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VCE Mathematical Methods ½
Combinations & Permutations [3.3]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 23
Supplementary Questions	Pg 24 – Pg 37

Section A: Compulsory Questions

Sub-Section [3.3.1]: Finding Permutations and Combinations

Question 1

Find the following:

a. ${}^3P_2$

$${}^3P_2 = \frac{3!}{(3-2)!} = 6$$

b. ${}^4P_2$

$${}^4P_2 = \frac{4!}{(4-2)!} = 12$$

c. ${}^3C_2$

$${}^3C_2 = \frac{3!}{2!\,(3-2)!} = 3$$

d. ${}^4C_2$

$${}^4C_2 = \frac{4!}{2!\,(4-2)!} = 6$$

Question 2

An affluent family has 4 luxury cars in their garage. However, the architect they hired to design their house was not particularly intelligent, and therefore only one car can fit in the driveway, meaning only one car can enter or exit the garage at a time.

a. How many different ways can all 4 cars exit the garage?

$$4! = 24$$

b. The family needs 2 cars for a holiday. How many different ways can 2 cars be selected?

$${}^4C_2 = \frac{4!}{2!\,(4-2)!} = 6$$

c. On Saturday afternoons, 3 cars are needed for various networking activities. How many ways can 3 of the cars exit the garage?

$${}^4P_3 = \frac{4!}{(4-3)!} = 24$$

Question 3

A bag of marbles has k marbles in it, where $k \in \mathbb{Z}^+$.

a. Find ${}^kP_2$. Express your answer as a quadratic in terms of k.

$${}^kP_2 = \frac{k!}{(k-2)!} = \frac{k(k-1)(k-2)!}{(k-2)!} = k(k-1) = k^2 - k$$

b. Find k if there are 6 different ways to pick 2 marbles out of the bag, one at a time.

$${}^kP_2 = 6 \Rightarrow k^2 - k = 6$$

$$\Rightarrow k^2 - k - 6 = 0$$

$$\Rightarrow (k-3)(k+2) = 0$$

$$\Rightarrow k = 3, -2, \text{ but } k \in \mathbb{Z}^+$$

$$\therefore k = 3$$

c. Find ${}^{k}C_{2}$. Express your answer as a quadratic in terms of k.

$${}^{k}C_{2} = \frac{k!}{2!(k-2)!} = \frac{k(k-1)(k-2)!}{2(k-2)!} = \frac{k(k-1)}{2} = \frac{k^2-k}{2}$$

d. Find k if there are 6 different ways that a group of 2 marbles can be chosen from the bag.

$${}^{k}C_{2} = \frac{k^2-k}{2} = 6 \Rightarrow k^2-k = 12$$

$$\Rightarrow k^2-k-12=0$$

$$\therefore k = 4, -3 \quad \text{but } k \in \mathbb{Z}^+$$

$$\therefore k = 4$$

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Question 4 Tech-Active.

In a game of poker, hands are dealt from a deck of 52 cards.

a. How many 2-card hands can be made?

$${}^{52}C_2 = \frac{52!}{2!\,(52-2)!} = 1326$$

```
In[1]:= Binomial[52, 2]
Out[1]= 1326
```

b. In a 7 player game, each hand is ranked based on which hands it beats and loses to. How many ways can 7 hands be ranked?

$${}^{7}P_7 = \frac{7!}{(7-7)!} = 5040$$

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Sub-Section [3.3.2]: Finding Composite/Restricted Permutations and Combinations

Question 5

A small field on a farm has 5 cows and 4 sheep. However, the field is overcrowded and 4 animals must be moved to a different field. How many different ways can the 4 animals be selected if:

a. There is exactly 1 cow in the group?

We are choosing 1 of 5 cows and the other 3 animals must be sheep:
$n = {}^5C_1 * {}^4C_3 = 5 * 4 = 20$

b. There is exactly 1 sheep in the group?

