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VCE Mathematical Methods ½
Probability [3.1]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 15

Section A: Compulsory Questions

Sub-Section [3.1.1]: Sample Space, Uncertainty and Equally Likely Events

Question 1

A bag contains red, blue, and green marbles. There are 4 red, 3 blue, and 5 green marbles. A marble is chosen at random.

Find the probability that the marble is green or blue.

The total number of marbles is 12, and there are 5 green marbles and 3 blue marbles, thus

$$\Pr(\text{Green or Blue}) = \frac{5+3}{12} = \frac{2}{3}$$

Question 2

A six-sided die is rolled, and the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.
Let event $A = \{2, 4, 6\}$ (rolling an even number) and event $B = \{1, 2, 3, 4\}$ (rolling a number less than 5).

Find $\Pr(A \cap B)$.

The intersection of A and B consists of outcomes in both sets:

$$A \cap B = \{2, 4\}$$

Since each outcome is equally likely, the probability is:

$$\Pr(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

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Question 3

A biased coin is tossed, and the probability of landing on heads is p. If two independent tosses are made, the probability of getting exactly one head is 0.48.

Find the possible value(s) of p.

The probability of getting exactly one head is:

$$\Pr(\text{One head}) = p(1-p) + (1-p)p = 2p(1-p)$$

We are given that:

$$2p(1-p) = 0.48$$

Solving the quadratic equation:

$$p - p^2 = 0.24$$

$$p^2 - p + 0.24 = 0$$

Using the quadratic formula:

$$p = \frac{1 \pm \sqrt{1-4(0.24)}}{2} = \frac{1 \pm \sqrt{0.04}}{2}$$

$$p = \frac{1 \pm 0.2}{2}$$

$$p = \frac{1.2}{2} = 0.6 \quad \text{or} \quad p = \frac{0.8}{2} = 0.4$$

Since p must be between 0 and 1, both values are valid solutions.

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Sub-Section [3.1.2]: Venn Diagrams and Karnaugh Tables

Question 4

For two events A and B, answer the following:

a. Given that $\Pr(A) = 0.4, \Pr(B) = 0.5$, and $\Pr(A \cap B) = 0.2$, find $\Pr(A \cup B)$.

Using the formula for the union of two events:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

Substituting the given values:

$$\Pr(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$$

b. In a certain experiment, $\Pr(A \cup B) = 0.7, \Pr(A) = 0.5$, and $\Pr(A \cap B) = 0.3$. Find $\Pr(B)$.

Rearranging the formula for the union of two events to solve for $\Pr(B)$:

$$\Pr(B) = \Pr(A \cup B) - \Pr(A) + \Pr(A \cap B)$$

Substituting the values:

$$\Pr(B) = 0.7 - 0.5 + 0.3 = 0.5$$

c. If $\Pr(A') = 0.6, \Pr(A \cap B) = 0.25$, and $\Pr(A \cup B) = 0.75$, find $\Pr(B)$.

Using the complement rule:

$$\Pr(A') = 1 - \Pr(A)$$

$$\Pr(A) = 1 - 0.6 = 0.4$$

Using the addition formula:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(B) = \Pr(A \cup B) - \Pr(A) + \Pr(A \cap B)$$

Substituting values:

$$\Pr(B) = 0.75 - 0.4 + 0.25 = 0.6$$

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Question 5

a. In a class of students, 40% play basketball, 50% play soccer, and 20% play both sports. A student is chosen at random.

Find the probability that the student plays either basketball or soccer.

Using the formula for probability of union:

$$\Pr(B \cup S) = \Pr(B) + \Pr(S) - \Pr(B \cap S)$$

Substituting the values:

$$\Pr(B \cup S) = 0.4 + 0.5 - 0.2 = 0.7$$

b. Students at a school are surveyed about whether they like chocolate or vanilla ice cream. 60% of the students like chocolate, 45% like vanilla, and 25% like both flavours. A student is selected at random.

Find the probability that the student likes at least one of the two flavours.

Using the formula for probability of union:

$$\Pr(C \cup V) = \Pr(C) + \Pr(V) - \Pr(C \cap V)$$

Substituting the values:

$$\Pr(C \cup V) = 0.6 + 0.45 - 0.25 = 0.8$$

c. A survey found that 70% of people use public transport while 55% use ride-sharing services. If 90% use at least one of the services, find the probability that a randomly chosen person uses both of the transport options.

