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VCE Mathematical Methods ½
Circular Function II [4.2]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
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Section A: Compulsory Questions

Sub-Section [4.2.1]: Fi

Head Tutor's Comment: For general solutions, answers may not necessarily look the same, but if the same values are represented then it is fine. i.e., As long as the general coefficient of $k\pi$ is the same number (can be positive or negative since k is all integers e.g., if a solution has $2k\pi$ then $-2k\pi$ is also fine) and the solution can reach the fixed value, then the general solution is valid (e.g $-\frac{\pi}{6}+2k\pi$ is the same as $\frac{11\pi}{6}+2k\pi$ as well as $-\frac{13\pi}{6}-2k\pi$).

Question 1

Find a general solution for x in the fo

a. $\sin\left(x-\frac{\pi}{3}\right)=\frac{1}{2}$

Reference angle: $\frac{\pi}{6},\frac{5\pi}{6}$

$$x-\frac{\pi}{3}=\frac{\pi}{6}+2k\pi,\frac{5\pi}{6}+2k\pi,k\in\mathbb{Z}$$

$$\therefore x=\frac{\pi}{2}+2k\pi,\frac{7\pi}{6}+2k\pi$$

b. $\cos(2x)=-\frac{1}{2}$

Reference angle: $\frac{2\pi}{3},\frac{4\pi}{3}$

$$2x=\frac{2\pi}{3}+2k\pi,\frac{4\pi}{3}+2k\pi,k\in\mathbb{Z}$$

$$\therefore 2x=\frac{\pi}{3}+k\pi,\frac{2\pi}{3}+k\pi$$

c. $\tan(-x)=1$

Reference angle: $\frac{\pi}{4}$

$$-x=\frac{\pi}{4}+k\pi,k\in\mathbb{Z}$$

$$x=-\frac{\pi}{4}-k\pi$$

d. $\sin\left(\frac{x}{2}\right) = -1$

Reference angle: $\frac{3\pi}{2}$

$$\frac{x}{2} = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$x = 3\pi + 4k\pi$$

e. $\cos(x + \pi) = 0$

Reference angle: $\frac{\pi}{2}, \frac{3\pi}{2}$

$$x + \pi = \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

$$x = -\frac{\pi}{2} + k\pi$$

f. $\tan(3x) = \sqrt{3}$

Reference angle: $\frac{\pi}{3}$

$$3x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$$

$$x = \frac{\pi}{9} + \frac{k}{3}\pi$$

Space for Personal Notes

Question 2

Find a general solution for x in the following expressions:

a. $2\sin\left(-3x-\frac{\pi}{6}\right)=\sqrt{3}$

$$\sin\left(-3x-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$$
$$-3x-\frac{\pi}{6}=\frac{\pi}{3}+2k\pi,\frac{2\pi}{3}+2k\pi,k\in\mathbb{Z}$$
$$-3x=\frac{\pi}{2}+2k\pi,\frac{5\pi}{6}+2k\pi$$
$$x=-\frac{\pi}{6}-\frac{2}{3}k\pi,-\frac{5\pi}{18}-\frac{2}{3}k\pi$$

b. $-\cos\left(2x+\frac{\pi}{3}\right)=\frac{1}{2}$

$$\cos\left(2x+\frac{\pi}{3}\right)=-\frac{1}{2}$$
$$2x+\frac{\pi}{3}=\frac{2\pi}{3}+2k\pi,\frac{4\pi}{3}+2k\pi,k\in\mathbb{Z}$$
$$2x=\frac{\pi}{3}+2k\pi,\pi+2k\pi$$
$$x=\frac{\pi}{6}+k\pi,\frac{\pi}{2}+k\pi$$

c. $1-\tan\left(\pi x+\frac{\pi}{4}\right)=0$

$$\tan\left(\pi x+\frac{\pi}{4}\right)=1$$
$$\pi x+\frac{\pi}{4}=\frac{\pi}{4}+k\pi,k\in\mathbb{Z}$$
$$\pi x=k\pi$$
$$x=k$$

