Website: contoureducation.com.au | Phone: 1800 888 300
Email: hello@contoureducation.com.au

VCE Mathematical Methods ½
Transformations Exam Skills [2.5]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2 – Pg 24
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Section A: Compulsory Questions

Sub-Section [2.5.1]: Apply Quick Method to Find Transformations

Question 1

Find the rule for the image of $f(x) = x^2$ under the transformations:

- A dilation by factor 3 from the x-axis.
- A translation 1 unit up.
- A translation of 3 units to the left.

$f(x) \mapsto 3x^2 \mapsto 3x^1 + 1 \mapsto 3(x+3)^2 + 1$

The image is $y = 3(x+3)^2 + 1$

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Question 2

Find the rule for the image of $f(x) = 2\sqrt{x+4} - 1$ under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (2x + 3, -y + 1)$.

$x' = 2x + 3 \implies x = \frac{x' - 3}{2}$.
So the image is given by

$$\begin{aligned} y &= -f\left(\frac{x-3}{2}\right) + 1 \\ &= -2\sqrt{\frac{x}{2} + \frac{5}{2}} + 1 + 1 \\ &= -\sqrt{2x + 10} + 2 \end{aligned}$$

Question 3

Describe a sequence of transformations that maps the graph of $y = 3^{2x+1} - 2$ onto the graph of $y = 1 - 3^x$.

Isolate the x part: $2x + 1 = x'$. Thus

- A dilation by factor 2 from the y-axis
- A translation 1 unit to the right

then looking at the y part we see $-(3^x - 2) - 1 = 1 - 3^x$, thus

- A reflection in the x-axis
- A translation 1 unit down.

Sub-Section [2.5.2]: Find Opposite Transformations

Question 4

Describe a sequence of transformations that map $f(x) = 2(x-1)^2 + 4$ to $y = x^2$.

- A dilation by factor $\frac{1}{2}$ from the x-axis
- A translation 1 unit to the left
- A translation 2 units down.

Question 5

The following sequence of transformations map the graph of $y = f(x)$ on the graph of $y = 2\sqrt{x-1} + 1$.

- A reflection in the y-axis
- A dilation by factor 2 from the x-axis
- A translation 2 units to the right

Find the rule of f.

We find f by applying the "opposite" transformations to y in the reverse order.
That is, a translation 2 units left, a dilation by factor 1/2 from the x-axis and a refleciton in the y-axis.
$2\sqrt{x-1}+1 \mapsto 2\sqrt{x+1}+1 \mapsto \sqrt{x+1}+\frac{1}{2} \mapsto \sqrt{1-x}+\frac{1}{2}$.

$$\boxed{f(x) = \sqrt{1-x} + \frac{1}{2}}$$

Question 6

Describe a sequence of transformations that map $f(x) = 3x^2 - 12x + 16$ to $y = x^2 - 2x + 2$.

Note that $f(x) = 3(x-2)^2 + 4$ and $y = (x-1)^2 + 1$. Thus the transformations are

- A dilation by factor $\frac{1}{3}$ from the x-axis
- A translation 1 unit to the left
- A translation $\frac{1}{3}$ units down.

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Sub-Section [2.5.3]: Apply Transformations of Functions to Find its Domain, Range, Transformed Points.

Question 7

Find the image of the point $A(2,5)$ under the transformation

$$T : \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = \left(2x-1, \frac{1}{2}(y-1)\right)$$

$$\boxed{\text{The image of } A \text{ under } T \text{ is } \left(4-1, \frac{1}{2}(5-1)\right) = (3,2)}$$

Question 8

Consider the function $f : [-3,1] \to \mathbb{R}, f(x) = x^2 - 4$. The sequence of transformations

- A dilation by factor 2 from the x-axis
- A dilation by factor 2 from the y-axis
- A translation 3 units to the left

Map the function f to the function g. Find the domain of g.

$$\boxed{[-3,1] \mapsto [-6,2] \mapsto [-9,-1]}$$

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Question 9

Consider the function $f : [-3, 2] \to \mathbb{R}, f(x) = x^2 + 4x - 1$. The sequence of transformations

- A dilation by factor 3 from the x-axis
- A dilation by factor 2 from the y-axis
- A translation 1 unit to the right
- A reflection in the x-axis

Map the function f to the function g. Find the range and domain of g.

