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VCE Mathematical Methods ½
Transformations Exam Skills [2.5]
Workbook

Outline:

Learning Objectives:

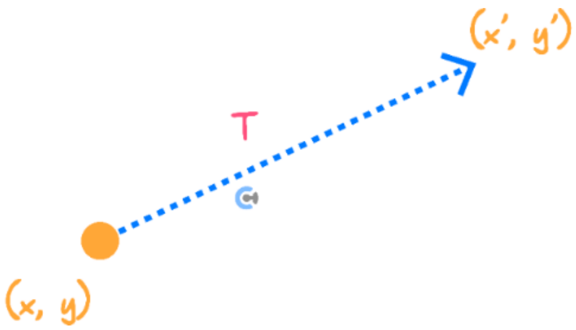
- ☐ **MM12 [2.5.1]** - Apply Quick Method to Find Transformations
- ☐ **MM12 [2.5.2]** - Find Opposite Transformations
- ☐ **MM12 [2.5.3]** - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points
- ☐ **MM12 [2.5.4]** - Find Transformations of the Inverse Functions $f(x)$
- ☐ **MM12 [2.5.5]** - Find Multiple Transformations for the Same Functions

Section A: Recap of Transformations

Sub-Section: Image and Pre-Image

What do we call an original coordinate and a transformed coordinate?

Image and Pre-Image



- The original coordinate is called the Pre-Image.
- The transformed coordinate is called the Image.

$$\text{Pre-Image: } (x, y)$$

$$\text{Image: } (x', y')$$

NOTE: The x' and y' notation will be used quite heavily!

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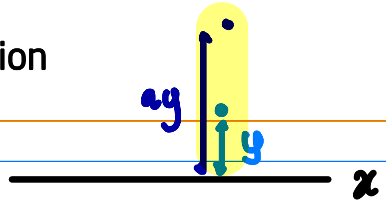
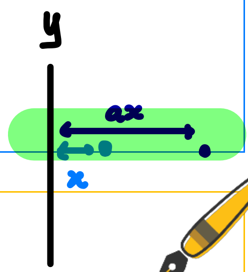
Sub-Section: Dilation



Dilation

Dilation by factor a from the x-axis: $y' = ay$

Dilation by factor b from the y-axis: $x' = bx$



NOTE: We are applying the transformations on (x, y) not (x', y').

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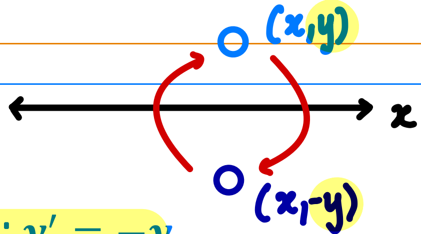
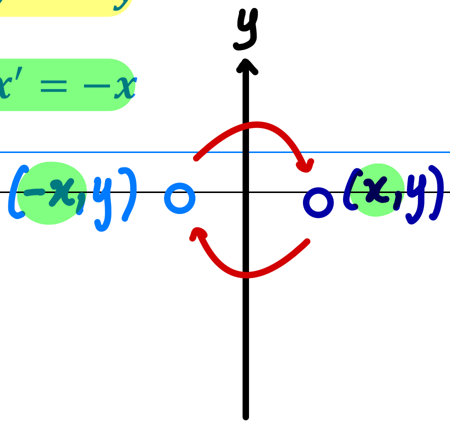
Sub-Section: Reflection



Reflection

Reflection in the x-axis: $y' = -y$

Reflection in the y-axis: $x' = -x$



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Sub-Section: Translation

$(x, y) \xrightarrow{+c} (x+c, y)$

x

Translation

Translation by c units in the positive direction of x-axis: $x' = x + c$

Translation by d units in the positive direction of y-axis: $y' = y + d$

$(x, y+d)$

$+d$

(x, y)

Question 1

Find the image (x', y') after applying the following transformations to (x, y).

Dilation by a factor 4 from the x-axis. $(x, 4y)$

Dilation by a factor 3 from the y-axis. $(3x, 4y)$

Translation by 3 units in the negative direction of the x-axis. $(3x-3, 4y)$

Translation by 6 units in the positive direction of the y-axis. $(3x-3, 4y+6)$

Key Takeaways

- ☑ The transformed point is called the image and is denoted by (x', y').
- ☑ The dilation factor is multiplied by the original coordinates.
- ☑ Reflection makes the original coordinates the negative of their original values.
- ☑ Translation adds a unit to the original coordinates.

Sub-Section: Basic Transformation of Points

Let's try to apply all types of transformations to a point!

Question 2

Find the image (x', y') after applying the following transformations to (x, y).

Dilation by a factor 2 from the x-axis. $(x, 2y)$

Dilation by a factor 4 from the y-axis. $(4x, 2y)$

Reflection in the x-axis. $(4x, -2y)$

Translation by 2 units in the negative direction of the x-axis. $(4x-2, -2y)$

Translation by 3 units in the positive direction of the y-axis. $(4x-2, -2y+3)$

NOTE: Order Is Important!

