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VCE Mathematical Methods ½
Probability [3.1]
Workbook

Outline:

- Highly recommended video series *Probabilities of Probabilities* by 3Blue1Brown on YouTube – It's still being added to by him, and definitely goes out of the course, but very interesting and high quality if you're curious/interested!!

Probabilities of Probabilities
http://bit.ly/3b1bprobability

Learning Objectives:

- [ ] **MM12 [3.1.1]** - Understand Probabilities in Terms of Favourable Outcomes From a Sample Space
- [ ] **MM12 [3.1.2]** -Basic Probability Operations. Use Venn diagrams and/or Karnaugh Maps and Apply the Addition Rule
- [ ] **MM12 [3.1.3]** - Understand the Meaning Behind Independent and Mutually Exclusive Events
- [ ] **MM12 [3.1.4]** - Understand Conditional Probability and Make use of Tree Diagrams

Section A: Introduction to Probability

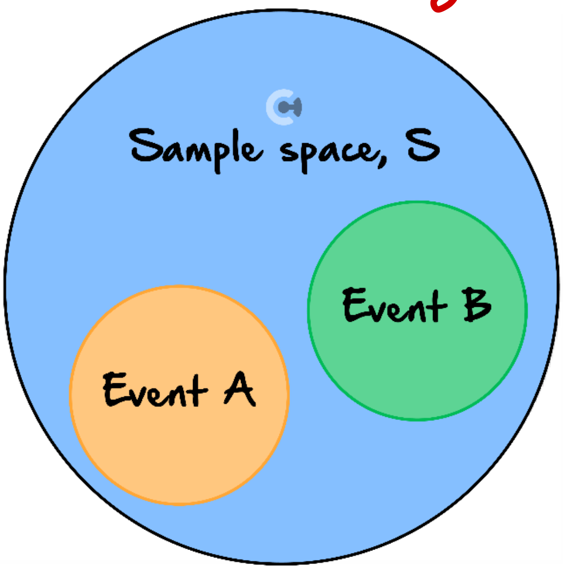
Sub-Section: Sample Space and Uncertainty

Discussion: What is probability?

→ : uncertainty

Exploration: Understanding Probability

- In probability, we are aiming to quantify uncertainty.



- Sample Space (ε) = Set of all possible outcomes.
- Probability of an event = Proportion of sample space that the event takes up.
- The probability of an event must **always** take a value between 0 & 1, inclusive.

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Definition

Sample Space (ε)

- The set of all possible outcomes in an experiment.
- For tossing two coins in a row, the sample space is:

$$\varepsilon = \{HH, TT, HT, TH\}$$

- For rolling a standard 6-sided dice, the sample space is:

$$\varepsilon = \{1, 2, 3, 4, 5, 6\}$$

- Total probability adds up to 1.

Discussion: What is wrong with the probability statement "I have a 60% chance of picking a dark chocolate from this bag and a 50% chance of picking a milk chocolate"?

Sum > 100%

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Question 1 Walkthrough.

Using the sample space for tossing two coins in a row, find:

a. The probability of both tosses being heads.

$\frac{1}{4}$

b. The probability of tossing exactly one head and one tail.

$\varepsilon = \{HH, HT, TH, TT\}$

$\frac{1}{2}$

c. The probability of tossing a head, and **then** a tail.

$\frac{1}{4}$

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Question 2

Using the sample space for rolling a standard 6-sided dice, find:

a. The probability of rolling a 6.

$\frac{1}{6}$

b. The probability of rolling a number less than 3.

$\varepsilon = \{1, 2, 3, 4, 5, 6\}$

$\frac{\{1,2\}}{\{1,..6\}} = \frac{1}{3}$ //

c. The probability of rolling an odd number.

$\frac{1}{2}$ //

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Sub-Section: Equally Likely Events

Discussion: How can we calculate probabilities if all events are equally likely? E.g., Each number on the dice.

→ No. of wanted outcomes

Count the no. of events

Calculating Probabilities for Equally Likely Outcomes

Definition

- When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{\text{No. of successful outcomes}}{\text{Total No. of outcomes}}$$

Question 3 Walkthrough.

You have signed up for a trial for experimental medication, in which participants are randomly assigned with 400 people in the test group (that receive the medication), and 500 in the control group (that receives a placebo).

What is the probability that you are receiving the actual medication?

$$\Pr(\text{Receiving medication}) = \frac{400}{900} = \frac{4}{9}$$

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Question 4

A fair 20-sided die (as shown) is rolled.

What is the probability that the result is a number divisible by 6?

$$\Pr(\text{Divisible by } 6) = \frac{3}{20}$$

Question 5 Extension.

Two fair six-sided dies are rolled.

What is the probability that the sum of the result is divisible by 3?

$$\Pr(\text{Div by } 3) = \frac{12}{36}$$

$$= \frac{1}{3}$$

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

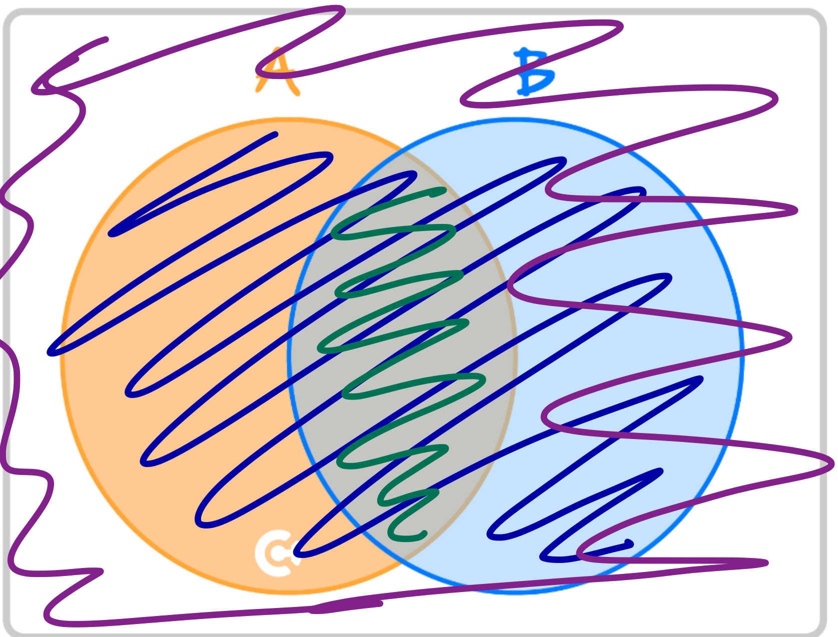
Section B: Venn Diagrams and Karnaugh Tables

Sub-Section: Union, Intersection, and Complement

Let's take a look at the relationship between two events!