Note that $f(x) = (x+2)^2 - 5$. $f(-3) = 1 - 5 = -4$ and $f(2) = 16 - 5 = 11$.
The range of f is $[-5, 11]$.
Thus the range of g is $[-33, 15]$ and the domain of g is $[-5, 5]$

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Sub-Section [2.5.4]: Find Transformations of the Inverse Functions

Question 10

Let $f : [0, \infty) \to \mathbb{R},\ f(x) = \sqrt{x}$.

f is mapped to function g, by a dilation by factor 2 from the x-axis and a translation 1 unit to the right.

Describe a sequence of transformations that map f^{-1} to g^{-1}.

A dilation by factor 2 from the y-axis followed by a translation 1 unit up.

Question 11

Consider the one-to-one functions, f and g. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x, y) = (2x + 4, y - 3)$ maps the function f to the function g.

Describe a sequence of transformations that map the function f^{-1} to the function g^{-1}.

- A dilation by factor 2 from the x-axis
- A translation 4 units up
- A translation 3 units left

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Question 12

Consider the functions $f : [0, \infty) \to \mathbb{R}, 2\sqrt{x} + 1$ and $g : [2, \infty) \to \mathbb{R}$, $g(x) = 4\sqrt{x-2} - 1$.

The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (ax + b, y + c)$ maps the function f^{-1} to the function g^{-1}.

The tranformations that take f to g are: a dilation by factor 2 from the x-axis, a translation 2 units right and a translation 3 units down.
Therefore f^{-1} is mapped to g^{-1} by a dilation by factor 2 from the y-axis, a translation 2 units up and a translation 3 units left.
Thus $a = 2, b = -3$ and $c = 2$.

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Sub-Section [2.5.5]: Find Multiple Transformations for the Same Functions

Question 13

Let $f(x) = x^3$ and $g(x) = 8x^3$.

a. State a dilation that maps $f(x)$ to $g(x)$.

A dilation by factor 8 from the x-axis

b. State a different dilation that maps $f(x)$ to $g(x)$.

A dilation by factor $\frac{1}{2}$ from the y-axis.

Question 14

Let $f(x) = (x - 1)^2 + 3$ and $g(x) = 4(x + 2)^2 + 1$.

a. Find a sequence of transformations that map $f(x)$ to $g(x)$, without using a dilation from the y-axis.

$f(x) \mapsto 4(x-1)^2 + 12 \mapsto 4(x+2)^2 + 12 \mapsto 4(x+2)^2 + 1$.
Therefore,

- A dilation by factor 4 from the x-axis
- A translation 3 units to the left
- A translation 11 units down.

b. Find a sequence of transformations that map $f(x)$ to $g(x)$, without using a dilation from the x-axis.

$f(x) \mapsto \left(2\left(x-\frac{1}{2}\right)\right)^2 + 3 \mapsto 4\left(x-\frac{1}{2}\right)^2 + 1 \mapsto 4(x+2)^2+1$

Therefore,

- A dilation by factor $\frac{1}{2}$ from the y-axis
- A translation 2 units down
- A translation $\frac{5}{2}$ units left.

OR alternatively

- A translation 5 units left
- A dilation by factor $\frac{1}{2}$ from the y-axis
- A translation 2 units down

Question 15

Let $f(x) = x^2 + 4x + 1$ and $g(x) = 9x^2 - 18x + 2$.

a. Find a sequence of transformations that map $f(x)$ to $g(x)$, without using a dilation from the y-axis.