We are choosing 1 of 4 sheep and the other 3 animals must be cows:
$n = {}^4C_1 * {}^5C_3 = 4 * 10 = 40$

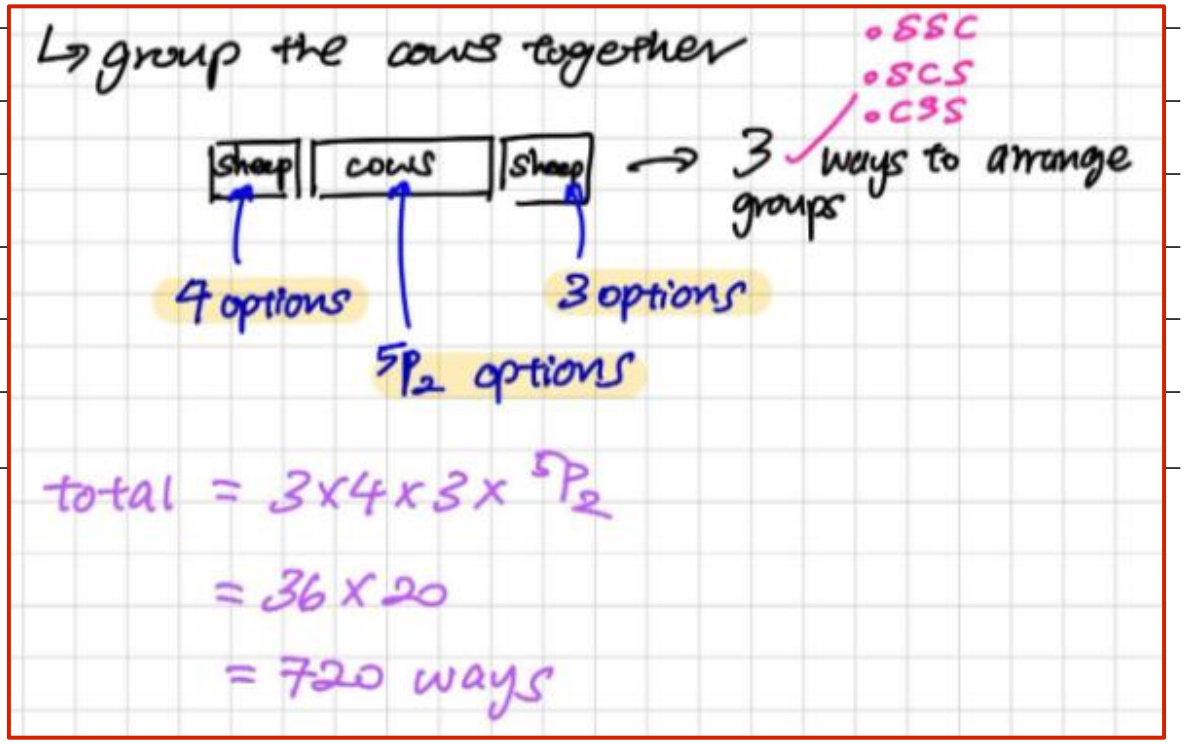
c. No animal can be the only one of its kind?

There has to be at least 2 cows and at least 2 sheep which means exactly 2 sheep and exactly 2 cows.
$n = {}^4C_2 * {}^5C_2 = 6 * 10 = 60$

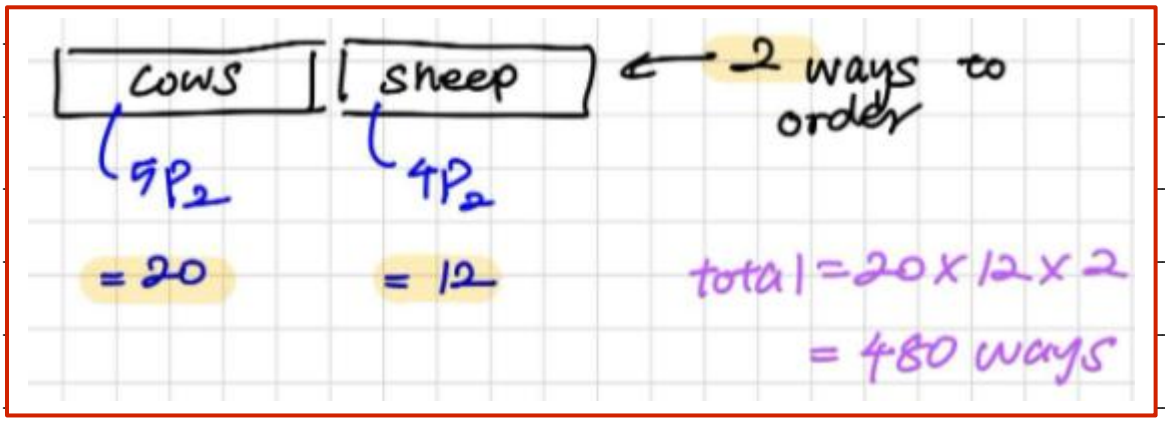
It is decided that 2 cows and 2 sheep will be moved, but not which cows or sheep. The animals are then brought to the new field one at a time.