- Apply the next transformation on top of everything that has already been done!

Sub-Section: The Order of Transformations

What is the order of transformations the same as?

The Order of Transformation

Order = BODMAS Order

Question 3

$\left(x, \frac{1}{2}y\right)$ $\left(x, \frac{1}{2}y+3\right)$

The series of transformations, "a dilation by a factor $\frac{1}{2}$ from the x-axis and a translation by 3 units up" yields the same result as the series of transformations, "a translation by a units up and a dilation by a factor b from the x-axis." Find the values of a and b.

$(x, y+a)$ $(x, b(y+a))$

$$\frac{1}{2}y+3 = b(y+a)$$

$$\frac{1}{2}(y+6) = b(y+a)$$

$$\therefore a=6,\ b=\frac{1}{2}$$

NOTE: Dilation factors don't change!

Sub-Section: Interpreting the Transformation of Points

Question 4

Dilate by factor __ from the x/y axis

Consider the transformation which maps:

Translate __ units L/R/U/D

$$x' = 3x + 6$$

$$y' = -2(y + 2)$$

write in full

a. State the transformation in DRT (Dilation, Reflection, Translation) order.

1. Dil 3 from y
2. Dil 2 from x
3. Reflection in x
4. 6 right
5. 4 down

Expanded:

$x' = 3x+6$

$y' = -2y-4$

b. State the transformation in the translation first order.

1. 2 right
2. 2 up
3. Dil 3 from y
4. Dil 2 from x
5. Reflection in x

Factorised:

$x' = 3(x+2)$

$y' = -2(y+2)$

Key Takeaways

- Transformations should be interpreted when x' and y' are isolated.
- The order of transformation follows the BODMAS order.
- To change the order of transformations, we either factorise or expand.

Sub-Section: Applying Transformations to Functions

Let's now work with functions!

Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- **Steps:**

1. Transform the points.
2. Make x and y the subjects.
3. Substitute them into the function.

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Question 5

a. $f(x) = x^2$

Dilation by factor 3 from the x-axis. $(x, 3y)$

Reflect in the y-axis. $(-x, 3y)$

Translate 3 units to the left. $(-x-3, 3y)$

Dilate by a factor of 5 from the y-axis. $(5(-x-3), 3y)$

$x' = -5x - 15$

$y' = 3y$

$x = \frac{x'+15}{-5}$

$y = \frac{y'}{3}$

$y = x^2$

$\frac{y'}{3} = \left(\frac{x'+15}{-5}\right)^2$

$\therefore y = 3\left(\frac{x+15}{-5}\right)^2$

b. $f(x) = \sqrt{x}$

Dilate by a factor of $\frac{1}{4}$ from the y-axis. $(\frac{1}{4}x, y)$

Dilate by a factor of 3 from the x-axis. $(\frac{1}{4}x, 3y)$

Translate 4 units to the left. $(\frac{1}{4}x - 4, 3y)$

Translate 1 unit up. $(\frac{1}{4}x - 4, 3y+1)$

Reflect in the y-axis. $(-(\frac{1}{4}x - 4), 3y+1)$

$x' = -\frac{1}{4}x + 4$

$y' = 3y + 1$

$x = -4(x' - 4)$

$y = \frac{y'-1}{3}$

$y = \sqrt{x}$

$\frac{y'-1}{3} = \sqrt{-4(x'-4)}$

$\therefore y = 3\sqrt{-4x+16} + 1$

$y = 6\sqrt{4-x} + 1$

Sub-Section: Finding the Applied Transformations

Now, let's go backwards!

Reverse Engineering

- **Steps:**

1. Add the dashes (‘) back to the transformed function.
2. Make $f(\ )$ the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make x' and y' the subjects and interpret the transformations.

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Your turn!

Question 6

State a series of transformations (in order) that allow $f(x)$ to be transformed into $g(x)$.

a. $f(x) = 2(x+1)^2 + 3$ and $g(x) = 6(x-4)^2 - 3$. 2

$y = 2(x+1)+3$ $\quad$ $y' = 6(x'-4)^2 - 3$

$\frac{y-3}{2} = (x+1)^2$ $\quad$ $\frac{y'+3}{6} = (x'-4)^2$

$\frac{y-3}{2} = \frac{y'+3}{6}$ $\quad$ $x+1 = x'-4$

$\therefore y' = 3y - 12$ $\quad$ $x' = x+5$

1. Dil 3 from x
2. 12 down

1. 5 right

b. $f(x) = 3(x-1)^2$ and $g(x) = \frac{1}{2}(2x+3)^2 + 1$.

