Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
AOS 3 Revision [3.0]
Contour Check Solutions

Contour Check

Section A: [3.1] - Probability (Checkpoints) (37 Marks)

Question 1 (11 marks)

A bag contains 4 red, 3 green and 5 blue balls.

A student randomly selects two balls, one after another, without replacement.

a. Find the probability that both balls are blue. (2 marks) **[3.1.1]**

$$\Pr(B \cap B) = \frac{5}{12} \times \frac{4}{11} \ \mathbf{1M} = \frac{20}{132} = \frac{5}{33} \ \mathbf{1A}$$

b. Find the probability that one ball is red and the other is green. (3 marks) **[3.1.1]**

$$\Pr(\text{Red then Green}) = \frac{4}{12} \times \frac{3}{11} \ \mathbf{1M} = \frac{12}{132}$$

$$\Pr(\text{Green then Red}) = \frac{3}{12} \times \frac{4}{11} \ \mathbf{1M} = \frac{12}{132}$$

$$\Pr(\text{One red, other green}) = \frac{24}{132} = \frac{2}{11} \ \mathbf{1A}$$

c. Find the probability that the first ball is green and the second is not green. (2 marks) **[3.1.1]**

$$\Pr(\text{Green then not green}) = \frac{3}{12} \times \frac{9}{11} \ \mathbf{1M} = \frac{27}{132} = \frac{9}{44} \ \mathbf{1A}$$

d. Given that the second ball drawn is red, what is the probability that the first was also red? (4 marks) **[3.1.1]**

$$\Pr(\text{Red 1} \mid \text{Red 2}) = \frac{\Pr(R_1 \cap R_2)}{\Pr(R_2)}$$

$$\Pr(R_1 \cap R_2) = \frac{4}{12} \times \frac{3}{11}\ \mathbf{1M} = \frac{12}{132}$$

$$\Pr(R_2) = \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{4}{11}\ \mathbf{1M}$$

$$= \frac{12 + 12 + 20}{132} = \frac{44}{132} = \frac{1}{3}$$

$$\Pr(R_1 \mid R_2) = \frac{12/132}{1/3}\ \mathbf{1M} = \frac{12}{44} = \frac{3}{11}\ \mathbf{1A}$$

Space for Personal Notes

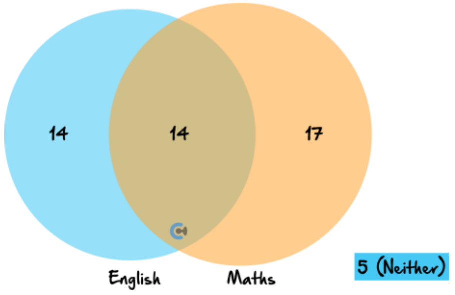
Question 2 (9 marks)

In a class of 50 students:

- 28 take English (E).
- 31 take Maths (M).
- 14 take both English and Maths.

A student is selected at random.

a. Complete a Venn diagram for this situation. (3 marks) **[3.1.2]**



1A for 14 left, **1A** for 17 right, **1A** for both 14 (intersection) and 5 (outside).

Total in both = 14

Only English = 28 − 14 = 14

Only Maths = 31 − 14 = 17

Neither = 50 − 14 − 14 − 17 = 5

b. What is the probability that a randomly selected student studies at least one of the two subjects? (2 marks) **[3.1.2]**

$$\Pr(\text{at least one}) = \frac{45}{50}\ \mathbf{1M} = \boxed{\frac{9}{10}}\ \mathbf{1A}$$

c. Given that a student studies English, find the probability that they also study Maths. (2 marks) **[3.1.2]**

$$\Pr(M \mid E) = \frac{\Pr(E \cap M)}{\Pr(E)} = \frac{14}{28}\mathbf{1M} = \boxed{\frac{1}{2}}\ \mathbf{1A}$$

d. Two students are selected at random without replacement. Find the probability that both study Maths. (2 marks) **[3.1.2]**

$$\text{Pr(both Maths)} = \frac{31}{50} \times \frac{30}{49} \ \mathbf{1M} = \frac{930}{2450} = \boxed{\frac{93}{245}} \ \mathbf{1A}$$

Space for Personal Notes

Question 3 (6 marks)

Let $\Pr(A) = 0.4$, $\Pr(B) = 0.5$ and $\Pr(A \cap B) = 0.2$.

a. Are events A and B independent? Justify. (2 marks) **[3.1.3]**

$\Pr(A) \times \Pr(B) = 0.4 \times 0.5 = 0.2 = \Pr(A \cap B)$**1M** ⇒"Yes, independent." **1A**

b. Are events A and B mutually exclusive? Justify. (2 marks) **[3.1.3]**

$\Pr(A \cap B) = 0.2 \neq 0$ **1M**

Not mutually exclusive. **1A**

c. Find $\Pr(A \cup B)$. (2 marks) **[3.1.3]**

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.4 + 0.5 - 0.2 \ \textbf{1M}\\ &= 0.7 \ \textbf{1A}\end{aligned}$$

Space for Personal Notes

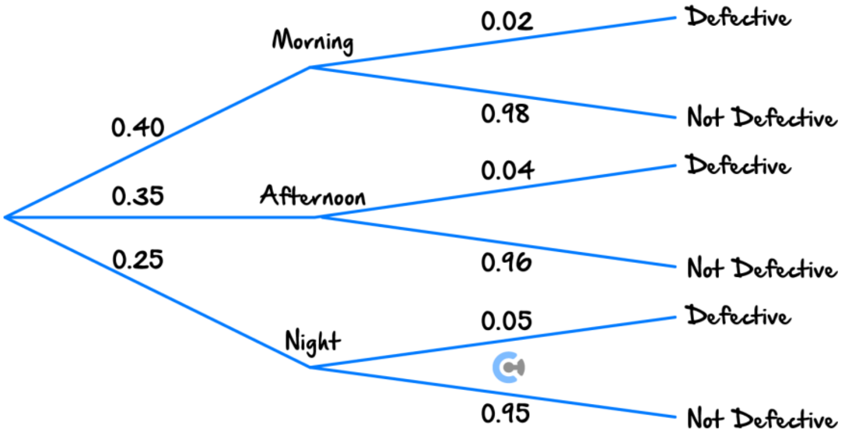
Question 4 (11 marks)

A machine produces widgets in three shifts:

- Morning: 40% of total output, 2% defective.
- Afternoon: 35% of total output, 4% defective.
- Night: 25% of total output, 5% defective.

A widget is chosen at random.

a. Draw a tree diagram showing all outcomes and probabilities. (3 marks) **[3.1.4]**



1A for morning tree, 1A for afternoon tree, 1A for night tree (with branches).

b. Calculate the probability that a widget is defective. (2 marks) **[3.1.4]**

$$\Pr(D) = 0.40 \times 0.02 + 0.35 \times 0.04 + 0.25 \times 0.05 \ \textbf{1M}$$
$$= 0.008 + 0.014 + 0.0125 = \boxed{0.0345} \ \textbf{1A}$$

c. Given that a widget is defective, find the probability it came from the afternoon shift. (2 marks) **[3.1.4]**

$$\Pr(A \mid D) = \frac{\Pr(D \cap A)}{\Pr(D)} = \frac{\Pr(A) \times \Pr(D \mid A)}{\Pr(D)}$$
$$\Pr(A \mid D) = \frac{0.35 \times 0.04}{0.0345} \ \textbf{1M}$$
$$= \frac{0.014}{0.0345}$$
$$= \boxed{\frac{28}{69}} \ \textbf{1A}$$

d. Suppose the defective rate for night is now unknown. If 40% of defective widgets are from the night shift, find the new defective rate for the night shift. (4 marks) **[3.1.4]**

Let the unknown defective rate for night be x.

