

## VCE Mathematical Methods ½ Transformations Exam Skills [2.5] Workbook

### Outline:



<b><u>Recap of Transformations</u></b>	Pg 2-12	<b><u>Transformations Exam Skills</u></b>	Pg 19-31
➤ Image and Pre-Image		➤ Quick Method	
➤ Dilation		➤ Finding Opposite Transformations	
➤ Reflection		➤ Finding Domain, Range, Points, and Tangents of Transformed Functions	
➤ Translation		➤ Finding Transformations of Inverse Functions	
➤ Basic Transformation of Points		➤ Multiple Pathways for the Same Transformation	
➤ The Order of Transformations			
➤ Interpreting the Transformation of Points		<b><u>Exam 1</u></b>	Pg 32-38
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### Learning Objectives:



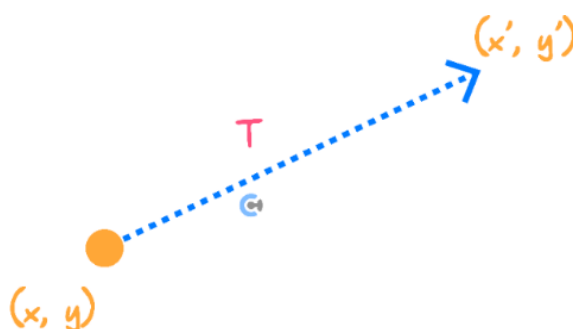
- MM12 [2.5.1] - Apply Quick Method to Find Transformations
- MM12 [2.5.2] - Find Opposite Transformations
- MM12 [2.5.3] - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points
- MM12 [2.5.4] - Find Transformations of the Inverse Functions  $f(x)$
- MM12 [2.5.5] - Find Multiple Transformations for the Same Functions

## Section A: Recap of Transformations

### Sub-Section: Image and Pre-Image

*What do we call an original coordinate and a transformed coordinate?*

#### Image and Pre-Image



- The original coordinate is called the \_\_\_\_\_.
- The transformed coordinate is called the \_\_\_\_\_.

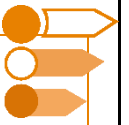
Pre-Image:  $(x, y)$

Image:  $(x', y')$

**NOTE:** The  $x'$  and  $y'$  notation will be used quite heavily!

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## Sub-Section: Dilation



### Dilation



Dilation by factor  $a$  from the  $x$ -axis:  $y' = ay$

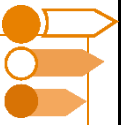
Dilation by factor  $b$  from the  $y$ -axis:  $x' = bx$

**NOTE:** We are applying the transformations on  $(x, y)$  not  $(x', y')$ .



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## Sub-Section: Reflection



### Reflection



Reflection in the  $x$ -axis:  $y' = -y$

Reflection in the  $y$ -axis:  $x' = -x$

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## Sub-Section: Translation



### Translation

Translation by  $c$  units in the positive direction of  $x$ -axis:  $x' = x + c$

Translation by  $d$  units in the positive direction of  $y$ -axis:  $y' = y + d$

### Question 1

Find the image  $(x', y')$  after applying the following transformations to  $(x, y)$ .

Dilation by a factor 4 from the  $x$ -axis.

Dilation by a factor 3 from the  $y$ -axis.

Translation by 3 units in the negative direction of the  $x$ -axis.

Translation by 6 units in the positive direction of the  $y$ -axis.



### Key Takeaways

- ✓ The transformed point is called the image and is denoted by  $(x', y')$ .
- ✓ The dilation factor is multiplied by the original coordinates.
- ✓ Reflection makes the original coordinates the negative of their original values.
- ✓ Translation adds a unit to the original coordinates.

## Sub-Section: Basic Transformation of Points



*Let's try to apply all types of transformations to a point!*



### Question 2

Find the image  $(x', y')$  after applying the following transformations to  $(x, y)$ .

Dilation by a factor 2 from the  $x$ -axis.

Dilation by a factor 4 from the  $y$ -axis.

Reflection in the  $x$ -axis.

Translation by 2 units in the negative direction of the  $x$ -axis.

