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VCE Mathematical Methods ½
Probability Exam Skills [3.2]
Workbook

Outline:

Learning Objectives:

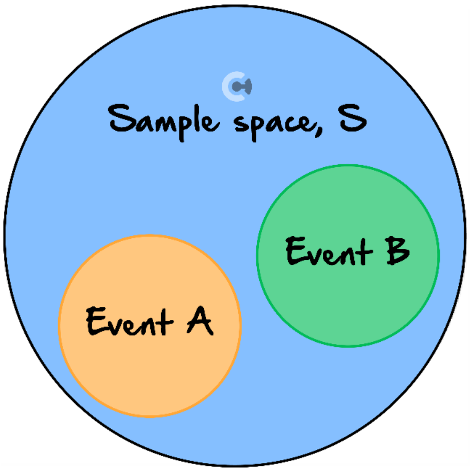
- ☐ **MM12 [3.2.1]** - Understand probabilities in terms of favourable outcomes from a sample space
- ☐ **MM12 [3.2.2]** -Basic probability operations. Use Venn diagrams and/or Karnaugh maps and apply the addition rule
- ☐ **MM12 [3.2.3]** - Understand the meaning behind independent and mutually exclusive events
- ☐ **MM12 [3.2.4]** - Understand conditional probability and make use of tree diagrams

Section A: Recap

If you were here last week skip to Section B – Warmup Test.

Exploration: Understanding Probability

- In probability, we are aiming to quantify uncertainty.



- Sample Space (ε) = Set of all possible outcomes.
- Probability of an event = Proportion of sample space that the event takes up.
- The probability of an event must **always** take a value between 0 and 1, inclusive.

Sample Space (ε)

- The set of all possible outcomes in an experiment.
- For tossing two coins in a row, the sample space is:

$$\varepsilon = \{HH, HT, TH, TT\}$$

- For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon = \{1, 2, 3, 4, 5, 6\}$$

- Total probability adds up to 1.

Question 1

Using the sample space for rolling a standard 6-sided dice, find:

a. The probability of rolling a 6.

$\frac{1}{6}$

b. The probability of rolling a number less than 3.

$\varepsilon = \{1, 2, 3, 4, 5, 6\}$

$\frac{1}{3}$

c. The probability of rolling an odd number.

$\frac{1}{2}$

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Sub-Section: Equally Likely Events

Discussion: How can we calculate probabilities if all events are equally likely? E.g., Each number on the dice.

↳ Count the no. of events

Calculating Probabilities for Equally Likely Outcomes

- When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{number\ of\ successful\ outcomes}{number\ of\ total\ outcomes}$$

Question 2

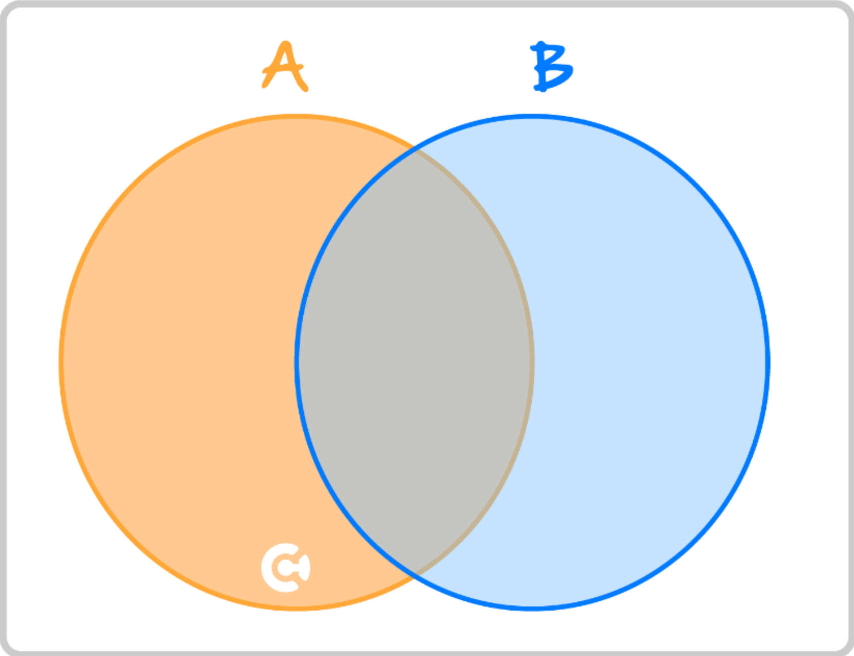
A fair 20-sided die (as shown) is rolled.

What is the probability that the result is a number divisible by 6?

$$\Pr(\text{Receiving medication}) = \frac{400}{900} = \frac{4}{9}$$

Definition

Union, Intersection, and Complement



- $A \cup B =$
  - Union of two events aka "or".
  - Equivalent to either event A **OR** event B **OR** BOTH occurring.
  - (Shade relevant area on Venn diagram above).
- $A \cap B =$
  - Intersection of two events aka "and".
  - Equivalent to both event A **AND** event B occurring.
  - (Shade relevant area on Venn diagram above).
- $A' =$
  - A complement aka "not".
  - Equivalent to event A NOT occurring.
  - E.g., if $A =$ dice rolled a 6, then $A' =$ dice rolled anything except a 6.

$$\mathbf{Pr}(A') = 1 - \mathbf{Pr}(A)$$

Question 3

Let event G = A student wears glasses, and an event F = A student is a female.

Write an expression for the probability of each of the following events, without doing any calculations.

a. A student is a female who wears glasses.

$F \cap G$

$\Rightarrow Pr(F \cap G)$

b. A student is either a female or doesn't wear glasses.

$F \cup^{?} G'$

$\Rightarrow Pr(F \cup G')$

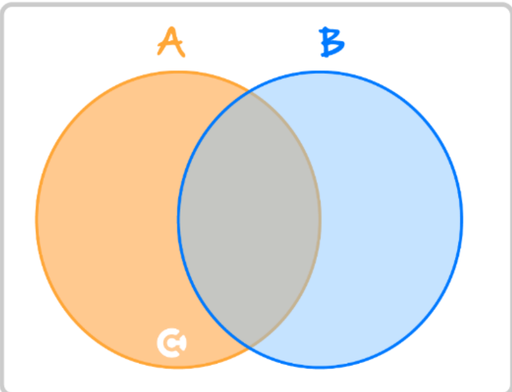
c. A student wears glasses and is not a female.

$G \cap F'$

$\Rightarrow Pr(G \cap F')$

Definition

Venn Diagram



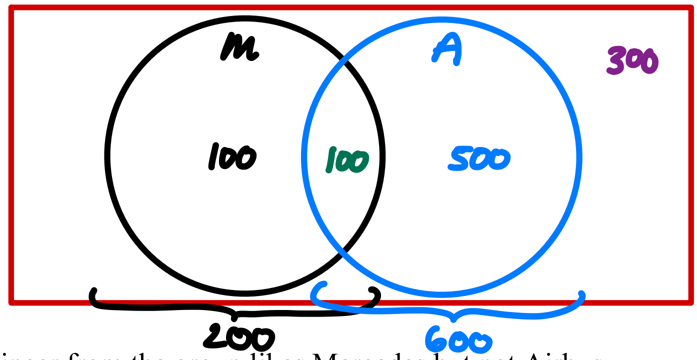
- Venn diagram is useful to visualise the two events.

Question 4

Out of 1000 engineers, 200 like Mercedes as an automaker. Out of the same 1000 engineers, 600 like Airbus as an aeroplane manufacturer. Exactly 100 engineers like both Mercedes and Airbus.

a. How many engineers neither like Mercedes, nor Airbus?