$\Pr(D) = 0.40 \times 0.02 + 0.35 \times 0.04 + 0.25 \times x$ **1M** $= 0.008 + 0.014 + 0.25x$

$$\Pr(N \mid D) = 0.40 = \frac{0.25x}{0.008+0.014+0.25x} \textbf{ 1M}$$

$$0.40 = \frac{0.25x}{0.022 + 0.25x}$$

$$0.40(0.022 + 0.25x) = 0.25x$$

$$\Rightarrow 0.0088 + 0.10x = 0.25x \textbf{ 1M}$$

$$\Rightarrow 0.0088 = 0.15x$$

$$\Rightarrow x = \frac{0.0088}{0.15} = \frac{22}{375}$$

New defective rate for night is approximately $\frac{22}{375}$. **1A**

Space for Personal Notes

Section B: [3.3] – Combinations and Permutations (Checkpoints)

Sub-Section [3.3.1]: Finding Permutations and Combinations

Question 5

Find the following:

a. ${}^{5}P_{2}$. **[3.3.1]**

$\boxed{20}$

b. ${}^{0}P_{0}$. **[3.3.1]**

$\boxed{1}$

c. $^{7}C_{4}$. **[3.3.1]**

$\boxed{35}$

d. $^{5}C_{4}$. **[3.3.1]**

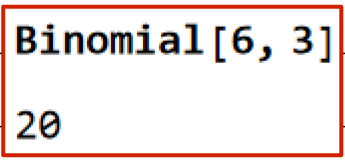
$\boxed{5}$

Space for Personal Notes

Question 6

Disaster has struck at Contour Glen Waverley. The paper is running out, and more is coming in tomorrow.
6 sets of booklets have to be printed, but there is only enough paper for 3 sets to be printed.

a. How many different sets of 3 can be chosen? **[3.3.1]**



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Binomial[6, 3]

20
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Now that 3 sets of booklets have been chosen, they need to be queued in the printer.

b. How many different ways can the chosen booklets be queued? **[3.3.1]**

$3! = 6$

Not to worry! Nayuta has saved the day by driving 10 reams of paper over from Contour Box Hill. Now all of the booklets can be printed.

c. How many different ways can all of the booklets be queued? **[3.3.1]**

$$\boxed{6! = 720}$$

Space for Personal Notes

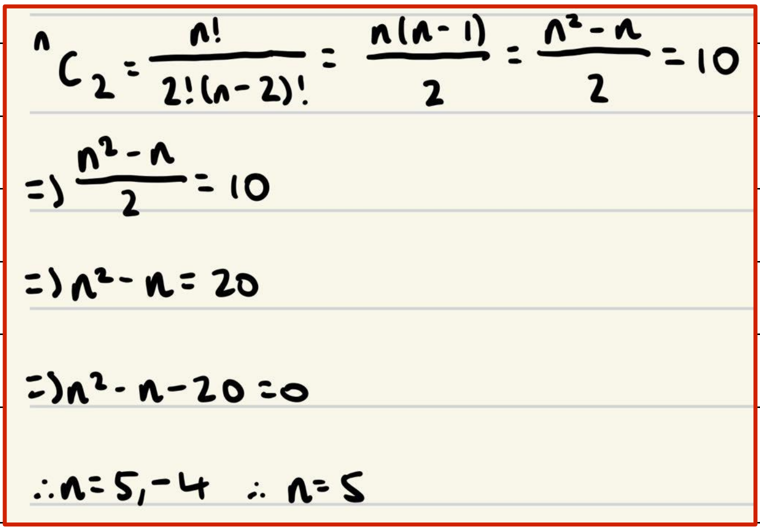
Question 7

At lunchtime, n students rush to be first in line at the canteen, where $n \in \mathbb{Z}^+$.

a. How many ways can the students queue? Give your answer in terms of n. **[3.3.1]**

$$\boxed{n!}$$

b. Within the n students, 2 of them are just in the queue to hang out with their friends, while the others are actually there to buy food. If there are 10 different ways that there can be a group of 2 students who aren't buying food, how many total students are in the queue? **[3.3.1]**



$$^nC_2 = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2} = \frac{n^2-n}{2} = 10$$

$$\Rightarrow \frac{n^2-n}{2} = 10$$

$$\Rightarrow n^2 - n = 20$$

$$\Rightarrow n^2 - n - 20 = 0$$

$$\therefore n = 5, -4 \quad \therefore n = 5$$

Space for Personal Notes

Sub-Section [3.3.2]: Finding Composite/Restricted Permutations and Combinations

Question 8

The letters in the word ‘METHODS’ are jumbled and rearranged.

How many ways can the letters be rearranged if:

a. There are no restrictions? **[3.3.2]**

$$\boxed{7! = 5040}$$

b. A vowel must be first? **[3.3.2]**

$$\boxed{2 \times 6! = 1440}$$

c. The vowels must be together? **[3.3.2]**

$$\boxed{2 \times 1 \times 6! = 1440}$$

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Question 9

A family of 3 children must share 6 different flavoured doughnuts. In order to prevent fighting, their mother ensures that they all receive an equal amount of doughnuts and take turns choosing.

a. How many different ways can the doughnuts be divided? **[3.3.2]**

$$\boxed{6! = 720}$$

b. If the eldest picks both of their doughnuts first, how many different selections can the middle child make if they make both of their selections next? **[3.3.2]**

$$\boxed{{}^4C_2 = 6}$$

c. The youngest child has a temper tantrum at this proposal, and so everyone agrees to let him pick his doughnuts first. Given that he takes a strawberry-flavoured one, how many different groups of 2 can the youngest child make? **[3.3.2]**

$\boxed{2}$

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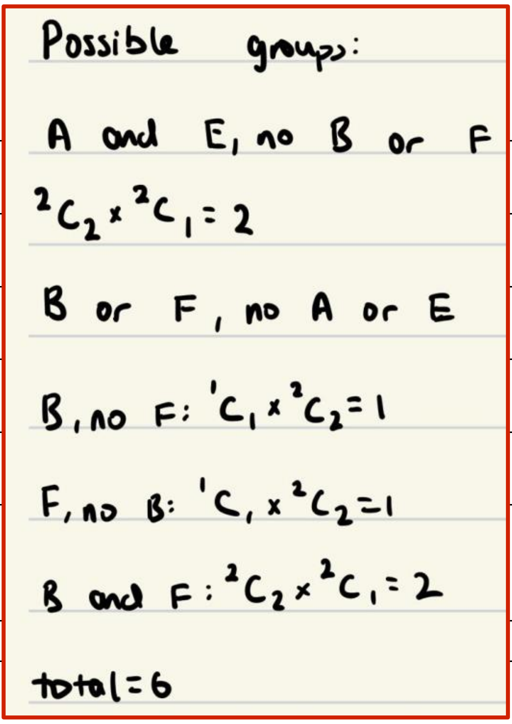
Question 10 [3.3.2]

A small errand requires a singular group of 3 to be made from within a small class of 6 students (Abbey, Ben, Charlie, Derek, Erica and Frank). The teacher has had enough of the students always working with the same people, and so they decide to make the groups. The teacher is quite well informed and knows that:

- Abbey and Ben recently broke up, so they cannot be in the group together.
- Frank and Ben are tight, so Frank will not take part in the group if Abbey is in the group.
- Abbey and Erica will not be part of the group if they are not together.

How many ways can the group be formed?



Possible groups:

A and E, no B or F

$^2C_2 \times {}^2C_1 = 2$

B or F, no A or E

B, no F: $^1C_1 \times {}^2C_2 = 1$

F, no B: $^1C_1 \times {}^2C_2 = 1$

B and F: $^2C_2 \times {}^2C_1 = 2$

total $= 6$

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Sub-Section [3.3.3]: Finding Probabilities using Counting Methods

Question 11

3 of the letters in the word 'CHANCE' are randomly selected. Find the probability that:

a. All 3 letters are consonants. **[3.3.3]**

$$\boxed{\begin{array}{c} {}^{4}C_3 = 3 \\ \text{Total} = {}^{6}C_3 = 20 \\ \Pr(3\text{ consonants}) = \dfrac{3}{20} \end{array}}$$

b. Both vowels are selected. **[3.3.3]**

$$\boxed{\begin{array}{c} {}^{2}C_2 * {}^{4}C_1 = 4 \\ \Pr(2\text{ vowels}) = \dfrac{4}{20} = \dfrac{1}{5} \end{array}}$$

c. Only 1 vowel is selected. **[3.3.3]**

$$^2C_1 * {}^4C_2 = 12$$

$$\Pr(1 \text{ vowel}) = \frac{12}{20} = \frac{3}{5}$$

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Question 12

When being assigned lockers, you and your 2 best friends really hope that you get adjacent lockers. There are 5 lockers to be assigned amongst 5 people. What is the probability that:

a. You get an end locker? **[3.3.3]**

$$\text{Total} = 5! = 120$$
$$\text{Ways} = 4 * 3 * 2 * 1 * 2 = 48$$
$$\Pr(\text{end}) = \frac{48}{120} = \frac{2}{5}$$

b. You and your 2 best friends get adjacent lockers? **[3.3.3]**

$$\text{Adjacent lockers} = 3! * 3! = 36$$
$$\Pr(\text{adjacent}) = \frac{36}{120} = \frac{3}{10}$$

c. You and your 2 best friends all have separated lockers? **[3.3.3]**

$$\text{Separated} = 3 * 2 * 2 * 1 * 1 = 12$$
$$\Pr(\text{separated}) = \frac{12}{120} = \frac{1}{10}$$