Definition

Union, Intersection, and Complement



- $A \cup B =$

  ↪ Union ⇒ "either", "or" ⇒ ALL of A and ALL of B

- $A \cap B =$

  ↪ intersection ⇒ "and" ⇒ in A AND B
  (be in both)

- $A' =$
  - Complement of A : Not in A
  -
  -

$$\Pr(A') = 1 - Pr(A)$$

Question 6

Let event $G =$ a student wears glasses, and an event $F =$ a student is a female.

Write an expression for the probability of each of the following events, without doing any calculations:

a. A student is a female who wears glasses.

$F \cap G$

$\hookrightarrow \therefore Pr(F \cap G)$ //

b. A student is either a female or doesn't wear glasses.

$F \cup G'$

$\hookrightarrow \therefore Pr(F \cup G')$ //

c. A student wears glasses and is not a female.

$G \cap F'$

$\hookrightarrow \therefore Pr(G \cap F')$ //

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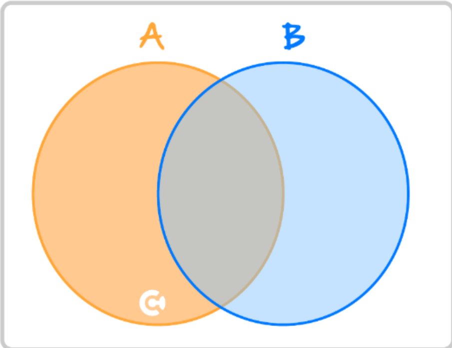
Sub-Section: Venn Diagram

Venn Diagram



- Venn diagram is useful to visualise the two events.

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Question 7 Walkthrough.

Let the probability that a random student wears glasses in a class be $\frac{1}{3}$, and the probability that a random student is female be $\frac{1}{2}$. The probability that a random student is a female who wears glasses is $\frac{1}{12}$.

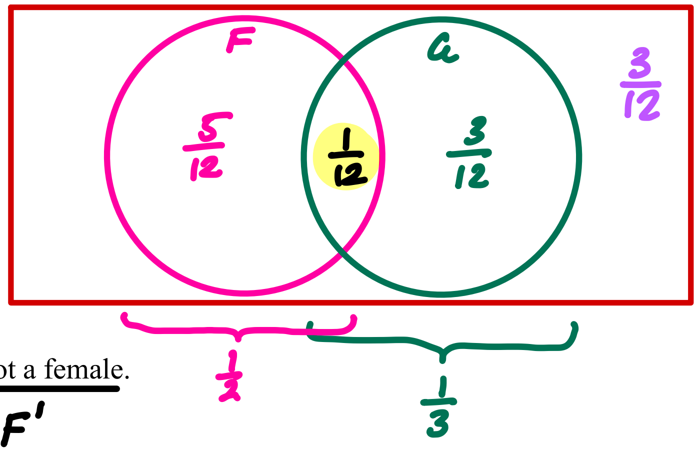
$F \cap G$

Find the probability of each of the following events:

a. A randomly picked student is a female who does not wear glasses.

$F \cap G'$

$Pr(F \cap G') = \frac{5}{12}$



b. A randomly picked student wears glasses and is not a female.

$G \cap F'$

$Pr(G \cap F') = \frac{3}{12} = \frac{1}{4}$

c. A randomly picked student is either a female or doesn't wear glasses.

$F \cup G'$

$Pr(F \cup G') = \frac{9}{12} = \frac{3}{4}$

NOTE: Drawing a Venn diagram can help a lot!

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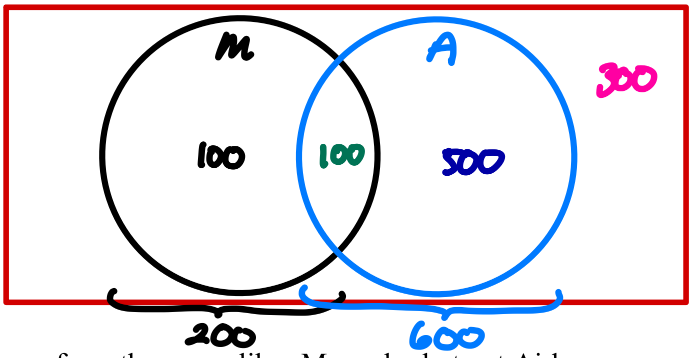
Question 8

Out of 1000 engineers, 200 like Mercedes as an automaker. Out of the same 1000 engineers, 600 like Airbus as an aeroplane manufacturer. Exactly 100 engineers like both Mercedes and Airbus.

a. How many engineers neither like Mercedes, nor Airbus?

300



b. Find the probability that a randomly picked engineer from the group likes Mercedes but not Airbus.

$M \cap A'$

$$\Pr(M \cap A') = \frac{100}{1000} = \frac{1}{10}$$

c. Find the probability that a randomly picked engineer from the group does not like Mercedes, but likes Airbus.

$M' \cap A'$

$$\Pr(M' \cap A) = \frac{500}{1000} = \frac{1}{2}$$

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Sub-Section: Karnaugh Tables

Are using Venn diagrams the only method to attempt the questions above?

Karnaugh Tables

- We can also represent probability problems using a Karnaugh Map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\mathbf{Pr}(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\mathbf{Pr}(A')$
	$\mathbf{Pr}(B)$	$\mathbf{Pr}(B')$	$\mathbf{1}$

  - The rows and columns add up to the last cell value.
- Remember the **total** probability must always add to 1.

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Question 9 Walkthrough.

The probability that a student will pass the final examination in both English and Methods is 0.5 and the probability of passing neither is 0.1.

If the probability of passing the English examination is 0.75, what is the probability of passing the Methods examination?

Use the Karnaugh map.

E = Passing English

M = Passing Methods

$Pr(M) = 0.65$

	E	E'	
M	0.5	0.15	0.65
M'	0.25	0.1	0.35
	0.75	0.25	1

Question 10

The probability that a person is tall and skinny is 0.4 and the probability of being neither is 0.1.

$T \cap S$

If the probability of being tall is 0.55, what is the probability of being skinny?

Use the Karnaugh map.

$\therefore Pr(S) = 0.75$

	S	S'	
T	0.4	0.15	0.55
T'	0.35	0.1	0.45
	0.75 ?	0.25	1

Question 11 Extension.