Note that $f(x) = (x+2)^2 - 3$ and $g(x) = 9(x-1)^2 - 7$

Therefore,

- A dilation by factor 9 from the x-axis
- A translation 3 units to the right
- A translation 20 units up.

b. Find a sequence of transformations that map $f(x)$ to $g(x)$, without using a dilation from the x-axis.

Note that $f(x) = (x+2)^2 - 3$ and $g(x) = 9(x-1)^2 - 7$

$f(x) \mapsto \left(3\left(x+\frac{2}{3}\right)\right)^2 - 3 \mapsto 9\left(x+\frac{2}{3}\right)^2 - 7 \mapsto 9(x-1)^2 - 7$

Therefore,

- A dilation by factor $\frac{1}{3}$ from the y-axis
- A translation 4 units down
- A translation $\frac{5}{3}$ units right.

OR alternatively

- A translation 5 units right.
- A dilation by factor $\frac{1}{3}$ from the y-axis
- A translation 4 units down

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Sub-Section: Exam 1 Questions

Question 16

Consider the transformation:

$$T : \mathbb{R}^2 \to \mathbb{R}^2, \quad T(x, y) = (2x + 1, 3y - 1)$$

a. Find the image of the point $P(2, 3)$ under T.

$P' = (4 + 1, 9 - 1) = (5, 8)$

b. Write out what the transformation T does in the order DRT.

- A dilation by factor 2 from the y-axis
- A dilation by factor 3 from the x-axis
- A translation 1 unit to the right
- A translation 1 unit down.

c. Find the image of the curve $y = x^2$ under the transformation T.

$x' = 2x + 1 \implies x = \dfrac{x' - 1}{2}$.

Thus image is $y = 3\left(\dfrac{x - 1}{2}\right)^2 - 1 = \dfrac{3}{4}(x - 1)^2 - 1$

Question 17

Let $f : \mathbb{R} \to \mathbb{R}, f(x) = x^2 - 9$.

a. Find the coordinates of all axes intercepts of f.

x-intercepts: $(-3, 0)$ and $(3, 0)$
y-intercept: $(0, -9)$

b. Let the graph of g be a transformation of the graph of f where the transformations have been applied in the following order:

- Dilation by a factor of $\frac{1}{2}$ from the y-axis.
- Dilation by a factor of 2 from the x-axis.
- Translation 1 unit to the left.

Find the rule for $g(x)$.

$$g(x) = 2f(2(x+1)) = 2(2(x+1))^2 - 18 = 8(x+1)^2 - 18$$

c. State the coordinates for the axes intercepts of g.

x-intercepts: $\left(-\frac{5}{2}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$
y-intercept: $(0, -10)$

Question 18

Consider the function $f(x) = \frac{2}{(x+1)^2} - 3$.

Apply the following sequence of transformations to $f(x)$.

- Dilation by a factor 3 from the x-axis
- Translated 2 units in the negative direction of the x-axis
- Reflection in the y-axis
- Translated 4 units in the positive direction of the y-axis
- Dilation by a factor of $\frac{1}{3}$ from the y-axis.

$f(x) \mapsto \frac{6}{(x+1)^2} - 9 \mapsto \frac{6}{(x+3)^2} - 9 \mapsto \frac{6}{(x-3)^2} - 9 \mapsto \frac{6}{(x-3)^2} - 5 \mapsto \frac{6}{(3x-3)^2} - 5.$

The image is $\frac{6}{(3(x-1))^2} - 5 = \frac{2}{3(x-1)^2} - 5$

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Question 19

Let $f(x) = \frac{1}{2x+2}$.

a. The transformation T_1 given by:

$$T_1 : \mathbb{R}^2 \to \mathbb{R}^2, T_1(x, y) = (x + a, by)$$

maps the graph of $y = f(x)$ onto the graph of $y = \frac{1}{x}$.

Find the values of a and b.

$f(x) = \dfrac{1}{2(x+1)}$.

So translation 1 unit to right and dilation by factor 2 from x-axis will map $f(x)$ to $\dfrac{1}{x}$.