Question 13

Subu and Sam have bought another bucket of KFC, once again containing their favourite combo of 5 nuggets, 3 original recipes, and 2 tenders. This time, they are inspecting each piece of chicken to see if they can recreate the recipe for themselves so they can stop spending so much money on KFC. To do this, they line each of the 10 pieces of chicken up on a bench. Find the probability that:

a. No nuggets are next to each other. **[3.3.3]**

$\text{Total} = 10!$

$\text{Nuggets Apart} = 5 * 5 * 4 * 4 * 3 * 3 * 2 * 2 * 1 * 1 * 2 = 2 * 5! * 5!$

$$\Pr(\text{nuggets apart}) = \frac{2 * 5! * 5!}{10!} = \frac{2 * 5 * 4 * 3 * 2 * 1}{10 * 9 * 8 * 7 * 6} = \frac{1}{126}$$

b. The 2 tenders are first in the line and the 3 original recipes are together. **[3.3.3]**

$\text{Tenders first and original together} = 2 * 1 * 3 * 2 * 1 * 6! = 12 * 6!$

$$\Pr(\text{tenders first and original together}) = \frac{12 * 6!}{10!} = \frac{1}{420}$$

c. The tenders are not next to each other. **[3.3.3]**

$$\Pr(\text{tenders apart}) = 1 - \Pr\,(\text{tenders together})$$

$$\text{tenders together} = 2 * 1 * 9!$$

$$\Pr(\text{tenders together}) = \frac{2 * 1 * 9!}{10!} = \frac{1}{5}$$

$$\therefore \Pr(\text{tenders apart}) = 1 - \frac{1}{5} = \frac{4}{5}$$

Space for Personal Notes

Section C: [3.4] - Combinations & Permutations Exam Skills (Checkpoints)

Sub-Section [3.4.1]: Applying Pascal's Triangle and Symmetrical Properties of Combinations

Question 14

a. Using Pascal's Triangle, determine the value of ${}^{n}C_{n}$ for $n \in \mathbb{Z}^{+}$. **[3.4.1]**

$\boxed{1}$

b. Find ${}^{6}C_{3} + {}^{6}C_{4}$ using Pascal's Triangle. Express your answer in the form ${}^{n}C_{r}$, where $n, r \in \mathbb{Z}^{+}$. **[3.4.1]**

$\boxed{{}^{7}C_{4}}$

c. Find ${}^{6}C_{2}$ without directly calculating using the symmetrical property of combinations. You may leave your answer in the form ${}^{n}C_{r}$, where $n, r \in \mathbb{Z}^{+}$. **[3.4.1]**

$\boxed{{}^{6}C_{2} = {}^{6}C_{6-2} = {}^{6}C_{4}}$

Space for Personal Notes

Question 15

Consider the expression ${}^nC_2 = 28$ where $n \in \mathbb{Z}^+\backslash\{1\}$.

a. Using the symmetrical property of combinations, state the value of ${}^nC_{n-2}$. **[3.4.1]**

$$\boxed{{}^nC_{n-2} = {}^nC_{n-(n-2)} = {}^nC_2 = 28}$$

b. Find n. **[3.4.1]**

$${}^nC_2 = \frac{n!}{2!(n-2)!} = 28$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2!(n-2)!} = 28$$

$$\Rightarrow \frac{n(n-1)}{2!} = 28$$

$$\Rightarrow n(n-1) = 56$$

$$\Rightarrow n^2 - n - 56 = 0$$

$$\Rightarrow (n-8)(n+7) = 0$$

$\therefore n = 8, -7$ but $n \in \mathbb{Z}^+\backslash\{1\}$

$\therefore n = 8$

c. Hence, find the value of ${}^{n+1}C_2$. **[3.4.1]**

$${}^{n+1}C_2 = {}^9C_2$$

$$= {}^8C_1 + {}^8C_2$$

$$= 8 + 28$$

$$= 36$$

Question 16

Contour is trying to find 2 Maths tutors to be assistant heads of maths because James has finally reached his limit. There are n sign-ups, where n is an integer larger than 2.

a. Find the smallest value of n for which ${}^{n}C_{2} = {}^{n}C_{n-2}$. **[3.4.1]**

$n - 2 \geq 0$ so $n \geq 2 \therefore n$ has a minimum value of 2.

b. Given that Subu is guaranteed the role because of his crazy teaching hours, how many possible ways can the 2 assistant heads be selected? Give your answer in terms of n. **[3.4.1]**

Subu and someone else:

${}^{1}C_{1} \times {}^{n-1}C_{1} = n-1$

c. Let $n = 10$. It is known that ${}^{10}C_{r} = {}^{10}C_{r^2-7r+18}$. Find all possible values of r, given that $r \in \mathbb{Z}^{+} \cup \{0\}$. **[3.4.1]**

Case 1:

$r = r^2 - 7r + 18$

$r^2 - 8r + 18 = 0$

$b^2 - 4ac = 64 - 72 = -8 \therefore$ no real solution

Case 2: symmetry:

$10 - r = r^2 - 7r + 18$

$r^2 - 6r + 8 = 0$

$(r-2)(r-4) = 0$

$\therefore r = 2, 4$

Sub-Section [3.4.2]: Finding Selections of Any Size

Question 17 [3.4.2]

James is organising another maths meeting. There are 6 possible attendees. Given that any number of attendees can attend or skip the meeting, how many possible attendee lists are possible?

$$\boxed{2^6 = 64}$$

Space for Personal Notes

Question 18

Your group of friends is planning your VCE results day hangout so everyone has something to look forward to. Everyone is discussing what movies to watch from a list of n movies. Any number of movies can be watched, including none if no one is feeling it.

a. Express the number of movie combinations in terms of n. **[3.4.2]**

$\boxed{2^n}$

b. How many movies are there to choose from if there are 128 possible movie combinations? **[3.4.2]**

$\boxed{2^n = 128 \rightarrow 2^n = 2^7 \therefore n = 7}$

c. In the event that at least one movie must be watched, how many possible selections are there now? **[3.4.2]**

$\boxed{\text{Possible selections} = total - {}^7C_0 = 128 - 1 = 127}$

Space for Personal Notes

Question 19

Sam is choosing snacks to bring to his secret snack stash at Contour. He has n different snacks to choose from, and he can pick any number of snacks, including none if he is feeling less cheeky that day.

a. One morning, Sam wakes up a little too smart and realises he has 2048 different snack combinations, but can't count how many snacks he has to choose from. How many different snacks does Sam have to choose from? **[3.4.2]**

$$2^n = 2048 \rightarrow 2^n = 2^{11} \therefore n = 11$$

b. Wanting to make sure he brings at least one snack to Contour, how many different snack combinations does Sam have now? **[3.4.2]**

$$\text{At least 1 snack} = total\ combinations - {}^{11}C_0 = 2047$$

c. Sam goes shopping before he goes to Contour and buys 2 more snacks, which makes him more likely to have something that Emily enjoys. He also wants to bring at least 3 snacks so that he definitely has something to give to Emily and something to eat for himself. How many different snack combinations are possible now? **[3.4.2]**

2 more snacks = 13 total

unrestricted ways = $2^{13} = 2048 \times 4 = 8196$

restricted ways = $8196 - {}^{13}C_0 - {}^{13}C_1 - {}^{13}C_2$

$= 8196 - 1 - 13 - \frac{13!}{2!(11!)}$

$= 8196 - 1 - 13 - 78$

$= 8104$ ways

Section D: [3.1 - 3.4] - Exam 1 Questions (Checkpoints) (21 Marks)

Question 20 (2 marks)

Sam no longer wants KFC after having a falling out with Subu because Sam ate all of the chicken nuggets. Now he goes to McDonald's but doesn't feel like any of the menu items so he decides to build his own burger. On top of the patty, the burger contains:

- 3 ingredients were chosen from lettuce, tomato, onion, pickles and cheese.
- 2 sauces chosen from ketchup, mustard, aioli, and BBQ sauce.