At the athletics carnival, the probability that Subu wins both the high jump and the long jump is w, and the probability that he wins neither event is 0.3. The probability that he wins the high jump is 0.6. Find the probability that he wins only one event in terms of w.

$\Pr(\text{1 event}) = \Pr(L \cap H') + \Pr(L' \cap H)$

$= 0.7 - w$

	H	H'	
L	w	0.1	$w + 0.1$
L'	$0.6 - w$	0.3	$0.9 - w$
	0.6	0.4	1

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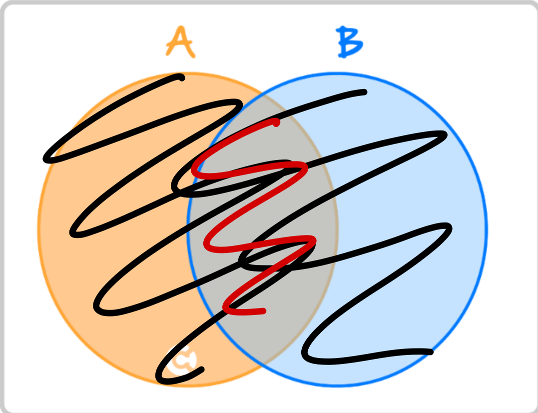
Sub-Section: Addition Rule

The Addition Rule



- When we add the probabilities of A and B, we count the outcomes contained in A **twice.**
- So, we must subtract one of them to get the probability of $A \cup B$.

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

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Question 12 Walkthrough.

If E and F are events such that $\Pr(E) = \frac{1}{4}$, $\Pr(F) = \frac{1}{2}$, $\Pr(E \cap F) = \frac{1}{8}$, find:

a. $\Pr(E \cup F)$

$$\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$$
$$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$
$$= \frac{2+4-1}{8} = \frac{5}{8}$$

~~**b.** $\Pr(\textit{neither } E \textit{ nor } F)$~~

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Question 13

A and B are events such that $\Pr(A) = 0.42$, $\Pr(B) = 0.48$ and $\Pr(A \cap B) = 0.16$.

Determine:

a. $\Pr(A')$

$\Pr(A') = 1 - 0.42$
$= 0.58$ //

b. $\Pr(B')$

$\Pr(B') = 1 - 0.48$
$= 0.52$ //

c. $\Pr(A \cup B)$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$= 0.42 + 0.48 - 0.16$
$= 0.9 - 0.16 = 0.74$ //

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Question 14 Extension.

A and B are events such that $\Pr(A \cup B) = a$, $\Pr(B) = 0.4$ and $\Pr(A \cap B) = b$.

Determine $\Pr(A')$ in terms of a and b .

$\because \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\therefore a = \Pr(A) + 0.4 - b$

$\hookrightarrow \Pr(A) = a + b - 0.4$

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$\therefore \Pr(A') = 1 - \Pr(A)$

$= 1 - (a + b - 0.4)$

$= 1.4 - a - b$

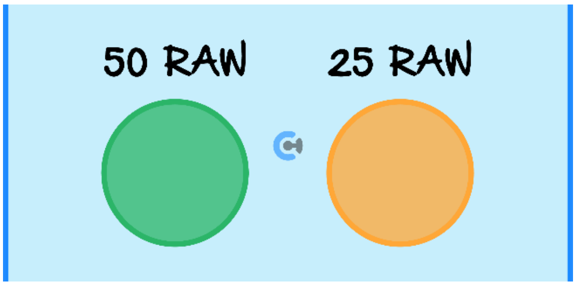
Section C: Mutually Exclusive Events and Independent Events

Sub-Section: Mutually Exclusive Events

What are mutually exclusive events?

Exploration: Mutually Exclusive Events

- Consider the event of getting 50RAW in Methods and 25RAW in Methods.



- Can these two events occur simultaneously? [Yes/No]
- These two events are hence called Mutually Exclusive.

Mutually Exclusive Events

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- Probability of both A and B happening together is 0

$$\Pr(A \cap B) = 0$$

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Question 15

State whether each of the following events would be mutually exclusive or inclusive.

Event 1	**Event 2**	**Exclusive / Inclusive**
Getting 50RAW in Methods.	Getting 48RAW in Methods.	Exclusive
Going to the beach.	Going to St. Kilda.	Inclusive

Question 16

$\Pr(E \cap F) = 0$

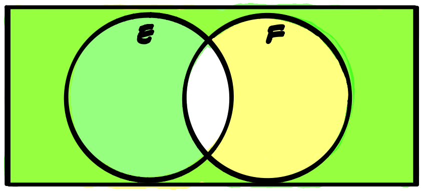
Events E and F are such that $\Pr(E' \cup F') = 0.25$. State whether E and F are mutually exclusive.

$\Pr(E' \cup F') = 1 - \Pr(E \cap F)$

$\therefore 0.25 = 1 - \Pr(E \cap F)$

$\Pr(E \cap F) = 0.75 \neq 0$

$\Pr(E \cap F)' =$



$\therefore$ E and F are not mutually exclusive

Question 17

$\Pr(A \cap B) = 0$

It is known that events A and B are mutually exclusive. ~~If $\Pr(A) = 0.2$ and $\Pr(B) = 0.5$~~, find $\Pr(A' \cup B')$.

$\Pr(A' \cup B') = 1 - \Pr(A \cap B)$

$= 1 - 0$

$= 1$

Sub-Section: Independent Events

What about independent events?

$Pr(H \cap H) = Pr(H) \cdot Pr(H)$
$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Exploration: Independent Event

- Consider the event of Qing (Contour Tutor) wearing a swimsuit and Subu teaching an MM12 Class.
- Are these two things related? [Yes/No]
- Hence, if you saw Qing walking down the road wearing a swimsuit, does that change the likelihood of Subu teaching a MM12 class? [Yes/No]
- These two events are called independent events as their likelihood doesn't affect each other.

Independent Events

- **Definition:** Independent events **do not affect** the likelihood of the other.
- **Mathematically**:

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

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Question 18

State whether each of the following events would be independent or dependent.

Event 1	Event 2	Independent / Dependent
Wearing a swimsuit.	Going to the beach.	Dependent
Doing homework for Methods.	Getting 50RAW in Methods.	Dependent
Doing homework for English.	Getting 50RAW in Methods.	Independent
Rolling a 6 on first dice roll.	Rolling a 6 on second dice roll.	Independent

Question 19

$Pr(B) = 1 - Pr(B') = 0.5$

$Pr(A \cap B) = Pr(A) \cdot Pr(B)$

Given that $\Pr(A) = 0.1$, $\Pr(B') = 0.5$, and $\Pr(A \cap B) = 0.03$, are the events A and B independent?

$$Pr(A \cap B) = Pr(A) \cdot Pr(B)$$

$$0.03 = 0.1 \times 0.5$$

$$0.03 \neq 0.05$$

$\therefore$ A and B are dependent events

NOTE: Use $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ to algebraically check independence.

