



Website: contoureducation.com.au | Phone: 1800 888 300

Email: hello@contoureducation.com.au

VCE Mathematical Methods ½ Combinations & Permutations [3.3] Workbook

Outline:



Introduction to Counting Methods

Pg 2-4

- Addition Principle
- Multiplication Principle

Arrangements (Permutations)

Pg 5-17

- Introduction to Arrangements
- General Arrangement
- Composite Arrangements
- Arrangements with Restrictions

Selections (Combinations)

Pg 18-22

- Introduction to Selections

Probability with Counting Method

Pg 23-29

- Probability with Arrangements
- Probability with Selections

Learning Objectives:

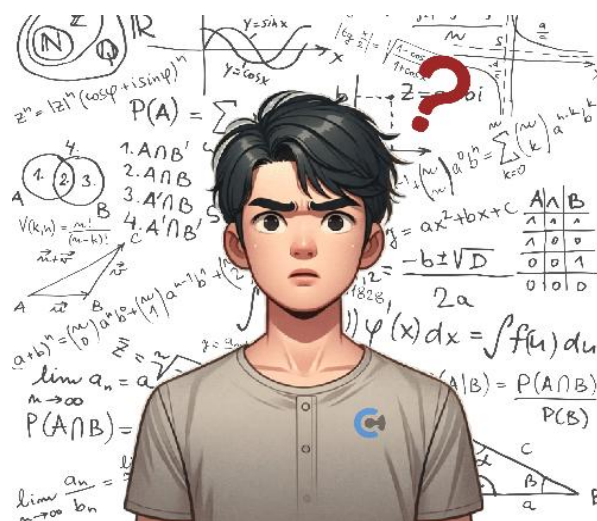
- MM12 [3.3.1] - Find Number of Permutations and Combinations
- MM12 [3.3.2] - Find Number of Permutations and Combinations with Restrictions/Composite
- MM12 [3.3.3] - Find Probabilities Using Counting Methods



Section A: Introduction to Counting Methods

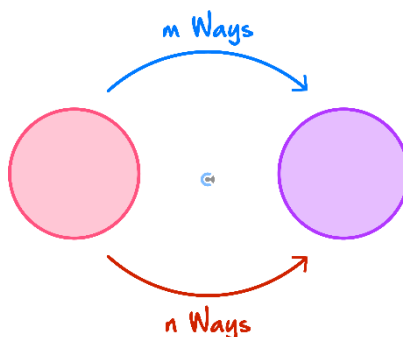
Sub-Section: Addition Principle

Context: Addition Principle



- Sam is choosing between his 4 pants and 3 shorts to go on his date with Emily.
- How many different options does he have? [7/12]
- We [added/multiplied] the options as Sam will choose one option [or/and] the other.

Addition Principle



- Associated with the use of the word "_____."

$$\text{Total Possibilities} = m + n$$

Question 1

A restaurant offers three vegan dishes or four vegetarian dishes.

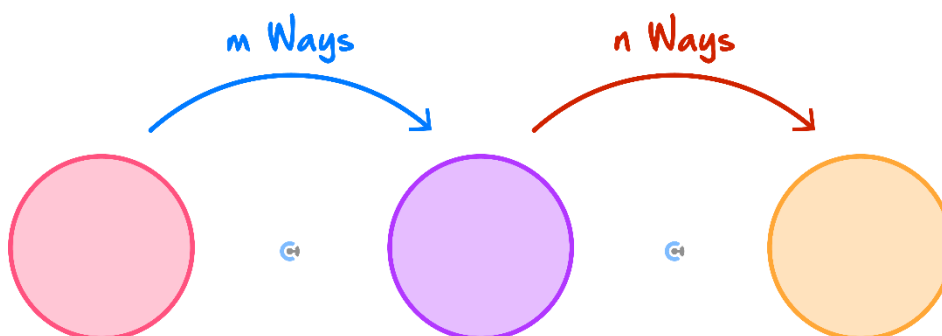
How many selections of one main meal does a customer have?