300



b. Find the probability that a randomly picked engineer from the group likes Mercedes but not Airbus.

$M \cap A'$

$$\Pr(M \cap A') = \frac{100}{1000} = \frac{1}{10}$$

c. Find the probability that a randomly picked engineer from the group does not like Mercedes, but likes Airbus.

$M' \cap A$

$$\Pr(M' \cap A) = \frac{500}{1000} = \frac{1}{2}$$

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Karnaugh Tables

- We can also represent probability problems using a Karnaugh Map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

- The rows and columns add up to the last cell value.

- Remember the **total** probability must always add to 1.

Question 5

$T \cap S$

The probability that a person is tall and skinny is 0.4 and the probability of being neither is 0.1.

If the probability of being tall is 0.55, what is the probability of being skinny?

Use the Karnaugh map.

$\Pr(S) =$

	S	S'	
T	0.4	0.15	0.55
T'		0.1	
	0.75	0.25	1

?

$\therefore \Pr(S) = 0.75$

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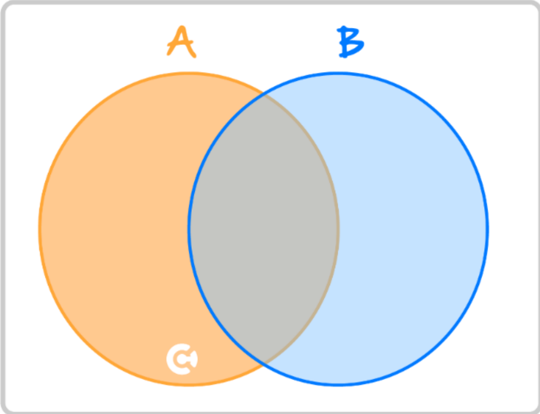
Definition

The Addition Rule



- When we add the probabilities of A and B, we count the outcomes contained in A **twice.**
- So, we must subtract one of them to get the probability of A.

$$\mathbf{Pr}(A \cup B) = \mathbf{Pr}(A) + \mathbf{Pr}(B) - \mathbf{Pr}(A \cap B)$$

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Question 6

A and B are events such that $\Pr(A) = 0.42$, $\Pr(B) = 0.48$ and $\Pr(A \cap B) = 0.16$.

Determine:

a. $\Pr(A')$.

$\Pr(A') = 1 - \Pr(A) = 1 - 0.42 = 0.58$

b. $\Pr(B')$.

$\Pr(B') = 1 - \Pr(B) = 1 - 0.48 = 0.52$

c. $\Pr(A \cup B)$.

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.42 + 0.48 - 0.16\\ &= 0.9 - 0.16\\ &= 0.74\end{aligned}$$

Definition

<u>Mutually Exclusive Events</u>

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- Probability of both A and B happening together is zero.

$$\Pr(A \cap B) = 0$$

Question 7

State whether each of the following events would be mutually exclusive or inclusive.

Event 1	**Event 2**	**Exclusive / Inclusive**
Getting 50RAW in Methods.	Getting 48RAW in Methods.	Exclusive
Going to the beach.	Going to St. Kilda.	Inclusive

Question 8

$\Pr(A \cap B) = 0$

It is known that events A and B are mutually exclusive. If $\Pr(A) = 0.2$ and $\Pr(B) = 0.5$, find $\Pr(A' \cup B')$.

$$\Pr(A' \cup B') = 1 - \Pr(A \cap B)$$
$$= 1 - 0$$
$$= 1$$

Definition

<u>Independent Events</u>

- **Definition:** Independent events **do not affect** the likelihood of the other.
- **Mathematically**:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Question 9

State whether each of the following events would be independent or dependent.

Event 1	**Event 2**	**Independent / Dependent**
Wearing a swimsuit.	Going to the beach.	dependent
Doing homework for Methods.	Getting 50RAW in Methods.	dependent
Doing homework for English.	Getting 50RAW in Methods.	independent
Rolling a 6 on first dice roll.	Rolling a 6 on second dice roll.	independent

Question 10

$\Pr(B) = 1 - \Pr(B')$
$= 0.5$

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

Given that $\Pr(A) = 0.1$, $\Pr(B') = 0.5$, and $\Pr(A \cap B) = 0.03$, are the events A and B independent?

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

$$0.03 = 0.1 \times 0.5$$

$$0.03 \neq 0.05$$

$\therefore$ A and B are dependent events

NOTE: Use $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ to algebraically check independence.

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Question 11

* Use the Addition Rule

Given that the events A and B are such that $\Pr(A) = \frac{1}{2}$, $\Pr(A \cup B) = \frac{3}{5}$ and $\Pr(B) = p$. Find p if they are:

a. Mutually Exclusive. $\Pr(A \cap B) = 0$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\frac{3}{5} = \frac{1}{2} + p - 0$$

$$p = \frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10} //$$

b. Independent. $\Pr(A \cap B) = \Pr(A) \cdot \Pr(B) = \frac{1}{2}p$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$$

$$\frac{1}{2}p = \frac{1}{10} \Rightarrow \therefore p = \frac{1}{5}$$

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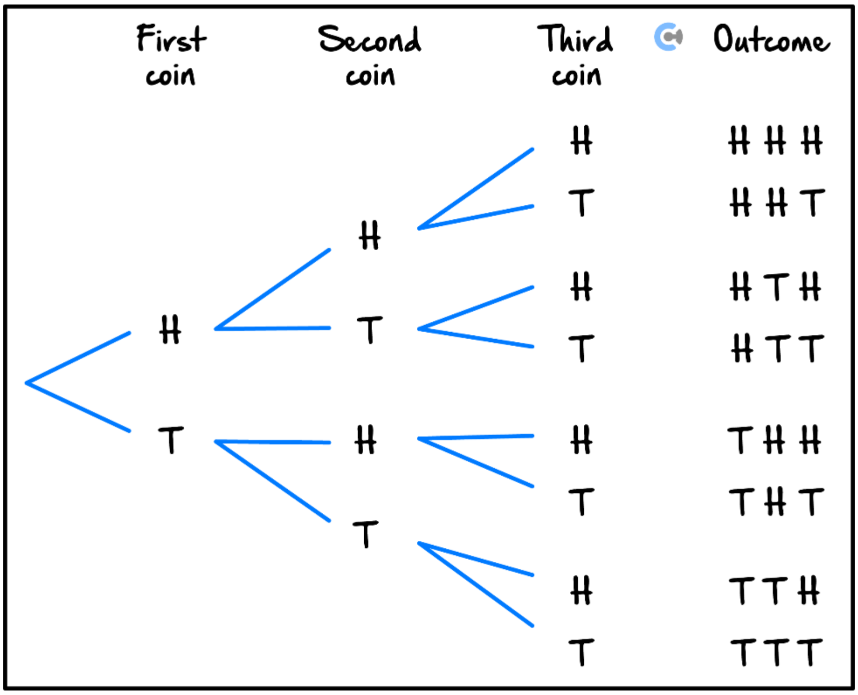
Section B: Tree Diagram and Conditional Probability

Tree Diagram



- Useful for multiple sequence of events.
- To calculate the probability of a sequence, we multiply **the probabilities along the relevant branches.**