d. How many ways can this occur if:

i. The cows are brought one after the other?



ii. The cows and sheep are brought one after the other?



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Question 6

5 boys and 5 girls are trying out for the starting lineup of a mixed volleyball team, which consists of 6 people.

a. How many ways can the team be selected if:

i. There are no restrictions?

$${}^{10}C_6 = \frac{10!}{6!\,(10-6)!} = \frac{10*9*8*7}{4*3*2*1} = 210$$

ii. There has to be an equal amount of boys and girls?

Choosing 3 girls from 5 and 3 boys from 5:

$$\text{Total} = {}^{5}C_3 * {}^{5}C_3 = \frac{5!}{3!(2!)} * \frac{5!}{3!(2!)} = 10 * 10 = 100$$

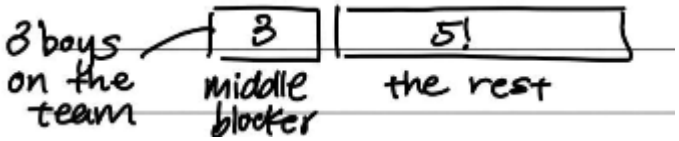
Given a team with an equal number of boys and girls is formed, the team has 6 different positions that must be assigned. Two of these positions are libero and middle blocker.

b. How many different ways can the positions be assigned if:

i. There are no restrictions?

$${}^6P_6 = \frac{6!}{(6-6)!} = 720$$

ii. The middle blocker must be a boy?



If middle blocker is boy only 3 the boys can be middle blockers, other positions don't matter:
Total $= 3 * 5 * 4 * 3 * 2 * 1 = 360$

iii. The middle blocker must be a boy AND the libero must be a girl?

3 people can be middle blockers, 3 people can be liberos, other positions don't matter:
Total $= 3 * 3 * 4 * 3 * 2 * 1 = 216$

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Question 7

Subu and Sam are craving KFC. Their favourite bucket consists of 3 original recipes, 2 tenders, and 5 nuggets. It can be assumed that each piece of chicken is unique (eating nugget 1 is different from eating nugget 2 for example). They are once again taking and eating chicken out of the bucket one at a time.

a. Ignoring who is eating the chicken, how many ways can all of the chicken be consumed in total? You may leave your answer in factorial form.

$10!$

b. Given that 3 nuggets are eaten first, how many different ways can the chicken be eaten if:

i. There are no further restrictions? You may leave your answer as a multiple of a factorial.

$\text{Total} = {}^5P_3 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 60 * 7!$

ii. The 2 tenders are the last 2 pieces of chicken taken out of the bucket? You may leave your answer as a multiple of a factorial.

$\text{Total} = {}^5P_3 * 5 * 4 * 3 * 2 * 1 * 2 * 1 = 120 * 5!$

Since it is bulking season, Subu and Sam get another bucket. But they decide to go about the eating differently this time: They each pick their pieces of chicken first before eating them.

Sam still really likes eating nuggets so he picks all of his chicken while Subu is in the bathroom.

c. How many different ways can Sam's chicken pile be made up if:

i. There are no further restrictions?

$\text{Total} = {}^{10}C_5 = 252$

ii. Sam gets at least 3 nuggets in his pile?

$\text{Total} = ({}^5C_3 \times {}^5C_2) + ({}^5C_4 \times {}^5C_1) + ({}^5C_5) = 126$

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Question 8 Tech-Active.