Space for Personal Notes

Sub-Section: Multiplication Principle



The Multiplication Principle



➤ Associated with the use of the word "_____."

$$\text{Total Possibilities} = m \times n$$

Question 2

Emily has two different pants, five different tops and three different pairs of shoes.

How many different choices does she have for a complete outfit?

Space for Personal Notes

Section B: Arrangements (Permutations)

Sub-Section: Introduction to Arrangements

What are arrangements?

Arrangements

➤ **Definition:** It is a study of a number of ways to _____ things.

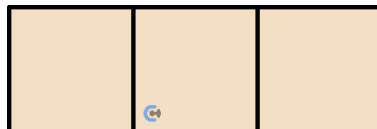
Discussion: How many ways can you arrange letters a, b, c ?

- _____
- _____
- _____
- _____
- _____
- _____

Is there a way to visualise the number of arrangements?

Box Diagram for Arrangements

➤ **Definition:** We can use it to write down a number of _____ for each position represented by each _____.



Question 3 Walkthrough.

3 different letters are ordered for a 3 letter word.

How many different words can you get?

Question 4

A family of 4 sits next to each other and is interested in a number of ways they could be seated.

How many different ways can the family of 4 sit in their 4 seats?

Space for Personal Notes



Calculator Commands: Factorial on Technology

➤ Mathematica

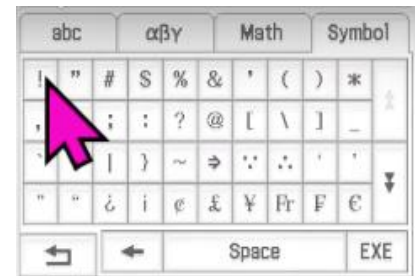
 Exclamation Mark

$x!$

➤ TI-Nspire

 Menu 51

➤ Casio Classpad



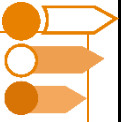
Question 5 Tech-Active.

A school is organising a photo session for 8 students. They are to stand in a single row for a group picture.

In how many different ways can the 8 students be arranged in a row?

Space for Personal Notes

Sub-Section: General Arrangement



What if we do not have enough spots for everything?



Exploration: General Arrangement



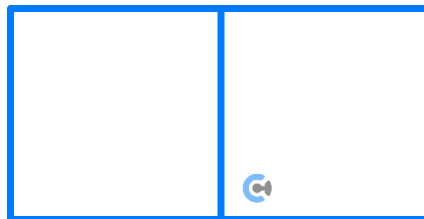
➤ Previously, we considered the case where **everything** was ordered.

How many ways can 5 people sit in 5 seats?



➤ What would happen if we don't have enough seats for everyone?

How many ways can 5 people sit in 2 seats?



➤ In summary:



➤ Let's generalise this for n people with only r many seats! ($n > r$).

🔄 The numerator represents a number of ways where we seat everyone.

It is given by $[n!, r!, (n - r)!]$.

🔄 The denominator represents a number of arrangements we missed out due to lack of a seat.

How many people aren't sitting? $[n, r, (n - r)]$

Hence, the denominator is given by $[n!, r!, (n - r)!]$

$$\text{Number of ways for } n \text{ people to sit in } r \text{ seats} = \frac{n!}{(n-r)!}$$

🔄 We call this ${}^n P_r$! Or Permutations!

Arrangement



➤ Generally:

Ways to arrange/order n many things for r spots = _____

➤ We call this ${}^n P_r$.

$${}^n P_r = \underline{\hspace{2cm}}$$

Space for Personal Notes

Question 6 Walkthrough.

James is trying to make a three-digit number by using the numbers 1, 2, 3, 4, 5, 6, 7 without repeating them.

How many different numbers can James have?

NOTE: We did **not** arrange all 7 numbers we had, as the number was three-digit.



Question 7

The teacher decides to pick 2 students from 10 student class and appoints them as a class captain and a vice-captain.

How many different ways could the teacher do this?

TIP: Only multiply the numbers after all the common factors are cancelled in the fraction.