$y = 3(x-1)^2$ $\quad$ $y' = \frac{1}{2}(2x'+3)^2 + 1$

$\frac{y}{3} = (x-1)^2$ $\quad$ $2(y'-1) = (2x'+3)^2$

$\frac{y}{3} = 2y'-2$ $\quad$ $x-1 = 2x'+3$

$y' = \frac{1}{6}y + 1$ $\quad$ $x' = \frac{1}{2}x - 2$

1. Dil $\frac{1}{6}$ from x
2. 1 up

1. Dil $\frac{1}{2}$ from y
2. 2 left

Key Takeaways

- ☑ We transform the coordinates first, then transform the function.
- ☑ To transform the function, replace its old variables with the new ones.
- ☑ To find the transformations, simply equate LHS with RHS after separating the transformations of x and y.

Section B: Warmup Test (15 Marks)

INSTRUCTION: 15 Marks. 15 Minutes Writing.

Question 7 (4 marks)

Consider the transformation T where $T: R^2 \to R^2, T(x, y) = (3x - 6, -2y + 4)$. Describe in words what the transformation T does with:

a. Dilation and reflections before translations. (1 mark)

1. Dil 3 from y
2. Dil 2 from x
3. Reflection in x
4. 6 left
5. 4 up

$x' = 3x - 6$

$y' = -2y + 4$

b. Translations before dilations and reflections. (2 marks)

1. 2 left
2. 2 down
3. Reflection in x
4. Dil 3 from y
5. Dil 2 from x

$x' = 3(x - 2)$

$y' = -2(y - 2)$

$(x, 2y)$

c. The series of transformations given by "a dilation by a factor of 2 from the x-axis, followed by a translation of 6 units up", yields the exact same result as the series of transformations given by "a translation by a units up, followed by a dilation by a factor of b from the x-axis".

$(x, y+a)$

Find the values of a and b. (1 mark) $(x, b(y+a))$

$(x, 2y+6)$

$2y+6 = b(y+a)$

$2(y+3) = b(y+a)$

$\therefore a=3,\ b=2$

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Question 8 (3 marks)

Consider the following function: $f(x) = (x+3)^2$

Apply the following transformations to the function above.

Dilation by a factor of $\frac{1}{2}$ from the y-axis. $(\frac{1}{2}x, y)$

Dilation by a factor of 3 from the x-axis. $(\frac{1}{2}x, 3y)$

Translation by 2 units in the negative direction of the x-axis. $(\frac{1}{2}x-2, 3y)$

Translation by 6 units in the positive direction of the y-axis. $(\frac{1}{2}x-2, 3y+6)$

Reflection in the y-axis. $(-(\frac{1}{2}x-2), 3y+6)$

$x' = -\frac{1}{2}x+2$ $\quad x = -2(x'-2)$

$y = (x+3)^2$

$\Downarrow$

$y' = 3y+6$ $\quad y = \frac{y'-6}{3}$ $\quad \frac{y'-6}{3} = (-2(x'-2)+3)^2$

$f(x) = 3(-2x+7)^2+6$

Question 9 (2 marks)

$(x, 3y)$

For the function $f(x) = \sqrt{2x+1}$, the function f is dilated by a factor of 3 from the x-axis, translated 2 units in the negative x-direction and then is reflected in the y-axis to produce the function g.

$(x-2, 3y)$ $(-(x-2), 3y)$

Find the rule for $g(x)$.

		$y = \sqrt{2x+1}$
$x' = -x+2$	$x = 2-x'$	$\Downarrow$
$y' = 3y$	$y = \frac{1}{3}y'$	$\frac{1}{3}y' = \sqrt{2(2-x')+1}$
		$\therefore g(x) = 3\sqrt{5-x}$

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Question 10 (3 marks)

Consider the following functions:

$$f(x) = \sqrt{x+3}$$

$$g(x) = -4\sqrt{4-2x} + 3$$

Find the set of transformations that maps $f(x)$ to $g(x)$.

f: $y = \sqrt{x+3}$ g: $y' = -4\sqrt{4-2x'} + 3$

$\frac{y'-3}{-4} = \sqrt{4-2x'}$

$\therefore y = \frac{y'-3}{-4}$ $\therefore x+3 = 4-2x'$

$y' = -4y+3$ $x' = -\frac{1}{2}x + \frac{1}{2}$

1. Dil 4 from x
2. Reflection in x
3. 3 up

1. Dil $\frac{1}{2}$ from y
2. Reflection in y
3. $\frac{1}{2}$ right

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Question 11 (3 marks)

Consider the following functions:

$$f_1(x) = x^3$$

$$f_2(x) = -4(2x+1)^3 - 5$$

Find the set of transformations that maps the function f_1 into f_2.

f_1: $y = x^3$

f_2: $y' = -4(2x'+1)^3 - 5$

$\frac{y'+5}{-4} = (2x'+1)^3$

$\therefore y = \frac{y'+5}{-4}$

$\therefore x = 2x'+1$

$y' = -4y - 5$

$x' = \frac{1}{2}x - \frac{1}{2}$

1. Dil 4 from x
2. Reflection in x
3. 5 down

1. Dil $\frac{1}{2}$ from y
2. $\frac{1}{2}$ left

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Section C: Transformations Exam Skills

Sub-Section: Quick Method

Let's try to do it more quickly!