Thus $a = 1$ and $b = 2$

b. The transformation T_2 given by

$$T_2 : \mathbb{R}^2 \to \mathbb{R}^2, T_2(x, y) = (c(x + d), y)$$

maps the graph of $y = \frac{1}{x}$ onto the graph of $y = f(x)$.

Find the values of c and d.

Want $y = \dfrac{1}{x}$ to map to $f(x) = \dfrac{1}{2(x+1)}$.

$x = 2(x'+1) \implies x' = \dfrac{1}{2}x - 1 = \dfrac{1}{2}(x-2)$.

Thus $c = \dfrac{1}{2}$ and $d = -2$.

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Question 20

The image of the curve $y = \sqrt{9 - x^2}$ under a transformation T, has the equation

$$y = \sqrt{32 - 4x - x^2} + 4$$

Find the transformations that makeup T, with dilations before translations.

$$y = \sqrt{9 - x^2} \text{ maps to } y' = \sqrt{36 - (x+2)^2} + 4$$
$$= 2\sqrt{9 - \frac{1}{4}(x+2)^2} + 4$$
$$= 2\sqrt{9 - \left(\frac{1}{2}(x+2)\right)^2} + 4$$

Therefore T can be described as:

- A dilation by factor 2 from the x-axis
- A dilation by factor 2 from the y-axis
- A translation 2 units to the left
- A translation 4 units up

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Sub-Section: Exam 2

Question 21

The graph of the function f passes through the point $(2, -6)$. If $h(x) = 3f(x-3)$, then the graph of the function h must pass through the point:

A. $(0, -6)$

B. $(-1, -18)$

C. $(5, -18)$

D. $(-1, -6)$

Question 22

The graph of the function $f : \mathbb{R} \to \mathbb{R}, f(x) = 2^x$, is reflected in the y-axis and then translated 3 units to the right and 2 units up. Which one of the following is the rule of the transformed graph?

A. $y = 2^{-x} + 2$

B. $y = 2^{-x+3} + 1$

C. $y = \left(\frac{1}{2}\right)^{x-3} + 2$

D. $y = \frac{1}{2} . 2^{-x+3} + 2$

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Question 23

The graph of the function g is obtained from the graph of the function:

$$f : [-2, 1] \to \mathbb{R}, f(x) = 2x^2 - 4x + 8,$$

by a dilation of factor 3 from the y-axis, followed by a dilation of factor $\frac{1}{3}$ from the x-axis, followed by a reflection in the x-axis, and finally followed by a translation of 2 units in the positive direction of the y-axis. The domain and range of g are respectively:

A. $[-6, 3]$ and $[-6, 3]$

B. $[-3, 6]$ and $[-3, 0]$

C. $[-6, 3]$ and $[-6, 0]$

D. $[-6, 3]$ and $[-3, 3]$

Question 24

Consider the functions $f(x) = \frac{1}{x-2} + 1$ and $g(x) = 2 - \frac{1}{x-1}$. If T transforms the graph of f onto the graph of g, then:

A. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (1 - x, y - 3)$

B. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x - 1, y - 3)$

C. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (x - 1, 3 - y)$

D. $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (1 - x, 3 - y)$

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Question 25

The image of the function $g(x) = x^3$ is $y = -5\left(\frac{x}{2}+2\right)^3$. The transformations that could have been applied are:

A. Reflection in the x-axis, then translation in the positive direction of the x-axis by 2 units followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

B. Reflection in the x-axis, then translation in the negative direction of the x-axis by 2 units, followed by a dilation from the x-axis by a factor of 5 and a dilation by factor 2 from the y-axis.

C. Reflection in the x-axis, then a dilation from the x-axis by a factor of 2, followed by a translation in the positive direction of the x-axis by 2 units, and finally a dilation from the y-axis by a factor of 2.

D. Reflection in the x-axis, then a dilation from the y-axis by a factor of $\frac{1}{2}$ followed by a translation in the negative direction of the x-axis by 2 units, and finally a dilation from the x-axis by a factor of $\frac{5}{2}$

Question 26

Consider the function $f : (-1, 2) \to \mathbb{R}, f(x) = (x - 1)^2(2x + 5)$.

a. State the range of f.