Question 20 Extension.

Let $\Pr(A') = \frac{3}{5}$, $\Pr(B') = \frac{7}{10}$, and $\Pr(A' \cap B') = \frac{2}{5}$. Are A and B independent? Show using calculations.

$$Pr(A) \cdot Pr(B) = Pr(A \cap B)$$

$$Pr(A') \cdot Pr(B') = Pr(A' \cap B')$$

independent events

$$\frac{3}{5} \cdot \frac{7}{10} = \frac{2}{5}$$

$$\frac{21}{50} \neq \frac{2}{5}$$

$\therefore$ A and B are dependent events//

What is the difference between a mutually exclusive event and an independent event?

Exploration: Understanding the difference between mutually exclusive events and independent events

- Remember the event of Qing wearing a swimsuit and Subu teaching a MM12 Class.
- We concluded that these two events are independent.
- However, are they also mutually exclusive? No!

  Meaning can they happen at the same time? [Yes/No]
- **In Summary**
  - Independent Events: Their likelihood does not affect each other, but they can occur at the same time.
  - Mutually Exclusive Events: They CANNOT happen at the same time.

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Question 21

★ Use the Addition Rule

Given that the events A and B are such that $\Pr(A) = \frac{1}{2}$, $\Pr(A \cup B) = \frac{3}{5}$ and $\Pr(B) = p$. Find p if they are:

a. Mutually Exclusive. $\Pr(A \cap B) = 0$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\frac{3}{5} = \frac{1}{2} + p - 0$$

$$p = \frac{3}{5} - \frac{1}{2}$$

$$p = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$$

b. Independent. $\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

$$= \frac{1}{2} \cdot p = \frac{1}{2}p$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p$$

$$\frac{1}{2}p = \frac{1}{10} \Rightarrow \therefore p = \frac{1}{5}$$

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Question 22

Given that the events A and B are such that $\Pr(A) = \frac{1}{4}$, $\Pr(A \cup B) = \frac{13}{25}$ and $\Pr(B) = p^2$, where $p > 0$. Find p if they are independent.

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B) = \frac{1}{4}p^2$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\frac{13}{25} = \frac{1}{4} + p^2 - \frac{1}{4}p^2$

$\frac{52}{100} - \frac{25}{100} = \frac{3}{4}p^2$

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$\frac{27}{100} = \frac{3}{4}p^2$

$\frac{108}{300} = p^2$

$\therefore p^2 = \frac{9}{25}$

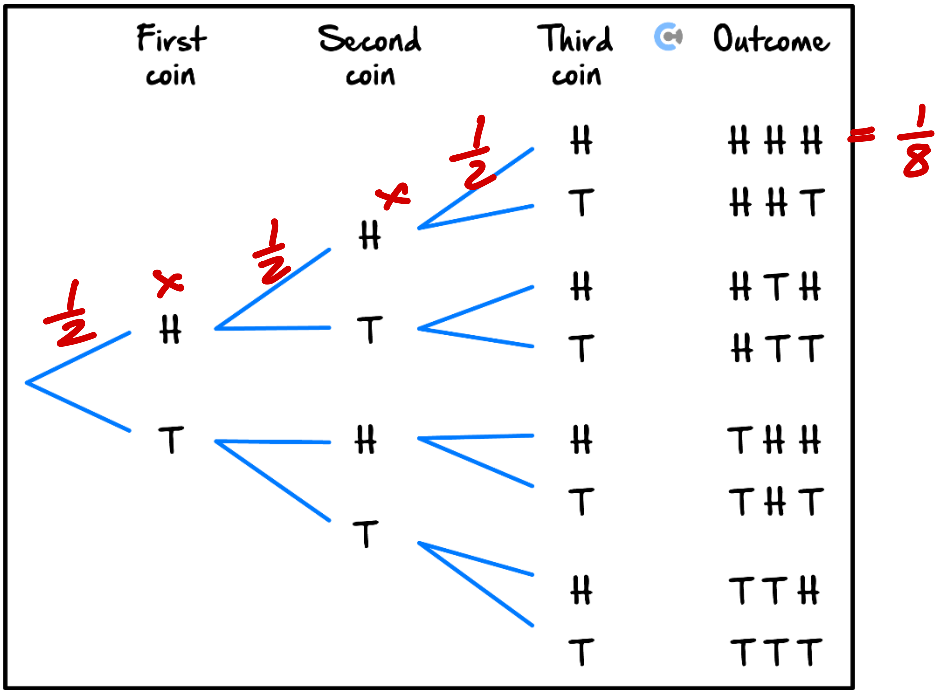
$p = \frac{3}{5}$ or $-\frac{3}{5}$

negative as $p > 0$

Section D: Tree Diagram and Conditional Probability

Sub-Section: Tree Diagram

Tree Diagram



- Useful for multiple sequence events.
- To calculate the probability of a sequence, we multiply the probabilities along the relevant branches.

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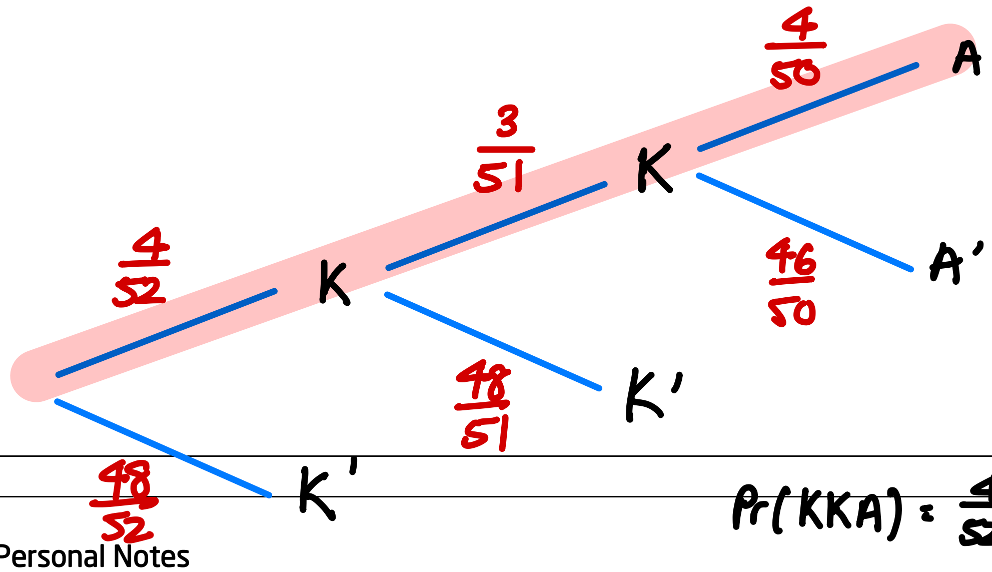
Question 23 Walkthrough.

decrease by 1

Three cards are drawn successively, without replacement from a pack of 52 well-shuffled cards.