A student committee of size 4 must be selected from a list of potential candidates, which has 9 girls and 7 boys on it. How many different ways can the committee be selected if:

a. There are no restrictions?

$^{16}C_4 = 1820$

Binomial[16, 4]

1820

b. There must be an equal amount of boys and girls?

$^{9}C_2 * {}^{7}C_2 = 756$

Binomial[9, 2] * Binomial[7, 2]

756

A committee of 2 girls and 2 boys is selected. 4 different leadership roles must be delegated, which includes planning and communication. How many ways can the roles be delegated with:

c. No restrictions?

$4! = 24$

d. A girl in charge of planning and a boy in charge of communication?

$\text{Ways} = 2 * 2 * 2 * 1 = 8$

Sub-Section [3.3.3]: Finding Probabilities using Counting Methods

Question 9

At the next upcoming school assembly, 4 subject heads have a presentation to make; the heads of english, maths, science, and humanities. The principal needs to decide the order in which the teachers will speak.

a. Find the total number of ways the speaking order can occur.

$4! = 24$

The principal knows that at least half of the school will fall asleep during the humanities teachers' speech and is trying to prevent this.

b. How many ways can the principal arrange the speech order so that the head of humanities goes last?

$3 * 2 * 1 * 1 = 6$

c. Hence, what is the probability that the history teacher does not cause the students to miss any other speeches?

$$\Pr(\text{humanities last}) = \frac{\text{humanities last}}{\text{total ways}} = \frac{6}{24} = \frac{1}{4}$$

Question 10

A student with time management struggles has 4 SACs tomorrow and hasn't revised for any of them. They know that they only have the time to cram for 2 subjects and are trying to decide which subjects to do.
Luckily, it is only $\frac{1}{2}$.

a. How many ways can the student select the 2 subjects to cram?

$$^4C_2 = \frac{4!}{2!\,(4-2)!} = 6$$

The student knows that their parents will be disappointed if they underperform in methods so they will definitely study for that. The student is also not the best at history and will drop it next year so they have decided not to study for that.

b. Find the number of ways the student can select 2 subjects with these conditions.

2
Methods and not history means ways $= 1 * 2 = 2$

c. Hence, find the probability that the student's parents are not disappointed.

$$\Pr(\text{not disappointed}) = \frac{\text{methods}}{\text{total}} = \frac{2}{6} = \frac{1}{3}$$

Question 11

The Contour top brass, consisting of 11 department heads (Angad, Amitav, Aaliyan, Caitlin, Emily, James, Jeff, Jonathan, Paul, Pranit, and Ramodh) have struggled to find a good time to have a department head's meeting due to their concerningly busy schedules. They have all begrudgingly settled on a time of 7 AM on a random Monday. All of the department heads rock up to the meeting severely in need of coffee. 3 people are needed to go and get coffee for everyone.

a. How many different groups of 3 people can go and get coffee?

$${}^{11}C_3 = \frac{11!}{3!\,(11-3)!} = \frac{11*10*9}{3*2*1} = 11*5*3 = 165$$

The first topic to be discussed is on academic logistics and so, Emily offers to be one of the 3 people to go get coffee. All of the subject heads (Angad, Aaliyan, Caitlin, James, Jeff, Jonathan, and Pranit) are needed for this part of the meeting or important points will be missed.

b. Given that Emily is one of the people getting coffee, how many different groups of 3 can be formed so that no important points are missed?

$$\text{Total} = {}^{1}C_1 * {}^{3}C_2 = 1*3 = 3$$

c. What is the probability that the coffee group does not cause any important points to be missed?

$$\Pr(\text{suitable group}) = \frac{\text{suitable groups}}{\text{total}} = \frac{3}{165} = \frac{1}{55}$$

James has offered to drive 3 other people to the meeting. His pretty fancy car can seat 2 in the front including himself and 2 in the back. James is not letting anyone else drive his nice car, so he must sit in the driver's seat. Angad and Amitav would like to tag along but are inseparable so they must sit together. Jeff likes nice cars and is happy if he is somewhere in the car. Everyone else doesn't mind.

d. What is the probability that everyone is happy with the seating arrangement in James' car if everyone is eligible to ride in James' car?

