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VCE Mathematical Methods ½
Combinations & Permutations Exam Skills [3.4]
Workbook

Outline:

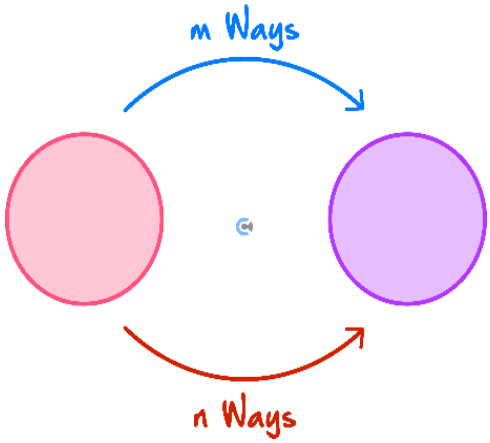
Learning Objectives:

- ☐ **MM12 [3.4.1]** - Applying Pascal's Triangle and Symmetrical Properties of Combinations
- ☐ **MM12 [3.4.2]** - Finding Selections of Any Size

Section A: Recap

If you were here last week, skip to Section B – Warmup Test.

Addition Principle



- Associated with the use of the word "**OR**."

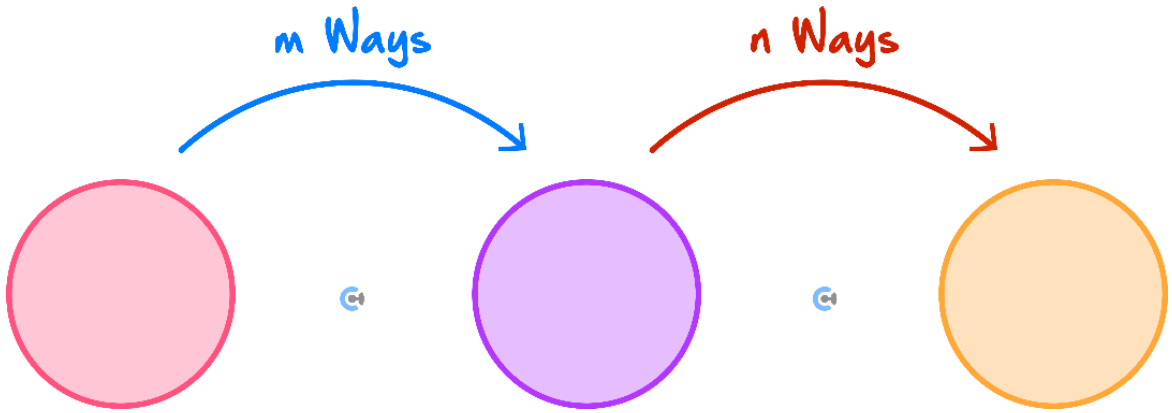
$$\text{Total Possibilities} = m + n$$

Question 1

A restaurant offers three vegan dishes or four vegetarian dishes.

How many selections of one main meal does a customer have?

The Multiplication Principle



- Associated with the use of the word "**AND**."

$$\text{Total Possibilities} = m \times n$$

Question 2

Emily has two different pants, five different tops and three different pairs of shoes.

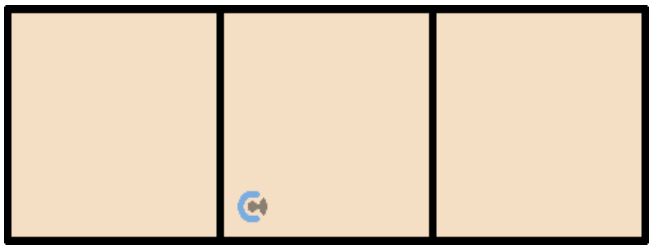
How many different choices does she have for a complete outfit?

Arrangements

- **Definition:** It is a study of a number of ways to arrange/order things.

Box Diagram for Arrangements

- **Definition:** We can use it to write down a number of arrangements for each position represented by each box.



Question 3

A family of 4 sits next to each other and is interested in a number of ways they could be seated.

How many different ways can the family of 4 sit in their 4 seats?

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Arrangement

- Generally:

$$\text{Ways to arrange/order } n \text{ many things for } r \text{ spots} = \frac{n!}{(n-r)!}$$

- We call this ${}^nP_r$.

$$^nP_r = \frac{n!}{(n-r)!}$$

Question 4

The teacher decides to pick 2 students from 10 student class and appoint them as a class captain and a vice-captain.