d. $-3\sin\left(\frac{3}{2}x+\frac{\pi}{3}\right)=\frac{3}{2}$

$$\sin\left(\frac{3}{2}x+\frac{\pi}{3}\right)=-\frac{1}{2}$$
$$\frac{3}{2}x+\frac{\pi}{3}=\frac{7\pi}{6}+2k\pi,\frac{11\pi}{6}+2k\pi,k\in\mathbb{Z}$$
$$\frac{3}{2}x=\frac{5\pi}{6}+2k\pi,\frac{3\pi}{2}+2k\pi$$
$$x=\frac{5\pi}{9}+\frac{4}{3}k\pi,\pi+\frac{4}{3}k\pi$$

e. $4\cos\left(-\pi x-\frac{\pi}{2}\right)+2=0$

$$\cos\left(-\pi x-\frac{\pi}{2}\right)=-\frac{1}{2}$$
$$-\pi x-\frac{\pi}{2}=\frac{2\pi}{3}+2k\pi,\frac{4\pi}{3}+2k\pi,k\in\mathbb{Z}$$
$$-\pi x=\frac{7\pi}{6}+2k\pi,\frac{11\pi}{6}+2k\pi$$
$$x=-\frac{7}{6}-2k,-\frac{11}{6}-2k$$

f. $\frac{1}{2}\tan\left(-2x+\frac{\pi}{2}\right)=-\frac{\sqrt{3}}{2}$

$$\tan\left(-2x+\frac{\pi}{2}\right)=-\sqrt{3}$$
$$-2x+\frac{\pi}{2}=\frac{2\pi}{3}+k\pi,k\in\mathbb{Z}$$
$$-2x=\frac{7\pi}{6}+k\pi$$
$$x=-\frac{7\pi}{12}-\frac{k}{2}\pi$$

Question 3

Solve the following equations under their domain restrictions. Express your answers as general solutions.

a. $\sin\left(-2x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$, $x\in[0,\infty)$

$$-2x+\frac{\pi}{4}=\frac{\pi}{4}+2k\pi,\frac{3\pi}{4}+2k\pi$$
$$-2x=2k\pi,\frac{\pi}{2}+2k\pi$$
$$x=-k\pi,-\frac{\pi}{4}-k\pi=-k\pi,\frac{3\pi}{4}-k\pi,k\in\mathbb{Z}^-\cup\{0\}$$

b. $-2\cos\left(3x+\frac{\pi}{6}\right)+1=0$, $x\in(-\infty,0]$

$$\cos\left(3x+\frac{\pi}{6}\right)=\frac{1}{2}$$
$$3x+\frac{\pi}{6}=\frac{\pi}{3}+2k\pi,\frac{5\pi}{3}+2k\pi$$
$$3x=\frac{\pi}{6}+2k\pi,\frac{3\pi}{2}+2k\pi$$
$$x=\frac{\pi}{18}+\frac{2}{3}k\pi,\frac{\pi}{2}+\frac{2}{3}k\pi,k\in\mathbb{Z}^-$$

c. $\tan\left(-\frac{5x}{3}-\frac{\pi}{3}\right)=\sqrt{3}$, $x\in[-2\pi,\infty)$

$$-\frac{5x}{3}-\frac{\pi}{3}=\frac{\pi}{3}+k\pi$$
$$-5x-\pi=\pi+3k\pi$$
$$-5x=2\pi+3k\pi$$
$$x=-\frac{2\pi}{5}-\frac{3}{5}k\pi,k\in\mathbb{Z}^-\cup\{0,1,2,3\}$$

Question 4 Tech-Active.

Solve the following equations under their domain restrictions. Express your answers as general solutions.

a. $\sin\left(3x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $x \in (0, \infty)$

```
Assuming[x > 0, Solve[Sin[3 x - π/4] == √2/2, x]] // Expand
```

$\left\{\left\{x \to \frac{\pi}{6} - \frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z} \,\&\&\, c_1 \leq 0\right\}, \left\{x \to -\frac{\pi}{3} - \frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z} \,\&\&\, c_1 \leq -1\right\}\right\}$

$\text{solve}\left(\sin\left(3\cdot x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, x\right) \mid x > 0$

$x = \frac{(2\cdot n1 - 1)\cdot\pi}{3} \text{ and } 2\cdot n1 - 1 > 0 \text{ and } n1 \geq 1 \text{ or } x = \frac{(4\cdot n1 + 1)\cdot\pi}{6} \text{ and } 4\cdot n1 + 1 > 0 \text{ and } n1 \geq 0$