The only suitable seating arrangements are James in the driver's seat, Jeff in the front passenger, and Angad and Amitav together in the back which has 2 possible configurations therefore there are only 2 total suitable configurations. The total configurations are James in the driver's seat and anyone else in the other seats so, total $= 1 * 10 * 9 * 8 = 720$ so the probability that everyone is happy is $\frac{2}{720} = \frac{1}{360}$.

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Question 12 Tech-Active.

Before preparing for his tutoring classes this week, Rei is trying to make a Spotify queue of 6 songs that will keep him energised for his work session. His playlist consists of 4 R&B songs, 3 rock songs, and 5 J-pop songs.

a. How many different Spotify queues can Rei make?

$$\frac{12!}{(12-6)!}$$

665 280

b. Rei feels like starting with rock to fire himself up. How many ways can the Spotify queue be organised if the first song has to be a rock song?

$$3 * \frac{11!}{(11-5)!} = 166320$$

$$3 * \frac{11!}{(11-5)!}$$

166 320

c. What is the probability that the first song in the queue is a rock song?

$$\frac{166320}{665280} = \frac{1}{4}$$

d. If 3 rock songs come on in a row, Rei will get bored and will no longer be energised. How many queues will bore Rei?

```
(*3 rock songs:*)
3*2*1*10!/(10-4)!

30240
```

e. What is the probability that Rei finds his Spotify queue boring?

$$\frac{30240}{665280} = \frac{1}{22}$$

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Section B: Supplementary Questions

Sub-Section [3.3.1]: Finding Permutations and Combinations

Question 13

Find the following:

a. ${}^5P_2$

20

b. ${}^0P_0$

1

c. ${}^7C_4$

35

d. ${}^5C_4$

5

Question 14

Disaster has struck at Contour Glen Waverley. The paper is running out and more is coming in tomorrow. 6 sets of booklets have to be printed but there is only enough paper for 3 sets to be printed.

a. How many different sets of 3 can be chosen?

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Binomial[6, 3]

20
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Now that 3 sets of booklets have been chosen, they need to be queued in the printer.

b. How many different ways can the chosen booklets be queued?

$3! = 6$

Not to worry! Nayuta has saved the day by driving 10 reams of paper over from Contour Box Hill. Now all of the booklets can be printed.

c. How many different ways can all of the booklets be queued?

$6! = 720$

Question 15

At lunchtime, n students rush to be first in line at the canteen, where $n \in \mathbb{Z}^+$.

a. How many ways can the students queue? Give your answer in terms of n.

$n!$

b. Within the n students, 2 of them are just in the queue to hang out with their friends while the others are actually there to buy food. If there are 10 different ways that there can be a group of 2 students who aren't buying food, how many total students are in the queue?

$${}^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2} = \frac{n^2-n}{2} = 10$$

$$\Rightarrow \frac{n^2-n}{2} = 10$$

$$\Rightarrow n^2 - n = 20$$

$$\Rightarrow n^2 - n - 20 = 0$$

$$\therefore n = 5, -4 \quad \therefore n = 5$$

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Sub-Section [3.3.2]: Finding Composite/Restricted Permutations and Combinations

Question 16

The letters in the word 'METHODS' are jumbled and rearranged.

How many ways can the letters be rearranged if:

a. There are no restrictions?

$7! = 5040$

b. A vowel must be first?

$2 * 6! = 1440$

c. The vowels must be together?

$2 * 1 * 6! = 1440$

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Question 17

A family of 3 children must share 6 different flavoured donuts. In order to prevent fighting, their mother ensures that they all receive an equal amount of doughnuts and take turns choosing.

a. How many different ways can the doughnuts be divided?

$${}^6C_2 \times {}^4C_2 \times {}^2C_2 = 15 \times 6 \times 1 = 90$$

b. If the eldest picks both of their doughnuts first, how many different selections can the middle child make if they make both of their selections next?

$${}^4C_2 = 6$$

c. The youngest child has a temper tantrum at this proposal, and so everyone agrees to let him pick his doughnuts first. Given that he takes a strawberry-flavoured one, how many different groups of 2 can the youngest child make?