Using the formula for probability of union:

$$\Pr(T \cup R) = \Pr(T) + \Pr(R) - \Pr(T \cap R)$$

Rearrange and substitute the values:

$$\Pr(T \cap R) = 0.7 + 0.55 - 0.9 = 0.35$$

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Question 6

In a survey of 200 students, they were asked whether they studied Mathematics (M) and/or Physics (P). The following probabilities are known:

- The probability that a randomly selected student studies Mathematics is x.
- The probability that a randomly selected student studies Physics is y.
- The probability that a student studies both subjects is 0.25.
- The probability that a student studies at least one of the two subjects is 0.85.
- The probability that a student studies Mathematics but not Physics is 0.4.

Find the values of x and y.

From the given data we have:
$\Pr(M) = x, \Pr(P) = y, \Pr(M \cap P) = 0.25, \Pr(M \cup P) = 0.85,$
$\Pr(M \cap P') = 0.4$

Using the addition rule:

$$\Pr(M \cup P) = \Pr(M) + \Pr(P) - \Pr(M \cap P)$$

Substituting the known values:

$$0.85 = x + y - 0.25$$

Rearrange:

$$x + y = 1.10 \quad (1)$$

We are also given that the probability of studying Mathematics but not Physics is 0.4:

$$\Pr(M \cap P') = \Pr(M) - \Pr(M \cap P)$$

$$0.4 = x - 0.25$$

Rearrange:

$$x = 0.65$$

Substituting $x = 0.65$ into Equation 1:

$$0.65 + y = 1.10$$

$$y = 0.45$$

therefore,

$$x = \Pr(M) = 0.65, \quad y = \Pr(P) = 0.45$$

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Sub-Section [3.1.3]: Independent and Mutually Exclusive Events

Question 7

Determine whether the following pairs of events are independent, mutually exclusive, both, or neither.

a. $\Pr(A) = 0.4, \Pr(B) = 0.3$, and $\Pr(A \cap B) = 0.12$.

Two events are independent if $\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$.

$$\Pr(A) \cdot \Pr(B) = (0.4)(0.3) = 0.12$$

Since $\Pr(A \cap B) = 0.12$, A and B are independent.
Events are mutually exclusive if $\Pr(A \cap B) = 0$. Since $\Pr(A \cap B) \neq 0$, they are not mutually exclusive.

b. $\Pr(A) = 0.5, \Pr(B) = 0.4$, and $\Pr(A \cap B) = 0$.

Since $\Pr(A \cap B) = 0$, events A and B are mutually exclusive.
Checking independence:

$$\Pr(A) \cdot \Pr(B) = (0.5)(0.4) = 0.2$$

Since $\Pr(A \cap B) \neq \Pr(A) \cdot \Pr(B)$, the events are not independent.

c. $\Pr(A) = 0.6, \Pr(B) = 0.5$, and $\Pr(A \cap B) = 0.3$.

Checking independence:

$$\Pr(A) \cdot \Pr(B) = (0.6)(0.5) = 0.3$$

Since $\Pr(A \cap B) = 0.3$, events A and B are independent.
Since $\Pr(A \cap B) \neq 0$, the events are not mutually exclusive.

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Question 8

a. Given $\Pr(A) = 0.2$ and $\Pr(B) = a$, find the value of a for which A and B are independent, given that $\Pr(A \cap B) = 0.08$.

For independence:

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

Substituting values:

$$0.08 = (0.2)a$$

$$a = \frac{0.08}{0.2} = 0.4$$

b. Given $\Pr(A) = 0.3$ and $\Pr(B) = a$, find the value of a for which A and B are mutually exclusive, given that $\Pr(A \cup B) = 0.5$.

For mutually exclusive events:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B)$$

Substituting values:

$$0.5 = 0.3 + a$$

$$a = 0.2$$

c. Given A and B are independent with $\Pr(A) = 0.4$ and $\Pr(B) = 0.5$, find $\Pr(A' \cap B')$.

Since A' and B' are also independent:

$$\Pr(A') = 1 - \Pr(A) = 1 - 0.4 = 0.6$$

$$\Pr(B') = 1 - \Pr(B) = 1 - 0.5 = 0.5$$

$$\Pr(A' \cap B') = \Pr(A') \cdot \Pr(B') = (0.6)(0.5) = 0.3$$

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Question 9

For two events A and B, it is given that:

- $\Pr(A) = a + 0.1$
- $\Pr(B) = 0.6$
- $\Pr(A \cap B) = a^2 - 0.2a$

The events A and B are independent. Find the possible value of a.