Translation by 3 units in the positive direction of the  $y$ -axis.

**NOTE: Order Is Important!**



➤ Apply the next transformation on top of everything that has already been done!

## Sub-Section: The Order of Transformations



*What is the order of transformations the same as?*



### The Order of Transformation



**Order = BODMAS Order**

### Question 3

The series of transformations, “a dilation by a factor  $\frac{1}{2}$  from the  $x$ -axis and a translation by 3 units up” yields the same result as the series of transformations, “a translation by  $a$  units up and a dilation by a factor  $b$  from the  $x$ -axis.” Find the values of  $a$  and  $b$ .

**NOTE:** Dilation factors don't change!





## Sub-Section: Interpreting the Transformation of Points

### Question 4

Consider the transformation which maps:

$$x' = 3x + 6$$

$$y' = -2(y + 2)$$

a. State the transformation in DRT (Dilation, Reflection, Translation) order.

b. State the transformation in the translation first order.

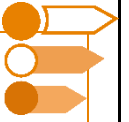
### Key Takeaways



- ✓ Transformations should be interpreted when  $x'$  and  $y'$  are isolated.
- ✓ The order of transformation follows the BODMAS order.
- ✓ To change the order of transformations, we either factorise or expand.



## Sub-Section: Applying Transformations to Functions



*Let's now work with functions!*



### Transformation of Functions



- The aim is to get rid of the old variables,  $x$  and  $y$ , and have the new variables,  $x'$  and  $y'$ , instead.

$$y = f(x) \rightarrow y' = f(x')$$

➤ **Steps:**

1. Transform the points.
2. Make  $x$  and  $y$  the subjects.
3. Substitute them into the function.

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**Question 5**

**a.**  $f(x) = x^2$

Dilation by factor 3 from the  $x$ -axis.Reflect in the  $y$ -axis.

Translate 3 units to the left.

Dilate by a factor of 5 from the  $y$ -axis.

**b.**  $f(x) = \sqrt{x}$

Dilate by a factor of  $\frac{1}{4}$  from the  $y$ -axis.Dilate by a factor of 3 from the  $x$ -axis.

Translate 4 units to the left.

Translate 1 unit up.

Reflect in the  $y$ -axis.

## Sub-Section: Finding the Applied Transformations



*Now, let's go backwards!*



### Reverse Engineering



#### ➤ Steps:

1. Add the dashes (') back to the transformed function.
2. Make  $f( )$  the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make  $x'$  and  $y'$  the subjects and interpret the transformations.

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*Your turn!*



### Question 6

State a series of transformations (in order) that allow  $f(x)$  to be transformed into  $g(x)$ .

a.  $f(x) = 2(x + 1)^2 + 3$  and  $g(x) = 6(x - 4)^2 - 3$ .

b.  $f(x) = 3(x - 1)^2$  and  $g(x) = \frac{1}{2}(2x + 3)^2 + 1$ .

### Key Takeaways



- ☒ We transform the coordinates first, then transform the function.
- ☒ To transform the function, replace its old variables with the new ones.
- ☒ To find the transformations, simply equate LHS with RHS after separating the transformations of  $x$  and  $y$ .

## Section B: Warmup Test (15 Marks)

INSTRUCTION: 15 Marks. 15 Minutes Writing.



### Question 7 (4 marks)

Consider the transformation  $T$  where  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (3x - 6, -2y + 4)$ . Describe in words what the transformation  $T$  does with:

a. Dilation and reflections before translations. (1 mark)

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b. Translations before dilations and reflections. (2 marks)

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- c. The series of transformations given by “a dilation by a factor of 2 from the  $x$ -axis, followed by a translation of 6 units up”, yields the exact same result as the series of transformations given by “a translation by  $a$  units up, followed by a dilation by a factor of  $b$  from the  $x$ -axis”.

Find the values of  $a$  and  $b$ . (1 mark)

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**Question 8** (3 marks)

Consider the following function:  $f(x) = (x + 3)^2$

Apply the following transformations to the function above.