Active Recall: Interpretation of Transformations

- When the new variables x' and y' are the subject, we can read the transformation directly.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the original variables x and y are the subject instead, we must read the transformation in the opposite way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

Quick Method

- The transformation of x in the function is represented in the opposite way in the final function.
- For applying transformation in a quick method:

Apply everything for x in the opposite direction, including the order!

- For interpreting transformation in a quick method:

Read everything for x in the opposite direction, including the order!

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Question 12 Walkthrough.

Apply the following transformations to $y = x^2$ using the quick method.

OPPOSITE

y

Dilation by a factor 3 from the x-axis. $3y$

$\frac{x-2}{2}$ Dilation by a factor 2 from the y-axis.

SAME

Reflection in the x-axis. $-3y$

$x-2$ Translation of 2 units right.

x

$\therefore y = -3\left(\frac{x-2}{2}\right)^2$

NOTE: For x, simply apply everything in the opposite way and order!

Your turn!

Question 13

Apply the following transformations to $y = \log_2(x)$ using the quick method. y

$-\frac{1}{3}(x+5)$ Dilation by a factor $\frac{1}{4}$ from the x-axis. $\frac{1}{4}y$

Dilation by a factor 3 from the y-axis.

$-(x+5)$ Reflection in the y-axis.

$x+5$ Translation of 5 units left.

Translation of 2 units up. $\frac{1}{4}y+2$

x

$$y = \frac{1}{4}\log_2\left(-\frac{1}{3}(x+5)\right) + 2$$

NOTE: For x, simply apply everything in the opposite way and order!

Now, interpreting transformations!

Question 14 Walkthrough.

$x = 3x' + 1 \Rightarrow 3x' = x - 1$

$x' = \frac{1}{3}(x-1)$

State the transformations required for $y = \sqrt{x}$ to transform into $y = 2\sqrt{3x+1} - 3$.

x:
1. 1 left
2. Dil $\frac{1}{3}$ from y

y:
1. Dil 2 from x
2. 3 down

NOTE: The order is opposite to BODMAS for x.

Your turn!

Question 15

State the transformation required for $y = 2^x$ to transform into $y = 5 \times 2^{3(x+1)} + 3$.

x:
1. Dil $\frac{1}{3}$ from y
2. 1 left

y:
1. Dil 5 from x
2. 3 up

Sub-Section: Finding Opposite Transformations

How can we undo transformations?

Analogy: Untying a Shoelace

- Sam is being silly and ties his shoelace when he was meant to take off his shoes at a chocolate restaurant that he's booked 3 years in advance.
- Which knot should he start untying first? [First Knot] / [Last Knot]
- Similarly, which transformations should we undo first? [First Transformation] / [Last Transformation]

Finding Opposite Transformations

- Order is revered.
- All transformations are opposite.

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Question 16

a. Find the transformation from $f(x) = 3(x+1)^2 - 6$ to $g(x) = x^2 + 3$.

$f \rightarrow g$:

1. Dil $\frac{1}{3}$ from x
2. 5 up
3. 1 right

b. Hence, state the transformation from $g(x)$ to $f(x)$.

$g \rightarrow f$:

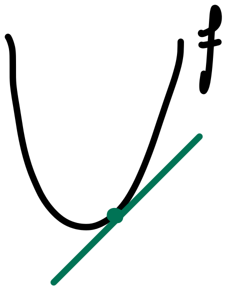
1. 1 left
2. 5 down
3. Dil 3 from x

Sub-Section: Finding Domain, Range, Points, and Tangents of Transformed Functions

Analogy: Function, Points, and Tangents

- Let's say your entire family decides to move 2 units right.

$$\textit{Family: Let's go 2 units right.}$$



- What does that mean for you?

You: 2 units right

- Similarly, if a function moves in a certain way, how should its points, tangents, domain, and range move? [Same way] / [Different way]

Finding Domain, Range, and Points of Transformed Functions

- Everything moves together as a function.
- **Steps:**

1. Find the transformations between two functions.
2. Apply the same transformations to domain, range, points, and tangents.

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Question 17 Walkthrough.

It is known that $f(x)$ has a domain of $[2,4]$ and a range of $(0,20]$.

The function has been transformed to $g(x) = -2f(x+3) + 5$.

a. State the transformation from $f(x)$ to $g(x)$.

$f \rightarrow g$:

1. Dil 2 from x
2. Reflection in x
3. 5 up
4. 3 left

b. State the domain of $g(x)$.

$x \in [2,4]$

3 left ↓

Dom : $x \in [-1,1]$

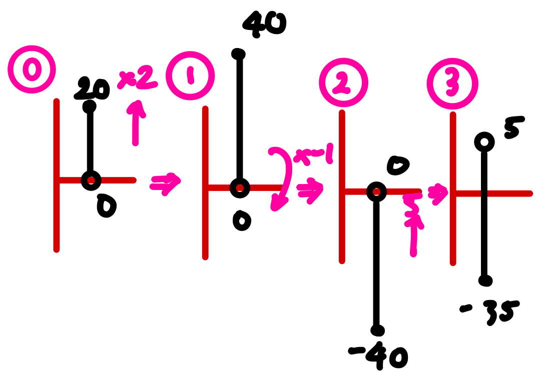
c. State the range of $g(x)$.