Draw the tree diagram and find the probability that the first two cards are kings and the third card drawn is an ace.



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$$Pr(KKA) = \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{4}{50}$$

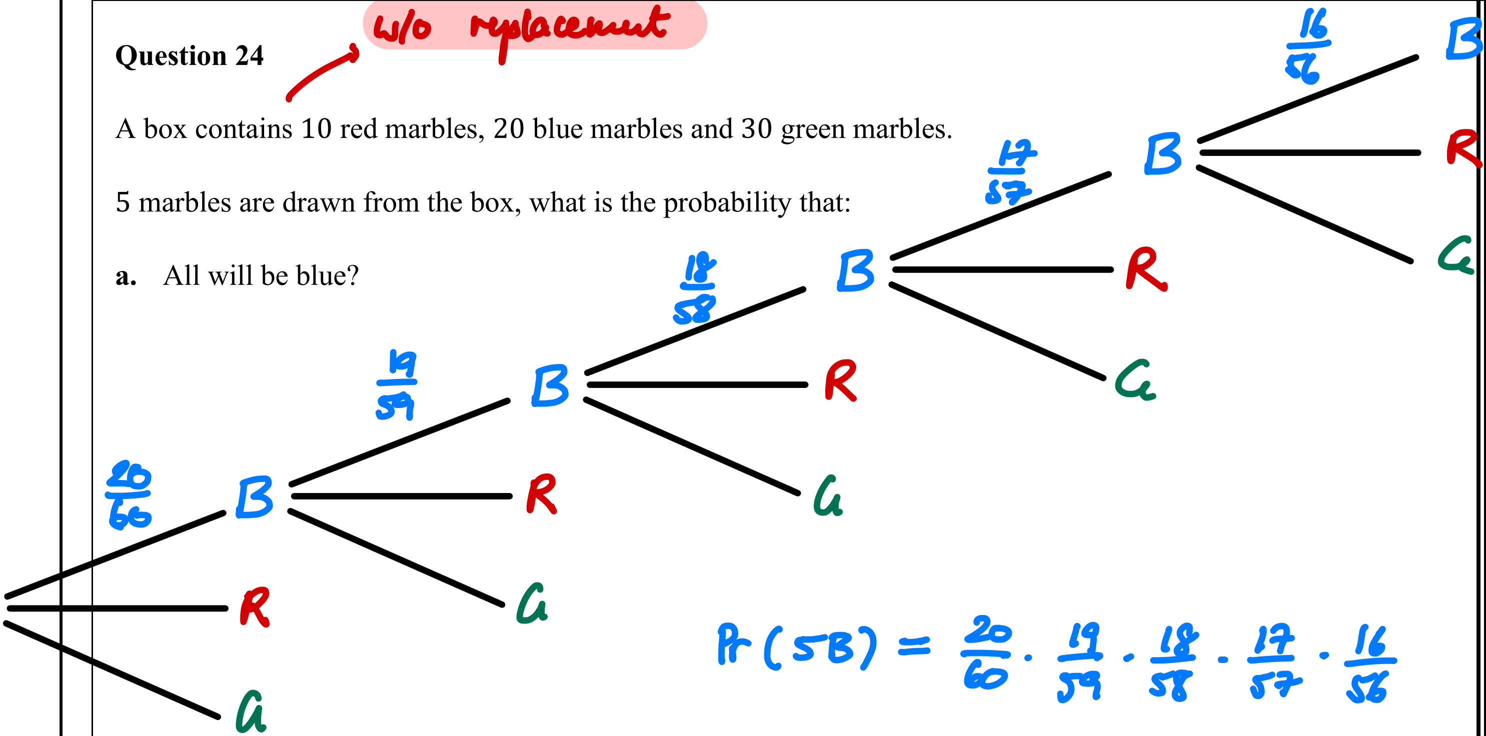
$$= \frac{2}{5525}$$

Question 24 w/o replacement

A box contains 10 red marbles, 20 blue marbles and 30 green marbles.

5 marbles are drawn from the box, what is the probability that:

a. All will be blue?



$$Pr(5B) = \frac{20}{60}\cdot\frac{19}{59}\cdot\frac{18}{58}\cdot\frac{17}{57}\cdot\frac{16}{56}$$

$$= \frac{34}{11{,}977}$$

b. At least one will be green?

$$Pr(G \geq 1) = 1 - Pr(G=0)$$

$$= 1 - \frac{30}{60}\cdot\frac{29}{59}\cdot\frac{28}{58}\cdot\frac{27}{57}\cdot\frac{26}{56}$$

