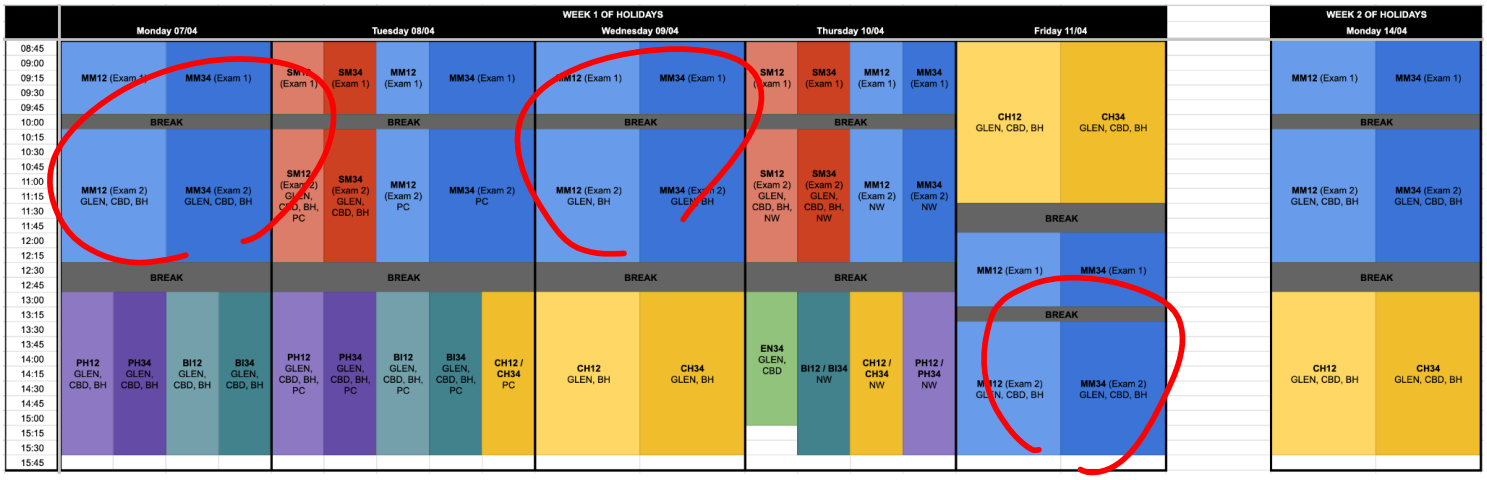
Subjects Running Mock Exams

- **MM12:** AOS 1, 2, 3
- **MM34:** AOS 1, 2
- **SM12:** AOS 1, 2, 3
- **SM34:** AOS 1, 2, 3
- **BI12:** AOS 1, 2 (Unit 1)
- **BI34:** AOS 1, 2 (Unit 3)
- **CH12:** AOS 1 & Half of AOS 2 (2.1-2.3)
- **CH34 (Normal):** AOS 1 & 2.1–2.4
- **CH34 (Flipped):** AOS 1 & 2.6–2.9
- **PH12:** AOS 1, 2, 3
- **PH34:** AOS 1, & 2.1
- **EN34:** Section A & Section B (No Section C)

Subjects NOT Running Mock Exams

- **MA07:** (Already run CATs)
- **MA08:** (Already run CATs)
- **MA09:** (Already run CATs)
- **MA10:** (Already run CATs)
- **EL34:** (New Subject)

Exam Schedule, Subject Specific Notes



[Link to Spreadsheet Here]

Subject Specific Notes

Chemistry: Running two sets of exams

- **Normal Exam:** Covers AOS 1 & 2.1–2.4
- **Flipped Exam:** Covers AOS 1 & 2.6–2.9

Booking: Students select either *Normal* or *Flipped* on the booking form. Both exams run at the same time and location. Students sit in the same room but receive different exams.