a. How many different burgers can Sam build if he can have multiple of the same ingredient or sauce if the order of the ingredients in the burger does matter (e.g. lettuce on top of cheese and cheese on top of lettuce are as different burgers)? (1 mark)

$$\boxed{5 * 5 * 5 * 4 * 4 = 2000 \text{ different burgers}}$$

b. How many different burgers can Sam build if all the ingredients and sauces must be different? (1 mark)

$$\boxed{5 * 4 * 3 * 4 * 3 = 720 \text{ different burgers}}$$

Space for Personal Notes

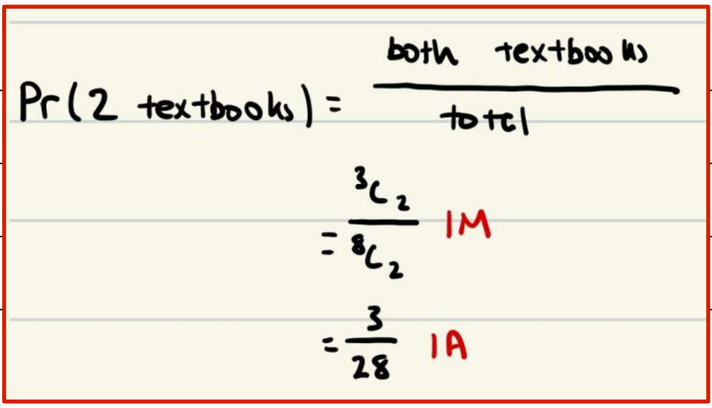
Question 21 (3 marks)

A student randomly selects 2 books from a shelf of 8 books in a library.

a. How many ways can the student select the 2 books? (1 mark)

$$\boxed{{}^{8}C_2 = \frac{8!}{2!\,(6-2)!} = 28}$$

b. If 3 of the books on the shelf are textbooks, what is the probability that both selected books are textbooks? (2 marks)



$$\Pr(2 \text{ textbooks}) = \frac{\text{both textbooks}}{\text{total}}$$

$$= \frac{{}^{3}C_2}{{}^{8}C_2} \quad \text{1M}$$

$$= \frac{3}{28} \quad \text{1A}$$

Space for Personal Notes

Question 22 (4 marks)

James wants to take a photo with the top 5 most senior members of the maths team. Everybody will stand in a line, but Sam and Subu still have beef after their falling out over KFC so they refuse to stand next to each other. How many different possible lines exist?

Sam or Subu on the left:

4	3	2	1	1

not Sam or Subu

Sam or Subu

∴ 2 × 3 × 3 × 2 = 36 possible lines

Same number of possibilities if Sam/Subu on the right

∴ 36 more possible outcomes

If Sam or Subu aren't on the left or right:

not Sam or Subu

3	2	2

Sam or Subu

3	2	1	everyone else

∴ 3 × 2 × 2 × 3 × 2 = 48 possibilities

∴ Total = 48 + 36 + 36 = 120 possible arrangements

1M – left and right cases 1M – realisation of composite middle case 1M – correct middle case 1A – answer

Space for Personal Notes

Question 23 (5 marks)

A lucky draw consists of 6 different prizes and a contestant randomly selects 2 prizes. 2 of the prizes are identical gift cards.

a. How many ways can the contestant select 2 prizes? (1 mark)

$$\boxed{{}^6C_2 = 15}$$

b. What is the probability that the contestant wins both gift cards? (2 marks)

$$\Pr(\text{both gift cards}) = \frac{{}^2C_2}{{}^6C_2} \quad 1M$$

$$= \frac{1}{15} \quad 1A$$

c. If the contestant wins at least one gift card, how many prize selections are possible? (2 marks)

$$\text{possibilities} = \left({}^2C_1 \times {}^4C_1\right) + \left({}^2C_2 \times {}^4C_0\right)$$

$$= 9$$

Space for Personal Notes

Question 24 (4 marks)

Khushi is choosing paint colours for a palette for a painting project. She has n different colours and can choose any number of them, including none.

a. If there are 64 possible palettes, find n. (1 mark)

$$\boxed{2^n = 64 \therefore n = 6}$$

b. Khushi realises that at least one colour of paint is needed to be able to paint. How many possible palettes can Khushi make now? (1 mark)

$$\boxed{64 - 1 = 63}$$

c. Khushi is gifted 2 new colours of paint by her friend, and so Khushi decides that she will use at least 3 colours on her palette. How many palettes can Khushi make now? (2 marks)

$$\boxed{\begin{array}{c}\text{2 more colours means } n_1 = 8 \\ \text{total possibilities} = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 \ = 256 - 1 - 8 - 28 = 219\end{array}}$$

Space for Personal Notes

Question 25 (3 marks)

A row of chairs contains 6 chairs. 3 students must be seated in this row. However:

- The students must all sit together in a single cluster.
- The other 3 chairs must stay empty.

Find the number of different ways the students can be seated in a row.

$\boxed{3}\boxed{2}\boxed{1}$ = 6 internal seating arrangements.

4 ways to choose 3 consecutive chairs in a row of 6

$\therefore$ Total arrangements = $6 \times 4 = 24$

Space for Personal Notes

Section E: [3.1 - 3.4] - Exam 2 Questions (Checkpoints) (87 Marks)

Question 26 (1 mark)

A 14-person club needs to form a 5-person leadership team out of the 14 members. Which of the following expressions is equivalent to the number of ways that this can happen?

A. ${}^{14}C_4 + {}^{14}C_5$

B. ${}^{14}C_5$

C. ${}^{14}C_9$

D. Both B and C.

Question 27 (1 mark)

A student is packing their bag after school and their locker has 8 things in it. The student can bring any number of these things home, including none. How many different packed bags are possible?

A. 64

B. 128

C. 256

D. 512

Question 28 (1 mark)

A group of 6 friends is deciding how they will sit on a theme park ride. Within the group, there is a couple who insist on sitting together. How many seating arrangements are possible?

A. 120

B. 240

C. 480

D. 720

Question 29 (1 mark)

A school randomly selects 3 parents from a committee of 10 to help organise an event.

What is the probability that a specific parent, Jennifer, is chosen?

A. $\frac{3}{10}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{10}$

Question 30 (1 mark)

A lock screen pin consists of 4 digits chosen from 0-9, where the first digit must be even and not 0, and digits cannot repeat. How many different pins are possible?

A. 10000

B. 5040

C. 2520

D. 2016

Question 31 (1 mark)

The letters of the word SOCIETY are placed at random in a row. The probability of getting a vowel is:

A. $\frac{2}{7}$

B. $\frac{3}{7}$

C. $\frac{4}{7}$

D. $\frac{5}{7}$

Question 32 (1 mark)

A girl calculates that the probability of her winning the first prize in a lottery is $\frac{8}{100}$. If 6,000 tickets are sold, how many tickets has she bought?

A. 400

B. 750

C. 480

D. 240

Question 33 (1 mark)

Three identical dice are rolled. What is the probability that the same number will appear on each of them?

A. $\frac{1}{6}$

B. $\frac{1}{36}$

C. $\frac{1}{18}$

D. $\frac{3}{28}$

Question 34 (1 mark)

A bag contains 5 brown and 4 white socks. George pulls out two socks. What is the probability that both socks are of the same colour?

A. $\frac{9}{20}$

B. $\frac{2}{9}$

C. $\frac{3}{20}$

D. $\frac{4}{9}$

Question 35 (1 mark)

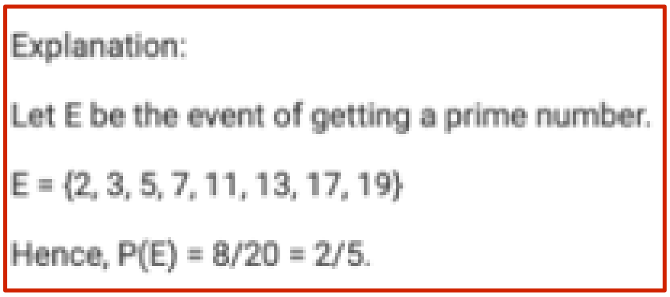
20 cards are numbered from 1 to 20. If one card is drawn at random, what is the probability that the number on the card is a prime number?

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. 5



Explanation:

Let E be the event of getting a prime number.

E = {2, 3, 5, 7, 11, 13, 17, 19}

Hence, P(E) = 8/20 = 2/5.

Question 36 (1 mark)

The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

A. 0.25

B. 0.145

C. $\frac{3}{20}$

D. None of the above

Question 37 (1 mark)

What is the probability that a number selected from the numbers $(1, 2, 3, \ldots\ldots\ldots\ldots, 15)$ is a multiple of 4?