Calculator Commands: Arrangements on Technology

➤ Mathematica

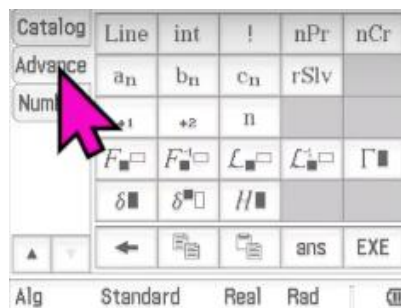
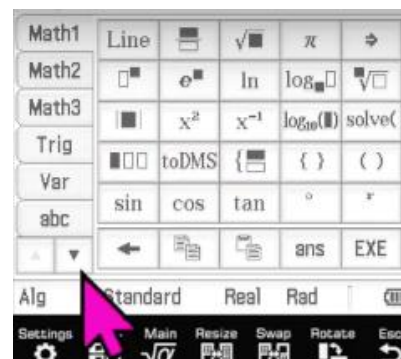
Just use factorial. Rip

➤ TI-Nspire

Menu 52

$${}^n P_r(n, r)$$

➤ Casio Classpad



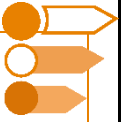
$${}^n P_r(n, r)$$

Question 8 Tech-Active.

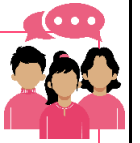
A painter is to paint the five circles of the Olympic flag. He cannot remember the colours to use for any of the circles, but he knows they should all be different. He has eight colours of paint available.

In how many ways can he paint the circles on the flag?

Sub-Section: Composite Arrangements



Discussion: How can we arrange a, b, c, d if a and b needs to be next to each other?



Composite Arrangements



- **Definition:** Occurs when an arrangement happens within another arrangement.
- **Steps:**
 - 🔄 Consider each group as one object and find the arrangements.
 - 🔄 Consider the arrangements within the groups and multiply.

Question 9 Walkthrough.

Consider a family of 4 which consists of a mum, dad, son, and daughter.

If it is known that the mum must sit next to the daughter and the dad must sit next to the son, how many different ways could they be sitting?

Question 10

Pranit wants to install seating plans for his rowdy class of 10 students. Fearing the backlash of the students, he lets them choose one friend to sit next to each other.

How many different seating plans could Pranit come up with?

Space for Personal Notes

Question 11 Tech-Active.

There are 12 animals in the animal farm; 4 dogs, 4 cats and 4 hamsters. Nayuta decides to label them with numbers ranging from 1-12. He decides to finish labelling all animals in the specific type of species before labelling animals from different species.

- a. How many different ways could the animals be numbered?
- b. If Nayuta decided to label the animals without considering their species, are there more ways to number them compared to **part a.** or less?

Space for Personal Notes

Sub-Section: Arrangements with Restrictions



Discussion: What do we have to consider when making a 4-digit number with 0, 1, 2, 3?



Arrangements with Restrictions



➤ **Definition:** The general principle to deal with restrictions is to:

🔄 Use the boxes.

🔄 Fill in the number of options for the slot that has the restriction first.

Question 12 Walkthrough.

How many different odd numbers between 1000 - 9999 exist, which only have the digits 1, 2, 3, 4, 5, 6, 7, 8 given each digit can only be used once?

TIP: Consider the number of options for the last digit. And do this before considering the rest!



Question 13

If no digit can be used more than once, find how many numbers that can be formed from the digits 2, 3, 4, 5, 6, 7, 8 that are:

- a. Three-digit numbers.
- b. Even three-digit numbers.
- c. Three-digit numbers greater than 700.

TIP: Target the digit with restrictions first.



Question 14 Tech-Active.

Four boys and two girls sit in a line on stools in front of a counter. Find the number of ways in which they can arrange themselves:

- a. If one of the boys, Raj, insists on being on (either) one of the ends.
- b. If the girls want to be at opposite ends of the line.

Space for Personal Notes

Section C: Selections (Combinations)

Sub-Section: Introduction to Selections

Discussion: Two different permutations AB and BA . Are they considered to be different selections?



