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VCE Mathematical Methods ½

Circular Functions I [4.1]

Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2-Pg 26

Section A: Compulsory Questions

TIP: Drawing a unit circle will really help!

Sub-Section [4.1.1]: Evaluating Exact Values for Sine, Cosine And Tangent

Question 1

Find:

a. $\sin\left(\frac{\pi}{3}\right)$

$$\boxed{\frac{\sqrt{3}}{2}}$$

b. $\cos(45°)$

$$\boxed{\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$$

c. $\tan\left(\frac{\pi}{6}\right)$

$$\boxed{\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$$

d. $\sin(150°)$

$$\boxed{\frac{1}{2}}$$

e. $\cos\left(\frac{3\pi}{2}\right)$

$$\boxed{0}$$

f. $\tan(225°)$

$$\boxed{1}$$

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Question 2

Find:

a. $\sin(10\pi)$

Remember for any angles greater than $360°/2\pi$ or less than 0, add / subtract multiples of $360°$ or 2π for sin and cos until you reach an angle within $[0,2\pi]$

$$\therefore \sin(10\pi) = \sin(0) = 0$$

b. $\cos(390°)$

$$= \cos(30°) = \frac{\sqrt{3}}{2}$$

c. $\tan(-210°)$

$$= \tan(150°) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

d. $\sin(-150°)$

$$\boxed{= \sin(210°) = -\frac{1}{2}}$$

e. $\cos\left(-\frac{5\pi}{4}\right)$

$$\boxed{= \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}}$$

f. $\tan\left(\frac{15\pi}{4}\right)$

$$\boxed{= \tan\left(\frac{7\pi}{4}\right) = -1}$$

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Question 3 Tech-Active.

Find the value of the following expressions, giving your answer to 3 decimal places if an exact answer is not possible.

a. $\sin\left(\frac{\pi}{9}\right)$

$\text{Sin}\left[\frac{\pi}{9}\right]$ // N

0.34202

Document Settings

Display Digits: Float 6

Angle: Radian

Alg Standard Real Rad

Casio / TI remember to switch between degree and radian.

b. $\cos(742°)$

Cos[742 °] // N

0.927184

c. $\tan\left(\frac{\pi}{12}\right)$

$\text{Tan}\left[\frac{\pi}{12}\right]$

$2 - \sqrt{3}$

d. $\sin(105°)$

Sin[105 °]

$\frac{1 + \sqrt{3}}{2\sqrt{2}}$

e. $\cos\left(-\frac{29\pi}{12}\right)$

```
Cos[-29π/12]
```

$$\frac{-1+\sqrt{3}}{2\sqrt{2}}$$

f. $\tan(111.3°)$

```
Tan[111.3°]
-2.56487
```

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Sub-Section [4.1.2]: Applying Identities to Evaluate Trigonometric Functions

Question 4

Given that $\sin(\theta) = \frac{3}{5}$ and θ is in the first quadrant, find:

a. $\cos(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2(\theta) = 1$

$\frac{9}{25} + \cos^2(\theta) = 1$

$\cos^2(\theta) = \frac{16}{25}$

$\cos(\theta) = \pm\frac{4}{5}$, but θ in Q1 $\therefore$ cos +ve

$\therefore \cos(\theta) = \frac{4}{5}$

b. $\tan(\theta)$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\frac{3}{5}}{\frac{4}{5}}$$

$$= \frac{3}{4}$$

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Question 5

Given that $\cos(\theta) = -\frac{5}{13}$ and θ is in the third quadrant, find:

a. $\sin(\theta)$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin(\theta) = \pm\sqrt{1 - \cos^2(\theta)}$$

$$= \pm\sqrt{1 - \frac{25}{169}}$$

$$= \pm\sqrt{\frac{144}{169}}$$

$$= \pm\frac{12}{13}, \quad \theta \text{ in } Q3 \therefore \sin -\text{ve}$$

$$\therefore \sin(\theta) = -\frac{12}{13}$$

b. $\tan(\theta)$

$$\frac{12}{5}$$

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Question 6

A point P lies on the unit circle such that its y-coordinate is $\frac{4}{5}$ and it lies in the second quadrant. Let θ be the angle that the line segment between P and the origin makes with the positive direction of the x-axis.

a. State an interval (a, b), where $b - a = \frac{\pi}{2}$, that θ could fall into.