How many different ways could the teacher do this?

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Composite Arrangements

- **Definition:** Occurs when an arrangement happens within another arrangement.
- **Steps:**
  1. Consider each group as one object and find the arrangements.
  2. Consider the arrangements within the groups and multiply.

Question 5

Pranit wants to install seating plans for his rowdy class of 10 students. Fearing the backlash of the students, he lets them choose one friend to sit next to each other.

How many different seating plans could Pranit come up with?

Arrangements with Restrictions

- **Definition:** The general principle to deal with restrictions is to:
  - **Use the boxes.**
  - **Fill in the number of options for the slot that has the restriction first.**

Selection

- Generally:

$$\text{Ways to select } r \text{ things from } n \text{ many things} = \frac{{}^{n}P_{r}}{r!}$$

- We call this ${}^{n}C_{r}$,

$$\boldsymbol{{}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!} = \frac{n!}{r!\,(n-r)!}}$$

- Where r = number of selection spots.

Question 6

How many ways are there to choose exactly two pets from a store with 8 dogs and 12 cats?

Probability with Arrangements

$$\Pr = \frac{n\,(Wanted\ Arrangements)}{n\,(Total\ Arrangements)}$$

Question 7

A four-digit number is to be made by arranging the numbers $1, 2, 3, 4$. We cannot use the same number twice.

a. How many four-digit numbers can be made in general?

b. How many four-digit even numbers can be made?

c. Hence, what is the probability that the four-digit number is an even number?

Probability with Selections

$$\Pr = \frac{n\,(Wanted\ Selections)}{n\,(Total\ Selections)}$$

Question 8

What is the probability that a team of four chosen at random from a group of eight friends, four males and four females, would consist of three women and one man?

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Section B: Warm Up Test (10 Marks)

Question 9 (4 marks)

a. How many different ways can the letters in the word **MATH** be arranged? (1 mark)

_____

_____

b. A committee of 3 people are to be chosen from a group of 6. How many different committees can be formed? (2 marks)

_____

_____

_____

_____

c. A school has 5 different sports trophies. In how many ways can these trophies be arranged on a shelf? (1 mark)

_____

_____

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Question 10 (2 marks)

A 4-digit PIN code is to be created using the digits $1, 2, 3, 4, 5, 6$.

How many different PIN codes are possible if:

a. Each digit can only be used once? (1 mark)

b. Digits may be repeated? (1 mark)

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Question 11 (4 marks)

a. A group of 7 students is forming a leadership team with a President, Vice President, and Secretary. How many different leadership teams are possible? (2 marks)

_____

_____

_____

_____

b. A teacher is selecting 3 students from a class of 8, but one particular student must be included. How many different ways can the teacher choose the 3 students? (2 marks)

_____

_____

_____

_____

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Section C: Exam Skills

Sub-Section: Pascal's Triangle and Selections

Exploration: Using Pascal's Triangle to Find ${}^nC_r$

- Once a Pascal's triangle is constructed (add two numbers on the top for the number below),
- We can use the Pascal's triangle to get the combination ${}^nC_r$.

- Try to add another row to the Pascal's triangle above and find ${}^5C_3$.

$${}^5C_3 = \_\_\_\_\_$$

Pascals Triangle and ${}^nC_r$

Question 12

Using Pascal's triangle, find ${}^6C_4$.

Sub-Section: Symmetrical Property of Selections

Exploration: Symmetrical Property

- Consider the number of ways we can select 5 pets from a group of 12 available pets.
  - So, we're asking "which 5 pets do we select, if order didn't matter?", i.e. ${}^{12}C_5$.
  - BUT that's the same as asking "which 7 pets did we NOT select, if order didn't matter?" i.e. ${}^{12}C_7$ which should intuitively evaluate to the same result - and mathematically does as well!

$$ {}^{n}C_r = \_\_\_\_\_ $$

Symmetrical Property

$$ {}^{n}C_r = {}^{n}C_{n-r} $$

Question 13

It is known that ${}^{40}C_3 = 9880$. Find ${}^{40}C_{37}$.

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Sub-Section: Selections of Any Size

Exploration: Selections of Any Size

- Consider a situation where we want to count all the combinations of **any size** r from a group of n objects.
- We can use the following shortcut to evaluate this quickly:

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = \_\_\_\_\_$$

Selection of Any Size

Definition

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

Question 14 Walkthrough.

Emma is selecting books to take on vacation. She has 5 books to choose from and can take any number of books, including the option of not taking any at all. How many different ways can she make her selection?