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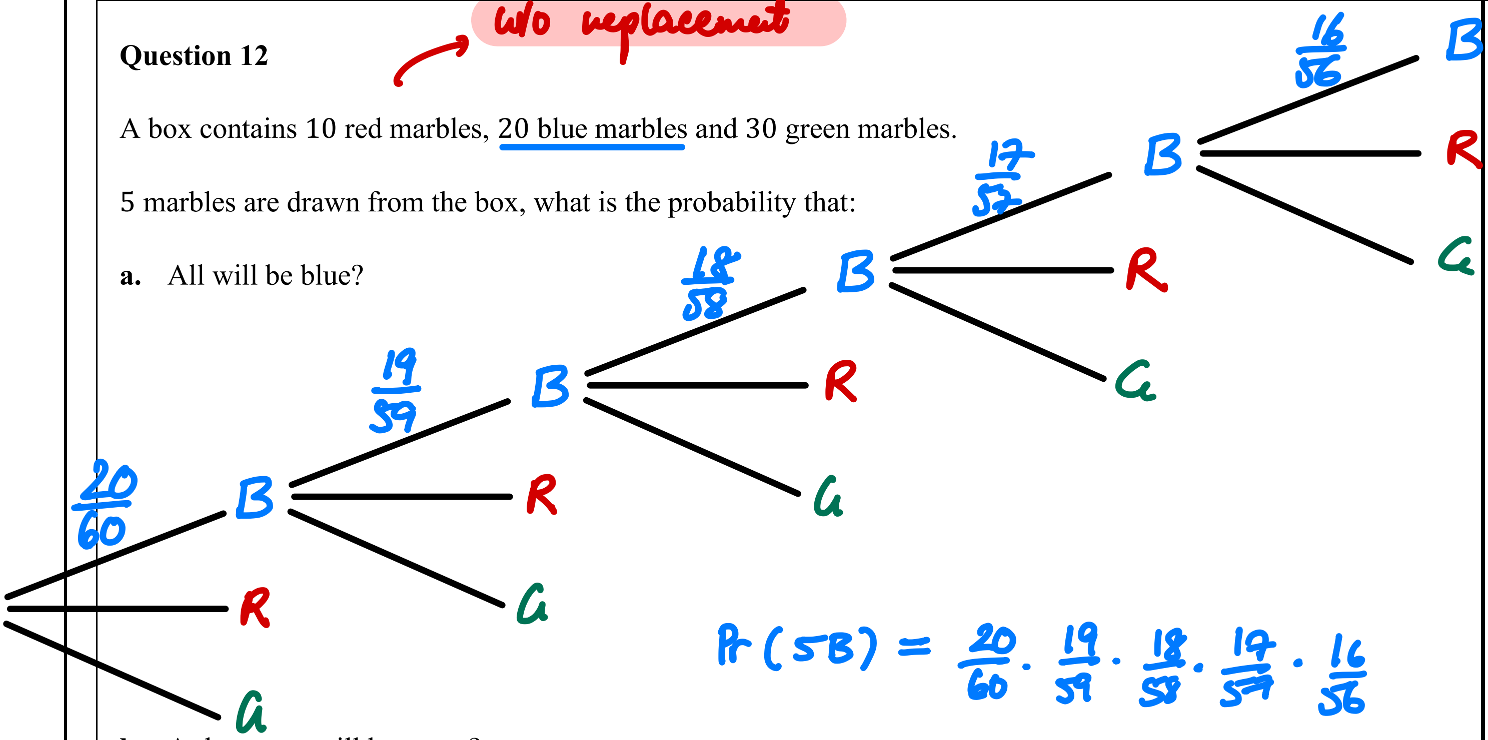
Question 12 w/o replacement

A box contains 10 red marbles, 20 blue marbles and 30 green marbles.

5 marbles are drawn from the box, what is the probability that:

a. All will be blue?



$$\Pr(5B) = \frac{20}{60} \cdot \frac{19}{59} \cdot \frac{18}{58} \cdot \frac{17}{57} \cdot \frac{16}{56}$$

$$= \frac{34}{11{,}977}$$

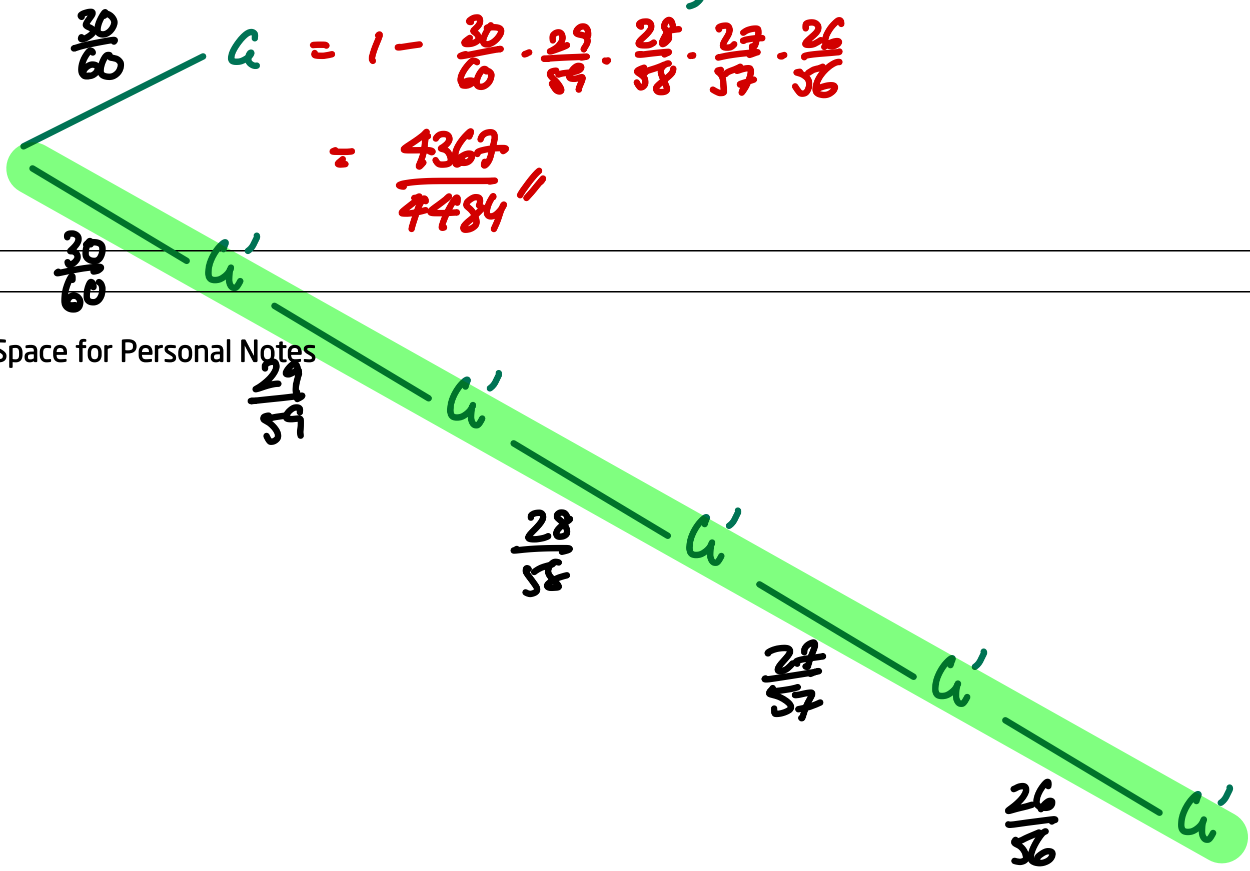
b. At least one will be green?

$$\Pr(G \geq 1) = 1 - \Pr(G = 0)$$

$$= 1 - \Pr(G'G'G'G'G')$$

$$= 1 - \frac{30}{60} \cdot \frac{29}{59} \cdot \frac{28}{58} \cdot \frac{27}{57} \cdot \frac{26}{56}$$

$$= \frac{4367}{4484}$$



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Exploration: Understanding Conditional Probabilities

- Consider two probabilities below:

Probability that:
A student randomly selected by James is male and wearing glasses.