$\left(\frac{\pi}{2}, \pi\right)$

b. Find $\sin(\theta)$.

Since $\sin(\theta)$ corresponds to the y-value of any point on the unit circle, $\sin(\theta) = \frac{4}{5}$.

c. Hence, find $\cos(\theta)$.

Rearranging $\sin^2(\theta) + \cos^2(\theta) = 1$ and solving for $\cos(\theta)$ gives $\cos(\theta) = \pm\frac{3}{5}$, but since θ is in Q2, $\cos(\theta)$ must be negative $\therefore \cos(\theta) = -\frac{3}{5}$.

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Question 7 Tech-Active.

Given that $\cos(\theta) = 0.4$ and θ is in the fourth quadrant, find:

a. $\sin(\theta)$

```
Solve[s^2 + (2/5)^2 == 1, s]
```

$$\left\{\left\{s \to -\frac{\sqrt{21}}{5}\right\}, \left\{s \to \frac{\sqrt{21}}{5}\right\}\right\}$$

$\sin(\theta) = \pm\frac{\sqrt{21}}{5}$, but θ is in the fourth quadrant so $\sin(\theta)$ must be negative, so $\sin(\theta) = -\frac{\sqrt{21}}{5}$.

b. $\tan(\theta)$

$$-\frac{\sqrt{21}}{2}$$

c. The coordinates of the point on the unit circle corresponding to the angle θ.

$$\left(0.4, -\frac{\sqrt{21}}{5}\right)$$

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Sub-Section [4.1.3]: Apply Supplementary and Complementary Relationships to Evaluate Trigonometric Functions

Question 8

Let $\sin(\theta) = \frac{3}{5}$, where θ is in the first quadrant.

Find:

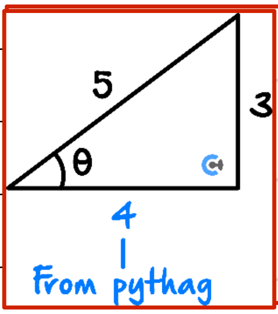
a. $\sin(\pi - \theta)$

$$= \sin(\theta) = \frac{3}{5}$$

b. $\cos\left(\frac{\pi}{2} + \theta\right)$

$$= -\sin(\theta) = -\frac{3}{5}$$

c. $\tan(\pi - \theta)$



$\tan(\theta) = \frac{3}{4}$

$\tan(\pi - \theta) = -\tan(\theta)$

$= -\frac{3}{4}$

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Question 9

Let $\sin(\theta) = \frac{8}{17}$, where θ is in the second quadrant.

Find:

a. $\cos\left(\frac{\pi}{2} - \theta\right)$

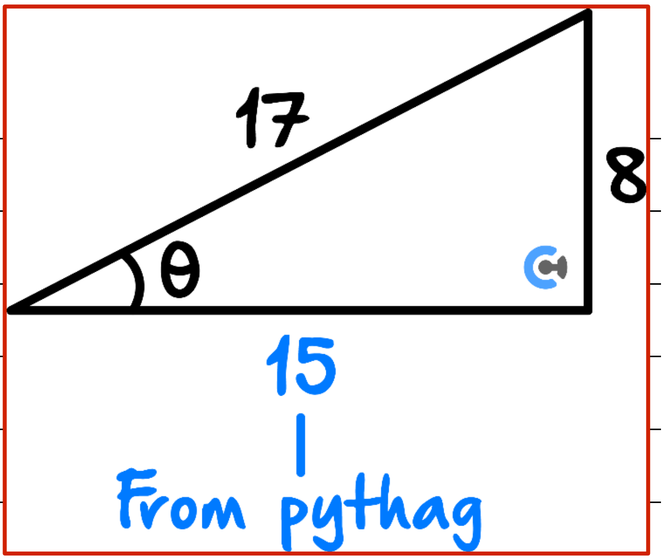
$$= \sin(\theta) = \frac{8}{17}$$

b. $\sin(\pi + \theta)$

$$= -\sin(\theta) = -\frac{8}{17}$$

c. $\tan(-\theta)$

$$= -\tan(\theta) = -\frac{8}{15}$$



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Question 10

Let α be an angle in the third quadrant such that $\sin(\alpha) = -\frac{5}{13}$ and β be an angle in the second quadrant such that $\cos(\beta) = -\frac{4}{5}$.