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Question 15

Nick is making a guest list for his party. He has 7 friends to choose from – and he can choose to invite any number of friends (or none of them at all). How many possible guest lists exist?

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Section D: Exam 1 (17 Marks)

Question 16 (7 marks)

a. How many different ways can the letters in the word **PEPPER** be arranged? (2 marks)

b. A box contains 4 red balls, 3 blue balls, and 2 green balls. How many ways can 3 balls be chosen if they must be different colours? (2 marks)

c. Five friends A, B, C, D, E are standing in a line. If A must be at the front, how many different ways can the friends stand in line? (1 mark)

d. A teacher is dividing 6 students into two groups of 3. How many ways can this be done? (2 marks)

Question 17 (10 marks)

A debate team of 2 speakers and 2 moderators is to be selected from a group of 7 speakers and 6 moderators.

a. Find the number of ways this can be done if two of the speakers are siblings. They both must either be included in the team or excluded altogether. (3 marks)

The digits of the number 1 224 687 can be rearranged to form many different 7-digit numbers.

b. How many of these 7-digit numbers are even? (3 marks)

c. How many different numbers between 20,000 and 30,000 can be created using 5 different digits from the set $\{1, 2, 4, 6, 7, 8\}$? (2 marks)

Oliver is decorating a shelf using a row of 8 decorative tiles. The tiles come in three colours: blue, red, and yellow.

Each tile placement is equally likely to be any of these three colours.

d. Find the probability that no two adjacent tiles share the same colour. Leave your answer in the form $\left(\frac{a}{b}\right)^c$, where a, b, c are positive integers. (2 marks)

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Section E: Tech-Active Exam Skills

Calculator Commands: Factorial on Technology

Mathematica

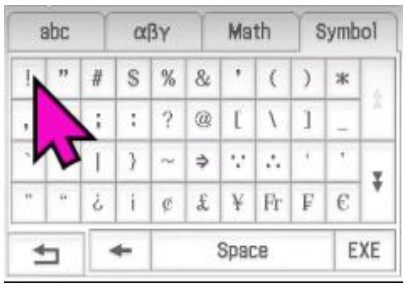
- Exclamation Mark

$x!$

TI-Nspire

- Menu 51

Casio Classpad



Calculator Commands: Arrangements on Technology

Mathematica

- FactorialPower

```
FactorialPower[n, r]
```

- OR make your own:

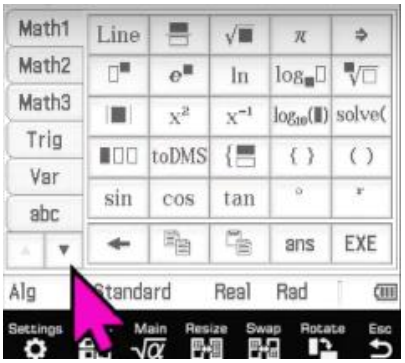
```
npr[n_, r_] := n! / (n - r)!
```

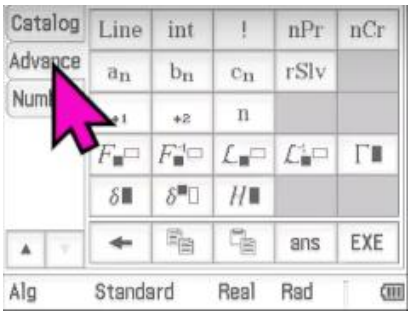
TI-Nspire

- Menu 52

${}^{n}P_{r}(n,r)$

Casio Classpad





${}^{n}P_{r}(n,r)$

Calculator Commands: Combinations of Technology

Mathematica

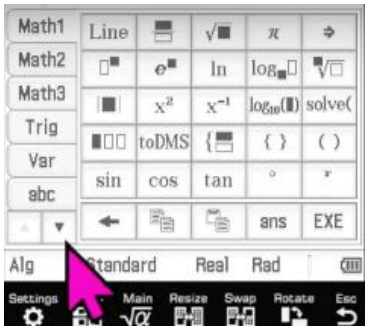
- Binomial $[n, r]$

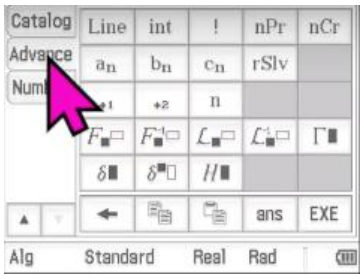
TI-Nspire

- Menu 53

$${}^{n}C_{r}\ (n, r)$$

Casio Classpad





$${}^{n}C_{r}\ (n, r)$$

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Section F: Exam 2 (37 Marks)

Question 18 (1 mark)

In how many ways can 3 different books be arranged on a shelf?