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{2}{15}$

D. $\frac{1}{3}$

Space for Personal Notes

Question 38 (1 mark)

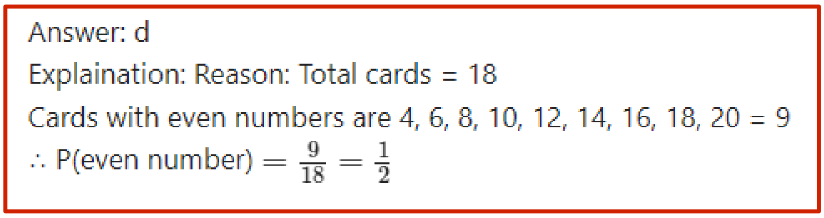
Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out from the bag at random. The probability that the number on the card taken out is an even number is:

A. $\frac{1}{20}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$



Answer: d

Explaination: Reason: Total cards = 18

Cards with even numbers are 4, 6, 8, 10, 12, 14, 16, 18, 20 = 9

$\therefore$ P(even number) $= \frac{9}{18} = \frac{1}{2}$

Question 39 (1 mark)

Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occurring together is:

A. 0.02

B. 0.20

C. 0.01

D. 0.10

Question 40 (1 mark)

How many different rearrangements are there of the letters in the word TATARS if the two A's are never adjacent?

A. 24

B. 120

C. 144

D. 180

Space for Personal Notes

Question 41 (1 mark)

In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together?

A. $4!$

B. $5!$

C. $4! + 5!$

D. $4! \times 5!$

Question 42 (1 mark)

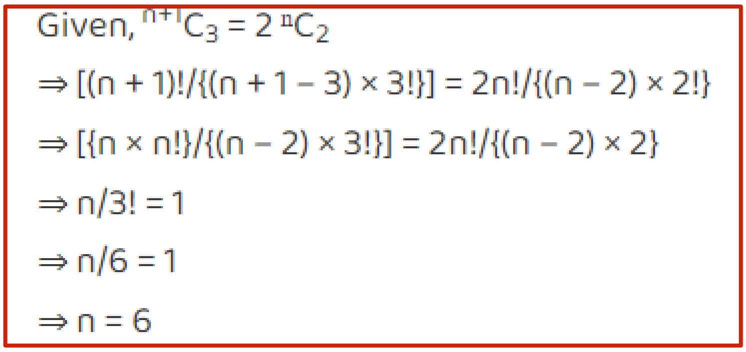
If $^{n+1}C_3 = 2\,^{n}C_2$, then the value of n is:

A. 3

B. 4

C. 5

D. 6



Given, $^{n+1}C_3 = 2\,^{n}C_2$

$\Rightarrow [(n + 1)!/\{(n + 1 - 3) \times 3!\}] = 2n!/\{(n - 2) \times 2!\}$

$\Rightarrow [\{n \times n!\}/\{(n - 2) \times 3!\}] = 2n!/\{(n - 2) \times 2\}$

$\Rightarrow n/3! = 1$

$\Rightarrow n/6 = 1$

$\Rightarrow n = 6$

Question 43 (1 mark)

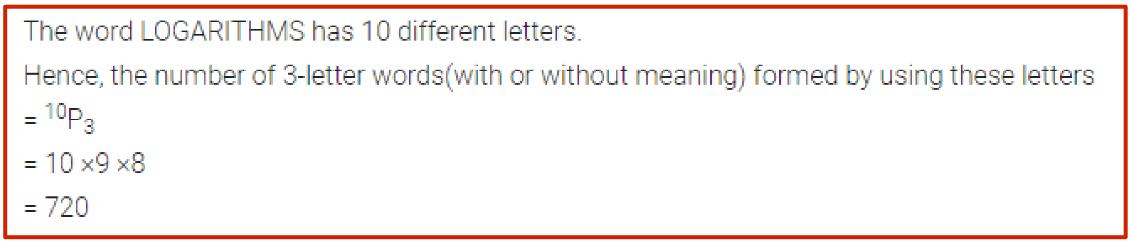
How many 3-letter words with or without meaning, can be formed out of the letters of the word, LOGARITHMS, if repetition of letters is not allowed?

A. 720

B. 420

C. None of these.

D. 5040



The word LOGARITHMS has 10 different letters.

Hence, the number of 3-letter words(with or without meaning) formed by using these letters

$= {}^{10}P_3$

$= 10 \times 9 \times 8$

$= 720$

Space for Personal Notes

Question 44 (1 mark)

From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?

A. 645

B. 564

C. 735

D. 756

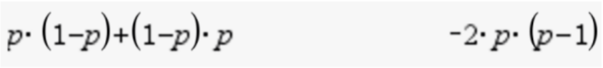
Space for Personal Notes

Question 45 (7 marks)

A biased coin has a probability p of landing on heads. The coin is tossed twice.

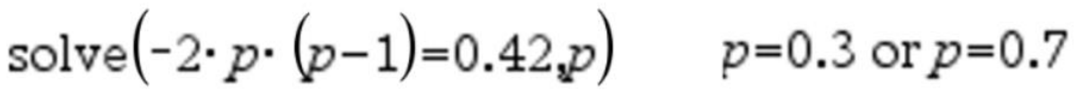
a. Find an expression for the probability of getting exactly one head. (2 marks) **[3.1.1]**

$$\mathrm{P}(1H) = \mathrm{P}(H,T) + \mathrm{P}(T,H)$$
$$= p(1-p) + (1-p)p \ \mathbf{1M}$$
$$= 2p(1-p) \ \mathbf{1M}$$



$p\cdot(1-p)+(1-p)\cdot p \qquad -2\cdot p\cdot(p-1)$

b. Given that the probability of getting exactly one head is 0.42, find the value(s) of p. (3 marks) **[3.1.1]**



$\mathrm{solve}(-2\cdot p\cdot(p-1)=0.42,p) \qquad p=0.3 \text{ or } p=0.7$

1M

1A each

c. Show that the total probability across all outcomes equals 1. (2 marks) **[3.1.1]**

$$\mathrm{P}(HH) = p^2, \mathrm{P}(1H) = 2p(1-p), \mathrm{P}(TT) = (1-p)^2$$
$$p^2 + 2p(1-p) + (1-p)^2 \ \mathbf{1M}$$
$$= p^2 + 2p(1-p) + 1 - 2p + p^2 \ \mathbf{1M}$$
$$= 1$$

Space for Personal Notes

Question 46 (16 marks)

At a university, 150 students were surveyed about their enrolment in two subjects:

- 92 students were enrolled in Data Science (D).
- 78 students were enrolled in Discrete Maths (M).
- 49 students were enrolled in both subjects.
- The rest were enrolled in neither.

Let:

- D = Enrolled in Data Science
- M = Enrolled in Discrete Maths

A student is selected at random.

a. Determine the number of students who:

i. Study only data science. (1 mark) **[3.1.2]**

$$\text{Only Data Science} = 92 - 49 = \boxed{43}\ \mathbf{1}\boldsymbol{A}$$

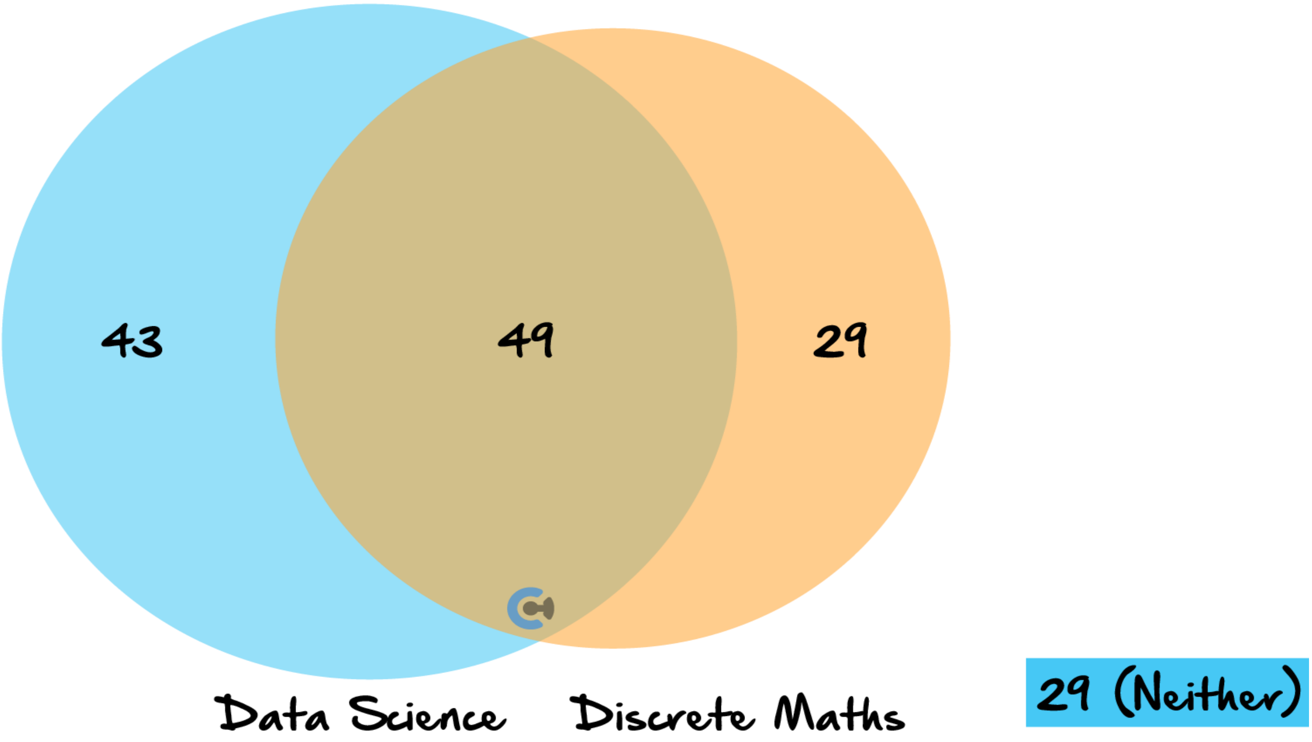
ii. Study only discrete maths. (1 mark) **[3.1.2]**

$$\text{Only Discrete Maths} = 78 - 49 = \boxed{29}\ \mathbf{1}\boldsymbol{A}$$

iii. Study neither subject. (1 mark) **[3.1.2]**

$$\text{Neither} = 150 - (43 + 29 + 49) = \boxed{29}\mathbf{1}\boldsymbol{A}$$

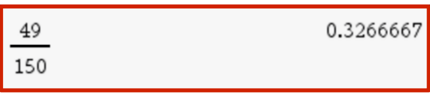
b. Represent this in a Venn diagram. (2 marks) **[3.1.2]**



1A for every two correct values

c. Find the probability, correct to four decimal places, that a randomly chosen student:

i. Studies both subjects. (1 mark) **[3.1.2]**



$\frac{49}{150}$ 0.3266667

1A for 0.3267

ii. Studies only one subject. (1 mark) **[3.1.2]**

$\frac{43+29}{150}$ 0.48

1A for 0.4800

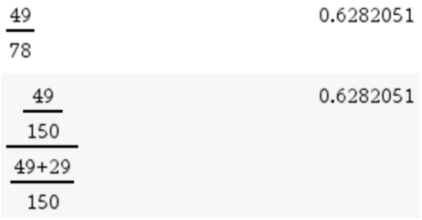
iii. Studies neither subject. (1 mark) **[3.1.2]**

$\frac{29}{150}$ 0.1933333

1A for 0.1933

iv. Studies Data Science given they study Discrete Maths. (2 marks) **[3.1.2]**

No need to use conditional formula



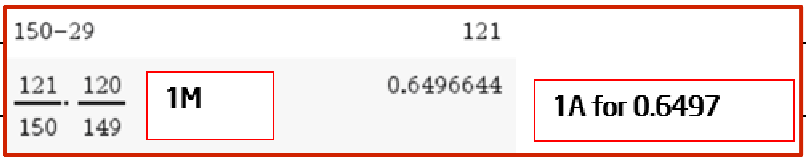
$\frac{49}{78}$ 0.6282051

$\frac{\frac{49}{150}}{\frac{49+29}{150}}$ 0.6282051

1M for either fractions, 1A for 0.6282

d. Two students are selected at random without replacement. Find the probability correct to four decimal places that:

i. Both study at least one subject. (2 marks) **[3.1.2]**



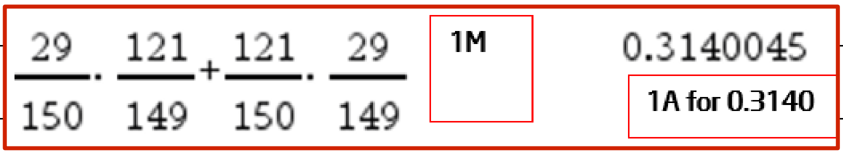
$150-29$ 121

$\frac{121}{150}\cdot\frac{120}{149}$ 0.6496644

1M

1A for 0.6497

ii. Exactly one of them studies only Discrete Maths. (2 marks) **[3.1.2]**

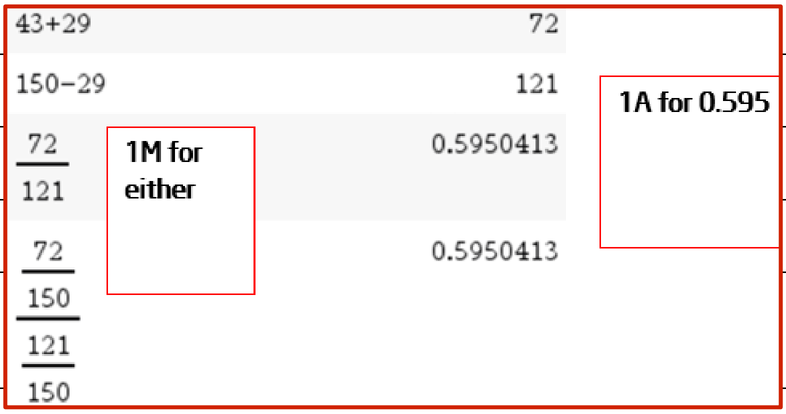


$\frac{29}{150}\cdot\frac{121}{149}+\frac{121}{150}\cdot\frac{29}{149}$ 0.3140045

1M

1A for 0.3140

e. Given that a student studies at least one subject, what is the probability that they study exactly one subject? Give your answer correct to three decimal places. (2 marks) **[3.1.2]**



$43+29$ 72

$150-29$ 121

$\frac{72}{121}$ 0.5950413

$\frac{\frac{72}{150}}{\frac{121}{150}}$ 0.5950413

1M for either

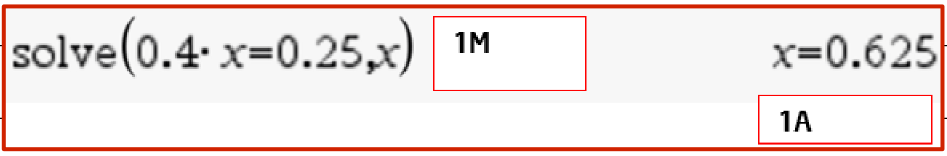
1A for 0.595

Space for Personal Notes

Question 47 (14 marks)

Let $\Pr(A) = 0.4$, $\Pr(B) = x$, $\Pr(A \cap B) = 0.25$

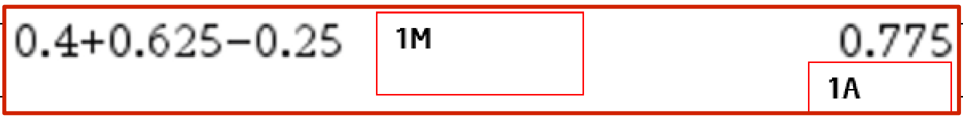
a. Find the value of x, if any, such that A and B are independent events. If not, justify why. (2 marks) **[3.1.3]**



$\text{solve}(0.4\cdot x=0.25,x)$ **1M**

$x=0.625$ **1A**

b. Find $\Pr(A \cup B)$, using your result from **part a.** (2 marks) **[3.1.3]**



$0.4+0.625-0.25$ **1M**

0.775 **1A**

c. Find the value of x, if any, such that A and B are mutually exclusive. If not, justify why. (2 marks) **[3.1.3]**

No such value of x such that A and B are mutually exclusive **1A**
$\Pr(A \cap B) = 0.25 \neq 0$ **1*M***

A new event C is defined such that $\text{P}(C) = 0.3$, and A and C are not independent.