Question 15

Consider letters A, B, C .

- a. State the number of ways we can arrange two letters from A, B, C .
- b. State the number of ways we can select two letters from A, B, C .

Okay, let's try generalising this!



Exploration: Selections

- Consider the following example.

Number of selections when we select two alphabets from A, B, C ?

- We can first solve for the number of arrangements.

Number of Arrangements = _____ = _____

How do we find number of selections?

- Let's consider the number of arrangements with the same selection.

EG: AB and BA

- How many different arrangements with same selection of letters? _____


Number of Selections = $\frac{\text{Number of Arrangements}}{\text{Number of Arrangements}}$

$= \frac{\text{Number of Arrangements}}{\text{Number of Arrangements}}$

- Let's generalise this for n people with only r many selected! ($n > r$)


-  The numerator represents number of ways where we arrange n people in r spots.

It is given by $[{}^nP_r, n!, r!]$.

-  The denominator represents number of arrangements in the same selections.

It is given by $[n!, r!, (n - r)!]$.

Number of Selections = $\frac{{}^nP_r}{r!} = \frac{n!}{r!(n-r)!}$

-  We call this nC_r ! Or combinations!



Selection

➤ Generally:

Ways to select r things from n many things = $\frac{{}^n P_r}{r!}$

➤ We call this ${}^n C_r$,

$${}^n C_r = \underline{\hspace{10em}}$$

➤ Where r = number of selection spots.

Question 16 Walkthrough.

A leadership team of three people is to be chosen from a group of eleven students. How many different teams are possible?

Discussion: Why do we divide by $r!$ again?



Question 17

How many ways are there to choose exactly two pets from a store with 8 dogs and 12 cats?

NOTE: We can treat dogs and cats the same although dogs are better.



Space for Personal Notes



Calculator Commands: Combinations of Technology

➤ Mathematica

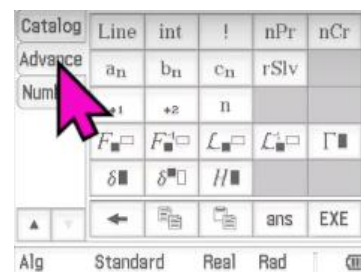
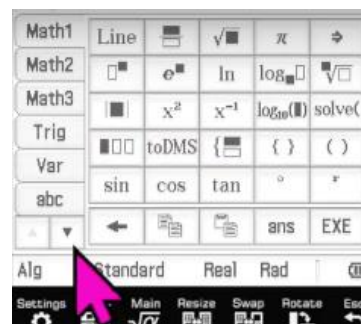
Binomial $[n, r]$

➤ TI-Nspire

Menu 53

$${}^nC_r(n, r)$$

➤ Casio Classpad



$${}^nC_r(n, r)$$

Question 18 Tech-Active.

A team of three boys and three girls is to be chosen from a group of nine boys and six girls. How many different teams are possible?

Section D: Probability with Counting Method

Sub-Section: Probability with Arrangements

Discussion: How would you find the probability of getting an even number from arrangements of 1, 3, 4, 5?



Probability with Arrangements



$$\text{Pr} = \frac{n(\text{Wanted Arrangements})}{n(\text{Total Arrangements})}$$

Space for Personal Notes

Question 19 Walkthrough.

Three-letter ‘words’ are to be made by arranging the letters of the word METHODS. We cannot use the same letter twice:

- a. How many three-letter words can be made from the letters METHODS?
- b. How many three-letter words start with the vowel?
- c. Hence, what is the probability that the word begins with a vowel?

NOTE: We get a different word depending on their arrangements. Hence, we use arrangements rather than selections!



ALSO NOTE: In combinations and permutations 'word' is used to mean an arrangement of letters, it does not have to be an actual English word.

Question 20

Four-digit number is to be made by arranging the numbers 1, 2, 3, 4. We cannot use the same number twice.