$y \in (0,20]$

Dil 2 from x $\in (0,40]$ $\times 2$

Reflection in x $\in [-40,0)$ $\times -1$

5 up $\in [-35,5)$ $+5$



Question 18

It is known that $f(x)$ has an x-intercept at $(3,0)$ and a range of $[-5,10)$.

The function has been transformed to $g(x) = 2f(2x-1)$.

a. State the transformation from $f(x)$ to $g(x)$.

$f \rightarrow g$:

1. Dil 2 from x
2. 1 right
3. Dil $\frac{1}{2}$ from y

b. State the x-intercept of $g(x)$.

$(3,0)$

1 right $(4,0)$

Dil $\frac{1}{2}$ from y $(2,0)$

c. State the range of $g(x)$.

$y \in [-5,10)$

Dil 2 from x $\in [-10,20)$

NOTE: Everything changes with respect to the transformations.

Sub-Section: Finding Transformations of Inverse Functions

REMINDER: Don't forget Inverse Relations.

Inverse functions swap x and y.

Discussion: If $f(x)$ moves 2 units right, where would $f^{-1}(x)$ go to?

$f(x)$: $x' = x + 2$
2 right

Swap x & y

$f^{-1}(x)$: $y' = y + 2$
2 up

Finding Transformation of Inverse Functions

$$f(x) \to f(x-2): 2 \text{ Right}$$

$$f^{-1}(x) \to f^{-1}(x) + 2: 2 \text{ Up}$$

- Steps:
  1. Find the transformation between two original functions.
  2. Inverse the transformations found in **1**.

Question 19 Walkthrough.

It is known that $f(x)$ has been transformed to $g(x) = 3f(x-4) + 1$.

State the transformations required for $f^{-1}(x)$ to transform to $g^{-1}(x)$.

$f \to g$:
1. Dil 3 from x
2. 1 up
3. 4 right

Swap x & y

$f^{-1} \to g^{-1}$:
1. Dil 3 from y
2. 1 right
3. 4 up

Active Recall: Steps on Finding Transformations of Inverse Functions

1. Find the transformations between two original functions.
2. Inverse (swap x & y) the transformations found in **1.**

Question 20

It is known that $f(x) = 2(x-1)^2 + 3$ has been transformed to $g(x) = 4(x+2)^2 - 1$.

State the transformations required for $f^{-1}(x)$ to transform to $g^{-1}(x)$.

swap x & y

$f \to g$:

1. Dil 2 from x
2. 7 down
3. 3 left

$f^{-1} \to g^{-1}$:

1. Dil 2 from y
2. 7 left
3. 3 down

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Sub-Section: Multiple Pathways for the Same Transformation

Discussion: Consider the transformations required for $f(x) = x^2$ to $g(x) = (2x)^2$. What happens if we take the factor of 2 inside the square bracket out?

Expanded:

$y = 4x^2$

1. Dil 4 from x

Factorised:

$y = (2x)^2$

1. Dil $\frac{1}{2}$ from y

Multiple Pathways

- Same transformations can be done differently by either putting it in or out of the $f(\ )$.
- Commonly, look for basic algebra, index, and log laws.

Question 21 Walkthrough.

Dil 4 from x

Find the transformation for $y = x^2$ to transform into $y = 4x^2$ by using a dilation from the y-axis.

$y = (2x)^2$

Dil $\frac{1}{2}$ from y

Question 22

Find the transformation for $y = (x+1)^3 - 2$ to transform into $y = 8x^3$ without using a dilation from the x-axis.

1. 1 right
2. Dil $\frac{1}{2}$ from y
3. 2 up

$\hookrightarrow$ $y' = (2x')^3$

NOTE: This skill is important for MCQ questions.

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Section D: Exam 1 (17 Marks)

Question 23 (2 marks)

The transformation $T: R^2 \rightarrow R^2, T(x, y) = (2x + 4, -y + 2)$ maps the function $f(x) = x^2$ to a function $g(x)$. Find the rule for $g(x)$.