$\text{solve}(\sin(3x - \pi/4) = \sqrt{2}/2, x) \mid x > 0$

$\left\{x > 0 \text{ and } x = \frac{2\cdot\pi\cdot\text{constn}(1)}{3} + \frac{\pi}{3}, x > 0 \text{ and } x = \frac{2\cdot\pi\cdot\text{constn}(2)}{3} + \frac{\pi}{6}\right\}$

$x = \frac{\pi}{6} + \frac{2}{3}k\pi, \frac{\pi}{3} + \frac{2}{3}k\pi, k \in \mathbb{Z}^+$

b. $\cos\left(2x + \frac{\pi}{3}\right) = -\frac{1}{2}$, $x \in (-\infty, 0)$

```
Assuming[x < 0, Solve[Cos[2 x + π/3] == -1/2, x]] // Expand
```

$\left\{\left\{x \to \frac{\pi}{6} - \pi c_1 \text{ if } c_1 \in \mathbb{Z} \,\&\&\, c_1 \geq 1\right\}, \left\{x \to -\frac{\pi}{2} - \pi c_1 \text{ if } c_1 \in \mathbb{Z} \,\&\&\, c_1 \geq 0\right\}\right\}$

$x = -\frac{5\pi}{6} - k\pi, -\frac{\pi}{2} - k\pi, k \in \mathbb{Z}^+$

c. $\tan\left(-\frac{3x}{2}+\frac{\pi}{6}\right)=\sqrt{3}, x \in (-\infty,-2\pi) \cup (3\pi,\infty)$

```
Assuming[x < -2 π, Solve[Tan[-3x/2 + π/6] == √3, x]] // Expand
```

$\left\{\left\{x \to -\frac{\pi}{9}-\frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z} \&\& c_1 \geq 3\right\}\right\}$

```
Assuming[x > 3 π, Solve[Tan[-3x/2 + π/6] == √3, x]] // Expand
```

$\left\{\left\{x \to -\frac{\pi}{9}-\frac{2\pi c_1}{3} \text{ if } c_1 \in \mathbb{Z} \&\& c_1 \leq -5\right\}\right\}$

$x=-\frac{\pi}{9}-\frac{2}{3}k\pi, k \in (-\infty,5] \cup [3,\infty)$

Space for Personal Notes

Sub-Section [4.2.2]: Solving Hidden Quadratic Equations for Trigonometric Functions

Question 5

Solve the following equations for x.

a. $\sin^2(x) - 2\sin(x) + 1 = 0$

let $a = \sin(x)$

$a^2 - 2a + 1 = 0$

$(a-1)^2 = 0$

$a = 1$

$\therefore \sin(x) = 1 \quad \therefore x = \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

b. $2\cos^2(x) - \cos(x) - 1 = 0,\ x \in [0, 2\pi]$

let $a = \cos(x)$

$2a^2 - a - 1 = 0$

$(2a+1)(a-1) = 0$

$a = -\frac{1}{2}, 1$

$\cos(x) = -\frac{1}{2}, 1 \implies x = \frac{2\pi}{3} + 2k\pi,\ \frac{4\pi}{3} + 2k\pi,\ 2k\pi,\ k \in \mathbb{Z}$

between $[0, 2\pi]$: $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

c. $\tan^2(x) - 3 = 0$

let $a = \tan(x)$

$a^2 - 3 = 0$

$(a - \sqrt{3})(a + \sqrt{3}) = 0$

$a = \pm\sqrt{3} \Rightarrow \tan(x) = \pm\sqrt{3}$

$\therefore\ x = \frac{\pi}{3} + k\pi, \frac{2\pi}{3} + k\pi,\ k \in \mathbb{Z}$

Question 6

Solve the following equations for x.

a. $2\sin^2(x) + 3\sin(x) - 2 = 0,\ x \in [-2\pi, 0]$

let $a = \sin(x)$

$2a^2 + 3a - 2 = 0$

$(2a - 1)(a + 2) = 0$

$a = \frac{1}{2}, -2 \Rightarrow \sin(x) = \frac{1}{2}, -2$

but $\sin(x) \neq -2$ since range is $[-1, 1]$

$\Rightarrow x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

between $[-2\pi, 0]$: $x = -\frac{11\pi}{6}, -\frac{7\pi}{6}$

b. $\cos^2\left(x - \frac{\pi}{3}\right) - 3\cos\left(x - \frac{\pi}{3}\right) + 2 = 0$