Find:

a. $\sin\left(\frac{\pi}{2} - \alpha\right)$

$\sin(\frac{\pi}{2} - \alpha) = \cos(\alpha)$

$\cos(\alpha) = \pm\sqrt{1 - \sin^2(\alpha)}$

$= \pm\frac{12}{13}$, α in Q3

$\therefore \cos$ -ve

$\therefore \sin(\frac{\pi}{2} - \alpha) = -\frac{12}{13}$

b. $\cos\left(\frac{\pi}{2} + \beta\right)$

$\cos(\frac{\pi}{2} + \beta) = -\sin(\beta)$

$\sin(\beta) = \pm\sqrt{1 - \cos^2(\beta)}$

$= \pm\frac{3}{5}$, β in Q2

$\therefore \sin$ +ve

$\therefore \cos(\frac{\pi}{2} + \beta) = -\frac{3}{5}$

c. $\tan\left(\frac{\pi}{2}+\alpha\right)\tan\left(\frac{\pi}{2}-\beta\right)$

$$\tan\left(\frac{\pi}{2}+\alpha\right) = -\frac{1}{\tan(\alpha)} = -\frac{\cos(\alpha)}{\sin(\alpha)} = -\frac{-\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$$

$$\tan\left(\frac{\pi}{2}-\beta\right) = \frac{1}{\tan(\beta)} = \frac{\cos(\beta)}{\sin(\beta)} = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$$

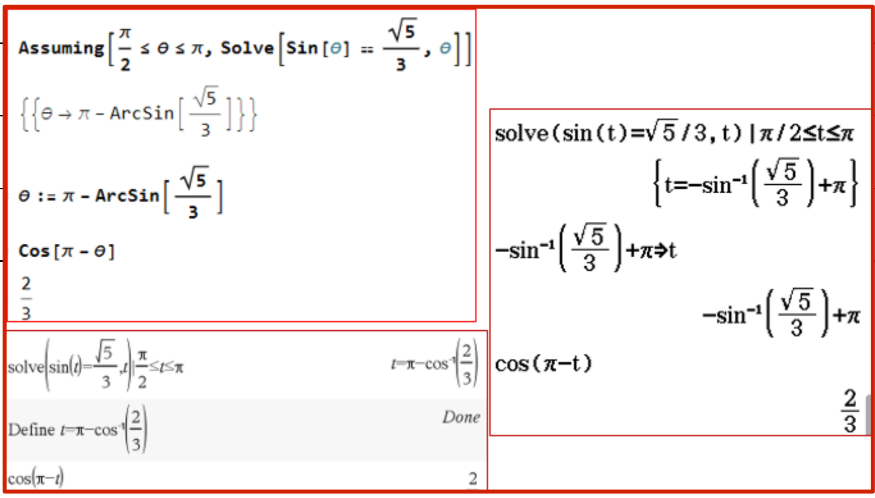
$$\Rightarrow \tan\left(\frac{\pi}{2}+\alpha\right)\tan\left(\frac{\pi}{2}-\beta\right) = -\frac{12}{5} \times -\frac{4}{3} = \frac{16}{5}$$

Question 11 Tech-Active.

Let $\sin(\theta) = \frac{\sqrt{5}}{3}$, where θ lies in the second quadrant.

Find:

a. $\cos(\pi - \theta)$



b. $\tan(-\theta)$

$\text{Tan}[-\theta]$ $\quad \frac{\sqrt{5}}{2}$

$\tan(-t) \quad \frac{\sqrt{5}}{2}$

$\tan(-t) \quad \frac{\sqrt{5}}{2}$

c. $\cos\left(\frac{\pi}{2}-\theta\right)$

$\text{Cos}\left[\frac{\pi}{2}-\theta\right]$ $\quad \frac{\sqrt{5}}{3}$

$\cos\left(\frac{\pi}{2}-t\right) \quad \frac{\sqrt{5}}{3}$

$\cos(\pi/2-t) \quad \frac{\sqrt{5}}{3}$

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Sub-Section [4.1.3]: Finding Particular Solutions for Trigonometric Functions

Question 12

Solve the following over the domain $x \in [0, 2\pi]$.