$x = \frac{x'-4}{2}$ $y' = -y+2$

$\therefore g(x) = -\left(\frac{x-4}{2}\right)^2 + 2$

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Question 24 (4 marks)

The following sequence of transformations:

- A translation 1 unit up.
- A translation 4 units left.
- A dilation by factor 3 from the x-axis.
- A dilation by factor $\frac{1}{2}$ from the y-axis.
- A reflection in the x-axis.

is applied to the function $f(x)$ so that $f(x)$ is mapped to $g(x) = \sqrt{x}$.

a. Find a sequence of transformations that maps $g(x)$ to $f(x)$. (2 marks)

$g \to f$:

1. Reflection in x
2. Dil 2 from y
3. Dil $\frac{1}{3}$ from x
4. 4 right
5. 1 down

b. Find the rule for $f(x)$. (2 marks)

$g \to f$:

1. Reflection in x — $-y$
2. Dil 2 from y
3. Dil $\frac{1}{3}$ from x — $-\frac{1}{3}y$
4. 4 right
5. 1 down — $-\frac{1}{3}y-1$

$\frac{1}{2}(x-4)$

$x-4$

x

y

$g(x) = \sqrt{x}$

$f(x) = -\frac{1}{3}\sqrt{\frac{1}{2}(x-4)} - 1$

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Question 25 (4 marks) CTS $(x-2)^2+1$

Consider the functions $f(x) = x^2 - 4x + 5$ and $g(x) = 9(x+1)^2 - 4$.

a. Find a sequence of three transformations in the order DTT that maps $f(x)$ to $g(x)$, and where the dilation is from the x-axis. (2 marks)

$f \to g$:

1. 3 left
2. Dil 9 from x
3. 13 down

b. Find a different sequence of transformations in the order DTT, where the dilation is from the y-axis, that also maps $f(x)$ to $g(x)$. (2 marks)

$f(x) = (x-2)^2 + 1 \qquad g(x) = (3x+3)^2 - 4$

$f \to g$:

1. Dil $\frac{1}{3}$ from y
2. $\frac{5}{3}$ left
3. 5 down

$3x'+3 = x-2$

$3x' = x-5$

$x' = \frac{1}{3}x - \frac{5}{3}$

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Question 26 (5 marks)

Consider the function $f(x) = 2\sqrt{(x-1)^2+3} - 2$ defined on the domain $[0, 4]$.

a. The function g is obtained by applying the following sequence of transformations to f.

$2(x-1)$ A dilation by factor $\frac{1}{2}$ from the y-axis. y

A dilation by factor 3 from the x-axis. $3y$

$x-1$ A translation 1 unit right.

A reflection in the x-axis. $-3y$

x

i. State the domain of g. (1 mark)

Dom $f \in [0,4]$

Dil $\frac{1}{2}$ from $y \in [0,2]$

1 right $\in [1,3]$

$\therefore$ Dom $g \in [1,3]$ //

ii. Find the rule for $g(x)$. (2 marks)

$f(x) = 2\sqrt{(x-1)^2+3} - 2$

$\therefore g(x) = -3\left(2\sqrt{(2(x-1)-1)^2+3} - 2\right)$

$g(x) = -6\sqrt{(2x-3)^2+3} + 6$ //

b. Let $h(x) = \sqrt{(x+\ 1)^2 + 3} + 1$. Write down a sequence of three transformations that map $f(x)$ to $h(x)$. (2 marks)

$f(x) = 2\sqrt{(x-1)^2 + 3} - 2$

$h(x) = \sqrt{(x+1)^2 + 3} + 1$

$f \to h$:

1. Dil $\frac{1}{2}$ from x
2. 2 up
3. 2 left

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Question 27 (2 marks)

Consider the function f with inverse function f^{-1}. The function f is transformed to the function g by the following sequence of transformations: A dilation by factor 3 from the x-axis and a translation 2 units down.

Write down the transformations that take f^{-1} to g^{-1}.

$f \to g$:

1. Dil 3 from x
2. 2 down

swap x & y

$f^{-1} \to g^{-1}$:

1. Dil 3 from y
2. 2 left

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Section E: Tech Active Exam Skills

Calculator Tip: Finding Transformed Functions

- Save the function as $f(x)$.
- Substitute the x and y in terms of x' and y'.
- Solve for y!
- Can also apply the transformations directly to $f(x)$. Must make sure you interpret the transformations correctly or you can easily make a mistake doing this.

Question 28 Tech-Active.

Apply the following transformations to $y = 2\sqrt{3x+6}$.

Dilation by a factor $\frac{1}{2}$ from the x-axis. $\left(x, \frac{1}{2}y\right)$

Dilation by a factor 3 from the y-axis. $\left(3x, \frac{1}{2}y\right)$

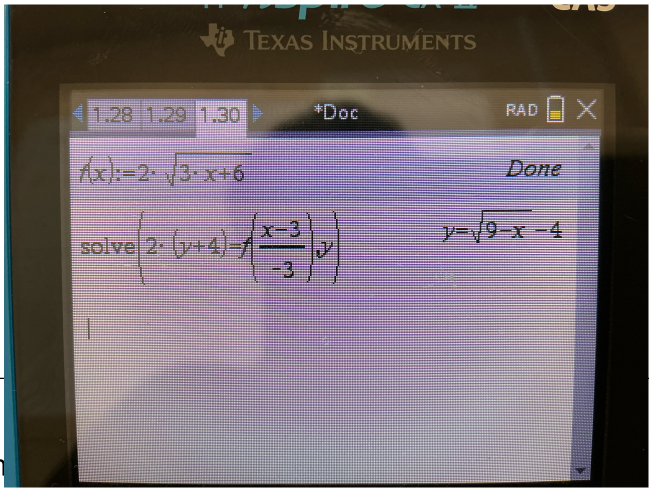
Reflection in the y-axis. $\left(-3x, \frac{1}{2}y\right)$