Probability that:
A male student randomly selected by James is wearing glasses.

- Which of the two probabilities tells us something that has already occurred? [First/Second]
- The [first/second] probability is called conditional probability.
- Conditional probability is when we know one event has already occurred.
- In the above example, for the second one, we already know that the student is male.

Question 13

State whether each of the following probabilities would be conditional or not.

Event	Conditional / Not
Probability of selecting a male JMSS student.	Not
Probability of selecting a JMSS student out of male students.	Conditional

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Question 14

A die is thrown twice, and the sum of the numbers appearing is observed to be 6. What is the probability that the number 4 has appeared at least once?

$Pr(\text{No. } 4 \text{ appears} \geq 1 \mid \text{Sum} = 6)$

$= \frac{2}{5}$

Dice 1

Dice 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Conditional Probability

- **Definition**: Probability of A given B.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

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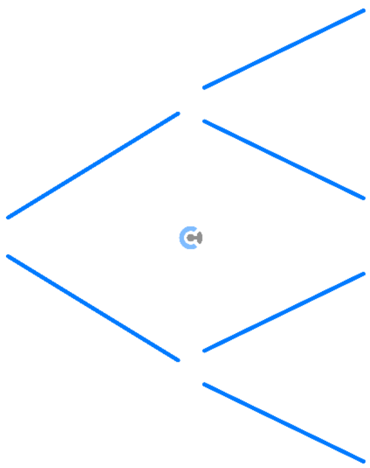
Question 15 Walkthrough.

If $\Pr(A) = \frac{2}{5}$, $\Pr(B) = \frac{3}{5}$, and $\Pr(A \cap B) = \frac{1}{5}$, evaluate $\Pr(A|B)$.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\left(\frac{1}{5}\right)}{\left(\frac{3}{5}\right)} = \frac{1}{3}$$

<u>Tree Diagram for Conditional Probability</u>



- Tree diagram is perfect for conditional probability as each branch is conditional probability.

$$\text{Each branch} = \Pr(\boldsymbol{leaf}|\boldsymbol{root})$$

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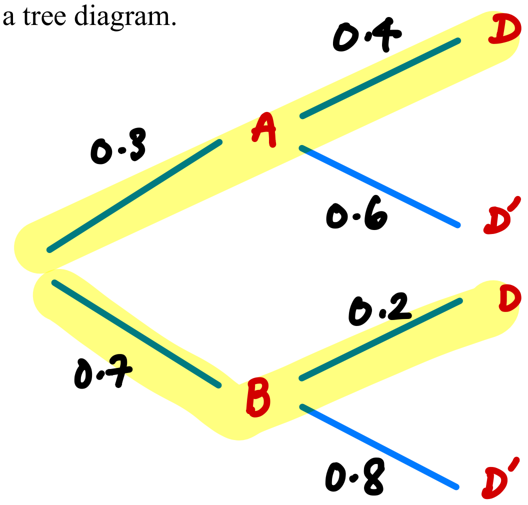
Question 16

A company manufactures laptops in two different factories: Factory A and Factory B. It is known that 30% of all laptops come from Factory A. Due to different quality standards, the probability of a laptop being **defective** from Factory A is 40%, while the probability of a laptop being **defective** from Factory B is 20%.

a. Construct a tree diagram.



b. What is the probability of a laptop battery being defective?

$$\Pr(D) = \Pr(A \cap D) + \Pr(B \cap D)$$
$$= 0.3 \times 0.4 + 0.7 \times 0.2$$
$$= 0.12 + 0.14$$
$$= 0.26$$

c. Given that a laptop is defective, what is the likelihood that it came from Factory A?

$$\Pr(A|D) = \frac{\Pr(A \cap D)}{\Pr(D)} = \frac{0.12}{0.26} = \frac{12}{26} = \frac{6}{13}$$

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Definition

Conditional Probability with Independent Events

$$\Pr(A|B) = \Pr(A)$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

Question 17

Let event A be "You draw a red card from a deck", and the event B be "It snows today".

a. Are A and B independent?

$\therefore$ Yes

b. Simplify the expression for $\Pr(A \mid B)$.

$\Pr(A|B) = \Pr(A)$

c. Hence, find this probability.

$= \frac{1}{2}$

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Section C: Warm Up Test (17 Marks)

INSTRUCTION: 17 Marks. 17 Minutes Writing.

Question 18 (5 marks)

Suppose that for events A and B, we know the following probabilities:

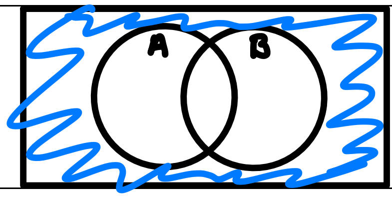
$$\mathrm{P}(A) = 0.4,\ \mathrm{P}(B) = 0.2 \text{ and } \mathrm{P}(A \cup B) = 0.5$$

Compute:

a. $\mathrm{P}(A')$. (1 mark)

$Pr(A') = 1 - Pr(A) = 0.6$ //

b. $\mathrm{P}(A' \cap B')$. (2 marks)



$Pr(A' \cap B') = 1 - Pr(A \cup B) = 0.5$ //

c. $\mathrm{P}(A \cap B)$. (2 marks)

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$0.5 = 0.4 + 0.2 - Pr(A \cap B)$

$\therefore Pr(A \cap B) = 0.1$ //

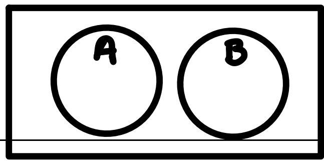
Question 19 (6 marks) $Pr(A \cap B) = 0$

Suppose that A and B are mutually exclusive events for which $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.6$. What is the probability that:

a. Either A or B occurs? (2 marks)

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$
$$= 0.3 + 0.6 - 0$$
$$= 0.9$$

b. A occurs but B does not? (2 marks)



$$Pr(A \cap B') = Pr(A) = 0.3$$

c. Both A and B occur? (~~2~~ 1 marks)

$$Pr(A \cap B) = 0$$

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Question 20 (6 marks)

Two hundred patients who had either shoulder surgery or knee surgery were asked whether they were satisfied or dissatisfied regarding the result of their surgery. The following table summarises their response.

Surgery	Satisfied	Dissatisfied	Total
Knee	70	35	105
Shoulder	60	35	95
Total	130	70	200

a. If one person from the 200 patients are selected at random, determine the probability that the person was satisfied ***GIVEN*** that the person had knee surgery. (2 marks)

$\Pr(\text{Satisfied} \mid \text{Knee Surgery}) = \dfrac{\Pr(S \cap K)}{\Pr(K)} = \dfrac{70}{105}$

$= \dfrac{2}{3}$ //

b. Was dissatisfied given that they had shoulder surgery? (2 marks)

$\Pr(\text{Dissatisfied} \mid \text{Shoulder}) = \dfrac{\Pr(D \cap S)}{\Pr(S)} = \dfrac{35}{95}$

$= \dfrac{7}{19}$ //

c. Had knee surgery given that they were dissatisfied? (2 marks)

$\Pr(\text{Knee Surgery} \mid \text{Dissatisfied}) = \dfrac{\Pr(K \cap D)}{\Pr(D)} = \dfrac{35}{70} = \dfrac{1}{2}$ //

Section D: Exam 1 Questions (24 Marks)

