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VCE Mathematical Methods ½
Circular Function II [4.2]
Workbook

Outline:

Learning Objectives:

- ☐ **MM12 [4.2.1]** - Solve General Solutions for Trigonometric Functions
- ☐ **MM12 [4.2.2]** - Solve Hidden Quadratic Equations for Trigonometric Functions

Section A: Particular and General Solutions

Sub-Section: Recap of Particular Solutions

Active Recall: Period of trigonometric function

$$\text{Period of } \sin(nx) \text{ and } \cos(nx) \text{ functions} = \_\_\_\_\_\_\_\_\_\_$$

$$\text{Period of } \tan(nx) \text{ functions} = \_\_\_\_\_\_\_\_\_\_$$

$$\text{where } n = \text{coefficient of } x \text{ and } n > 0$$

Discussion: How often would the solution to $\sin(x) = \frac{1}{2}$ repeat?

Active Recall: Particular Solutions

- Solving trigonometric equations **for finite solutions.**
- **Steps**

1. Make the trigonometric function the subject.
2. Find the necessary ____________ for one period.
3. Solve for x by equating the necessary angles to the ______________ of the trigonometric functions.
4. Add and subtract the ______________ to find all other solutions in the domain.

Question 1 Walkthrough.

Solve the following equation for x over the domain specified:

$$2\cos(2x) + \sqrt{2} = 0 \text{ for } x \in [0, 2\pi]$$

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Question 2

Solve the following equations for x over the domains specified:

a. $\sin(3x) = -1$ for $x \in [-\pi, \pi]$.

b. $2\sin\left(2x - \frac{\pi}{2}\right) - 1 = 0$ for $x \in [0, 2\pi]$.

Question 3 Walkthrough.

Solve the following equations for x over the domains specified:

$$3\tan(2x - \pi) - 3\sqrt{3} = 0 \text{ for } x \in [0, 2\pi]$$

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Discussion: Why do we need to find one angle only for tangents?

Question 4

Solve the following equation for x over the domain specified:

$$\sqrt{3}\tan\left(x-\frac{\pi}{4}\right)+1=0 \text{ for } x\in(0,3\pi)$$

Sub-Section: General Solutions

Discussion: How many solutions would there be for $x \in R$?

General Solutions

- Finding ______________ solutions to a trigonometric equation.
- **Steps**
  1. Make the trigonometric function the subject.
  2. Find the necessary angle for one period.
  3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
  4. Add $Period \cdot n$ where $n \in Z$.

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Question 5 Walkthrough.

Find the general solutions to the following equations:

$$2\sin\left(2x+\frac{\pi}{2}\right)-1=0$$

<u>Active Recall:</u> **General Solutions**

- **Steps**

1. Make the trigonometric function the subject.
2. Find the necessary ____________ for one period.
3. Solve for x by equating the necessary angles to the inside of the trigonometric functions.
4. Add ______________________ where $n \in Z$.

Question 6

Find the general solutions to the following equations:

a. $-2\sin\left(3x+\frac{\pi}{4}\right)=\sqrt{2}$

b. $2\cos\left(2x+\frac{\pi}{6}\right)=1$

c. $4\sin\left(3x - \frac{\pi}{6}\right) = 2$

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Question 7 Walkthrough.

Find the general solutions to the following equations:

$$\tan\left(\frac{1}{2}x - \pi\right) - \frac{1}{\sqrt{3}} = 0$$

NOTE: We only need to find one angle for tangents!

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Question 8

Find the general solutions to the following equations:

a. $\sqrt{3} - \tan\left(2\left(x + \frac{\pi}{3}\right)\right) = 0$

b. $2\tan\left(2x - \frac{\pi}{4}\right) = 2$

c. $\sqrt{3}\tan\left(3x - \frac{\pi}{6}\right) = 1$

Sub-Section: Equivalent General Solutions

Discussion: Is $3 + 6k, k \in Z$ the same as $9 + 6k, k \in Z$?

Multiple Forms of a General Solution

$$a + Period \cdot n = b + Period \cdot n$$

If the difference of a and b is a multiple of period.

Question 9 Walkthrough.

Which one of the following is **not** the same as the rest?