let $a = \cos(x - \frac{\pi}{3})$

$a^2 - 3a + 2 = 0$

$(a-2)(a-1) = 0$

$a = 1, 2 \Rightarrow \cos(x) = 1, 2$

$\cos(x) \neq 2$ as range is $[-1, 1]$

$x = 2k\pi, k \in \mathbb{Z}$

c. $\tan^2\left(x + \frac{\pi}{2}\right) + \tan\left(x + \frac{\pi}{2}\right) = 0$

let $a = \tan(x + \frac{\pi}{2})$

$a^2 + a = 0$

$a(a+1) = 0$

$a = -1, 0 \Rightarrow \tan(x + \frac{\pi}{2}) = -1, 0$

$\Rightarrow x + \frac{\pi}{2} = \frac{3\pi}{4} + k\pi, k\pi, k \in \mathbb{Z}$

$\therefore x = \frac{\pi}{4} + k\pi, -\frac{\pi}{2} + k\pi$

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Question 7

Solve the following equations for x.

a. $2\sin^2(x) + 3\sin(-x) + 1 = 0$, $x \in [-\pi, \pi]$

$2\sin^2(x) + 3\sin(-x) + 1 = 2\sin^2(x) - 3\sin(x) + 1$

let $a = \sin(x)$

$(2a-1)(a-1) = 0$

$a = \frac{1}{2}, 1 \Rightarrow \sin(x) = \frac{1}{2}, 1$

$\Rightarrow x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

between $[-\pi, \pi]$: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

b. $2\cos^2\left(\frac{\pi}{2} - x\right) - \cos\left(x + \frac{\pi}{2}\right) - 1 = 0$

$2\cos^2\left(\frac{\pi}{2} - x\right) - \cos\left(x + \frac{\pi}{2}\right) - 1 = 2\sin^2(x) - (-\sin(x)) - 1$

$= 2\sin^2(x) + \sin(x) - 1$

let $a = \sin(x)$

$2a^2 + a - 1 = 0$

$(2a-1)(a+1) = 0$

$\Rightarrow a = -1, \frac{1}{2} \Rightarrow \sin(x) = -1, \frac{1}{2}$

$x = \pi + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$

c. $\tan^2(x) + \cos^2(x) + \sin^2(x) = 2,\ x < 0$

$\tan^2(x) + \underbrace{\cos^2(x) + \sin^2(x)}_{1} = 2$

$\tan^2(x) + 1 = 2$

$\tan^2(x) = 1$

$\tan(x) = \pm 1$

$x = \frac{\pi}{4} + k\pi,\ \frac{3\pi}{4} + k\pi,\ k \in \mathbb{Z}$

$\therefore x = \frac{\pi}{4} + \frac{k}{2}\pi$

Question 8 Tech-Active.

Solve the following equations for x, giving answers that are not a rational multiple of π to 2 decimal places.

a. $3\sin^2\left(2x - \frac{\pi}{3}\right) + 5\sin\left(2x - \frac{\pi}{3}\right) + 2 = 0,\ x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

```
Assuming[-π/2 ≤ x ≤ 3π/2, Solve[3 Sin[2x - π/3]^2 + 5 Sin[2x - π/3] + 2 == 0, x]]
{{x → -π/12}, {x → -π/12}, {x → 11π/12}, {x → 11π/12}, {x → 1/12 (-π - 6 ArcCos[2/3])}, {x → 1/12 (11π - 6 ArcCos[2/3])}, {x → 1/12 (-π + 6 ArcCos[2/3])}, {x → 1/12 (11π + 6 ArcCos[2/3])}}
```

```
Assuming[-π/2 ≤ x ≤ 3π/2, Solve[3 Sin[2x - π/3]^2 + 5 Sin[2x - π/3] + 2 == 0, x]] // N
{{x → -0.261799}, {x → -0.261799}, {x → 2.87979}, {x → 2.87979}, {x → -0.682334}, {x → 2.45926}, {x → 0.158735}, {x → 3.30033}}
```

```
solve(3*(sin(2x-π/3))^2+5*sin(2x-π/3)+2=0,x)|-π/2≤x≤3*π/2
{x=-0.2617993878,x=2.879793266,x=-0.6823337231,x=0.1587349475,x=2.459258931,x=3.300327601}
```