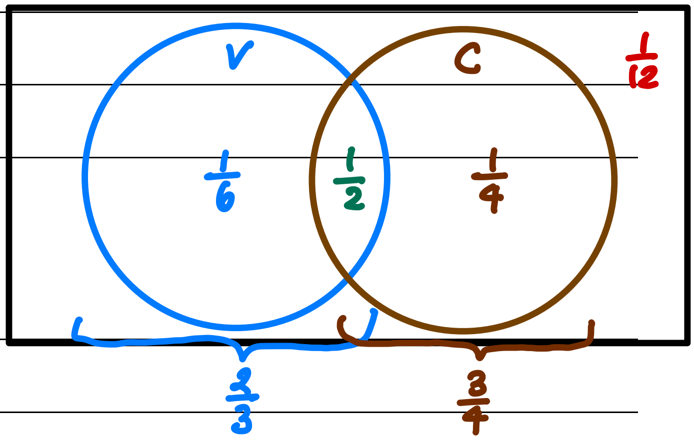
Question 21 (6 marks)

The probability that a random Contour student likes vanilla flavoured ice cream is $\frac{2}{3}$ and the probability that they like chocolate ice cream is $\frac{3}{4}$. The probability that they like both ice cream flavours is $\frac{1}{2}$. Find the probability that a randomly picked Contour student likes:

a. Vanilla ice cream but not chocolate ice cream. (1 mark)

$V \cap C'$

$Pr(V \cap C') = \frac{1}{6}$



b. Neither chocolate nor vanilla ice cream. (2 marks)

$C' \cap V'$

$Pr(C' \cap V') = \frac{1}{12}$

c. Vanilla ice cream given that they like chocolate ice cream. (2 marks)

$V \mid C$

$Pr(V \mid C) = \dfrac{Pr(V \cap C)}{Pr(C)} = \dfrac{\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)} = \frac{2}{3}$ //

d. Are the events that a student likes vanilla ice cream and a student likes chocolate ice cream independent? (1 mark)

$Pr(V \cap C) = Pr(V) \cdot Pr(C)$

$\frac{1}{2} = \frac{2}{3} \cdot \frac{3}{4}$

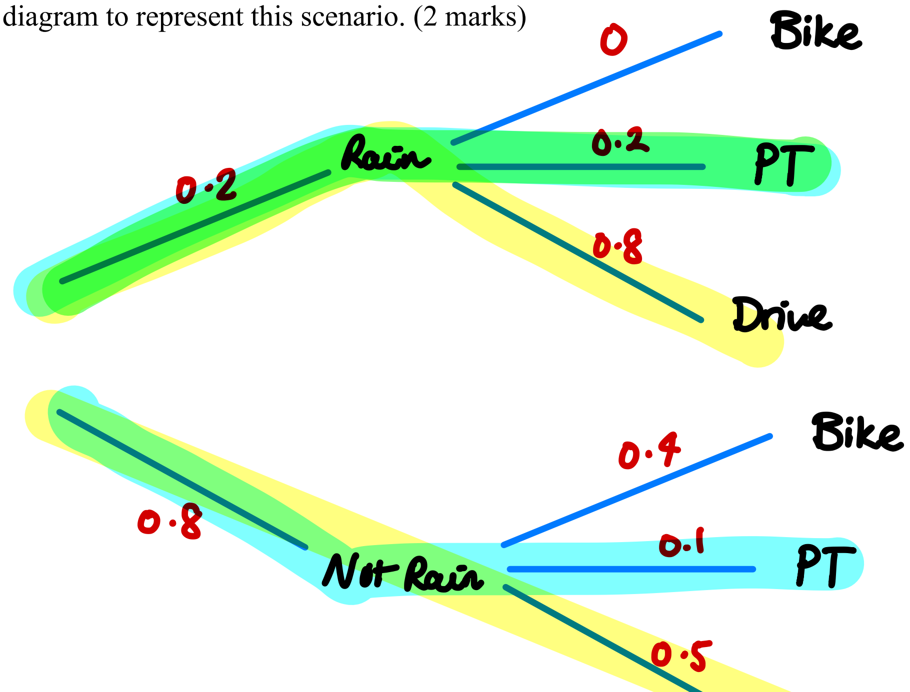
$\frac{1}{2} = \frac{2}{4}$ ✓

∴ V and C are independent

Question 22 (7 marks)

The way Joseph travels to work depends on whether it is raining or not. If there is no rain; there is a 40% chance that he bikes, a 10% chance that he uses public transport and a 50% chance that he drives. If it is raining; there is a 80% chance that he drives and a 20% chance that he uses public transport. The chance that it rains on any given day is 20%.

a. Construct a tree diagram to represent this scenario. (2 marks)



b. What is the probability that Joseph drives to work? (2 marks)

Pr(Drive)

$Pr(Drive) = Pr(R \cap D) + Pr(R' \cap D)$

$= 0.2 \times 0.8 + 0.8 \times 0.5$

$= 0.16 + 0.4 = 0.56$ //

c. Given that it is not raining, what is the probability that Joseph drives to work? (1 mark)

$Pr(Drive \mid Not\ Raining) = \frac{1}{2}$ //

d. Given that Joseph uses public transport to get to work, what is the probability that it was raining? (2 marks)

$$\Pr(\text{Raining} \mid PT) = \frac{\Pr(R \cap PT)}{\Pr(PT)} = \frac{0.2 \times 0.2}{0.2 \times 0.2 + 0.8 \times 0.1}$$

$$= \frac{0.04}{0.04 + 0.08} = \frac{1}{3} //$$

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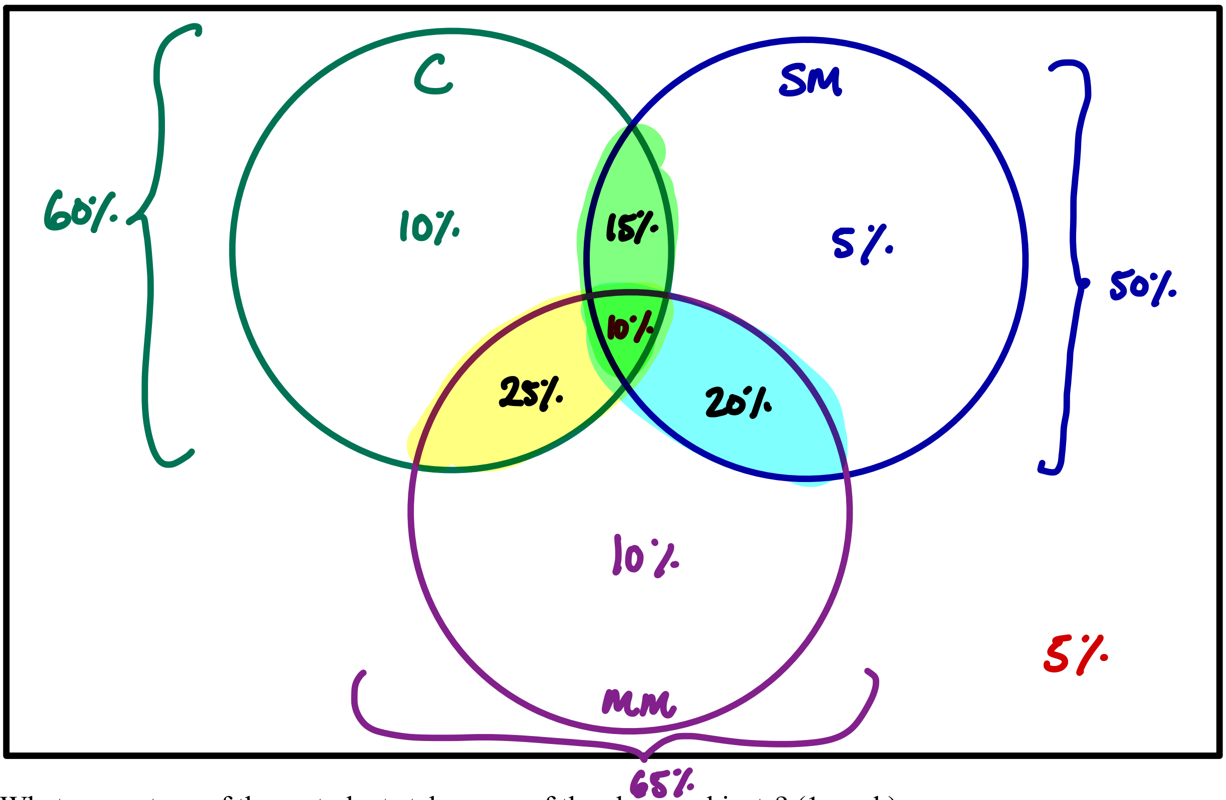
Question 23 (7 marks)