English:

Mock Exam Duration: 2 hours writing + 10 minutes reading (instead of 3 hours writing + 15 minutes reading)

Content Note: Section C (Argument Analysis) is excluded as most students haven't covered it yet. Reading time is reduced since there's no Section C passage.

Additional Things For Students to Bring:

- **MM12:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
- **MM34:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
- **SM12:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
- **SM34:** TI/Casio/Mathematica and Bound Reference for Exam 2 (only VCAA Formula Sheet Provided)
- **BI12:** N/A
- **BI34:** N/A
- **CH12:** Scientific Calculator (only VCAA Data Book Provided)
- **CH34:** Scientific Calculator (only VCAA Data Book Provided)
- **PH12:** Scientific Calculator, Cheat Sheet (only VCAA Formula Sheet Provided)
- **PH34:** Scientific Calculator, Cheat Sheet (only VCAA Formula Sheet Provided)
- **EN34:** Dictionary

Frequently Asked Questions (FAQ)

How Exams Run & What to Bring

What to Bring: Pens, pencils, eraser, ruler, calculator (if permitted), form of ID (to verify identity)

Materials Provided on Day: All other materials (exam paper, MCQ sheet, VCAA formula/data sheets) will be provided.

Arrival Time: Arrive at least 15 minutes before the exam starts.

Exam Conditions: Exams are invigilated and run under strict exam conditions.

Exam Quality: Exams are carefully constructed to cover key learning objectives and are thoroughly proofread for high quality.

Student Attendance for Mock Exams

Mock Exam Eligibility: Students can only sit mock exams for subjects they are enrolled in at Contour Education (e.g., enrolled in Chemistry only = Chemistry exam only).

Number of Mock Exams Run: Students only need to sit **one** exam per subject - the different times are all the same exam - just different availability.

Attendance: Attendance is **compulsory** unless students are on holiday or have valid reasons.

Student Reschedules

Reschedules: If a student can't attend, an online copy and solutions will be available one week later on the LMS. Their exams will not be marked by us.

Walkthrough Videos: A video walkthrough will be uploaded to the LMS, explaining marking criteria and solutions.

Marking & Reviewing Exam on LMS

Marking & Results: Exams are scanned, marked, and returned as PDFs via email—usually within 2 weeks.

Walkthrough Videos: A video walkthrough will be uploaded to the LMS, explaining marking criteria and solutions.

The Exact Values Table

x	$0\ (0^\circ)$	$\frac{\pi}{6}\ (30^\circ)$	$\frac{\pi}{4}\ (45^\circ)$	$\frac{\pi}{3}\ (60^\circ)$	$\frac{\pi}{2}\ (90^\circ)$
$\sin(x)$	$\frac{\sqrt{0}}{2}$ 0	$\frac{\sqrt{1}}{2}$ $\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$ $\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$ 1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

↳ $\frac{\sin}{\cos}$

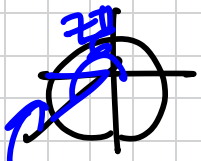
$\pi = 180^\circ$

warm-up

(a) $\sin\left(\frac{7\pi}{6}\right)$ walk-through

↓

quad Q R S
not sign

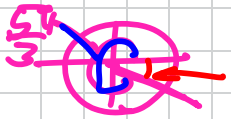


$\frac{7\pi}{6} - \pi = \frac{1}{6}$

$= -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$

(b) $\tan\left(\frac{5\pi}{3}\right)$



Q R S
4 2q −ve

$-\frac{5\pi}{3}$

$= \frac{\pi}{3}$

S | A
T | C

$= -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

(c) $\cos\left(\frac{3\pi}{4}\right)$

Q R S
2 T (−)

$= -\cos\left(\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}$

Section A: Particular and General Solutions

Sub-Section: Recap of Particular Solutions

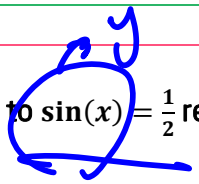
Active Recall: Period of trigonometric function