From the graph of f we see that the range is [0,12)

b. The following sequence of transformations, T, map the graph of f onto the graph of g.

- A dilation by a factor of 2 from the x-axis, followed by,
- A translation of 3 units down and 4 units left, followed by,
- A reflection in the y-axis.

i. State the rule of g.

$$g(x) = 2f(-x+4) - 3 = -4x^3 + 50x^2 - 192x + 231$$

ii. State the domain of g.

We apply the transformation $x \mapsto -(x-4)$ onto the interval $(-1, 2)$ to get the domain of g.
Thus the domain of g is $(2, 5)$.

iii. State the range of g.

We apply the transformation $y \mapsto 2y - 3$ onto the interval $(0, 12)$ to get the range of g.
Thus the range of g is $(-3, 21)$

iv. Find the image of the point $(1, 0)$ under T.

$(3, -3)$

Let g be a function with the same rule as f but defined for all $x \in \mathbb{R}$.

That is $g : \mathbb{R} \to: \mathbb{R}, g(x) = f(x)$.

c. A transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$, $S(x, y) = (a - x, b - y)$ maps the graph of g onto itself.

Determine the values of a and b.

$x' = a - x \implies x = a - x'$
$y' = b - y \implies y = b - y'$

$$b - y = f(a - x) \implies y = b - f(a - x)$$
$$y = -2a^3 - a^2 + 8a + b - 5 + \left(6a^2 + 2a - 8\right) x + (-6a - 1)x^2 + 2x^3$$

and the original is

$$y = 2x^3 + x^2 - 8x + 5$$

We compare coefficient to conclude that

$$a = -\frac{1}{3} \quad \text{and} \quad b = \frac{343}{27}$$

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Question 27

Consider the function $f : [-4, 4] \to \mathbb{R}, f(x) = x^2 - 9$

a. Consider the transformation $T(x, y) = \left(2x + 4, \frac{1}{3}y - 2\right)$. Find the transformed function of $y = f(x)$ under the transformation T. State the new domain and range also.

$x' = 2x + 4 \implies x = \frac{1}{2}(x' - 4)$

$y' = \frac{1}{3}y - 2 \implies y = 3(y' + 2)$

New function: $y = \frac{1}{3}f\left(\frac{1}{2}(x - 4)\right) - 2 = \frac{1}{12}(x - 4)^2 - 5$

New domain: $[-8 + 4, 8 + 4] = [-4, 12]$

Range is $\left[-5, \frac{1}{3}\right]$

Let g be the function that is the image of f under T.

b. Find a transformation $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ that maps the function g to the function f.

The function T_1 "undos" the transformation T.

$x = 2x' + 4 \implies x' = \frac{1}{2}(x - 4)$

$y = \frac{1}{3}y' - 2 \implies y' = 3(y + 2)$

Therefore, $T_1(x, y) = \left(\frac{1}{2}(x - 4), 3(y + 2)\right)$

c. A function h is such that applying the transformation T to h maps it to the function f. Find the rule and domain for the function h.

We can find h by applying the transformation T_1 to f.

$x' = \frac{1}{2}(x-4) \implies x = 2x' + 4$

$y' = 3(y+2) \implies y = \frac{1}{3}y' - 2$

Domain of h: $\boxed{[-4,0]}$

$h(x) = 3f(2x+4) + 6 = 12x^2 + 48x + 27 = 12(x+2)^2 - 21$

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Section B: Supplementary Questions

Sub-Section [2.5.1]: Apply Quick Method to Find Transformations

Question 28

Find the image of the graph of $y = x^2$ under the transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2,\ T(x, y) = (1 - 2x, y + 5)$.

Apply the transformation $x \to 1 - 2x$ in an opposite way, replacing x with $\frac{1-x}{2}$. After applying y-axis transformations as well, we get: $y = \left(\frac{1-x}{2}\right)^2 + 5$.