Dilation by a factor of  $\frac{1}{2}$  from the  $y$ -axis.

Dilation by a factor of 3 from the  $x$ -axis.

Translation by 2 units in the negative direction of the  $x$ -axis.

Translation by 6 units in the positive direction of the  $y$ -axis.

Reflection in the  $y$ -axis.

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**Question 9** (2 marks)

For the function  $f(x) = \sqrt{2x + 1}$ , the function  $f$  is dilated by a factor of 3 from the  $x$ -axis, translated 2 units in the negative  $x$ -direction and then is reflected in the  $y$ -axis to produce the function  $g$ .

Find the rule for  $g(x)$ .

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**Question 10** (3 marks)

Consider the following functions:

$$f(x) = \sqrt{x + 3}$$

$$g(x) = -4\sqrt{4 - 2x} + 3$$

Find the set of transformations that maps  $f(x)$  to  $g(x)$ .

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**Question 11** (3 marks)

Consider the following functions:

$$f_1(x) = x^3$$

$$f_2(x) = -4(2x + 1)^3 - 5$$

Find the set of transformations that maps the function  $f_1$  into  $f_2$ .

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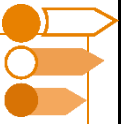
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## Section C: Transformations Exam Skills

### Sub-Section: Quick Method



*Let's try to do it more quickly!*



#### Active Recall: Interpretation of Transformations



- When the new variables  $x'$  and  $y'$  are the subject, we can read the transformation directly.

$$x' = x + 5 \rightarrow 5 \text{ right}$$

- When the original variables  $x$  and  $y$  are the subject instead, we must read the transformation in the opposite way.
- This includes the order of transformation!

$$x = x' - 5 \rightarrow 5 \text{ right}$$

#### Quick Method



- The transformation of  $x$  in the function is represented in the opposite way in the final function.
- For applying transformation in a quick method:

**Apply everything for  $x$  in the opposite direction,  
including the order!**

- For interpreting transformation in a quick method:

**Read everything for  $x$  in the opposite direction,  
including the order!**

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**Question 12 Walkthrough.**

Apply the following transformations to  $y = x^2$  using the quick method.

Dilation by a factor 3 from the  $x$ -axis.

Dilation by a factor 2 from the  $y$ -axis.

Reflection in the  $x$ -axis.

Translation of 2 units right.

**NOTE:** For  $x$ , simply apply everything in the opposite way and order!



*Your turn!*



### Question 13

Apply the following transformations to  $y = \log_2(x)$  using the quick method.

Dilation by a factor  $\frac{1}{4}$  from the  $x$ -axis.

Dilation by a factor 3 from the  $y$ -axis.

Reflection in the  $y$ -axis.

Translation of 5 units left.

Translation of 2 units up.

**NOTE:** For  $x$ , simply apply everything in the opposite way and order!



*Now, interpreting transformations!*



### Question 14 Walkthrough.

State the transformations required for  $y = \sqrt{x}$  to transform into  $y = 2\sqrt{3x + 1} - 3$ .

**NOTE:** The order is opposite to BODMAS for  $x$ .



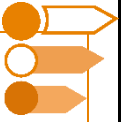
*Your turn!*



### Question 15

State the transformation required for  $y = 2^x$  to transform into  $y = 5 \times 2^{3(x+1)} + 3$ .

## Sub-Section: Finding Opposite Transformations



*How can we undo transformations?*



### Analogy: Untying a Shoelace



- Sam is being silly and ties his shoelace when he was meant to take off his shoes at a chocolate restaurant that he's booked 3 years in advance.
- Which knot should he start untying first? [First Knot] / [Last Knot]
- Similarly, which transformations should we undo first? [First Transformation] / [Last Transformation]

### Finding Opposite Transformations



- Order is \_\_\_\_\_.
- All transformations are \_\_\_\_\_.

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**Question 16**

a. Find the transformation from  $f(x) = 3(x + 1)^2 - 6$  to  $g(x) = x^2 + 3$ .

b. Hence, state the transformation from  $g(x)$  to  $f(x)$ .