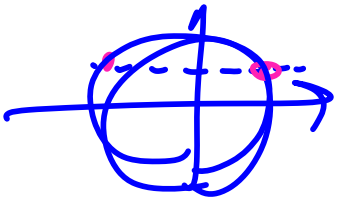
Period of $\sin(nx)$ and $\cos(nx)$ functions = $\frac{2\pi}{n}$

Period of $\tan(nx)$ functions = $\frac{\pi}{n}$

where n = coefficient of x and $n > 0$

Discussion: How often would the solution to $\sin(x) = \frac{1}{2}$ repeat?





1 cycle : 2 + period

Active Recall: Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add and subtract the period to find all other solutions in the domain.

sin( )

Question 1 Walkthrough.

QR angle period

quad, for one period, reference angle, add/subtract

Solve the following equation for x over the domain specified:

$$2\cos(2x) + \sqrt{2} = 0 \text{ for } x \in [0, 2\pi]$$

$$\cos(2x) = -\frac{\sqrt{2}}{2}$$

quadrant

quad 2 & 3

reference angle (table)

$\text{ref} = \frac{\pi}{4}$

angles (1 cycle)

$2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$2x = \frac{3\pi}{4}, \frac{5\pi}{4}$

$x = \frac{3\pi}{8}, \frac{5\pi}{8}$

quad 1: arg = ref

quad 2: arg = π − ref

quad 3: arg = π + ref

quad 4: arg = 2π − ref

period $\frac{2\pi}{2} = \pi$

$x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$

+π

Space for Personal Notes

Question 2

Solve the following equations for x over the domains specified:

a. $\sin(3x) = -1$ for $x \in [-\pi, \pi]$.

```
In[3]:= Reduce[Sin[3 x] == -1 && -Pi ≤ x ≤ Pi]
Out[3]= x == -5 π/6 || x == -π/6 || x == π/2
```

b. $2\sin\left(2x - \frac{\pi}{2}\right) - 1 = 0$ for $x \in [0, 2\pi]$.

$\sin\left(2x - \frac{\pi}{2}\right) = \frac{1}{2}$

quadrant → 1 & 2

ref : $\frac{\pi}{6}$

↳ angle (1 cycle)

$2x - \frac{\pi}{2} = \frac{\pi}{6}$ (1), $\pi - \frac{\pi}{6}$ (2)

$2x - \frac{\pi}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

± period

↳ $\frac{2\pi}{n} = \frac{2\pi}{2} = \pi$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ $\frac{4\pi}{3}$ $\frac{5\pi}{3}$

$+\pi$

Question 3 Walkthrough.

Solve the following equations for x over the domains specified:

$$3\tan(2x-\pi)-3\sqrt{3}=0 \text{ for } x\in[0,2\pi]$$

$$\tan(2x-\pi)=\frac{3\sqrt{3}}{3}$$

$$\tan(2x-\pi)=\sqrt{3}$$

quad → 1 & 3

ref → $\frac{\pi}{3}$

angle

① ③

$$2x-\pi=\frac{\pi}{3},\ \pi+\frac{\pi}{3}$$

$$2x-\pi=\frac{\pi}{3},\ \frac{4\pi}{3}$$

$$2x=\frac{4\pi}{3},\ \frac{7\pi}{3}$$

$$x=\frac{2\pi}{3},\ \frac{7\pi}{6}$$

Period : $\frac{\pi}{n}=\frac{\pi}{2}$

$$x=\frac{\pi}{6},\ \cancel{\frac{2\pi}{3}},\ \frac{2\pi}{3},\ \frac{7\pi}{6},\ \cancel{\frac{7\pi}{6}},\ \frac{5\pi}{3}$$

$+\frac{\pi}{2}$ $-\frac{\pi}{2}$

Space for Personal Notes

Discussion: Why do we need to find one angle only for tangents?