Question 29

Describe a sequence of transformations that maps the graph of $y = x^3$ onto the graph of $y = 2(3x + 2)^3 - 3$.

In the equation x is replaced with $3x + 2$, so we apply those transformations in reverse;

- Translate 2 units left.
- Dilate by a factor of $\frac{1}{3}$ from the y-axis.

Then the y-transformations as normal;

- Dilate by a factor of 2 from the x-axis.
- Translate 3 units down.

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Question 30

Find the image of the graph of $y = \log_2(x)$ under the following sequence of transformations:

- A dilation by a factor of 3 from the x-axis, followed by,
- A translation of 2 units left and 3 units up, followed by,
- A reflection in the y-axis, followed by,
- A dilation by a factor of 5 from the y-axis.

The last 3 transformations apply to x, and applying them in reverse gives:

$$x \to \frac{1}{5}x \to -\frac{1}{5}x \to -\frac{1}{5}x + 2$$

Applying the y-axis transformation in order gives:

$$y \to 3y + 3$$

As such, the rule for the image of our graph after the transformations is:

$$y = 3\left(\log_2\left(-\frac{1}{5}x + 2\right)\right) + 3$$

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Sub-Section [2.5.2]: Find Opposite Transformations

Question 31

Describe a sequence of transformations that maps the graph of $y = 4(x-2)^2 - 3$ onto the graph of $y = x^2$.

1. Translate 2 units left.
2. Translate 3 units up
3. Dilate by a factor of $\frac{1}{4}$ from the x-axis.

Question 32

The transformation, $T : \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(2x + 3, \frac{1}{3}y - 4\right)$ maps the graph of $y = f(x)$ onto the graph of $y = x^3$.

Find the rule of f.

Let (x', y') be the image of any point (x, y) under T on the graph of $y = f(x)$.

We can substitute $x' = 2x + 3$ and $y' = \frac{1}{3}y - 4$ into the equation $y' = (x')^3$ to get:

$$\frac{1}{3}y - 4 = (2x+3)^3 \to y = f(x) = 3(2x+3)^3 + 12$$

Question 33

The following sequence of transformations maps the graph of f onto the graph of $y = \sqrt{x}$, for $x \in (2, \infty)$:

A dilation by a factor of 3 from the x-axis, followed by,

A translation of 2 units left and 4 units up, followed by,

A reflection in both the x-axis and the y-axis.

State the rule and domain of f.

From the transformation, we can see that:
$(x, y) \to (x, 3y) \to (x - 2, 3y + 4) \to (2 - x, -3y - 4)$
Let (x', y') be the image of any point (x, y) under T on the graph of $y = f(x)$.
Therefore $y' = \sqrt{x'}$, and as such substituting the transformed values of y and x (y' and x'), we get:

$$-3y - 4 = \sqrt{2 - x} \to y = f(x) = -\frac{\sqrt{2 - x}}{3} - \frac{4}{3}$$

Now to get the domain of f, we simply use the equation
$x' > 2 \to 2 - x > 2 \to x < 0$ therefore, the domain of f is $(-\infty, 0)$.

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Sub-Section [2.5.3]: Apply Transformations of Functions to Find its Domain, Range, Transformed Points

Question 34

The function $f: \mathbb{R} \to \mathbb{R}$ has a range of $[2, \infty]$.

The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (5 - 2x, 3 + y)$ maps the graph of f onto the graph of g. State the domain and range of g.

Apply T to both the domain and range.
Since, x is any real number $5 - 2x$ can be any real number
Since, $y \geq 2$, as a result $y + 3 \geq 5$.
Therefore, $dom(g) = \mathbb{R}$ and $ran(g) = [5, \infty]$.

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Question 35

The function $f : (-\infty, -1) \to \mathbb{R}$ has a range of $(-2, \infty)$.

Describe a sequence of transformations that maps the graph of f onto a graph of a function with a domain of $[0, \infty]$ and a range of $(-\infty, 2)$.