$$= \frac{4367}{4484}$$

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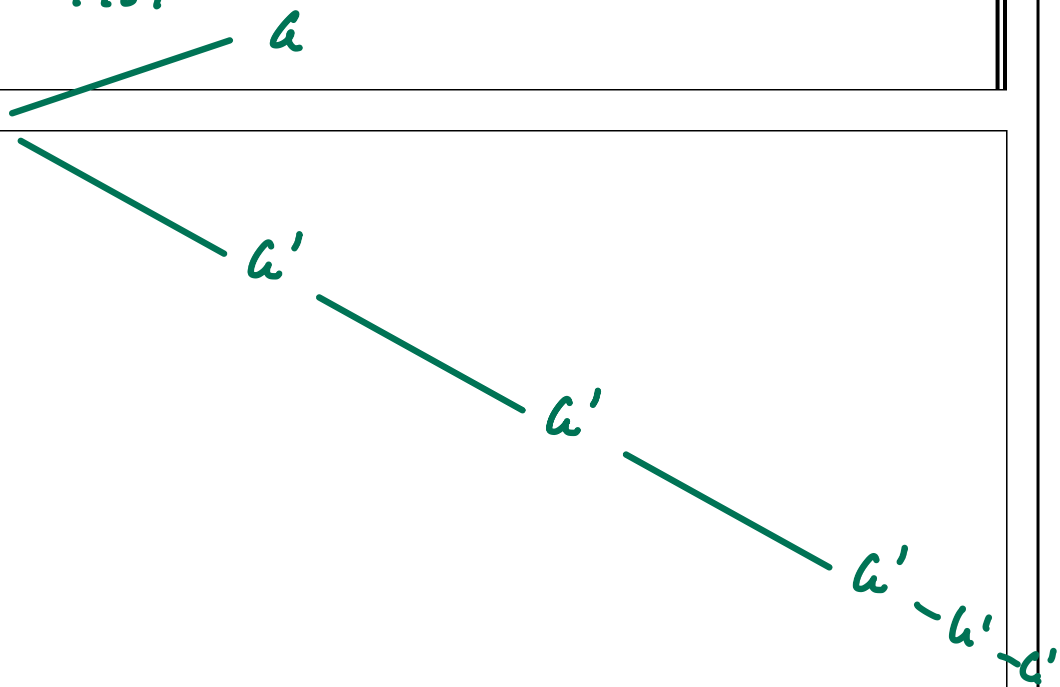


Question 25 Extension.

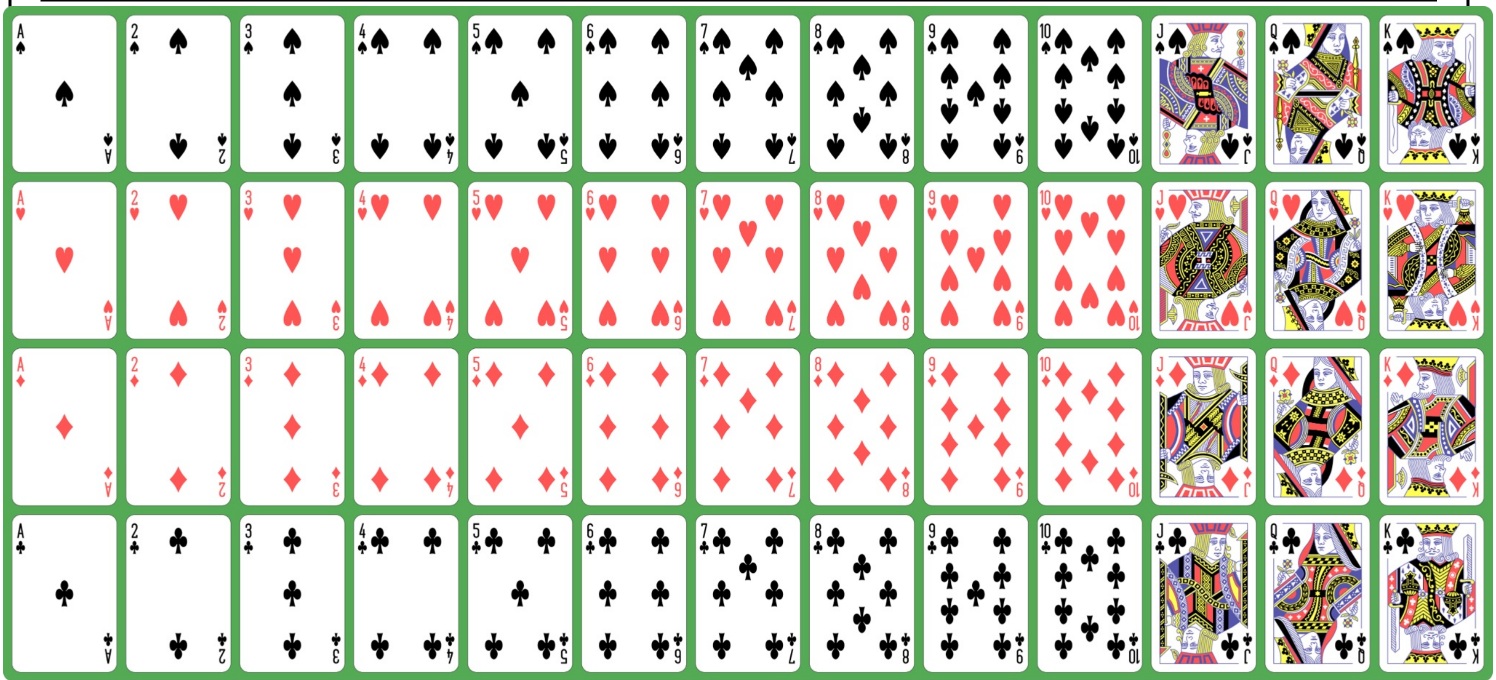
Three cards are drawn successively, without replacement from a pack of 52 well-shuffled cards.

Find the probability that **at least** one face card is drawn (Jack, Queen, King), given that the first card drawn is red.

$\Pr(\geq 1 \text{ Face Cards} \mid \text{First Card is Red}) = 1 - \Pr(0 \text{ Face Cards} \mid \text{FC is Red})$

$= 1 - \frac{20}{26} \cdot \frac{39}{51} \cdot \frac{38}{50}$

$= \frac{47}{85}$ //



Sub-Section: Conditional Probability

What is conditional probability? Any guesses?

Exploration: Understanding Conditional Probabilities

- Consider two probabilities below:

Probability that:
A student randomly selected by James is male and wearing glasses.

Probability that:
A male student randomly selected by James is wearing glasses.

- Which of the two probabilities tells us something that has already occurred? [First/Second]
- The [first/second] probability is called Conditional probability.
- Conditional probability is when we know one event has already occurred.
- In the above example, for the second one, we already know that the student is male.

Question 26

State whether each of the following probabilities would be conditional or not.

Event	Conditional / Not
Probability of selecting a male JMSS student.	Not
Probability of selecting a JMSS student out of male students.	Conditional

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Question 27

A die is thrown twice, and the sum of the numbers appearing is observed to be 6. What is the probability that the number 4 has appeared at least once?

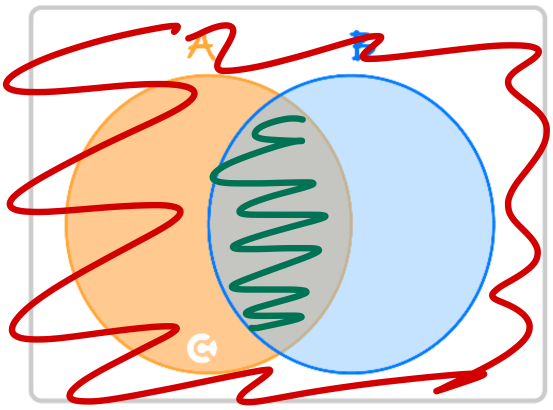
$\Pr(4 \text{ at least once} \mid \text{Sum} = 6) = \frac{2}{5}$ //

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

How can we visualise the conditional probability $\Pr(A|B)$ using Venn diagrams?