$\text{solve}\left(3\cdot\left(\sin\left(2\cdot x-\frac{\pi}{3}\right)\right)^2+5\cdot\sin\left(2\cdot x-\frac{\pi}{3}\right)+2=0,x\right)\Big|\frac{-\pi}{2}\le x\le\frac{3\cdot\pi}{2}$

$x=\frac{3\cdot\sin^{-1}\left(\frac{2}{3}\right)-2\cdot\pi}{6}$ or $x=\frac{-\pi}{12}$ or $x=\frac{-\left(3\cdot\sin^{-1}\left(\frac{2}{3}\right)-\pi\right)}{6}$ or $x=\frac{3\cdot\sin^{-1}\left(\frac{2}{3}\right)+4\cdot\pi}{6}$ or $x=\frac{11\cdot\pi}{12}$ or $x=\frac{-\left(3\cdot\sin^{-1}\left(\frac{2}{3}\right)-7\cdot\pi\right)}{6}$

$\text{solve}\left(3\cdot\left(\sin\left(2\cdot x-\frac{\pi}{3}\right)\right)^2+5\cdot\sin\left(2\cdot x-\frac{\pi}{3}\right)+2=0,x\right)\Big|\frac{-\pi}{2}\le x\le\frac{3\cdot\pi}{2}$

$x=-0.682334$ or $x=-0.261799$ or $x=0.158735$ or $x=2.45926$ or $x=2.87979$ or $x=3.30033$

```
solve(3*(sin(2x-π/3))^2+5*sin(2x-π/3)+2=0,x)|-π/2≤x≤3*π/2
```

$\left\{x=\frac{-\pi}{12},x=\frac{11\cdot\pi}{12},x=\frac{\sin^{-1}\left(\frac{2}{3}\right)}{2}-\frac{\pi}{3},x=\frac{-\sin^{-1}\left(\frac{2}{3}\right)}{2}+\frac{\pi}{6},x=\frac{\sin^{-1}\left(\frac{2}{3}\right)}{2}+\frac{2\cdot\pi}{3},x=\frac{-\sin^{-1}\left(\frac{2}{3}\right)}{2}+\frac{7\cdot\pi}{6}\right\}$

$x = -\frac{\pi}{12}, \frac{11\pi}{12}, -0.68, 2.46, 0.16, 3.30$

b. $-4\cos^2\left(-x+\frac{\pi}{4}\right)+3\cos\left(x-\frac{\pi}{4}\right)+1=0,\ x>0$

```
Assuming[x > 0, Solve[-4 Cos[-x + π/4]^2 + 3 Cos[x - π/4] + 1 == 0, x]] // Expand
{{x → π/4 - 2 π c1 if c1 ∈ ℤ && c1 ≤ 0}, {x → π/4 - ArcCos[-1/4] - 2 π c1 if c1 ∈ ℤ && c1 ≤ -1}, {x → π/4 + ArcCos[-1/4] - 2 π c1 if c1 ∈ ℤ && c1 ≤ 0}}
```

```
Assuming[x > 0, Solve[-4 Cos[-x + π/4]^2 + 3 Cos[x - π/4] + 1 == 0, x]] // Expand // N
{{x → 0.785398 - 6.28319 c1 if c1 ∈ ℤ && c1 ≤ 0.}, {x → -1.03808 - 6.28319 c1 if c1 ∈ ℤ && c1 ≤ -1.}, {x → 2.60887 - 6.28319 c1 if c1 ∈ ℤ && c1 ≤ 0.}}
```

$$x=\frac{\pi}{4}+2k\pi, 5.32+2k\pi, 2.61+2k\pi, k\in\mathbb{Z}^+\cup\{0\}$$

c. $2\tan^2\left(\frac{3x}{2}+\frac{\pi}{2}\right)+3\tan\left(\frac{3x}{2}+\frac{\pi}{2}\right)-5=0,\ x<2\pi$

```
Assuming[x < 2 π, Solve[2 Tan[3 x/2 + π/2]^2 + 3 Tan[3 x/2 + π/2] - 5 == 0, x]] // Expand
{{x → 2/3 ArcCot[5/2] + 2 π c1/3 if c1 ∈ ℤ && c1 ≤ 2}, {x → -π/6 + 2 π c1/3 if c1 ∈ ℤ && c1 ≤ 3}}
```

```
Assuming[x < 2 π, Solve[2 Tan[3 x/2 + π/2]^2 + 3 Tan[3 x/2 + π/2] - 5 == 0, x]] // Expand // N
{{x → 0.253671 + 2.0944 c1 if c1 ∈ ℤ && c1 ≤ 2.}, {x → -0.523599 + 2.0944 c1 if c1 ∈ ℤ && c1 ≤ 3.}}
```

$$x=-\frac{\pi}{6}+\frac{2}{3}k\pi, -1.84+\frac{2}{3}k\pi, k\in\mathbb{Z}^-\cup\{0,1,2,3\}$$