quad 1 & 2

~~3 & 4~~

Question 4

Solve the following equation for x over the domain specified:

$$\sqrt{3}\tan\left(x-\frac{\pi}{4}\right)+1=0 \text{ for } x\in(0,3\pi)$$

$\tan(x-\frac{\pi}{4}) = -\frac{1}{\sqrt{3}}$

quad 2

ref : $\frac{\pi}{6}$

angle (1 cycle)

$x-\frac{\pi}{4} = \pi-\frac{\pi}{6}$

$x-\frac{\pi}{4} = \frac{5\pi}{6}$

$x = \frac{5\pi}{6}+\frac{\pi}{4}$

$x = \frac{10\pi}{12}+\frac{3\pi}{12}$

$x = \frac{13\pi}{12}$

± period

$\frac{\pi}{n} = \pi$

$x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{25\pi}{12}, \frac{37\pi}{12}$ ✗

$+\pi$ $-\pi$

Sub-Section: General Solutions

Discussion: How many solutions would there be for $x \in R$?

General Solutions

- Finding Infinite solutions to a trigonometric equation.
- **Steps**

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add $Period \cdot n$ where $n \in Z$.

Space for Personal Notes

Question 5 Walkthrough.

Find the general solutions to the following equations:

$$2\sin\left(2x+\frac{\pi}{2}\right)-1=0$$

$\sin(2x+\frac{\pi}{2})=\frac{1}{2}$

quad 1 & 2

ref $\frac{\pi}{6}$

angle (1 cycle)

① ②

$2x+\frac{\pi}{2}=\frac{\pi}{6}, \pi-\frac{\pi}{6}$

$2x+\frac{\pi}{2}=\frac{\pi}{6}, \frac{5\pi}{6}$

$2x=-\frac{\pi}{3}, \frac{\pi}{3}$

$x=-\frac{\pi}{6}, \frac{\pi}{6}$

± period

$\frac{2\pi}{2}=\pi$

$x=-\frac{\pi}{6}+\pi n$

$x=\frac{\pi}{6}+\pi n$

$n\in Z$

Active Recall: General Solutions

- Steps

1. Make the trigonometric function the subject.
2. Find the necessary angle for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add period · n where $n \in Z$.

Question 6

Find the general solutions to the following equations:

a. $-2\sin\left(3x+\frac{\pi}{4}\right)=\sqrt{2}$

$\sin\left(3x+\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$

Q: 3 & 4

R: $\frac{\pi}{4}$

angles

$3x+\frac{\pi}{4}=\pi+\frac{\pi}{4},\ 2\pi-\frac{\pi}{4}$

$3x+\frac{\pi}{4}=\frac{5\pi}{4},\ \frac{7\pi}{4}$

$3x=\pi,\ \frac{3\pi}{2}$

$x=\frac{\pi}{3},\ \frac{\pi}{2}$

period: $\frac{2\pi}{n}=\frac{2\pi}{3}$

$x=\frac{\pi}{3}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$

$x=\frac{\pi}{2}+\frac{2\pi}{3}n,\ n\in\mathbb{Z}$

b. $2\cos\left(2x+\frac{\pi}{6}\right)=1$

$\cos\left(2x+\frac{\pi}{6}\right)=\frac{1}{2}$

Q: 1, 4

R: $\frac{\pi}{3}$

$2x+\frac{\pi}{6}=\frac{\pi}{3},\ \frac{5\pi}{3}$

$2x=\frac{\pi}{6},\ \frac{3\pi}{2}$

$x=\frac{\pi}{12},\ \frac{3\pi}{4}$

period $\frac{2\pi}{2}=\pi$

$x=\frac{\pi}{12}+\pi n,\ n\in\mathbb{Z}$

$x=\frac{3\pi}{4}+\pi n,\ n\in\mathbb{Z}$

c. $4\sin\left(3x-\frac{\pi}{6}\right)=2$

$\sin\left(3x-\frac{\pi}{6}\right)=\frac{1}{2}$

Q: 1, 2

R: $\frac{\pi}{6}$

$3x-\frac{\pi}{6}=\frac{\pi}{6}, \frac{5\pi}{6}$

$3x=\frac{\pi}{3}, \pi$

$x=\frac{\pi}{9}, \frac{\pi}{3}$

$\frac{2\pi}{3}$

$x=\frac{\pi}{9}+\frac{2\pi}{3}n, n\in\mathbb{Z}$

$x=\frac{\pi}{3}+\frac{2\pi}{3}n, n\in\mathbb{Z}$

Space for Personal Notes

Question 7 Walkthrough.

Find the general solutions to the following equations:

$$\tan\left(\frac{1}{2}x-\pi\right)-\frac{1}{\sqrt{3}}=0$$

```
In[14]:= Solve[Tan[1/2 x - Pi] - 1/Sqrt[3] == 0] // Expand

Out[14]= {{x -> Pi/3 + 2 Pi c1 if c1 ∈ Z}}
```

NOTE: We only need to find one angle for tangents!

Space for Personal Notes

Question 8

Find the general solutions to the following equations:

a. $\sqrt{3} - \tan\left(2\left(x + \frac{\pi}{3}\right)\right) = 0$

In[15]:= `Solve[ √3 - Tan[2 (x + Pi / 3)] == 0] // Expand`

Out[15]= $\left\{\left\{x \to -\frac{\pi}{6} + \frac{\pi c_1}{2} \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$

b. $2\tan\left(2x - \frac{\pi}{4}\right) = 2$ $2\tan\left(2x - \frac{\pi}{4}\right) = 2$

In[17]:= `Solve[2 Tan[2 x - Pi / 4] == 2] // Expand`

Out[17]= $\left\{\left\{x \to \frac{\pi}{4} - \frac{\pi c_1}{2} \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$

$\sqrt{3}\tan\left(3x - \frac{\pi}{6}\right) = 1$

c. $\sqrt{3}\tan\left(3x - \frac{\pi}{6}\right) = 1$

In[19]:= `Solve[ √3 Tan[3 x - Pi / 6] == 1] // Expand`

Out[19]= $\left\{\left\{x \to \frac{\pi}{9} - \frac{\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$

Sub-Section: Equivalent General Solutions

Discussion: Is $3 + 6k, k \in Z$ the same as $9 + 6k, k \in Z$?

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Question 9 Walkthrough.

Which one of the following is **not** the same as the rest?

A. $\frac{5\pi}{6} + \frac{\pi}{3}n, n \in Z$

B. $\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

C. $-\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

D. $\frac{5\pi}{3} + \frac{\pi}{3}n, n \in Z$

Space for Personal Notes

NOTE: Very important for multiple choice questions in VCAA exams!

Question 10

Which one of the following is **not** the same as the rest?

A. $\frac{2\pi}{3}+\frac{\pi}{4}n, n \in Z$

B. $\frac{5\pi}{8}+\frac{\pi}{4}n, n \in Z$

C. $-\frac{\pi}{3}+\frac{\pi}{4}n, n \in Z$

D. $\frac{7\pi}{6}+\frac{\pi}{4}n, n \in Z$

Space for Personal Notes

Section B: Advanced Trigonometric Algebra

Sub-Section: General Solutions with Domain Restrictions

Discussion: What is the main difference between the general and particular solution questions?

∞ Finite

Question 11 Walkthrough.