For the independence to be satisfied we must have that

$$(a + 0.1) \times 0.6 = a^2 - 0.2a$$
$$0.6a + 0.06 = a^2 - 0.2a$$
$$a^2 - 0.8a - 0.06 = 0$$
$$\left(a - \frac{2}{5}\right)^2 = \frac{22}{100}$$
$$a - \frac{2}{5} = \pm\frac{\sqrt{22}}{10}$$
$$a = \frac{2}{5} \pm \frac{\sqrt{22}}{10}$$

note that $\frac{2}{5} < \frac{\sqrt{22}}{10} < \frac{1}{2}$ and since probabilities are between 0 and 1 it must be that $a = \frac{2}{5} + \frac{\sqrt{22}}{10}$

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Sub-Section [3.1.4]: Tree Diagram and Conditional Probability

Question 10

a. Given $\Pr(A \cap B) = 0.2$ and $\Pr(B) = 0.5$, find $\Pr(A|B)$.

By the conditional probability formula:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Substituting the values:

$$\Pr(A|B) = \frac{0.2}{0.5} = 0.4$$

b. Given $\Pr(A \cap B) = 0.15$ and $\Pr(A) = 0.6$, find $\Pr(B|A)$.

Using the conditional probability formula:

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$$

$$\Pr(B|A) = \frac{0.15}{0.6} = 0.25$$

c. Given $\Pr(A') = 0.4$, $\Pr(B') = 0.5$, and $\Pr(A' \cap B') = 0.3$, find $\Pr(A'|B')$.

$$\Pr(A'|B') = \frac{\Pr(A' \cap B')}{\Pr(B')}$$

$$\Pr(A'|B') = \frac{0.3}{0.5} = 0.6$$

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Question 11

For each of the following, determine the requested probability given additional conditions.

a. Given $\Pr(A) = 0.4, \Pr(B) = 0.5$, and A and B are independent, find $\Pr(B'|A)$.

Since A and B are independent:

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B) = (0.4)(0.5) = 0.2$$

Now,

$$\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.4 - 0.2 = 0.2$$

So,

$$\Pr(B'|A) = \frac{\Pr(A \cap B')}{\Pr(A)} = \frac{0.2}{0.4} = 0.5$$

b. Given $\Pr(A) = 0.3, \Pr(B) = 0.7$, and A and B are mutually exclusive, find $\Pr(A'|B)$.

Since mutually exclusive events satisfy $\Pr(A \cap B) = 0$, we have:

$$\Pr(A'|B) = \frac{\Pr(A' \cap B)}{\Pr(B)}$$

Since $\Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B) = 0.7 - 0 = 0.7$:

$$\Pr(A'|B) = \frac{0.7}{0.7} = 1$$

c. Given $\Pr(A) = 0.6, \Pr(B) = 0.5$, and $\Pr(A \cap B) = 0.3$, find $\Pr(B'|A)$.

$$\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.6 - 0.3 = 0.3$$

$$\Pr(B'|A) = \frac{\Pr(A \cap B')}{\Pr(A)}$$

$$\Pr(B'|A) = \frac{0.3}{0.6} = 0.5$$

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Question 12

A company produces batteries from three different factories. The percentage of batteries that come from each factory are: 40% from Factory X, 35% from Factory Y, and 24% from Factory Z.

The percentage of defective batteries from each factory is:

- $\frac{1}{5}$ of X's batteries.
- $\frac{1}{7}$ of Y's batteries.
- $\frac{1}{8}$ of Z's batteries.

a. Find the probability that a randomly chosen battery is defective.

We have that

$$\Pr(D) = \Pr(D|X)\Pr(X) + \Pr(D|Y)\Pr(Y) + \Pr(D|Z)\Pr(Z)$$

$$\Pr(D) = \left(\frac{1}{5} \times 0.4\right) + \left(\frac{1}{7} \times 0.35\right) + \left(\frac{1}{8} \times 0.24\right)$$

$$= 0.08 + 0.05 + 0.03 = 0.16$$

b. If a battery is defective, find the probability that it came from the Factory X.