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Sub-Section: The 'Final Boss'

Question 9

Consider the function $f(x) = 2\sin^2\left(2x - \frac{\pi}{4}\right) - 3\sin\left(2x - \frac{\pi}{4}\right) + 1$.

a. Express the x-values of the x-intercepts of $f(x)$ using a general solution.

let $a = \sin\left(2x - \frac{\pi}{4}\right)$

$2a^2 - 3a + 1 = 0$, solve for a

$(2a-1)(a-1) = 0 \Rightarrow a = \frac{1}{2}, 1$

$\Rightarrow \sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{2}, 1$

$\therefore x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

b. Hence, find the coordinates of the x-intercepts in the interval:

i. $x \in [0, 2\pi]$

$$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$$

ii. $x \in [-2\pi, 0]$

$$\boxed{x = -\frac{11\pi}{6}, -\frac{3\pi}{2}, -\frac{7\pi}{6}}$$

iii. $x \in [0, \infty)$

$$\boxed{x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}^+}$$

Space for Personal Notes

Section B: Supplementary Questions

Sub-Section [4.2.1]: Finding General Solutions of Trigonometric Functions

Question 10

Find a general solution for x in the following expressions:

a. $\sin\left(x + \frac{\pi}{6}\right) = -\frac{1}{2}$

Reference angle: $\frac{7\pi}{6}, \frac{11\pi}{6}$

$x + \frac{\pi}{6} = \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$x = \pi + 2k\pi, \frac{5\pi}{3} + 2k\pi$

b. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

Reference angle: $\frac{\pi}{6}, \frac{11\pi}{6}$

$\frac{x}{2} = \frac{\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$x = \frac{\pi}{3} + 4k\pi, \frac{11\pi}{3} + 4k\pi$

c. $\tan(2x) = -1$

Reference angle: $\frac{3\pi}{4}$

$$2x = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$x = \frac{3\pi}{8} + \frac{k}{2}\pi$$

d. $\sin(x - \pi) = 0$

Reference angle: $0, \pi$

$$x - \pi = k\pi, k \in \mathbb{Z}$$

$$x = (k+1)\pi = k\pi$$

e. $\cos(3x) = 1$

Reference angle: 0

$$3x = 2k\pi, k \in \mathbb{Z}$$

$$x = \frac{2k}{3}\pi$$

f. $\tan\left(x+\frac{\pi}{2}\right)=\sqrt{3}$

Reference angle: $\frac{\pi}{3}$

$$x+\frac{\pi}{2}=\frac{\pi}{3}+k\pi,\ k\in\mathbb{Z}$$

$$x=-\frac{\pi}{6}+k\pi$$

Question 11

Find a general solution for x in the following expressions:

a. $-2\sin\left(-x+\frac{\pi}{2}\right)+1=0$

$$\sin\left(-x+\frac{\pi}{2}\right)=\frac{1}{2}$$

$$-x+\frac{\pi}{2}=\frac{\pi}{6}+2k\pi,\frac{5\pi}{6}+2k\pi,k\in\mathbb{Z}$$

$$-x=-\frac{\pi}{3}+2k\pi,\frac{\pi}{3}+2k\pi$$

$$x=\frac{\pi}{3}-2k\pi,-\frac{\pi}{3}-2k\pi$$

b. $\cos\left(-2x-\frac{\pi}{6}\right)=-\frac{1}{2}$

$$-2x-\frac{\pi}{6}=\frac{2\pi}{3}+2k\pi,\frac{4\pi}{3}+2k\pi,k\in\mathbb{Z}$$
$$-2x=\frac{5\pi}{6}+2k\pi,\frac{3\pi}{2}+2k\pi$$
$$x=-\frac{5\pi}{12}-k\pi,-\frac{3\pi}{4}-k\pi$$