## Sub-Section: Finding Domain, Range, Points, and Tangents of Transformed Functions



### Analogy: Function, Points, and Tangents



- Let's say your entire family decides to move 2 units right.

*Family: Let's go 2 units right.*

- What does that mean for you?

*You:* \_\_\_\_\_

- Similarly, if a function moves in a certain way, how should its points, tangents, domain, and range move? [Same way] / [Different way]

### Finding Domain, Range, and Points of Transformed Functions



- Everything moves together as a function.
- Steps:
  1. Find the transformations between two functions.
  2. Apply the same transformations to domain, range, points, and tangents.

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**Question 17 Walkthrough.**

It is known that  $f(x)$  has a domain of  $[2,4]$  and a range of  $(0,20]$ .

The function has been transformed to  $g(x) = -2f(x + 3) + 5$ .

**a.** State the transformation from  $f(x)$  to  $g(x)$ .

**b.** State the domain of  $g(x)$ .

**c.** State the range of  $g(x)$ .

**Question 18**

It is known that  $f(x)$  has an  $x$ -intercept at  $(3, 0)$  and a range of  $[-5, 10)$ .

The function has been transformed to  $g(x) = 2f(2x - 1)$ .

a. State the transformation from  $f(x)$  to  $g(x)$ .

b. State the  $x$ -intercept of  $g(x)$ .

c. State the range of  $g(x)$ .

**NOTE:** Everything changes with respect to the transformations.



## Sub-Section: Finding Transformations of Inverse Functions

**REMINDER:** Don't forget Inverse Relations.

Inverse functions swap  $x$  and  $y$ .

**Discussion:** If  $f(x)$  moves 2 units right, where would  $f^{-1}(x)$  go to?

### Finding Transformation of Inverse Functions

$$f(x) \rightarrow f(x - 2): 2 \text{ Right}$$

$$f^{-1}(x) \rightarrow f^{-1}(x) + 2: 2 \text{ Up}$$

► **Steps:**

1. Find the transformation between two original functions.
2. Inverse the transformations found in 1.

### Question 19 Walkthrough.

It is known that  $f(x)$  has been transformed to  $g(x) = 3f(x - 4) + 1$ .

State the transformations required for  $f^{-1}(x)$  to transform to  $g^{-1}(x)$ .



### Active Recall: Steps on Finding Transformations of Inverse Functions

1. Find the \_\_\_\_\_ between two original functions.
2. \_\_\_\_\_ the transformations found in 1.

### Question 20

It is known that  $f(x) = 2(x - 1)^2 + 3$  has been transformed to  $g(x) = 4(x + 2)^2 - 1$ .

State the transformations required for  $f^{-1}(x)$  to transform to  $g^{-1}(x)$ .

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## Sub-Section: Multiple Pathways for the Same Transformation



**Discussion:** Consider the transformations required for  $f(x) = x^2$  to  $g(x) = (2x)^2$ . What happens if we take the factor of 2 inside the square bracket out?



### Multiple Pathways



- Same transformations can be done differently by either putting it in or out of the  $f( )$ .
- Commonly, look for basic algebra, index, and log laws.

### **Question 21 Walkthrough.**

Find the transformation for  $y = x^2$  to transform into  $y = 4x^2$  by using a dilation from the  $y$ -axis.

**Question 22**

Find the transformation for  $y = (x + 1)^3 - 2$  to transform into  $y = 8x^3$  without using a dilation from the  $x$ -axis.

**NOTE:** This skill is important for MCQ questions.



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**Section D: Exam 1 (17 Marks)****Question 23** (2 marks)

The transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $T(x, y) = (2x + 4, -y + 2)$  maps the function  $f(x) = x^2$  to a function  $g(x)$ . Find the rule for  $g(x)$ .

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**Question 24** (4 marks)

The following sequence of transformations:

- A translation 1 unit up.
- A translation 4 units left.
- A dilation by factor 3 from the  $x$ -axis.
- A dilation by factor  $\frac{1}{2}$  from the  $y$ -axis.
- A reflection in the  $x$ -axis.

is applied to the function  $f(x)$  so that  $f(x)$  is mapped to  $g(x) = \sqrt{x}$ .