We have that

$$\Pr(X|D) = \frac{\Pr(D|X)\Pr(X)}{\Pr(D)}$$

$$= \frac{(0.2)(0.4)}{0.16} = 0.5$$

c. Suppose now that only $\frac{1}{m}$ of X's batteries are defective. Given that 30% of all defective batteries come from Factory X, determine the value of m.

Let $\Pr(D|X) = \frac{1}{m}$. Then:

$$\Pr(X|D) = \frac{\Pr(D|X)\Pr(X)}{\Pr(D)} = 0.3$$

Solving for m:

$$\frac{1}{m} = \frac{0.3 \times 0.16}{0.4} = 0.12$$

$$m = \frac{25}{3}$$

Sub-Section: The 'Final Boss'

Question 13

A special deck of 20 cards consist of 4 red, 6 blue, 5 green, and 5 yellow cards, numbered 1 to 20. A game involves drawing one card at random and flipping a fair coin.

Assume that a card's number and its colour are independent.

a.

i. Find the probability that a randomly drawn card is blue or even-numbered.

There are 6 blue cards: $\Pr(\text{blue}) = \frac{6}{20}$. There are 10 even-numbered cards: $\Pr(\text{even}) = \frac{10}{20} = \frac{1}{2}$. $\Pr(\text{blue} \cap \text{even}) = \frac{3}{20}$.
Then

$$\Pr(\text{blue} \cup \text{even}) = \Pr(\text{blue}) + \Pr(\text{even}) - \Pr(\text{both})$$

$$= \frac{6}{20} + \frac{10}{20} - \frac{3}{20} = \frac{13}{20}$$

ii. Given that a drawn card is not yellow, find the probability that it is red.

Given that the card is not yellow, 15 cards remain. The probability that the card is red is:

$$\Pr(\text{red}|\text{not yellow}) = \frac{\Pr(\text{red and not yellow})}{\Pr(\text{not yellow})}$$

$$= \frac{4}{15}$$

Define the following events:

- A: The event that the card is red or blue.
- B: The event that the card has a prime number.

Also, suppose that the cards have been shuffled in a non-random way so that the colour and number on the card are **no longer** independent events.

b.

i. Find $\Pr(A)$ and $\Pr(B)$.

$\Pr(A) = \Pr(\text{red}) + \Pr(\text{blue}) = \frac{4}{20} + \frac{6}{20} = \frac{10}{20} = \frac{1}{2}$

Prime numbers in 1 to 20: $\{2, 3, 5, 7, 11, 13, 17, 19\}$ (8 total), so $\Pr(B) = \frac{8}{20} = \frac{2}{5}$.

ii. It is known that $\Pr(B \mid A) = \frac{3}{5}$. Determine $\Pr(A \cap B)$.

$\frac{3}{5} = \frac{\Pr(A \cap B)}{1/2}$.

$\Pr(A \cap B) = \frac{3}{10}$.

iii. Show that events A and B are not mutually exclusive.

Events are mutually exclusive if $\Pr(A \cap B) = 0$. Since $\Pr(A \cap B) = 0.3 \neq 0$, the events are **not** mutually exclusive.

iv. If a card drawn is prime, find the probability that it is red or blue.

Using conditional probability:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$
$$= \frac{0.3}{0.4} = \frac{3}{4}$$

A card is drawn, and a coin is flipped. If the coin lands on heads, the card is returned, and another card is drawn. If the coin lands on tails, the card is kept.

c.

i. What is the probability that the same card is drawn twice?

The probability that the same card is drawn twice happens if heads occurs, and then the same card is drawn again.

$$\Pr(\text{same card}) = \Pr(H) \times \Pr(\text{same card on second draw})$$
$$= \frac{1}{2} \times \frac{1}{20} = \frac{1}{40}$$

ii. If a red card was drawn first, find the probability that the second card is not red.

We have that

$$\begin{aligned}\Pr(\text{2nd not red}) &= \Pr(\text{2nd not red} \mid \text{heads}) \times \Pr(\text{heads}) \\ &\quad + \Pr(\text{2nd not red} \mid \text{tails}) \times \Pr(\text{tails}) \\ &= \frac{16}{20} \times \frac{1}{2} + \frac{16}{19} \times \frac{1}{2} \\ &= \frac{2}{5} + \frac{8}{19} \\ &= \frac{78}{95}\end{aligned}$$

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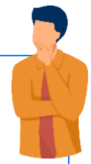

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