c. $3\tan(3x+\pi)+\sqrt{3}=0$

$$\tan(3x+\pi)=-\frac{\sqrt{3}}{3}$$
$$3x+\pi=\frac{5\pi}{6}+k\pi,k\in\mathbb{Z}$$
$$3x=-\frac{\pi}{6}+k\pi$$
$$x=-\frac{\pi}{18}+\frac{k}{3}\pi$$

d. $1+\sin\left(4x+\frac{\pi}{4}\right)=0$

$$\sin\left(4x+\frac{\pi}{4}\right)=-1$$
$$4x+\frac{\pi}{4}=\frac{3\pi}{2}+2k\pi,k\in\mathbb{Z}$$
$$4x=\frac{5\pi}{4}+2k\pi$$
$$x=\frac{5\pi}{16}+\frac{k}{2}\pi$$

e. $-\cos\left(\pi x - \frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$

$$\cos\left(\pi x - \frac{\pi}{2}\right) = -\frac{\sqrt{2}}{2}$$
$$\pi x - \frac{\pi}{2} = \frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi, k \in \mathbb{Z}$$
$$\pi x = \frac{5\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi$$
$$x = \frac{5}{4} + 2k, \frac{7}{4} + 2k$$

f. $\tan\left(-x - \frac{\pi}{3}\right) = -1$

$$-x - \frac{\pi}{3} = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}$$
$$-x = \frac{13\pi}{4} + k\pi$$
$$x = -\frac{13\pi}{4} - k\pi$$

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Question 12

Find the general solution for x in the following expressions:

a. $\sin\left(3x + \frac{\pi}{6}\right) = \frac{1}{2}$, $x > 0$

$$3x + \frac{\pi}{6} = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$
$$3x = 2k\pi, \frac{2\pi}{3} + 2k\pi$$
$$x = \frac{2k\pi}{3}, \frac{2\pi}{9} + \frac{2k}{3}\pi, k \in \mathbb{Z}^+$$

b. $\cos\left(-2x + \frac{\pi}{4}\right) = 0$, $x > 0$

$$-2x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$$
$$-2x = \frac{\pi}{4} + k\pi$$
$$x = -\frac{\pi}{8} - \frac{k}{2}\pi, k \in \mathbb{Z}^-$$

c. $\tan\left(4x - \frac{\pi}{2}\right) = \sqrt{3}$, $x < 0$

$$4x - \frac{\pi}{2} = \frac{\pi}{3} + k\pi$$
$$4x = -\frac{\pi}{6} + k\pi$$
$$x = -\frac{\pi}{24} + \frac{k}{4}\pi, k \in \mathbb{Z}^- \cup \{0\}$$

Sub-Section [4.2.2]: Solving Hidden Quadratic Equations for Trigonometric Functions

Question 13

Solve the following equations for x.

a. $\sin^2(x) + 2\sin(x) - 3 = 0$

let $a = \sin(x)$

$a^2 + 2a - 3 = 0$

$(a-1)(a+3) = 0$

$a = 1, -3 \quad \therefore \sin(x) = 1, \ \sin(x) \neq -3$

$\therefore x = \frac{\pi}{2} + 2k\pi, \ k \in \mathbb{Z}$

b. $\cos^2(x) - 5\cos(x) + 4 = 0$, $x \in [0, 2\pi]$

let $a = \cos(x)$

$a^2 - 5x + 4 = 0$

$(a-1)(a-4) = 0$

$\cos(x) = 1$

$x = 2k\pi, \ k \in \mathbb{Z}$

$\therefore x = 0, 2\pi$

c. $\tan^2(x) - \left(1+\sqrt{3}\right)\tan(x) + \sqrt{3} = 0$

let $a = \tan(x)$

$a^2 - (1+\sqrt{3})a + \sqrt{3} = 0$

$(a-1)(a-\sqrt{3}) = 0$

$a = 1, \sqrt{3} \Rightarrow \tan(x) = 1, \sqrt{3}$

$x = \frac{\pi}{4} + k\pi, \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$

Question 14

Solve the following equations for x.