Solve the following trigonometric equation:

$$\sin\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

∞ ✓

quad: 1 & 2

ref: $\frac{\pi}{4}$

angles

$2x+\frac{\pi}{4}=\frac{\pi}{4}$ (1), $\pi-\frac{\pi}{4}$ (2)

$2x+\frac{\pi}{4}=\frac{\pi}{4}, \frac{3\pi}{4}$

$2x=0, \frac{\pi}{2}$

$x=0, \frac{\pi}{4}$

period

$\frac{2\pi}{n}=\pi$

$x=0+\pi n \quad n\in Z^+\cup\{0\}$

$x=\frac{\pi}{4}+\pi n \quad n\in Z^+\cup\{0\}$

General Solution with Domain Restriction

$$E.G\ \text{trig}\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\ \text{for}\ x\geq 0$$

- We can have infinite solutions for a restricted domain.
- The value of n is also restricted.

Space for Personal Notes

Your Turn!

Question 12

Solve the following trigonometric equations:

a. $\cos\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$ for $x<0$.

Q: 1 & 4

R: $\frac{\pi}{3}$

$2x-\frac{\pi}{6}=\frac{\pi}{3}, 2\pi-\frac{\pi}{3}$

$2x-\frac{\pi}{6}=\frac{\pi}{3}, \frac{5\pi}{3}$

$2x=\frac{\pi}{2}, \frac{11\pi}{6}$

$x=\frac{\pi}{4}, \frac{11\pi}{12}$

period $=\frac{2\pi}{n}=\pi$

$x=\frac{\pi}{4}+\pi n$, $n\in\mathbb{Z}^-$

$x=\frac{11\pi}{12}+\pi n$

b. $2\sin\left(3x+\frac{\pi}{3}\right)=\sqrt{3}$ for $x>0$.

$\sin\left(3x+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$

Q: 1 & 2

R: $\frac{\pi}{3}$

$3x+\frac{\pi}{3}=\frac{\pi}{3}, \frac{2\pi}{3}$

$3x=0, \frac{\pi}{3}$

$x=0, \frac{\pi}{9}$

period $=\frac{2\pi}{3}$

$x=\frac{2\pi}{3}n$, $n\in\mathbb{Z}^+$

$x=\frac{\pi}{9}+\frac{2\pi}{3}n$, $n\in\mathbb{Z}^+\cup\{0\}$

c. $\tan\left(2x-\frac{\pi}{4}\right)+\sqrt{3}=0$ for $x \leq 0$.

$\tan\left(2x-\frac{\pi}{4}\right)=-\sqrt{3}$

Q: 2

R: $\frac{\pi}{3}$

$2x-\frac{\pi}{4}=\pi-\frac{\pi}{3}$

$2x-\frac{\pi}{4}=\frac{2\pi}{3}$

$2x=\frac{11\pi}{12}$

$x=\frac{11\pi}{24}+\frac{\pi}{2}n,\ n\in\mathbb{Z}$

NOTE: This was assessed in a VCAA exam!

Space for Personal Notes

Sub-Section: Hidden Quadratics

Let's have a look at hidden quadratics for circular functions!

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A = f(x)$

Question 13 Walkthrough.

Solve the following for the values of x:

$$\sin^2\left(x+\frac{\pi}{3}\right) + \sin\left(x+\frac{\pi}{3}\right) = 2, 0 \le x \le 3\pi$$

$A = \sin(x+\frac{\pi}{3})$

$A^2 + A = 2$

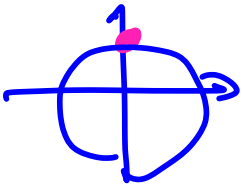
$A^2 + A - 2 = 0$

$(A+2)(A-1) = 0$

$A = -2, 1$

$\sin(x+\frac{\pi}{3}) = -2$, Reject

$\sin(x+\frac{\pi}{3}) = 1$



$x + \frac{\pi}{3} = \frac{\pi}{2}$

$x = \frac{\pi}{6}$

period $= \frac{2\pi}{n} = 2\pi$

$x = \frac{\pi}{6}, \frac{13\pi}{6}$

$+2\pi$

NOTE: sin and cos are between -1 and 1.