a. $\sin(x) = \frac{1}{2}$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

b. $\cos(x) = -\frac{1}{2}$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

c. $\tan(x) = 1$

$$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$$

d. $\sin(x) = -\frac{\sqrt{2}}{2}$

$$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$$

e. $\cos(x) = \frac{\sqrt{2}}{2}$

$$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$$

f. $\tan(x) = -\sqrt{3}$

$$\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{3}}$$

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Question 13

Solve the following over the domain $x \in [0, 2\pi]$.

a. $2\sin(2x) - \sqrt{3} = 0$

$\sin(2x) = \frac{\sqrt{3}}{2}$

$\Rightarrow 2x = \frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$x = \frac{\pi}{6} + k\pi, \frac{\pi}{3} + k\pi$

in $x \in [0, 2\pi]$: $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

b. $3 + 4\cos\left(3x - \frac{\pi}{4}\right) = 1$

$\cos(3x - \frac{\pi}{4}) = -\frac{1}{2}$

$3x - \frac{\pi}{4} = \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$3x = \frac{2\pi}{3} + \frac{\pi}{4} + 2k\pi, \frac{4\pi}{3} + \frac{\pi}{4} + 2k\pi$

$= \frac{11\pi + 24k\pi}{12}, \frac{19\pi + 24k\pi}{12}$

$= \frac{11\pi + 24k\pi}{36}, \frac{19\pi + 24k\pi}{36}$

in $x \in [0, 2\pi]$: $x = \frac{11\pi}{36}, \frac{19\pi}{36}, \frac{35\pi}{36}, \frac{43\pi}{36}, \frac{59\pi}{36}, \frac{67\pi}{36}$

c. $-2\tan\left(4x+\frac{\pi}{6}\right)+1=3$

$\tan\left(4x+\frac{\pi}{6}\right)=-1$

$4x+\frac{\pi}{6}=\frac{3\pi}{4}+k\pi, k\in\mathbb{Z}$

$4x=\frac{3\pi}{4}-\frac{\pi}{6}+k\pi$

$=\frac{7\pi+12k\pi}{12}$

$x=\frac{7\pi+12k\pi}{48}$

in $x\in[0,2\pi]$: $x=\frac{7\pi}{48},\frac{19\pi}{48},\frac{31\pi}{48},\frac{43\pi}{48},\frac{55\pi}{48},\frac{67\pi}{48},\frac{79\pi}{48},\frac{91\pi}{48}$

d. $2\cos\left(\frac{x}{2}\right)-\sqrt{2}=0$

$$\boxed{x=\frac{\pi}{2}}$$

e. $1 - 2\sin\left(3x + \frac{\pi}{3}\right) = 3$

$$\boxed{x = \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{4}}$$

f. $2\tan(5x) + 1 = 3$

$$\boxed{x = \frac{\pi}{20}, \frac{\pi}{4}, \frac{9\pi}{20}, \frac{13\pi}{20}, \frac{17\pi}{20}, \frac{21\pi}{20}, \frac{5\pi}{4}, \frac{29\pi}{20}, \frac{33\pi}{20}, \frac{37\pi}{20}}$$

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Question 14

Solve the following over the domain $x \in [-2\pi, 0]$.

a. $\sqrt{3}\sin(2x) + \cos(2x) = 0$

$\sqrt{3}\sin(2x) = -\cos(x)$

$\frac{\sin(2x)}{\cos(2x)} = -\frac{1}{\sqrt{3}}$

$\tan(2x) = -\frac{1}{\sqrt{3}}$

$2x = \frac{5\pi}{6} + k\pi, \ k \in \mathbb{Z}$

$\Rightarrow x = \frac{5\pi + 6k\pi}{12}$

in $x \in [-2\pi, 0]$: $x = -\frac{19\pi}{12}, -\frac{13\pi}{12}, -\frac{7\pi}{12}, -\frac{\pi}{12}$

b. $\sin(\pi - 3x) = \sin\left(\frac{\pi}{2} + 3x\right)$

$\sin(3x) = \cos(3x)$

$\Rightarrow \tan(3x) = 1$

$\therefore 3x = \frac{\pi}{4} + k\pi, \ k \in \mathbb{Z}$

$\therefore x = \frac{(1 + 4k)\pi}{12}$

in $x \in [-2\pi, 0]$: $x = -\frac{23\pi}{12}, -\frac{19\pi}{12}, -\frac{5\pi}{4}, -\frac{11\pi}{12}, -\frac{7\pi}{12}, -\frac{\pi}{4}$

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Question 15 Tech-Active.