A. 3

B. 6

C. 9

D. 8

Question 19 (1 mark)

A committee of 4 members is to be selected from a group of 10 people. How many different committees can be formed?

A. 40

B. 210

C. 120

D. 5040

Question 20 (1 mark)

In how many ways can the letters of the word **APPLE** be arranged?

A. 60

B. 120

C. 30

D. 240

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Question 21 (1 mark)

How many different 4-digit numbers can be formed using the digits 1, 2, 3, 4, and 5, if repetition is **not** allowed?

A. 60

B. 240

C. 120

D. 625

Question 22 (1 mark)

A student can choose any number of books from a collection of 6 different books, including the option of choosing none. How many different selections can be made?

A. 15

B. 32

C. 64

D. 720

Question 23 (1 mark)

Which of the following is an equivalent expression for ${}^{100}C_{27}$?

A. ${}^{100}C_{73} + {}^{99}C_{73}$

B. ${}^{100}C_{77}$

C. ${}^{100}C_{73}$

D. ${}^{99}C_{73}$

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Question 24 (1 mark)

A code consists of 3 letters followed by 2 digits. If the letters are chosen from A, B, C, D, and E without repetition, and the digits are chosen from 0 to 9 with repetition allowed, how many such codes can be formed?

A. 5000

B. 6000

C. 12000

D. 10000

Question 25 (1 mark)

A school committee is to be formed with a president, a vice president, and 3 general members from a pool of 10 students. How many different committees can be formed?

A. 2520

B. 3024

C. 5040

D. 15120

Question 26 (1 mark)

Eight people are to be seated in a row, but two specific people must **not** sit next to each other. In how many ways can this be done?

A. $7!$

B. $8!$

C. $8! - 7!$

D. $8! - 7! \times 2!$

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Question 27 (1 mark)

A team of 5 is to be selected from a group of 8 girls and 6 boys, with the condition that at least 2 boys must be included. How many ways can this be done?

A. 540

B. 1526

C. 924

D. 1260

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Question 28 (9 marks)

A committee of 6 people is to be selected from a group of 5 men and 8 women.

a. Determine the number of ways this can be done in each of the following cases:

i. If the number of women in the committee exceeds the number of men. (4 marks)

ii. If the committee must include exactly 3 men and 3 women, but two specific men refuse to be on the same committee. (3 marks)

One particular committee consists of 5 women and 1 man.

b. In how many ways can the members of this specific committee be arranged in a line if the man is not positioned at either end? (2 marks)

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Question 29 (10 marks)

A library has 7 different history books, 5 different science books, and 8 different literature books available for borrowing. A student chooses 6 books from these 20.

a. In how many ways can the student make their selection if they borrow at most 3 history books and no more than 2 books from each of the other categories? (4 marks)

The student borrows 3 history books, 1 science book, and 2 literature books and arranges them on a shelf that has space for 10 books.

b. In how many ways can the books be arranged on the shelf if the history books must be placed together with no gaps, the literature books must also be together with no gaps, and all empty spaces are grouped together? (3 marks)

c. In how many ways can the books be arranged on the shelf if the literature books are positioned at each end and there are no gaps between any of the books?

Note that the position of empty space on the shelf matters. (3 marks)

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Question 30 (8 marks) **Extension.**

Seven fair dice, each with faces numbered 1 to 6, are rolled and arranged in a row.

a. Determine the number of possible ways to arrange them if the sum of the numbers on the two dice at the ends of the row is exactly 4. (3 marks)

Nine different books are to be distributed among Alex, Rei, and Subu in such a way that each person receives an odd number of books.

b. Find the number of ways this distribution can be carried out. (5 marks)

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Contour Check

☐ Learning Objective: [3.4.1] - Applying Pascal's triangle and symmetrical properties of combinations

Key Takeaways

☐ Pascal's Triangle and ${}^nC_r$:

- ○ A new entry in Pascal's triangle is found by adding the two entries _____ from the previous row.

☐ Symmetrical Property:

$$ {}^nC_r = \_\_\_\_\_ $$

☐ Learning Objective: [3.4.2] - Finding selections of any size

Key Takeaways

☐ Selection of Any Size:

$$ {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = \_\_\_\_\_ $$

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