Question 14

Solve the following for the values of x:

a. $2\cos^2(2x) + 5\cos(2x) = 3, 0 \leq x \leq 2\pi$

$A = \cos(2x)$

$2A^2 + 5A = 3$

$2A^2 + 5A - 3 = 0$

$(2A - 1)(A + 3) = 0$

$A = \frac{1}{2}, \cancel{-3}$

$\cos(2x) = \frac{1}{2}$

$\cos(2x) = \frac{1}{2}$

Q: 1, 4

R: $\frac{\pi}{3}$

$2x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ period: π

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\frac{7\pi}{6}, \frac{11\pi}{6}$

$+\pi$

b. $4\tan^2\left(x - \frac{\pi}{4}\right) - 3\tan^2\left(x - \frac{\pi}{4}\right) = 1, -\pi \leq x \leq \pi$

$A = \tan(x - \frac{\pi}{4})$

$4A^2 - 3A^2 = 1$

$A^2 = 1$

$A = \pm 1$

$\tan(x - \frac{\pi}{4}) = 1$

quad: 1

ref: $\frac{\pi}{4}$

$\tan(x - \frac{\pi}{4}) = -1$

quad: 2

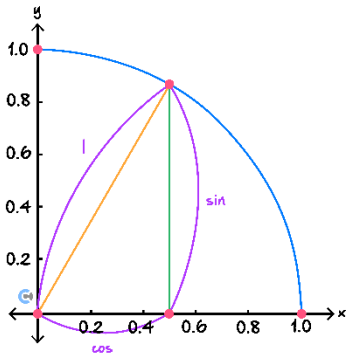
ref: $\frac{\pi}{4}$

$x - \frac{\pi}{4} = \frac{\pi}{4}$ period $= \pi$ $x - \frac{\pi}{4} = \pi - \frac{\pi}{4}$

$x = -\frac{\pi}{2}, \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \cancel{\frac{5\pi}{2}}$ $x = -\pi, 0, \pi, \cancel{2\pi}$

$-\pi$ $+2\pi$

REMINDER: Pythagorean Identity



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

- Can be used for finding one trigonometry function by using the other.

Question 15 Extension.

Find the general solution to the following equation:

$$-4\sin^2(3x) + 6\cos(3x) = 0$$

```
In[46]:= Solve[-4 Sin[3 x]^2 + 6 Cos[3 x] == 0, x, Reals] // Expand
```

$$\text{Out[46]= } \left\{\left\{x \to -\frac{\pi}{9} + \frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z}\right\}, \left\{x \to \frac{\pi}{9} + \frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$$

TIP: $\sin^2(\theta) = 1 - \cos^2(\theta)$

Contour Check

Learning Objective: [4.2.1] – Solve general solutions for trigonometric functions

Key Takeaways

General Solutions

- Finding infinite solutions to a trigonometric equation.
- **Steps**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add period n where $n \in Z$.
- If there is a domain restriction, only step 4 changes, and we need to be more careful in specifying what values n can take.

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Learning Objective: [4.2.2] – Solve hidden quadratic equations for trigonometric functions

Key Takeaways

- Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

Let $A =$ f(x)

- May need to use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) =$ 1

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45 + raw scores, 99 + ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
Book via bit.ly/contour-methods-consult-2025 (or QR code below). One active booking at a time (must attend before booking the next).	Message +61 440 138 726 with questions. Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726

CONTOUREDUCATION

Website: contoureducation.com.au | **Phone:** 1800 888 300 | **Email:** hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support

Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45 + raw scores, 99 + ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

1-on-1 Video Consults	Text-Based Support
Book via bit.ly/contour-methods-consult-2025 (or QR code below). One active booking at a time (must attend before booking the next).	Message +61 440 138 726 with questions. Save the contact as "Contour Methods".

Booking Link for Consults
bit.ly/contour-methods-consult-2025

Number for Text-Based Support
+61 440 138 726