Both functions swap ∞ signs therefore a reflection in both axes are required;

- Reflect in the x-axis.
- Reflect in the y-axis.

After applying the above the domain is now $[1, \infty]$ and the range is now $(-\infty, 2)$. Therefore;

- Translate 1 unit left (to fix domain).

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Question 36

Consider the function, $f : [-2, \infty] \to R, f(x) = 3\sqrt{x+2} - 5$.

The following sequence of transformations maps the graph of f onto the graph of g:

- A reflection in the x-axis, followed by,
- A dilation by a factor of 3 from the x-axis, followed by,
- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 3 units up and 2 units left.

State the domain and range of g.

The domain of f is $[-2, \infty]$ and the range is $[-5, \infty]$.
Under the above transformations,

$$(x, y) \to (x, -y) \to \left(\frac{1}{2}x, -3y\right) \to \left(\frac{1}{2}x - 2, 3 - 3y\right)$$

Now apply those transformations to the domain and range:

$$\text{dom}(f) = [-2, \infty] \therefore \text{dom}(g) = \left[\frac{1}{2}(-2) - 2, \infty\right) = [-3, \infty)$$

$$\text{ran}(f) = [-5, \infty] \therefore \text{ran}(g) = (-\infty, 3 - 3(-5)] = (-\infty, 18]$$

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Sub-Section [2.5.4]: Find Transformations of Inverse Functions

Question 37

Consider the function, $f: \mathbb{R}\{1\} \to \mathbb{R}, f(x) = \frac{2}{x-1} + 4$. The transformation, $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x, y) = (x + a, y + b)$ maps the graph of f onto the graph of its inverse function. Find the values of a and b.

The horizontal asymptote of f is $y = 4$, whilst the horizontal asymptote of f^{-1} is $y = 1$.
Therefore, a translation of 3 units down is needed for the graph of $f, \therefore b = -3$.
The vertical asymptote of f is $x = 1$, whilst the vertical asymptote of f^{-1} is $x = 4$.
Therefore, a translation of 3 units right is needed for the graph of $f, \therefore a = 3$.

Question 38

Consider the one-to-one functions, $f(x)$ and $g(x)$. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2,\ T(x, y) = (3 - x, 2y + 7)$ maps the graph of f onto the graph of g.

Describe a sequence of transformations that maps the graph of f^{-1} onto the graph of g^{-1}.

Swap x and y in the equation of T to get a transformation $S: \mathbb{R}^2 \to \mathbb{R}^2, S(x, y) = (2x + 7, 3 - y)$ that maps the graph of f^{-1} onto the graph of g^{-1}. It is possible to read off a sequence of transformations from here using DRT;

- A dilation by a factor of 2 from the y-axis then,
- A reflection in the x-axis then,
- A translation of 7 units right and 3 units up.

Question 39

Let $f: [1, \infty] \to \mathbb{R}, f(x) = 3x^2 - 6x + 8$ and $g: [-3, \infty] \to \mathbb{R},\ g(x) = \sqrt{x\ +\ 3} + 4$.

Describe a sequence of transformations that maps the graph of f onto the graph of g^{-1}.

First find the rule for g^{-1} by solving $g(y) = x$ for y;
$\sqrt{y+3} + 4 = x \to x - 4 = \sqrt{y+3} \to y = (x-4)^2 - 3$

The domain of g^{-1} is the range of g which is $[4, \infty)$.
Similarly the range of g^{-1} is $[-3, \infty]$.

By completing the square of $f(x)$, we get $f(x) = 3(x-1)^2 + 5$
Now we can transform:

1. Translate 5 units down $f_1(x) = 3(x-1)^2$.
2. Dilate by a factor of $\frac{1}{3}$ from the x-axis $f_2(x) = (x-1)^2$.
3. Translate 3 units to the right $f_3(x) = (x-4)^2$.
4. Translate 3 units down $g^{-1}(x) = (x-4)^2 - 3$.