5

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Question 18

A small errand requires a singular group of 3 to be made from within a small class of 6 students (Abbey, Ben, Charlie, Derek, Erica, and Frank). The teacher has enough of the students always working with the same people and so, they decide to make the groups. The teacher is quite well informed and knows that:

- Abbey and Ben recently broke up, so they cannot be in the group together.
- Frank and Ben are tight, so Frank will not take part in the group if Abbey is in the group.
- Abbey and Erica will not be part of the group if they are not together.

How many ways can the group be formed?

Possible groups:

A and E, no B or F

${}^2C_2 \times {}^2C_1 = 2$

B or F, no A or E

B, no F: ${}^1C_1 \times {}^2C_2 = 1$

F, no B: ${}^1C_1 \times {}^2C_2 = 1$

B and F: ${}^2C_2 \times {}^2C_1 = 2$

total = 6

Space for Personal Notes

Sub-Section [3.3.3]: Finding Probabilities using Counting Methods

Question 19

3 of the letters in the word 'CHANCE' are randomly selected. Find the probability that:

a. All 3 letters are consonants.

$${}^4C_3 = 4$$
$$\text{Total} = {}^6C_3 = 20$$
$$\Pr(3 \text{ consonants}) = \frac{1}{5}$$

b. Both vowels are selected.

$${}^2C_2 * {}^4C_1 = 4$$
$$\Pr(2 \text{ vowels}) = \frac{4}{20} = \frac{1}{5}$$

c. Only 1 vowel is selected.

$${}^2C_1 * {}^4C_2 = 12$$
$$\Pr(1 \text{ vowel}) = \frac{12}{20} = \frac{3}{5}$$

Space for Personal Notes

Question 20

When being assigned lockers, you and your 2 best friends really hope that you get adjacent lockers. There are 5 lockers to be assigned amongst 5 people. What is the probability that:

a. You get an end locker?

$$\text{Total} = 5! = 120$$
$$\text{Ways} = 4 * 3 * 2 * 1 * 2 = 48$$
$$\Pr(\text{end}) = \frac{48}{120} = \frac{2}{5}$$

b. You and your 2 best friends get adjacent lockers?

$$\text{Adjacent lockers} = 3! * 3! = 36$$
$$\Pr(\text{adjacent}) = \frac{36}{120} = \frac{3}{10}$$

c. You and your 2 best friends all have separated lockers?

$$\text{Separated} = 3 * 2 * 2 * 1 * 1 = 12$$
$$\Pr(\text{separated}) = \frac{12}{120} = \frac{1}{10}$$

Space for Personal Notes

Question 21

Subu and Sam have bought another bucket of KFC, once again containing their favourite combo of 5 nuggets, 3 original recipes, and 2 tenders. This time, they are inspecting each piece of chicken to see if they can recreate the recipe for themselves so they can stop spending so much money on KFC. To do this, they line each of the 10 pieces of chicken up on a bench. Find the probability that:

a. No nuggets are next to each other.

$$\text{Total} = 10!$$
$$\text{Nuggets Apart} = 5 * 5 * 4 * 4 * 3 * 3 * 2 * 2 * 1 * 1 * 2 = 2 * 5! * 5!$$
$$\Pr(\text{nuggets apart}) = \frac{2 * 5! * 5!}{10!} = \frac{2 * 5 * 4 * 3 * 2 * 1}{10 * 9 * 8 * 7 * 6} = \frac{1}{126}$$

b. The 2 tenders are first in the line and the 3 original recipes are together.

$$\text{Tenders first and original together} = 2 * 1 * 3 * 2 * 1 * 6! = 12 * 6!$$
$$\Pr(\text{tenders first and original together}) = \frac{12 * 6!}{10!} = \frac{1}{420}$$

c. The tenders are not next to each other.

$$\Pr(\text{tenders apart}) = 1 - \Pr(\text{tenders together})$$

$$\text{tenders together} = 2 * 1 * 9!$$

$$\Pr(\text{tenders together}) = \frac{2 * 1 * 9!}{10!} = \frac{1}{5}$$

$$\therefore \Pr(\text{tenders apart}) = 1 - \frac{1}{5} = \frac{4}{5}$$

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