A. $\frac{5\pi}{6} + \frac{\pi}{3}n, n \in Z$

B. $\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

C. $-\frac{\pi}{2} + \frac{\pi}{3}n, n \in Z$

D. $\frac{5\pi}{3} + \frac{\pi}{3}n, n \in Z$

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NOTE: Very important for multiple choice questions in VCAA exams!

Question 10

Which one of the following is **not** the same as the rest?

A. $\frac{2\pi}{3} + \frac{\pi}{4}n, n \in Z$

B. $\frac{5\pi}{8} + \frac{\pi}{4}n, n \in Z$

C. $-\frac{\pi}{3} + \frac{\pi}{4}n, n \in Z$

D. $\frac{7\pi}{6} + \frac{\pi}{4}n, n \in Z$

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Section B: Advanced Trigonometric Algebra

Sub-Section: General Solutions with Domain Restrictions

Discussion: What is the main difference between the general and particular solution questions?

Question 11 Walkthrough.

Solve the following trigonometric equation:

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \text{ for } x \geq 0$$

<u>General Solution with Domain Restriction</u>

$$E.G\ \text{trig}\left(2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\text{ for } x\geq 0$$

- We can have infinite solutions for a restricted domain.
- The value of n is also restricted.

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Your Turn!

Question 12

Solve the following trigonometric equations:

a. $\cos\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}$ for $x < 0$.

b. $2\sin\left(3x + \frac{\pi}{3}\right) = \sqrt{3}$ for $x > 0$.

c. $\tan\left(2x - \frac{\pi}{4}\right) + \sqrt{3} = 0$ for $x \leq 0$.

NOTE: This was assessed in a VCAA exam!

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Sub-Section: Hidden Quadratics

Let's have a look at hidden quadratics for circular functions!

Hidden Quadratics

$$af(x)^2 + bf(x) + c = 0$$

$$\text{Let } A = f(x)$$

Question 13 Walkthrough.

Solve the following for the values of x:

$$\sin^2\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{\pi}{3}\right) = 2, 0 \leq x \leq 3\pi$$

NOTE: $\sin$ and $\cos$ are between -1 and 1.

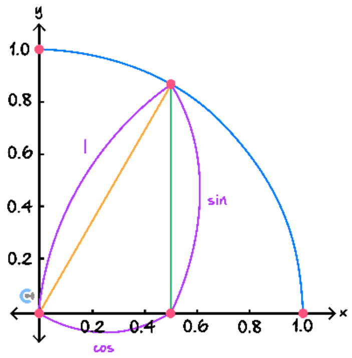
Question 14

Solve the following for the values of x:

a. $2\cos^2(2x) + 5\cos(2x) = 3, 0 \leq x \leq 2\pi$

b. $4\tan^2\left(x - \frac{\pi}{4}\right) - 3\tan^2\left(x - \frac{\pi}{4}\right) = 1, -\pi \leq x \leq \pi$

REMINDER: Pythagorean Identity



$$\sin^2(\theta) + \cos^2(\theta) = 1$$

- Can be used for finding one trigonometry function by using the other.

Question 15 Extension.

Find the general solution to the following equation:

$$-4\sin^2(3x) + 6\cos(3x) = 0$$

TIP: $\sin^2(\theta) = 1 - \cos^2(\theta)$

Contour Check

Learning Objective: [4.2.1] - Solve general solutions for trigonometric functions

Key Takeaways

- **General Solutions**
  - Finding ______________ solutions to a trigonometric equation.
  - **Steps**
    1. Make the trigonometric function the subject.
    2. Find the necessary ____________ for one period.
    3. Solve for x by equating the necessary angles to the ______________ of the trigonometric functions.
    4. Add ___________________________ where $n \in Z$.
  - If there is a domain restriction, only step 4 changes, and we need to be more careful in specifying what values n can take.
- **Multiple Forms of a General Solution**

$$a + Period \cdot n = b + Period \cdot n$$

If the ______________________ of a and b is a multiple of period.

☐ Learning Objective: [4.2.2] – Solve hidden quadratic equations for trigonometric functions

Key Takeaways

☐ **Hidden Quadratics**

$$af(x)^2 + bf(x) + c = 0$$

Let $A =$________

- May need to use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) =$_____

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