Translation of 3 units right. $\left(-3x+3, \frac{1}{2}y\right)$

Translation of 4 units down. $\left(-3x+3, \frac{1}{2}y-4\right)$

$x' = -3x+3 \Rightarrow x = \frac{x'-3}{-3}$

$y' = \frac{1}{2}y - 4 \Rightarrow y = 2(y'+4)$



Calculator Tip: Mathematica UDF

- ApplyTransformList[]

ApplyTransformList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in the chronological order.

```
ApplyTransformList[x^2, {x, y}, {x - 1, 2 x, y + 3}]
```

$$4 + x + \frac{x^2}{4}$$

```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 x, y + 3}]
```

-3 + f[2 (-1 + x)]

```
ApplyTransformInvList[Sin[x], {x, y}, {x - π / 2, 2 y, y - 1 }]
```

$$\text{Sin}\left[\frac{x}{2}\right]^2$$

- ApplyTransformInvList[]

ApplyTransformInvList[*f[x]* , *{x, y}*, *list of transforms*]
Applies the list of transforms to f[x] in reverse order and as the inverse to the transforms of *ApplyTransformList.*

In[ ]:=
```
ApplyTransformInvList[x^2, {x, y}, {x - 1, 2 * x, y + 3}]
```
Out[ ]=
$$1 - 8 x + 4 x^2$$

In[ ]:=
```
ApplyTransformInvList[f[x], {x, y}, {x - 1, 2 * x, y + 3}]
```
Out[ ]=
-3 + f[2 (-1 + x)]

In[ ]:=
```
ApplyTransformInvList[2 * Cos[x] - 1, {x, y}, {x - Pi / 2, 2 * y, y - 1 }]
```
Out[ ]=
Sin[x]

Calculator Tip: TI UDF

- transform()

Transform a Function

$$transform\left(\sin(x),x,\left\{x-\frac{\pi}{2},2\cdot y,y-1\right\}\right)$$

▶ Translation $\frac{\pi}{2}$ units along the neg. x−dir.

$\cos(x)$

▶ Dilation by factor of 2 from the x−axis

$2\cdot\cos(x)$

▶ Translation −1 unit along the neg. y−dir.

$2\cdot\cos(x)-1$

Overview:

Apply any sequence of transformations to a function. The program will display the transformed function after each step.

Input:

```
transform(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a (horizontal or vertical) matrix of expressions or a list of expressions

- transform_inv()

Invert a Transformation

$$transform\_inv\left(x^2,x,\{x-1,2\cdot x,y+3\}\right)$$

▶ Inverted Transformations:

$\left\{y-3,\frac{x}{2},x+1\right\}$

▶ Translation −3 units along the neg. y−dir.

x^2-3

▶ Dilation by factor of $\frac{1}{2}$ from the y−axis

$4\cdot x^2-3$

▶ Translation 1 unit along the pos. x−dir.

$4\cdot x^2-8\cdot x+1$

Overview:

Find the preimage of a function under a list of transformations. The program will display the list of inverted transformations and the transformed function after each step.

Input:

```
transform_inv(<function>, <variable>,
      <list of transformations>)
```

Other notes:

- The list of transformations can either be presented in a row or column matrix, or a list of expressions

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Section F: Exam 2 (15 Marks)

Question 29 (1 mark)

Let $f: [0,4] \to \mathbb{R}, f(x) = x^2 + 4$. The graph of f is transformed by a reflection in the x-axis, followed by a dilation of factor 2 from the y-axis, then a dilation by a factor of 2 from the x-axis. The resulting graph is defined by:

A. $g : [0, 4] \to \mathbb{R}, g(x) = -\frac{x^2}{8} - 8$

B. $g: [0,8] \to \mathbb{R}, g(x) = -\frac{x^2}{16} - 4$

C. $g : [0, 8] \to \mathbb{R}, g(x) = -\frac{x^2}{8} - 4$

D. $g: [0,4] \to \mathbb{R}, g(x) = -\frac{x^2}{4} - 8$

Question 30 (1 mark)

The point $P(2, 4)$ lies on the graph of f. The point $Q(4, 12)$ lies on the graph of h. A transformation that maps the graph of f to the graph of h also maps the point P to the point Q. The relationship between f and h could be given by:

A. $h(x) = \frac{1}{2} f(x + 2)$

B. $h(x) = 2f(x - 2)$

C. $h(x) = 3f(x - 2)$

D. $h(x) = 3f(x + 2)$

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Question 31 (1 mark)

A transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$, which maps the curve with equation $y = 4x^2$ onto the curve with equation $y = (x-1)^2 + 3$, has the rule:

A. $T(x, y) = (2x + 1, y + 3)$

B. $T(x, y) = \left(\frac{x}{2} + 1, y + 3\right)$

C. $T(x, y) = (x - 1, 4y + 3)$

D. $T(x, y) = \left(\frac{x}{2} + 2, y + 3\right)$

$y = \sqrt{\quad} + 6$

1\. 3 left
2\. Dil $\frac{1}{2}$ from x
3\. 6 down

1\.
2\.
3\.

Question 32 (1 mark)

A sequence of transformations is applied to create the image rule $y = 2\sqrt{x-3} + 6$ from the original function $y = \sqrt{x}$, in an appropriate order, could be:

A. A dilation by a factor of 4 from the x-axis, a dilation by factor 2 from the y-axis, a translation 3 units to the left, and finally a translation of 6 units up.

B. A dilation by a factor of 2 from the x-axis, a translation 3 units to the left, and finally a translation of 6 units up.

C. A dilation by a factor of $\frac{1}{4}$ from the y-axis, a translation 3 units to the right, and finally a translation 6 units up.

D. A dilation by a factor of 2 from the x-axis, followed by a reflection in the y-axis, a translation 2 units right, and finally a translation of 3 units up.

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Question 33 (1 mark)

If the graphs of $y = h(x)$ and $y = k(x)$ intersect at (p, q), then the graphs of $y = 3h(2x)$ and $y = 3k(2x)$ intersect at:

A. $\left(2p, \frac{q}{3}\right)$

B. $\left(\frac{p}{3}, 2q\right)$

C. $\left(\frac{p}{2}, 3q\right)$

D. $(3p, \frac{q}{2})$

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Question 34 (10 marks)

Consider the functions:

$$f: \mathbb{R} \longrightarrow \mathbb{R}, f(x) = (x+1)^2(x-2)$$

$$g: \mathbb{R} \longrightarrow \mathbb{R}, g(x) = x^3 - 3x + 2$$

a.

i. Factorise $g(x)$. (1 mark)

g(x)=(x-1)^2 (x+2)

ii. Find the rule for the image of f, if f is reflected in the y-axis. (1 mark)

f(-x)=-(x+2) (1-x)^2

iii. Hence or otherwise, describe a sequence of **reflections** that map the graph of f onto the graph of g. (2 marks)

Note that (1-x)^2=(x-1)^2.

1. A reflection in the y-axis, followed by,
2. A reflection in the x-axis.

iv. Describe a single translation that maps the graph of f onto the graph of g. (1 mark)

Note that f(x)=x^3-3x-2.
Required translation is a shift 4 units up.

Consider the following transformations:

$$T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, T(x,y) = (2x-1, 3y+2)$$

$$S: \mathbb{R}^2 \longrightarrow \mathbb{R}^2, S(x,y) = (-x+2, 2y-3)$$

b.

i. Find the rule for the image of f after it has undergone the transformation T. (2 marks)

Solution: Let $f_T(x)$ be the image of f under the transformation T.

$$\begin{aligned} f_T(x) = 3f\left(\frac{1}{2}(x+1)\right)+2 &= 3\left(\frac{x+1}{2}-2\right)\left(\frac{x+1}{2}+1\right)^2+2 \\ &= \frac{1}{8}\left(3x^3+9x^2-27x-65\right) \end{aligned}$$

ii. Hence, find the rule for the image of f after it has undergone the transformation T followed by the transformation S. (1 mark)

We apply the transformation S to $f_T(x)$.

$$f_{TS}(x) = 2f_T(2-x)-3 = \frac{1}{4}\left(-3x^3+27x^2-45x-71\right)$$

c. Find the coordinates of the point $P(u, v)$, if the image of the point P under T and S is the same. (2 marks)

Solution: Solve the equations

$$2x - 1 = -x + 2$$
$$3y + 2 = 2y - 3$$

$x = 1$ and $y = -5$. Therefore, $P(1, -5)$

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Contour Checklist

Learning Objective: [2.5.1] - Apply Quick Method to Find Transformations

Key Takeaways

- For applying transformations in the quick method:
  - Apply everything for x in the opposite direction, including the order!
- For interpreting transformations in the quick method:
  - Read everything for x in the opposite direction, including the order!

Learning Objective: [2.5.2] – Find Opposite Transformations

Key Takeaways

- Order is reversed.
- All transformations are in the opposite direction.

Learning Objective: [2.5.3] – Apply Transformations of Functions to Find Their Domain, Range, Transformed Points

Key Takeaways

- Everything moves together as a function.
- **Steps:**
  1. Find the transformations between two functions.
  2. Apply the same transformations to domain, range, and points.

Learning Objective: [2.5.4] – Find Transformations of the Inverse Functions $f(x)$

Key Takeaways

- **Steps:**
  1. Find the transformations between the two original functions.
  2. Swap x & y for the transformations found in **1**.

Learning Objective: [2.5.5] – Find Multiple Transformations for the Same Functions

Key Takeaways

- Same transformations can be done for x or y by either putting it in or out of the $f()$.
- Commonly, look for basic algebra, index, and log laws.

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VCE Mathematical Methods ½

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