Exploration: Derivation of the formula for Conditional Probability

- Consider $\Pr(A|B)$: Probability of A happening when B has already occurred.



- Cross out the section of the Venn diagram that cannot occur!
- Out of all possibilities, where does A occur? [A / $A \cap B$ / B]

- Hence, the probability of $\mathbf{Pr}(A|B)$ $\underline{Pr(A \cap B)}$ out of $\underline{Pr(B)}$.

$$\mathbf{Pr}(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Conditional Probability

- **Definition**: Probability of A given B.

$$\mathbf{Pr}(A|B) = \frac{\mathbf{Pr}(A \cap B)}{\mathbf{Pr}(B)}$$

Question 28 Walkthrough.

If $\Pr(A) = \frac{2}{5}$, $\Pr(B) = \frac{3}{5}$, and $\Pr(A \cap B) = \frac{1}{5}$, evaluate $\Pr(A|B)$.

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{\left(\frac{1}{5}\right)}{\left(\frac{3}{5}\right)} = \frac{1}{3} //$$

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Question 29

If $\Pr(A) = \frac{7}{13}$, $\Pr(B) = \frac{9}{13}$, and $\Pr(A \cap B) = \frac{4}{13}$, evaluate $\Pr(A|B)$.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\left(\frac{4}{13}\right)}{\left(\frac{9}{13}\right)} = \frac{4}{9}$$

Question 30 Extension.

If $\Pr(A) = \frac{2}{5}$, $\Pr(B) = \frac{1}{2}$, and $\Pr(B|A) = \frac{1}{4}$, evaluate $\Pr(A|B)$.

$$\Pr(B|A) = \frac{\Pr(B \cap A)}{\Pr(A)} = \frac{1}{4}$$

$$\rightarrow \Pr(B \cap A) = \frac{1}{4}\Pr(A)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\left(\frac{1}{10}\right)}{\frac{1}{2}} = \frac{1}{5}$$

$$= \frac{2}{20} = \frac{1}{10}$$

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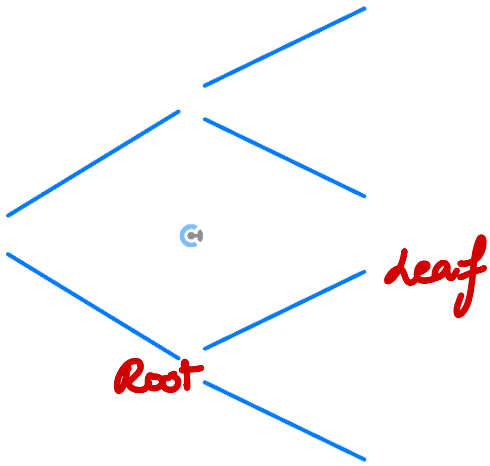
Sub-Section: Using Tree Diagram for Conditional Probabilities

Can we use a tree diagram for conditional probability questions?

Tree Diagram for Condition Probability



- Tree diagram is perfect for conditional probability as each branch is conditional.

Each branch = Pr(Leaf | Root)

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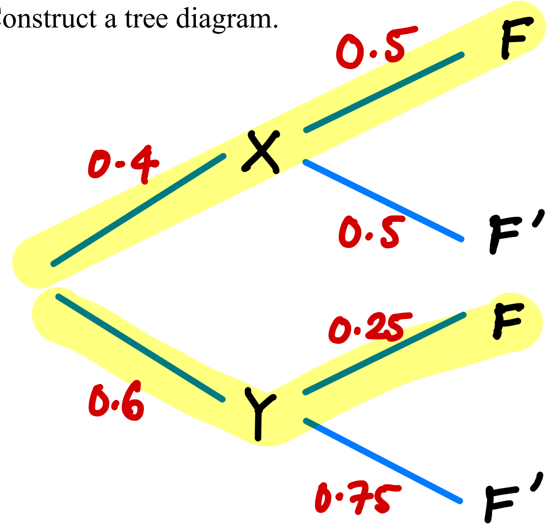
Question 31 Walkthrough.

Batteries are produced from Factories X and Y. It is known that 40% of all batteries come from the Factory X. The quality control of the Factory X is poor, and the probability of batteries produced from the Factory X being faulty is 50%, while Factory Y halves the chance.

a. Construct a tree diagram.



b. What is the probability of a faulty battery being produced?

$$\Pr(F) = \Pr(X \cap F) + \Pr(Y \cap F)$$
$$= 0.4 \times 0.5 + 0.6 \times 0.25$$
$$= 0.2 + 0.15 = 0.35$$

c. Given that the battery ~~came from factory Y~~ is faulty, what is the probability that it ~~is faulty~~ came from Y?

Given

$\Pr(F|Y)$ OR $\Pr(Y|F)$?

$$\Pr(Y|F) = \frac{\Pr(Y \cap F)}{\Pr(F)}$$
$$= \frac{0.15}{0.35} = \frac{15}{35}$$
$$= \frac{3}{7}$$

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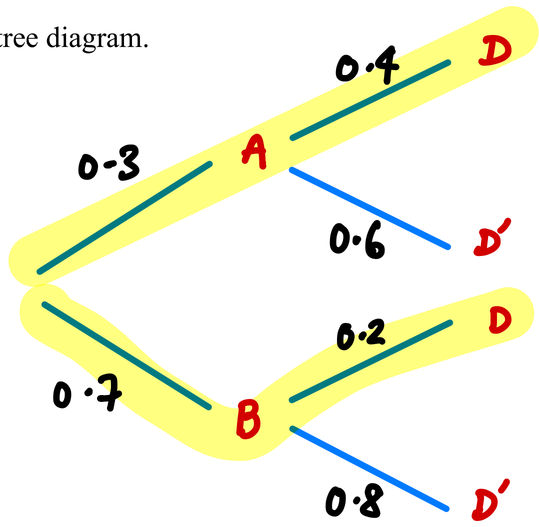
Question 32

A company manufactures laptops in two different factories: Factory A and Factory B. It is known that 30% of all laptops come from Factory A. Due to different quality standards, the probability of a laptop being **defective** from Factory A is 40%, while the probability of a laptop being **defective** from Factory B is 20%.