- a.** Find a sequence of transformations that maps  $g(x)$  to  $f(x)$ . (2 marks)

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**b.** Find the rule for  $f(x)$ . (2 marks)

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**Question 25** (4 marks)

Consider the functions  $f(x) = x^2 - 4x + 5$  and  $g(x) = 9(x + 1)^2 - 4$ .

- a. Find a sequence of three transformations in the order DTT that maps  $f(x)$  to  $g(x)$ , and where the dilation is from the  $x$ -axis. (2 marks)

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- b. Find a different sequence of transformations in the order DTT, where the dilation is from the  $y$ -axis, that also maps  $f(x)$  to  $g(x)$ . (2 marks)

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**Question 26** (5 marks)


Consider the function  $f(x) = 2\sqrt{(x - 1)^2 + 3} - 2$  defined on the domain  $[0, 4]$ .

**a.** The function  $g$  is obtained by applying the following sequence of transformations to  $f$ .

 A dilation by factor  $\frac{1}{2}$  from the  $y$ -axis.

 A dilation by factor 3 from the  $x$ -axis.

 A translation 1 unit right.

 A reflection in the  $x$ -axis.

**i.** State the domain of  $g$ . (1 mark)

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**ii.** Find the rule for  $g(x)$ . (2 marks)

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- b. Let  $h(x) = \sqrt{(x + 1)^2 + 3} + 1$ . Write down a sequence of three transformations that map  $f(x)$  to  $h(x)$ . (2 marks)

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**Question 27** (2 marks)

Consider the function  $f$  with inverse function  $f^{-1}$ . The function  $f$  is transformed to the function  $g$  by the following sequence of transformations: A dilation by factor 3 from the  $x$ -axis and a translation 2 units down.

Write down the transformations that take  $f^{-1}$  to  $g^{-1}$ .

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## Section E: Tech Active Exam Skills



### Calculator Tip: Finding Transformed Functions

- Save the function as  $f(x)$ .
- Substitute the  $x$  and  $y$  in terms of  $x'$  and  $y'$ .
- Solve for  $y'$ !
- Can also apply the transformations directly to  $f(x)$ . Must make sure you interpret the transformations correctly or you can easily make a mistake doing this.

### Question 28 Tech-Active.

Apply the following transformations to  $y = 2\sqrt{3x + 6}$ .

Dilation by a factor  $\frac{1}{2}$  from the  $x$ -axis.

Dilation by a factor 3 from the  $y$ -axis.

Reflection in the  $y$ -axis.

Translation of 3 units right.

Translation of 4 units down.



## Calculator Tip: Mathematica UDF

### ➤ ApplyTransformList[]

**ApplyTransformList[  $f[x]$ , { $x, y$ }, list of transforms ]**

Applies the list of transforms to  $f[x]$  in the chronological order.

**ApplyTransformList[ $x^2$ , { $x, y$ }, { $x - 1$ ,  $2x$ ,  $y + 3$ }]**

$$4 + x + \frac{x^2}{4}$$

**ApplyTransformInvList[ $f[x]$ , { $x, y$ }, { $x - 1$ ,  $2x$ ,  $y + 3$ }]**

$$-3 + f[2(-1 + x)]$$

**ApplyTransformInvList[Sin[ $x$ ], { $x, y$ }, { $x - \pi/2$ ,  $2y$ ,  $y - 1$ }]**

$$\sin\left[\frac{x}{2}\right]^2$$

### ➤ ApplyTransformInvList[]

**ApplyTransformInvList[  $f[x]$ , { $x, y$ }, list of transforms ]**

Applies the list of transforms to  $f[x]$  in reverse order and as the inverse to the transforms of *ApplyTransformList*.