a. $2\sin^2\left(x+\frac{\pi}{3}\right) + \sin\left(x+\frac{\pi}{3}\right) - 1 = 0$

let $a = \sin(x + \frac{\pi}{3})$

$2a^2 + a - 1 = 0$

$(2a-1)(a+1) = 0$

$a = \frac{1}{2}, -1 \Rightarrow \sin(x+\frac{\pi}{3}) = \frac{1}{2}, -1$

$\therefore x + \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\therefore x = -\frac{\pi}{6} + 2k\pi, \frac{\pi}{2} + 2k\pi, \frac{7\pi}{6} + 2k\pi$

b. $\cos^2\left(2x - \frac{\pi}{4}\right) + 3\cos\left(2x - \frac{\pi}{4}\right) + 2 = 0,\ x \in [-2\pi, 0]$

let $a = \cos(2x - \frac{\pi}{4})$

$a^2 + 3a + 2 = 0$

$(a+2)(a+1) = 0$

$a = -1, -2 \therefore \cos(2x - \frac{\pi}{4}) = -1,\ \cos(2x - \frac{\pi}{4}) \neq -2$

$2x - \frac{\pi}{4} = \pi + 2k\pi,\ k \in \mathbb{Z}$

$2x = \frac{5\pi}{4} + 2k\pi \quad \therefore x = \frac{5\pi}{8} + k\pi$

in $[-2\pi, 0]$: $x = -\frac{11\pi}{8}, -\frac{3\pi}{8}$

c. $\tan^2(\pi x) - 1 = 0,\ x \in [-1,1]$

let $a = \tan(\pi x)$

$a^2 - 1 = 0 \quad \therefore a = \pm 1$

$\therefore \tan(\pi x) = \pm 1$

$\pi x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in \mathbb{Z}$

$x = \frac{1}{4} + \frac{k}{2}$

in $[-1, 1]$: $x = -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}$

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Question 15

Solve the following equations for x.

a. $\cos^2\left(-2x+\frac{\pi}{6}\right)+2=1-2\sin\left(-2x+\frac{2\pi}{3}\right)$

$\cos^2(-2x+\frac{\pi}{6})+2\sin(-2x+\frac{\pi}{6}+\frac{\pi}{2})+1=0$

$\cos^2(-2x+\frac{\pi}{6})+2\cos(-2x+\frac{\pi}{6})+1=0$

let $a=\cos(-2x+\frac{\pi}{6})$

$a^2+2a+1=0 \Rightarrow (a+1)^2=0$

$\therefore a=-1 \therefore \cos(-2x+\frac{\pi}{6})=0$

$\therefore -2x+\frac{\pi}{6}=\pi+2k\pi, k\in\mathbb{Z}$

$\therefore -2x=\frac{5\pi}{6}+2k\pi$

$\therefore x=-\frac{5\pi}{12}-k\pi$

b. $\cos^2\left(\frac{3x}{2}-\frac{\pi}{2}\right)-2\sin\left(\frac{3x}{2}\right)-3=0,\ x>0$

$\cos^2(\frac{3x}{2}-\frac{\pi}{2})-2\sin(\frac{3x}{2})-3=0$

$\sin^2(\frac{3x}{2})-2\sin(\frac{3x}{2})-3=0$

let $a=\sin(\frac{3x}{2})$

$a^2-2a-3=0 \Rightarrow (a-3)(a+1)=0$

$\therefore a=3,-1 \Rightarrow \sin(\frac{3x}{2})=-1$

$\therefore \frac{3x}{2}=\frac{3\pi}{2}+2k\pi$

$\therefore x=\pi+\frac{4k}{3}\pi, k\in\mathbb{Z}^+$

c. $\sin^2\left(x+\frac{\pi}{4}\right) - 2\sin\left(x+\frac{\pi}{4}\right) + \cos^2\left(-\left(x+\frac{\pi}{4}\right)\right) = 0$

$\sin^2\left(x+\frac{\pi}{4}\right) - 2\sin\left(x+\frac{\pi}{4}\right) + \cos^2\left(-\left(x+\frac{\pi}{4}\right)\right) = 0$

$\sin^2\left(x+\frac{\pi}{4}\right) + \cos^2\left(x+\frac{\pi}{4}\right) - 2\sin\left(x+\frac{\pi}{4}\right) = 0$

$1 - 2\sin\left(x+\frac{\pi}{4}\right) = 0$

$\sin\left(x+\frac{\pi}{4}\right) = \frac{1}{2}$

$x+\frac{\pi}{4} = \frac{\pi}{6}+2k\pi, \frac{5\pi}{6}+2k\pi, \ k \in \mathbb{Z}$

$\therefore x = -\frac{\pi}{12}+2k\pi, \frac{7\pi}{12}+2k\pi$

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