It is known that $\text{P}(A \cap C) = 0.08$.

d. Show that A and C are not independent events. (1 mark) **[3.1.3]**

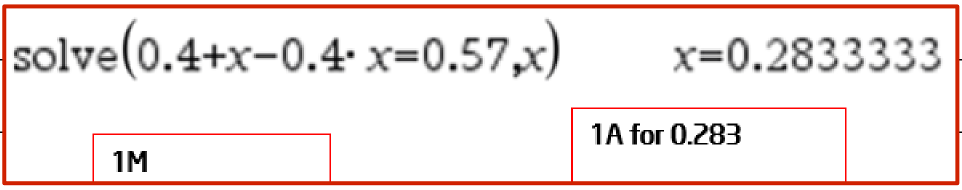
$0.4 \times 0.3 = 0.12 \neq 0.08$ **1*M***

e. Find the probability that neither A nor C occurs. (2 marks) **[3.1.3]**

$1-(0.4+0.3-0.08)$ **1M**

1A 0.38

f. Suppose that $\Pr(A \cup B) = 0.57$ and we do not know $\Pr(A \cap B)$. It is known that A and B are independent. Find the value of x correct to three decimal places. (2 marks) **[3.1.3]**



$\text{solve}(0.4+x-0.4\cdot x=0.57,x)$ $x=0.2833333$

1M

1A for 0.283

g. Given A and B are events such that $\Pr(A) + \Pr(B) > 1$. Show that A and B cannot be mutually exclusive (3 marks) **[3.1.3]**

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A \cup B) + \Pr(A \cap B) = \Pr(A) + \Pr(B)$$

As $\Pr(A) + \Pr(B) > 1$, $\Pr(A \cup B) + \Pr(A \cap B) > 1$ **1*M***

But Pr $(A \cup B) \leq 1$(since no probability can exceed 1)

So, the only way $\Pr(A \cup B) + \Pr(A \cap B) > 1$ is if $\Pr(A \cap B) > 0$

1M explanation

$\therefore \Pr(A \cap B) > 0$ **1*M*** $\Rightarrow$ A and B are not mutually exclusive

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Question 48 (11 marks)

A security screening system has two levels:

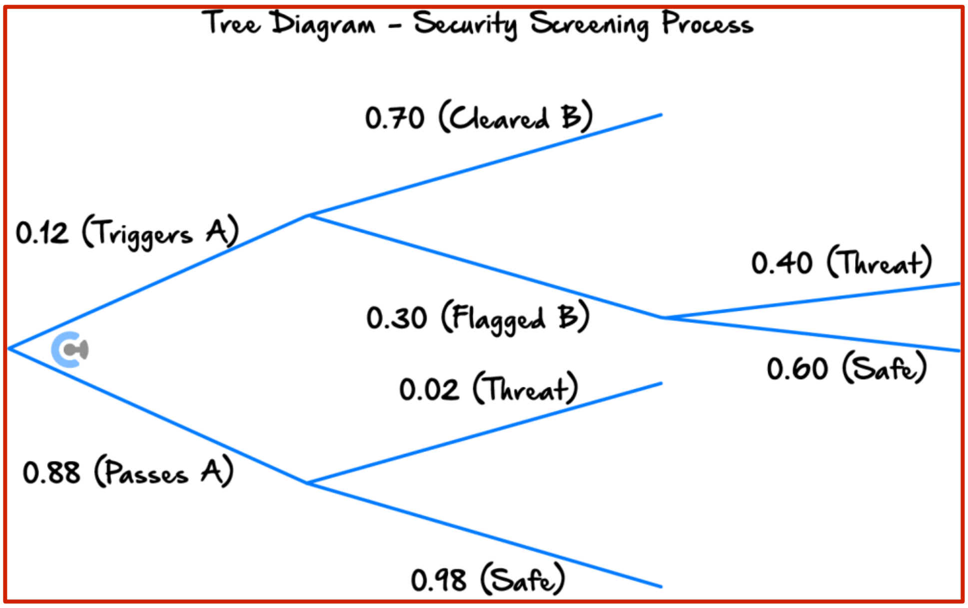
- All passengers first go through Scanner A. If a passenger sets off Scanner A, they go to Scanner B for a second check.
- If a passenger does not trigger Scanner A, they are allowed through without further checks.

It is known that:

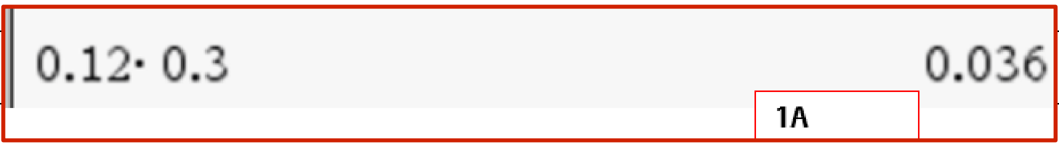
- 12% of passengers trigger Scanner A.
- Of those who go to Scanner B, 70% are cleared, and 30% are flagged.
- Of those who pass Scanner A (i.e., 88%), only 2% are actually threats.
- Of those flagged by Scanner B, 40% are real threats.

A passenger is selected at random.

a. Draw a clearly labelled tree diagram showing all outcomes. (3 marks) **[3.1.4]**



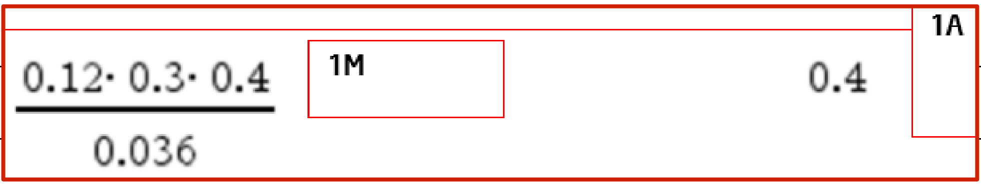
b. Calculate the probability that a passenger is flagged. (1 mark) **[3.1.4]**



$0.12 \cdot 0.3$ 0.036

1A

c. Given a passenger is flagged, calculate the probability they are actually a threat. (2 marks) **[3.1.4]**

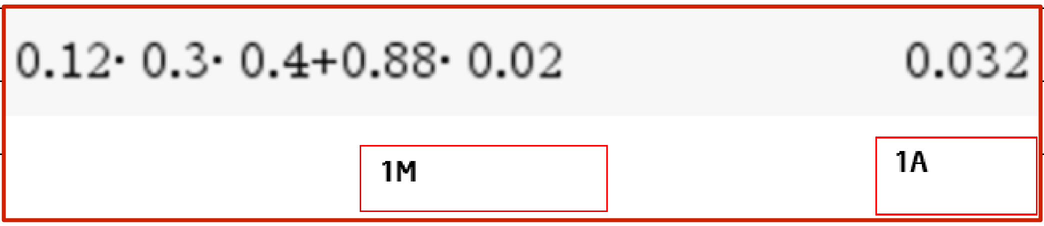


$\frac{0.12\cdot 0.3\cdot 0.4}{0.036}$ 0.4

1M

1A

d. Calculate the probability that a randomly selected passenger is a real threat. (2 marks) **[3.1.4]**



$0.12\cdot 0.3\cdot 0.4+0.88\cdot 0.02$ 0.032

1M

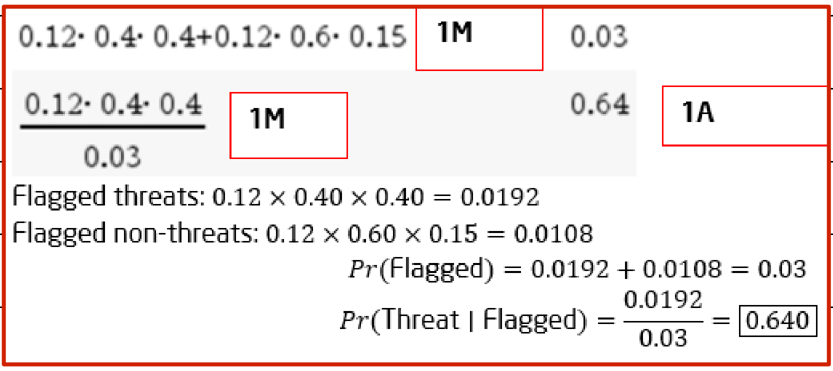
1A

e. The airport introduces a new AI system to reduce false positives at Scanner B. Now:

- 15% of non-threats are incorrectly flagged (false positives).
- 40% of actual threats are still correctly flagged (true positives, unchanged).

Assume all other rates stay the same.