a. Construct a tree diagram.



b. What is the probability of a laptop battery being defective?

$$\Pr(D) = \Pr(A \cap D) + \Pr(B \cap D)$$
$$= 0.3 \times 0.4 + 0.7 \times 0.2$$
$$= 0.12 + 0.14$$
$$= 0.26$$

c. Given that a laptop is defective, what is the likelihood that it came from Factory A?

$$\Pr(A \mid D) = \frac{\Pr(A \cap D)}{\Pr(D)} = \frac{0.12}{0.26} = \frac{6}{13}$$

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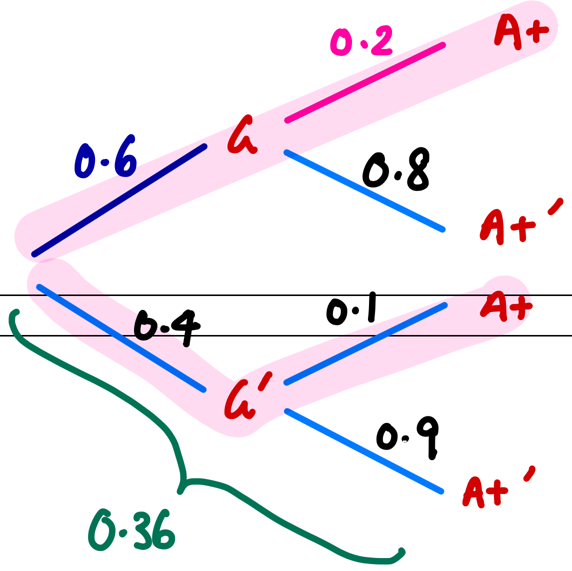
Question 33 Extension.

Student exam scores depend on whether they got good sleep the night before. The probability of scoring an A + given they had a good sleep is 0.2. The probability of having a bad sleep **and** not scoring an A + is 0.36. Find the probability of a student scoring an A + if 60% of students get good sleep the night before the exam.

$Pr(A+|G) = 0.2$

$Pr(G' \cap A+') = 0.36$



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$Pr(A+) = Pr(G \cap A+) + Pr(G' \cap A+)$

$= 0.6 \times 0.2 + 0.4 \times 0.1$

$= 0.12 + 0.04$

$= 0.16$

Sub-Section: Conditional Probability and Independence

Active Recall: Independent Events

- When A and B are independent events:

$$\Pr(A \cap B) = \underline{\Pr(A) \cdot \Pr(B)}$$

Exploration: Derivation of Independence Formula

Pr(A | B)

- Let's say A and B are independent events.
  - Does the given condition (B) affect the probability of A? [Yes/No]
  - Hence, what does $\Pr(A|B)$ equal to?

$$\Pr(A|B) = \underline{\Pr(A)}$$

- Hence,

$$\frac{\Pr(A \cap B)}{\Pr(B)} = \underline{\Pr(A)}$$

- Rearrange, what do you get?

$$\Pr(A \cap B) = \underline{\Pr(A) \cdot \Pr(B)}$$

  - We get the independent probability formula!

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Definition

Conditional Probability with Independent Events

$$\Pr(A|B) = \underline{\Pr(A)}$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

Question 34

Let event A be "You draw a red card from a deck", and the event B be "It snows today".

a. Are A and B independent?

Yes!

b. Simplify the expression for $\Pr(A \mid B)$.

$\Pr(A|B) = \Pr(A)$

c. Hence, find this probability.

$\Pr(A) = \frac{1}{2}$

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Contour Check

☐ Learning Objective: [3.1.1] – Understand probabilities in terms of favourable outcomes from a sample space

Key Takeaways

- ☐ The sample space is the set of all possible outcomes in an experiment.
- ☐ Total probability must always add to 1.
- ☐ When there is some number of equally likely outcomes, the probability of a "successful" outcome can be calculated as:

$$\Pr(success) = \frac{\text{No. of successful outcomes}}{\text{Total no. of outcomes}}$$

☐ Learning Objective: [3.1.2] – Basic probability operations. Use Venn diagrams and/or Karnaugh maps and apply the addition rule

Key Takeaways

- ☐ If A and B are events, then we interpret the following operations as:
- ☐ $A \cup B =$ A or B
- ☐ $A \cap B =$ A and B
- ☐ $A' =$ Not A

$$\Pr(A') = 1 - \Pr(A)$$

Karnaugh Tables

- We can represent probability problems using a Karnaugh Map.

	B	B'	
A	$\Pr(A \cap B)$	$\Pr(A \cap B')$	$\Pr(A)$
A'	$\Pr(A' \cap B)$	$\Pr(A' \cap B')$	$\Pr(A')$
	$\Pr(B)$	$\Pr(B')$	1

- **The addition rule**

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

- **Learning Objective: [3.1.3] – Understand the meaning behind independent and mutually exclusive events**

Key Takeaways

Mutually Exclusive Events

- Two events A and B are **mutually exclusive** if they cannot occur at the same time.
- Probability of both A and B happening together is zero.

$$\Pr(A \cap B) = 0$$

Independent Events

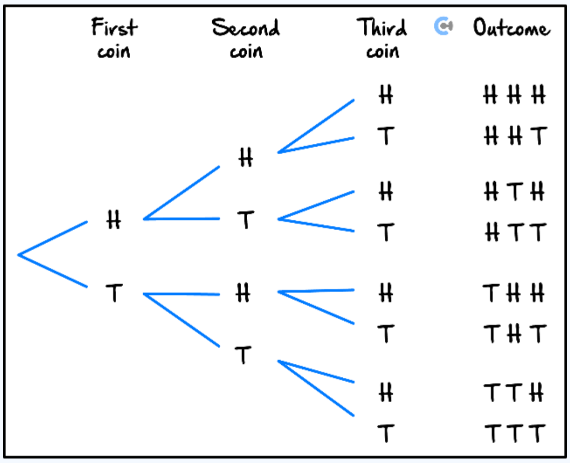
- **Definition:** Independent events **do not affect** the likelihood of the other.
- **Mathematically:**

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Learning Objective: [3.1.4] - Understand conditional probability and make use of tree diagrams

Key Takeaways

Tree Diagram



- Useful for multiple sequence events.
- To calculate the probability of a sequence, we multiply the probabilities along the relevant branches.

Conditional Probability

- **Definition**: Probability of A given B.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Conditional Probability with Independent Events

$$\Pr(A|B) = \Pr(A)$$

- If A and B are independent, the given condition does **not** affect the probability of the event.

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