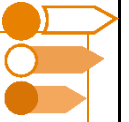
- a. How many four-digit numbers can be made in general?
- b. How many four-digit even numbers can be made?
- c. Hence, what is the probability that the four-digit number is an even number?

NOTE: That was an obvious answer but make sure to follow these steps for harder questions!

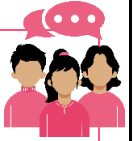


Space for Personal Notes

Sub-Section: Probability with Selections



Discussion: How would you find the probability of selecting A and B out of 4 letters A, B, C and D ?



Probability with Selections



$$\text{Pr} = \frac{n(\text{Wanted Selections})}{n(\text{Total Selections})}$$

Space for Personal Notes

Question 21 Walkthrough.

We decide to make a team of 4 people from the pool of 4 males and 4 females.

- a. How many different selections of three women and one man is possible?
- b. How many different teams of 4 can we make from the selection?
- c. What is the probability that a team of 4 would consist of three women and one man?

Space for Personal Notes

Question 22

What is the probability that a team of four chosen at random from a group of eight friends, four males and four females, would consist of three women and one man?

TIP: Find the number of selections for three women and one man first.



Space for Personal Notes

Question 23 Tech-Active.

We decide to select a team of 11 from the pool of 5 bowlers and 12 non-bowlers.

- a. How many different possible teams can be selected?
- b. In how many ways can the team include exactly 4 bowlers?
- c. Hence, what is the probability of having a team of 4 bowlers?

Space for Personal Notes



Contour Check

☐ Learning Objective: [3.3.1] - Find number of permutations and combinations

Key Takeaways

☐ Box Diagram for Arrangements

- ☐ **Definition:** We can use it to write down number of _____ for each position represented by each _____.

☐ Arrangement

- ☐ Generally:

Ways to arrange/order n many things for r spots = _____

- ☐ We call this nP_r .

$${}^nP_r = \underline{\hspace{2cm}}$$

☐ Selection

- ☐ Generally:

Ways to select r things from n many things = $\frac{{}^nP_r}{r!}$

- ☐ We call this nC_r .

$${}^nC_r = \underline{\hspace{2cm}}$$

- ☐ Where r = number of selection spots.

- **Learning Objective: [3.3.2] - Find number of permutations and combinations with restrictions/composite**

Key Takeaways

- **Composite Arrangements**

- **Definition:** Occurs when an arrangement happens within another arrangement.

- **Steps:**

- Consider each group as _____ object and find the arrangements.

- Consider the _____ within the groups and multiply.

- **Arrangements with Restrictions**

- **Definition:** The general principle to deal with restrictions is to:

- Use a box diagram.

- Fill in the number of options for the slot that has the restriction _____.

□ Learning Objective: [3.3.3] - Find probabilities using counting methods

Key Takeaways

□ Probability with Arrangements

$$\text{Pr} = \frac{n(\textit{Wanted Arrangements})}{\underline{\hspace{2cm}}}$$

□ Probability with Selections

$$\text{Pr} = \frac{n(\textit{Wanted Selections})}{\underline{\hspace{2cm}}}$$



Website: contoureducation.com.au | Phone: 1800 888 300 | Email: hello@contoureducation.com.au

VCE Mathematical Methods ½

Free 1-on-1 Support



Be Sure to Make The Most of These (Free) Services!

- Experienced Contour tutors (45+ raw scores, 99+ ATARs).
- For fully enrolled Contour students with up-to-date fees.
- After school weekdays and all-day weekends.

| <u>1-on-1 Video Consults</u> | <u>Text-Based Support</u> |
|--|---|
| <ul style="list-style-type: none">➤ Book via bit.ly/contour-methods-consult-2025 (or QR code below).➤ One active booking at a time (must attend before booking the next). | <ul style="list-style-type: none">➤ Message +61 440 138 726 with questions.➤ Save the contact as "Contour Methods". |

[Booking Link for Consults](https://bit.ly/contour-methods-consult-2025)
bit.ly/contour-methods-consult-2025



[Number for Text-Based Support](tel:+61440138726)
[+61 440 138 726](tel:+61440138726)