In[\*]:=

**ApplyTransformInvList[ $x^2$ , { $x, y$ }, { $x - 1$ ,  $2 * x$ ,  $y + 3$ }]**

Out[\*]:=

$$1 - 8x + 4x^2$$

In[\*]:=

**ApplyTransformInvList[ $f[x]$ , { $x, y$ }, { $x - 1$ ,  $2 * x$ ,  $y + 3$ }]**

Out[\*]:=

$$-3 + f[2(-1 + x)]$$

In[\*]:=

**ApplyTransformInvList[ $2 * \cos[x] - 1$ , { $x, y$ }, { $x - \pi/2$ ,  $2 * y$ ,  $y - 1$ }]**

Out[\*]:=

$$\sin[x]$$





### Calculator Tip: TI UDF

➤ transform()

#### Transform a Function

$$\text{transform}\left(\sin(x), x, \left\{x - \frac{\pi}{2}, 2 \cdot y, y - 1\right\}\right)$$

► Translation  $\frac{\pi}{2}$  units along the neg. x-dir.

$$\cos(x)$$

► Dilation by factor of 2 from the x-axis

$$2 \cdot \cos(x)$$

► Translation -1 unit along the neg. y-dir.

$$2 \cdot \cos(x) - 1$$

#### Overview:

Apply any sequence of transformations to a function. The program will display the transformed function after each step.

#### Input:

transform(<function>, <variable>,  
<list of transformations>)

#### Other notes:

➤ The list of transformations can either be presented in a (horizontal or vertical) matrix of expressions or a list of expressions

➤ transform\_inv()

#### Invert a Transformation

$$\text{transform\_inv}(x^2, x, \{x - 1, 2 \cdot x, y + 3\})$$

► Inverted Transformations:

$$\left\{y - 3, \frac{x}{2}, x + 1\right\}$$

► Translation -3 units along the neg. y-dir.

$$x^2 - 3$$

► Dilation by factor of  $\frac{1}{2}$  from the y-axis

$$4 \cdot x^2 - 3$$

► Translation 1 unit along the pos. x-dir.

$$4 \cdot x^2 - 8 \cdot x + 1$$

#### Overview:

Find the preimage of a function under a list of transformations. The program will display the list of inverted transformations and the transformed function after each step.

#### Input:

transform\_inv(<function>, <variable>,  
<list of transformations>)

#### Other notes:

➤ The list of transformations can either be presented in a row or column matrix, or a list of expressions

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## Section F: Exam 2 (15 Marks)

### Question 29 (1 mark)

Let  $f: [0, 4] \rightarrow \mathbb{R}, f(x) = x^2 + 4$ . The graph of  $f$  is transformed by a reflection in the  $x$ -axis, followed by a dilation of factor 2 from the  $y$ -axis, then a dilation by a factor of 2 from the  $x$ -axis. The resulting graph is defined by:

A.  $g: [0, 8] \rightarrow \mathbb{R}, g(x) = -\frac{x^2}{8} - 8$

B.  $g: [0, 8] \rightarrow \mathbb{R}, g(x) = -\frac{x^2}{16} - 4$

C.  $g: [0, 8] \rightarrow \mathbb{R}, g(x) = -\frac{x^2}{8} - 4$

D.  $g: [0, 4] \rightarrow \mathbb{R}, g(x) = -\frac{x^2}{4} - 8$

### Question 30 (1 mark)

The point  $P(2, 4)$  lies on the graph of  $f$ . The point  $Q(4, 12)$  lies on the graph of  $h$ . A transformation that maps the graph of  $f$  to the graph of  $h$  also maps the point  $P$  to the point  $Q$ . The relationship between  $f$  and  $h$  could be given by:

A.  $h(x) = \frac{1}{2} f(x + 2)$

B.  $h(x) = 2f(x - 2)$

C.  $h(x) = 3f(x - 2)$

D.  $h(x) = 3f(x + 2)$

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**Question 31** (1 mark)

A transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ , which maps the curve with equation  $y = 4x^2$  onto the curve with equation  $y = (x - 1)^2 + 3$ , has the rule:

- A.  $T(x, y) = (2x + 1, y + 3)$
- B.  $T(x, y) = \left(x + 1, \frac{1}{4}y + 3\right)$
- C.  $T(x, y) = (x - 1, 4y + 3)$
- D.  $T(x, y) = \left(\frac{x}{2} + 2, y + 3\right)$