In a random sample of VCE Contour students, it was found that: 65% take Math Methods, 50% take Specialist Maths, 60% take Chemistry, 30% take both Methods and Specialist, 25% take Specialist and Chemistry, 35% take Methods and Chemistry and 10% take all three subjects.

a. Draw a Venn diagram to illustrate the information above. (4 marks)



b. What percentage of these students take none of the above subjects? (1 mark)

Pr(No Subjects) = 5%

c. What percentage of these students take exactly two of the above subjects? (1 mark)

Pr(2 Subjects) = 25% + 15% + 10% = 60% //

d. What percentage of these studens take *at least* two of the above subjects? (1 mark)

Pr (≥ 2 subjects) = 60% + 10% = 70% //

Question 24 (4 marks)

If A and B are events such that $\Pr(A \cup B) = \frac{3}{4}$, $\Pr(A \cap B) = \frac{1}{6}$, and $\Pr(A) > \Pr(B)$.

a. Are events A and B mutually exclusive? (1 mark)

↪ No! As $\Pr(A \cap B) \neq 0$! //

b. Given that events A and B are independent. Find the values for $\Pr(A)$ and $\Pr(B)$. (3 marks)

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

$\frac{1}{6} = \Pr(A) \cdot \Pr(B)$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\frac{3}{4} = a + b - \frac{1}{6}$

Let $a = \Pr(A)$ & $b = \Pr(B)$:

$a + b = \frac{3}{4} + \frac{1}{6} = \frac{9+2}{12} = \frac{11}{12}$

$ab = \frac{1}{6} \Rightarrow a = \frac{1}{6b} \ldots (1)$

$a + b = \frac{11}{12} \ldots (2)$

$\therefore \frac{1}{6b} + b = \frac{11}{12}$ $\quad \times 12b$

$2 + 12b^2 = 11b$

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$12b^2 - 11b + 2 = 0$

$(4b - 1)(3b - 2) = 0$

$\therefore b = \frac{1}{4}$ or $b = \frac{2}{3}$

$a = \frac{1}{6b}$

$= \frac{1}{\left(\frac{6}{4}\right)}$

$a = \frac{2}{3}$

$\therefore a = \frac{1}{4}$

reject as $a > b$

$\therefore \Pr(A) = \frac{2}{3}$ & $\Pr(B) = \frac{1}{4}$

Section E: Exam 2 Questions (31 Marks)

Question 25 (1 mark)

$2^8 = (4)^4 = 16^2 = 256$

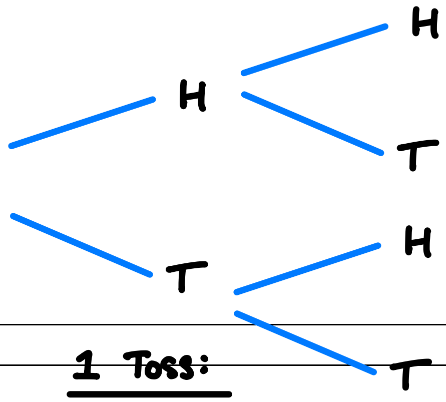
An experiment is ran where 8 coins are tossed simultaneously. The size of the sample space for this experiment is:

A. 2

B. 128

C. 256

D. 512



1 Toss: 2^1

2 Toss: 2^2

Question 26 (1 mark)

Subu buys a x tickets to a random raffle, where every ticket has an equal chance of winning first prize. A total of 5,500 tickets are sold and Subu calculates that he has a 20% chance of winning the raffle.

Determine the value of x.

A. 555

B. 1100

C. 2000

D. 1000

20% of 5500 tickets = 1100

Question 27 (1 mark)

16 cards are numbered from 1 to 16. If a card is drawn at random, what is the probability that the number on the card is a prime number?

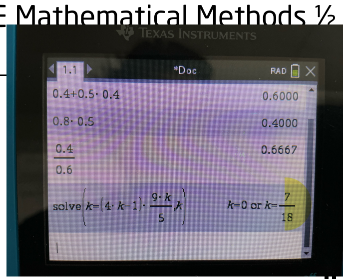
A. $\frac{5}{16}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{2}{5}$

$$\frac{\{2, 3, 5, 7, 11, 13\}}{\{1, \ldots\ldots, 16\}} = \frac{6}{16} = \frac{3}{8}$$



Question 28 (1 mark)

For events A and B, $\Pr(A \cap B) = k$, $\Pr(A' \cap B) = \frac{4k}{5}$ and $\Pr(A \cap B') = 3k - 1$.

If A and B are independent, then the value of k is:

A. 0

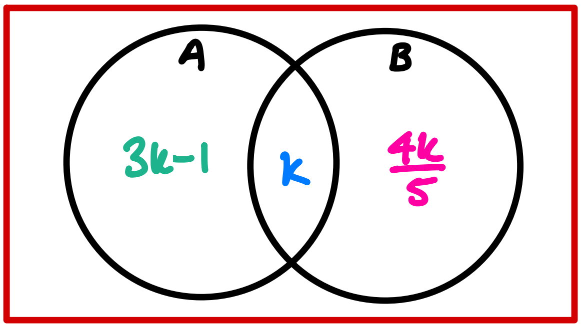
B. $\frac{9}{20}$

C. $\frac{7}{18}$

D. $\frac{5}{13}$

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

$k = (4k-1) \cdot \frac{9k}{5}$



Question 29 (1 mark)

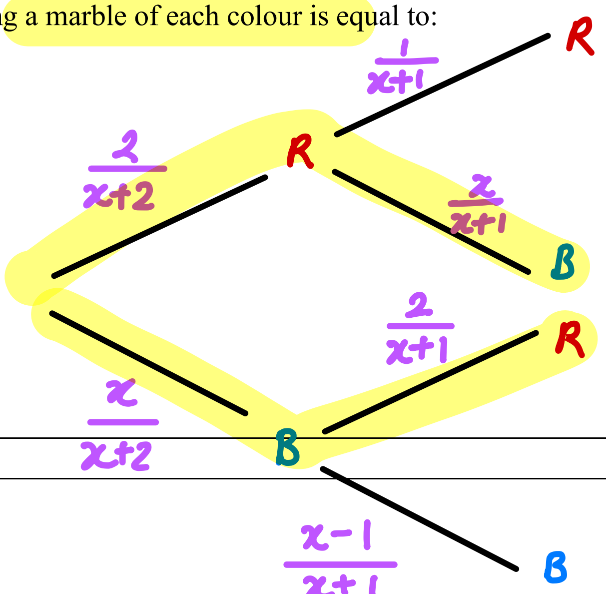
A bag contains two red marbles and x blue marbles. Two marbles are randomly drawn from the bag without replacement. The probability of drawing a marble of each colour is equal to:

A. $\frac{2x}{(1+x)(2+x)}$

B. $\frac{4x}{(1+x)(2+x)}$

C. $\frac{x}{1+x}$

D. $\frac{4+x}{(2+x)(3+x)}$



$= \frac{2x}{(x+1)(x+2)}$
$+$
$= \frac{2x}{(x+1)(x+2)}$
$= \frac{4x}{(x+1)(x+2)}$

Question 30 (1 mark)

Four fair coins are tossed at the same time. The outcome for each coin is independent of the outcome for any other coin. The probability that there is an equal number of heads and tails, given that there is at least one head, is:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{2}{5}$

$\Pr(2H2T \mid H \geq 1) = \frac{6}{15} = \frac{2}{5}$

$2^4 = 16$

HHHH
⋮
TTTT

$H \geq 1 = 15$ combos

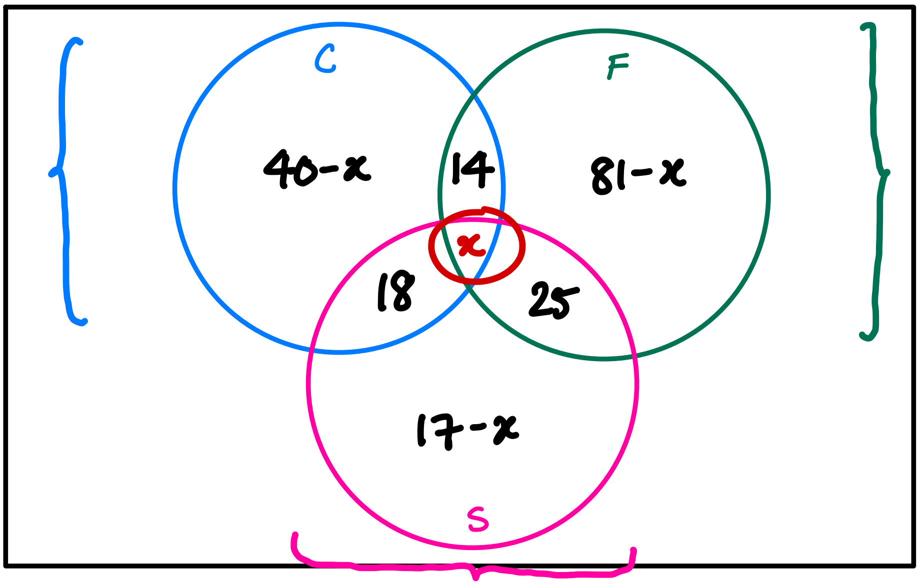
6: HHTT, HTHT, THHT, THTH, HTTH, TTHH

Question 31 (8 marks)

A restaurant receives feedback from customers in three main categories: food quality (F), staff friendliness (S), and restaurant cleanliness (C). In total, 185 customer feedback responses were recorded in a certain month. The feedback breakdown is as follows:

- 120 customers gave feedback about the food.
- 60 customers gave feedback about the staff.
- 72 customers gave feedback about the cleanliness.
- 25 customers gave feedback about the food and the staff, but not the cleanliness.
- 14 customers gave feedback about the food and the cleanliness, but not the staff.
- 18 customers gave feedback about the staff and the cleanliness, but not the food.
- The number of customers who gave feedback about all three categories is an unknown x.

a. Draw a Venn diagram to illustrate the above information. Express numbers in terms of x, where appropriate. (4 marks)



b. Determine the number of customers who gave feedback about all three categories. (2 marks)

$195 - 2x = 185$

$2x = 10$

$\therefore x = 5$

3 Categories = 5 ppl

c. Determine the probability that a random response gave feedback about at least two categories. (2 marks)

$$\Pr(\geq 2 \text{ Categories}) = \frac{14+18+25+x}{185} = \frac{62}{185}$$

Space for Personal Notes

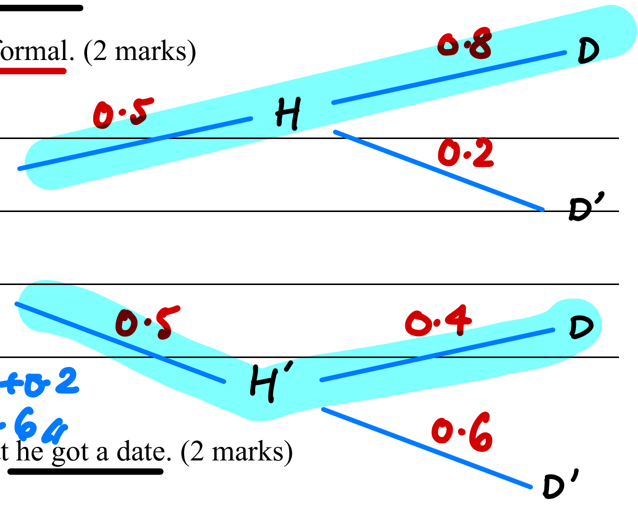
Question 32 (11 marks)

Caleb is considering getting a haircut to increase his chances of getting a date for the upcoming year 11 formal. The probability that Caleb gets a haircut *is* 0.5. If Caleb gets a haircut, the probability that he gets a date is 0.8. If he does not get a haircut, the probability that he gets a date is 0.4.

a. Calculate the probability that Caleb got a date to the formal. (2 marks)

Let H = getting a haircut

Let D = getting a date



$Pr(D) = Pr(H \cap D) + Pr(H' \cap D)$

$= 0.5 \times 0.8 + 0.5 \times 0.4 = 0.4 + 0.2$

$= 0.6$

b. Find the probability that Caleb got a haircut given that he got a date. (2 marks)

$Pr(H|D) = \dfrac{Pr(H \cap D)}{Pr(D)} = \dfrac{0.4}{0.6} = \dfrac{2}{3}$

c. State whether the events "getting a date" and "getting a haircut" are independent or not. (1 mark)

D H

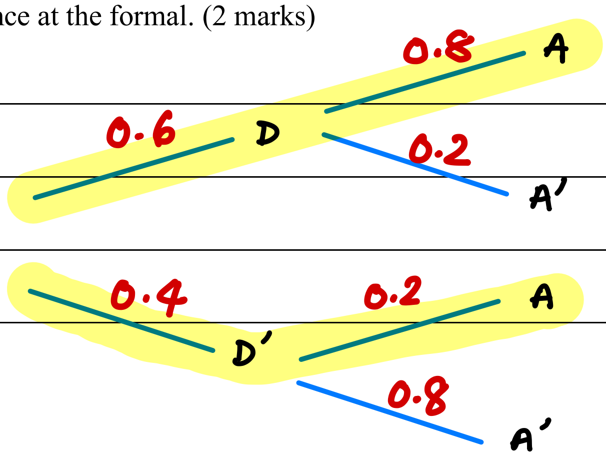
$Pr(H \cap D) = Pr(H) \cdot Pr(D)$

$0.4 \neq 0.5 \times 0.6$

∴ H & D are dependent

At the formal, if Caleb has a date there is a 80% chance that he will be asekd to dance, however if he does not have date there is only a 20% chance that he will be asked to dance.

d. Calculate the probability that Caleb was asked to dance at the formal. (2 marks)



$Pr(A) = Pr(D \cap A) + Pr(D' \cap A)$

$= 0.6 \times 0.8 + 0.4 \times 0.2$

$= 0.48 + 0.08$

$= 0.56$

e. Given that Caleb was asked to dance, find the probability that he got a date. (2 marks)

$$Pr(D|A) = \frac{Pr(D \cap A)}{Pr(A)} = \frac{0.48}{0.56} = \frac{48}{56} = \frac{12}{14} = \frac{6}{7}$$