Solve the following over the domain $x \in [0,2\pi]$, giving solutions to 3 decimal places.

a. $\sin(2x) + \cos(2x) = \frac{\sqrt{2}}{2}$

```
Assuming[0 ≤ x ≤ 2 π, Solve[Sin[2 x] + Cos[2 x] == √2/2, x]] // N
{{x → 3.01069}, {x → 6.15229}, {x → 0.916298}, {x → 4.05789}}
```

$$\text{solve}\left(\sin(2\cdot x)+\cos(2\cdot x)=\frac{\sqrt{2}}{2},x\right)|0\le x\le 2\cdot\pi$$

$x=0.916298$ or $x=3.01069$ or $x=4.05789$ or $x=6.15229$

```
solve(sin(2*x)+cos(2*x)=√2/2,x)|0≤x≤2*π
    {x=0.9162978573,x=3.01069296,x=4.057890511,x=6.152285613}
```

b. $\sin\left(x - \frac{\pi}{12}\right) - \cos\left(3x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{4}$

```
Assuming[0 ≤ x ≤ 2 π, Solve[Sin[x - π/12] - Cos[3 x + π/4] == √3/4, x]] // N
{{x → 3.2936}, {x → 0.371588}, {x → 1.4865}, {x → 2.17869}}
```

c. $\tan^2\left(\frac{x}{3} + \frac{\pi}{16}\right) = 7$

```
Assuming[0 ≤ x ≤ 2 π, Solve[Tan[x/3 + π/16]^2 == 7, x]] // N
{{x → 5.20744}, {x → 3.03924}}
```

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Sub-Section: The 'Final Boss'

Question 16

a. Find $\sin\left(\frac{\pi}{6}\right)$.

$$\frac{1}{2}$$

b. Hence, using the Pythagorean identity, find $\cos\left(\frac{\pi}{6}\right)$.

The Pythagorean identity must be used here:

$$\sin^2(x) + \cos^2(x) = 1$$

$$\cos(x) = \pm\sqrt{1 - \sin^2(x)}$$

$$\cos\left(\frac{\pi}{6}\right) = \pm\sqrt{1 - \frac{1}{4}} = \pm\frac{\sqrt{3}}{2}$$

$\frac{\pi}{6}$ in Q1 $\therefore \cos$ *is positive* $\therefore \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

c. Given that $\tan(\theta) = \frac{5}{12}$ and θ lies in quadrant 3, find:

i. $\sin(\theta)$

$\frac{\sin(\theta)}{\cos(\theta)} = \frac{5}{12}$

$\Rightarrow \cos(\theta) = \frac{12}{5}\sin(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\Rightarrow \sin^2(\theta) + \frac{144}{25}\sin^2(\theta) = 1$

$\Rightarrow \frac{169}{25}\sin^2(\theta) = 1$

$\Rightarrow \sin^2(\theta) = \frac{25}{169}$

$\therefore \sin(\theta) = \pm\frac{5}{13}$, Q3 $\therefore$ sin −ve

$\therefore \sin(\theta) = -\frac{5}{13}$

ii. $\cos(\theta)$

$\frac{-\frac{5}{13}}{\cos(\theta)} = \frac{5}{12}$

$\Rightarrow \frac{\cos(\theta)}{-\frac{5}{13}} = \frac{12}{5}$

$\therefore \cos(\theta) = -\frac{12}{13}$

iii. $\sin\left(\frac{\pi}{2} - \theta\right)$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) = -\frac{12}{13}$

iv. $\cos\left(\frac{\pi}{2}+\theta\right)$

$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin(\theta) = \frac{5}{13}$

d. Solve $\sin(x) + 2\cos(x) + \cos(x)\tan(x) = 0$ over the interval $x \in [-\pi, \pi]$.

$\sin(x) + 2\cos(x) + \cancel{\cos(x)}\frac{\sin(x)}{\cancel{\cos(x)}} = 0$

$\sin(x) + 2\cos(x) + \sin(x) = 0$

$2\sin(x) + 2\cos(x) = 0$

$2\sin(x) = -2\cos(x)$

$\frac{2\sin(x)}{2\cos(x)} = -1$

$\tan(x) = -1$

$x = \frac{3\pi}{4} + k\pi$

$\therefore x = -\frac{\pi}{4}, \frac{3\pi}{4}$

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