**Question 32** (1 mark)

A sequence of transformations is applied to create the image rule  $y = 2\sqrt{x - 3} + 6$  from the original function  $y = \sqrt{x}$ , in an appropriate order, could be:

- A. A dilation by a factor of 4 from the  $x$ -axis, a dilation by factor 2 from the  $y$ -axis, a translation 3 units to the left, and finally a translation of 6 units up.
- B. A dilation by a factor of 2 from the  $x$ -axis, a translation 3 units to the left, and finally a translation of 6 units up.
- C. A dilation by a factor of  $\frac{1}{4}$  from the  $y$ -axis, a translation 3 units to the right, and finally a translation 6 units up.
- D. A dilation by a factor of 2 from the  $x$ -axis, followed by a reflection in the  $y$ -axis, a translation 2 units right, and finally a translation of 3 units up.

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**Question 33** (1 mark)

If the graphs of  $y = h(x)$  and  $y = k(x)$  intersect at  $(p, q)$ , then the graphs of  $y = 3h(2x)$  and  $y = 3k(2x)$  intersect at:

- A.  $(2p, \frac{q}{3})$
- B.  $(\frac{p}{3}, 2q)$
- C.  $(\frac{p}{2}, 3q)$
- D.  $(3p, \frac{q}{2})$

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**Question 34** (10 marks)

Consider the functions:

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = (x + 1)^2(x - 2)$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = x^3 - 3x + 2$$

**a.**

- i.** Factorise  $g(x)$ . (1 mark)

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- ii.** Find the rule for the image of  $f$ , if  $f$  is reflected in the  $y$ -axis. (1 mark)

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- iii.** Hence or otherwise, describe a sequence of **reflections** that map the graph of  $f$  onto the graph of  $g$ . (2 marks)

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- iv.** Describe a single translation that maps the graph of  $f$  onto the graph of  $g$ . (1 mark)

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Consider the following transformations:

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (2x - 1, 3y + 2)$$

$$S: \mathbb{R}^2 \rightarrow \mathbb{R}^2, S(x, y) = (-x + 2, 2y - 3)$$

**b.**

- i.** Find the rule for the image of  $f$  after it has undergone the transformation  $T$ . (2 marks)

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- ii.** Hence, find the rule for the image of  $f$  after it has undergone the transformation  $T$  followed by the transformation  $S$ . (1 mark)

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- c. Find the coordinates of the point  $P(u, v)$ , if the image of the point  $P$  under  $T$  and  $S$  is the same. (2 marks)

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## Contour Checklist

### ☐ Learning Objective: [2.5.1] - Apply Quick Method to Find Transformations

#### Key Takeaways

- ☐ For applying transformations in the quick method:
  - ☐ Apply everything for  $x$  in the \_\_\_\_\_ direction, including the order!
- ☐ For interpreting transformations in the quick method:
  - ☐ Read everything for  $x$  in the opposite direction, including the \_\_\_\_\_!

### ☐ Learning Objective: [2.5.2] - Find Opposite Transformations

#### Key Takeaways

- ☐ Order is \_\_\_\_\_.
- ☐ All transformations are in the \_\_\_\_\_ direction.



**□ Learning Objective: [2.5.3] - Apply Transformations of Functions to Find Their Domain, Range, Transformed Points**

**Key Takeaways**

- Everything moves together as a function.
- Steps:
  1. Find the \_\_\_\_\_ between two functions.
  2. Apply the \_\_\_\_\_ transformations to domain, range, and points.

**Learning Objective: [2.5.4] - Find Transformations of the Inverse Functions  $f(x)$**

**Key Takeaways**

- Steps:
  1. Find the \_\_\_\_\_ between the two original functions.
  2. \_\_\_\_\_ the transformations found in 1.

**Learning Objective: [2.5.5] - Find Multiple Transformations for the Same Functions**

**Key Takeaways**

- Same transformations can be done \_\_\_\_\_ by either putting it in or out of the  $f()$ .
- Commonly, look for basic algebra, index, and log laws.

## VCE Mathematical Methods ½

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