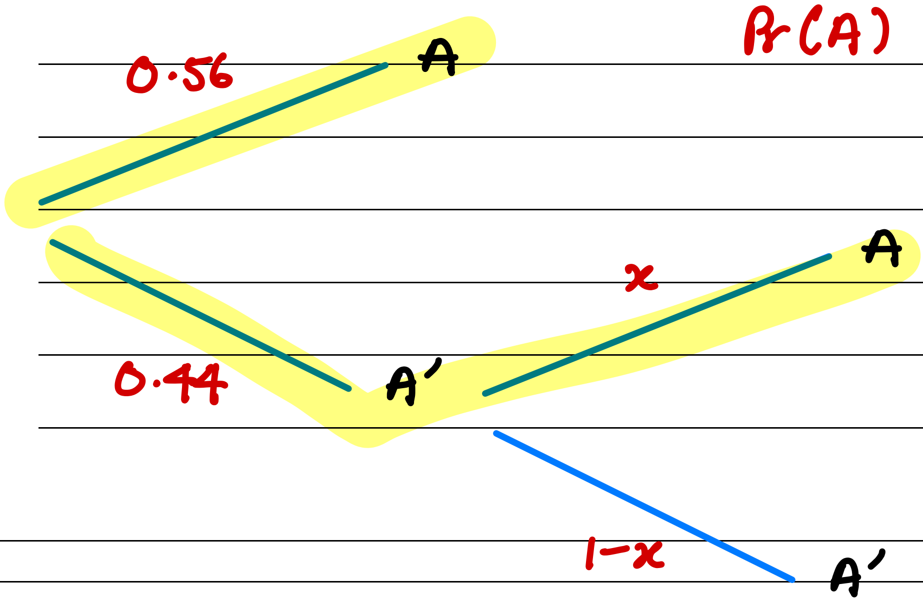
If Caleb has not been asked to dance, he decides to put on a very impressive solo dance act. After Caleb finishes his solo dance, there is probabilty of x that he is asked to a dance.

f. By the end of the night, there is a 95% chance that Caleb has been asked to dance by somebody. Find the value of x. (2 marks)

$$Pr(A) = 0.56 + 0.44x = 0.95$$

$$0.44x = 0.39$$

$$\therefore x = \frac{39}{44}$$



Space for Personal Notes

Question 33 (6 marks)

In his year 12 studies, Rei would spend either 0, 1, 2, 3 or 4 hours on homework every night. The probabilities are shown in the table below, where k is a real number.

H # Hours	0	1	2	3	4
Probability	$10k^2$	$2k$	$3k$	$3k$	$20k^2 - k$

$\underbrace{\hphantom{10k^2 + 2k + 3k + 3k + 20k^2-k}}_{1}$

a. Find the value of k. (3 marks)

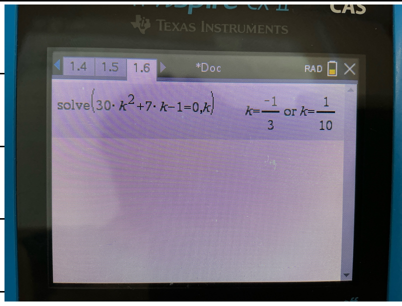
$\because$ Sum Probability $= 1$

$10k^2 + 2k + 3k + 3k + 20k^2 - k = 1$

$30k^2 + 7k - 1 = 0$

$\therefore k = -\frac{1}{3}$ or $k = \frac{1}{10}$

reject as $k \geq 0$



b. Find the probabiilty that Rei spends at most 1 hour studying on a given night. (1 mark)

$H \leq 1$

$Pr(H \leq 1) = Pr(H=0) + Pr(H=1)$

$= 10k^2 + 2k = \frac{10}{100} + \frac{2}{10} = \frac{3}{10}$

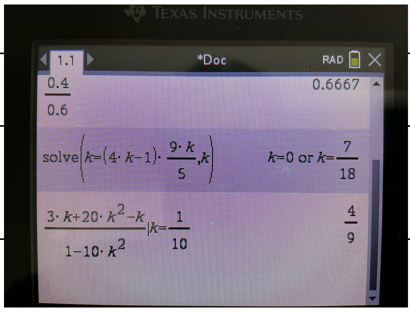
c. Find the probability that Rei spends **more** than 2 hours studying given that he spends at least 1 hour studying. (2 marks)

$H > 2 \quad | \quad H \geq 1$

$Pr(H>2 \mid H \geq 1) = \frac{Pr(H>2 \cap H \geq 1)}{Pr(H \geq 1)}$

$= \frac{Pr(H>2)}{Pr(H \geq 1)} = \frac{3k + 20k^2 - k}{1 - 10k^2}$

$= \frac{4}{9}$



Contour Check

☐ Learning Objective: [3.2.1] – Understand probabilities in terms of favourable outcomes from a sample space

Key Takeaways

- ☐ The sample space is the set of all possible outcomes in an experiment.
- ☐ Total probability must always add to 1.
- ☐ When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{\text{No. of successful outcomes}}{\text{No. of total outcomes}}$$

☐ Learning Objective: [3.2.2] – Basic probability operations. Use Venn diagrams and/or Karnaugh maps and apply the addition rule

Key Takeaways

- ☐ If A and B are events, then we interpret the following operations as:
- ☐ $A \cup B =$ A or B
- ☐ $A \cap B =$ A and B
- ☐ $A' =$ Not A

$$\Pr(A') = 1 - \Pr(A)$$

Karnaugh Tables

- We can represent probability problems using a Karnaugh Map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

- **The addition rule:**

$$\Pr(A \cup B) = \underline{\Pr(A)+\Pr(B)-\Pr(A\cap B)}$$

Learning Objective: [3.2.3] – Understand the meaning behind independent and mutually exclusive events

Key Takeaways

Mutually Exclusive Events

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- Probability of both A and B happening together is zero.

$$\Pr(A \cap B) = \underline{0}$$

Independent Events

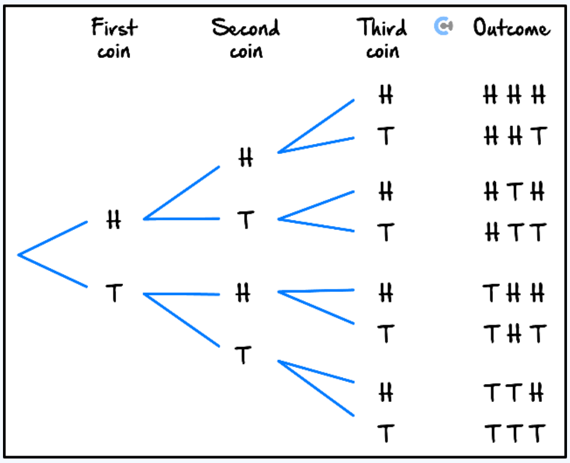
- **Definition:** Independent events **do not affect** the likelihood of the other.
- **Mathematically:**

$$\Pr(A \cap B) = \underline{\Pr(A)\times\Pr(B)}$$

Learning Objective: [3.2.4] - Understand conditional probability and make use of tree diagrams

Key Takeaways

Tree Diagram



- Useful for ultiple sequence events.
- To calculate the probability of a sequence, we multiply the probabilities along the relevant branches.

Conditional Probability

- **Definition**: Probability of A given B.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Conditional Probability with Independent Events

$$\Pr(A|B) = \Pr(A)$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

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