Find the percentage of flagged passengers that are actually threats. (3 marks) **[3.1.4]**



$0.12\cdot 0.4\cdot 0.4+0.12\cdot 0.6\cdot 0.15$ 1M 0.03

$\frac{0.12\cdot 0.4\cdot 0.4}{0.03}$ 1M 0.64 1A

Flagged threats: $0.12\times 0.40\times 0.40=0.0192$

Flagged non-threats: $0.12\times 0.60\times 0.15=0.0108$

$$Pr(\text{Flagged})=0.0192+0.0108=0.03$$

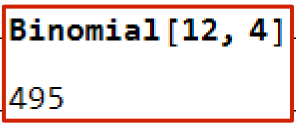
$$Pr(\text{Threat}\mid\text{Flagged})=\frac{0.0192}{0.03}=\boxed{0.640}$$

Space for Personal Notes

Question 49 (9 marks)

Joseph is preparing a set of practice SAC questions for Methods from a set of pre-existing questions. There are 12 pre-existing questions, and Joseph has to choose 4 of them.

a. How many ways can these 4 questions be selected? (1 mark)

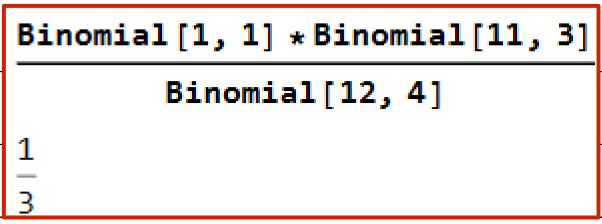


```
Binomial[12, 4]
495
```

b. Once selected, the 4 questions must be ordered. How many ways can this be done? (1 mark)

```
4!
24
```

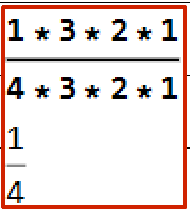
c. What is the probability of a specific question, Question 7, being included in the final selection? (2 marks)



$$\frac{\text{Binomial}[1, 1] * \text{Binomial}[11, 3]}{\text{Binomial}[12, 4]}$$

$$\frac{1}{3}$$

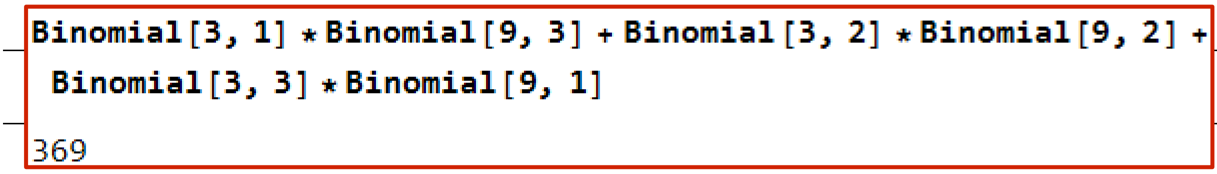
d. If Question 7 is selected, what is the probability that it is chosen to go first in the final order? (2 marks)



$$\frac{1 * 3 * 2 * 1}{4 * 3 * 2 * 1}$$

$$\frac{1}{4}$$

e. Of the 12 questions, 3 of them are on probability. Joseph wants at least one probability question to be on the final question set. How many selections are now possible? (3 marks)



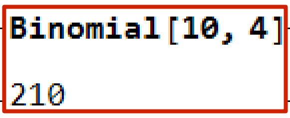
```
Binomial[3, 1] * Binomial[9, 3] + Binomial[3, 2] * Binomial[9, 2] +
 Binomial[3, 3] * Binomial[9, 1]

369
```

Space for Personal Notes

Question 50 (11 marks)

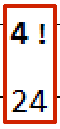
A school is forming a relay team.

a. There are 10 candidates available. How many ways can a 4-person team be formed? (1 mark)



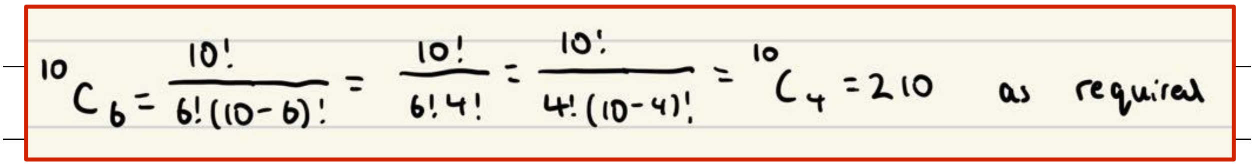
```
Binomial[10, 4]
210
```

b. Once the team is selected, a running order must be decided. How many running orders are possible? (1 mark)



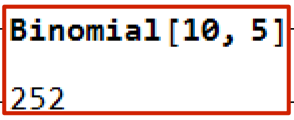
```
4!
24
```

c. Instead of choosing 4 candidates, the school chooses to eliminate 6 candidates. Show that the number of ways to do this is equal to your answer from **part a.** (2 marks)



$${}^{10}C_6 = \frac{10!}{6!(10-6)!} = \frac{10!}{6!4!} = \frac{10!}{4!(10-4)!} = {}^{10}C_4 = 210 \quad \text{as required}$$

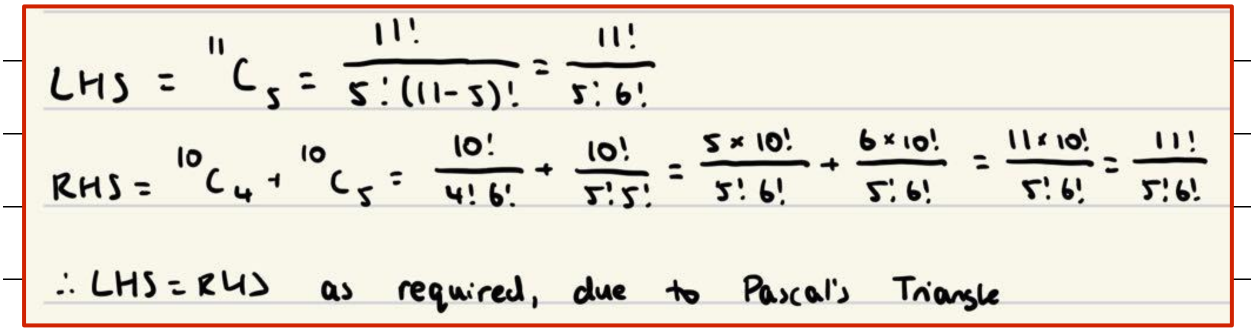
d. The school decides that a reserve should be picked for the team, bringing the total number of team members up to 5. How many possible relay teams can be formed now? (1 mark)



```
Binomial[10, 5]
252
```

A last-minute candidate joins, increasing the total number of candidates to 11.

e. Show that the new amount of possible teams with a reserve is equal to the sum of your answer from **part a.** and **part c.**, and state the principle that this property is derived from. (3 marks)

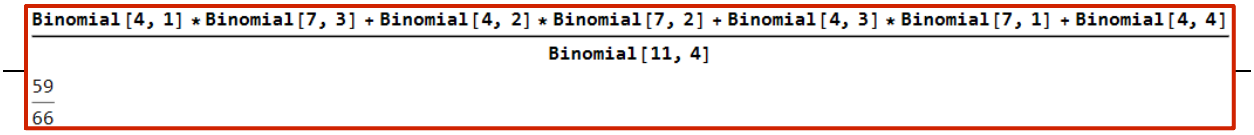


LHS = $^{11}C_5 = \frac{11!}{5!(11-5)!} = \frac{11!}{5!\,6!}$

RHS = $^{10}C_4 + {}^{10}C_5 = \frac{10!}{4!\,6!} + \frac{10!}{5!\,5!} = \frac{5 \times 10!}{5!\,6!} + \frac{6 \times 10!}{5!\,6!} = \frac{11 \times 10!}{5!\,6!} = \frac{11!}{5!\,6!}$

∴ LHS = RHS as required, due to Pascal's Triangle

f. At least one of the candidates chosen for the team must be an experienced runner. If 4 of the candidates are experienced runners, what is the probability that a randomly chosen team meets the requirements? (3 marks)



$$\frac{\text{Binomial}[4, 1] * \text{Binomial}[7, 3] + \text{Binomial}[4, 2] * \text{Binomial}[7, 2] + \text{Binomial}[4, 3] * \text{Binomial}[7, 1] + \text{Binomial}[4, 4]}{\text{Binomial}[11, 4]}$$

$$\frac{59}{66}$$

Space for Personal Notes

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