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Sub-Section [2.5.5]: Find Multiple Transformations for the Same Functions

Question 40

Describe a sequence of transformations that map the graph of $f(x) = 4(x-3)^2 + 5$ to $g(x) = x^2$ without using a dilation from the x-axis.

1. Translate 5 units down and 3 units left.
2. Dilate by a factor of 2 from the y-axis.

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Question 41

Consider the functions $f(x) = x^2 - 8x + 10$ and $g(x) = 4(x+2)^2 - 5$. Find 2 different sets of transformations, one using a dilation from the x-axis and one using a dilation from the y-axis to map the graph of $f(x)$ to the graph of $g(x)$.

Write $f(x)$ in T.P form: $f(x) = (x-4)^2 - 6$.

Set 1:

1. Dilate by a factor of 4 from x-axis.
2. Translate 19 units up.
3. Translate 6 units left.

Set 2:

1. Translate 4 units right $f_1(x) = x^2 - 6$.
2. Dilate by a factor of $\frac{1}{2}$ from y-axis $f_2(x) = (2x)^2 - 6$.
3. Translate 2 units left $f_3(x) = 4(x+2)^2 - 6$.
4. Translate 1 unit up $g(x) = 4(x+2)^2 - 5$.

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Question 42

Consider the functions $f(x) = x^2 + 6x + 7$ and $g(x) = 16x^2 - 32x + 6$. Find 2 different sequences of 3 transformations, one using a dilation from the x-axis and one using a dilation from the y-axis to map the graph of $f(x)$ to the graph of $g(x)$.

Convert both functions to TP form:
$f(x) = (x+3)^2 - 2$, $g(x) = 16(x-1)^2 - 26$

Set 1:
1. Dilate by a factor of 16 from the x-axis.
2. Translate 6 units up.
3. Translate 4 units right.

Set 2:
1. Translate 7 units right $f_1(x) = (x-4)^2 - 2$.
2. Dilate by a factor of $\frac{1}{4}$ from the y-axis.
   $f_2(x) = (4x-4)^2 - 2 = (4)^2(x-1)^2 - 2 = 16(x-1)^2 - 2$
3. Translate 24 units down.

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Sub-Section: Exam 1 Questions

Question 43

Consider the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = \left(\frac{1}{2}x - 3, 4y + 2\right)$.

a. Find the image of the point $(4, 1)$ under T.

$$P' = \left(\frac{1}{2}(4) - 3, 4(1) + 2\right) = (-1, 6)$$

b. Write out what the transformation T does in the order DRT.

1. Dilation by a factor of $\frac{1}{2}$ from the y-axis.
2. Dilation by a factor of 4 from the x-axis.
3. Translation 3 units to the left.
4. Translation 2 units up.

c. Find the image of the curve $y = x^3$ under the transformation T. Give your answer in the form $y = a(x + b)^3 + c$.

$$x' = \frac{1}{2}x - 3 \to x = 2(x' + 3).$$

Therefore, image is $y = 4\left(2(x + 3)\right)^3 + 2 = 32(x + 3)^3 + 2$.

Question 44

Consider the function $f: \mathbb{R} \to \mathbb{R},\ f(x) = 4x^2 - 16$.

a. Find the coordinates of all axes intercepts of f.

x-intercepts: $(2, 0)$ and $(-2, 0)$
y-intercept: $(0, -16)$

b. Let the graph of g be a transformation of the graph of f where the transformations have been applied in the following order:

1. Dilation by a factor of 2 from the y-axis.

2. Dilation by a factor of 3 from the x-axis.

3. Translation 6 units to the right.

Find the rule for $g(x)$.

$$g(x) = 3f\left(\frac{1}{2}(x-6)\right) = 3\left(4\left(\frac{1}{2}(x-6)\right)^2 - 16\right) = 3(x-6)^2 - 48$$

c. State the coordinates of the axes intercepts of g.

x-intercepts: $(2, 0)$ and $(10, 0)$
y-intercept: $(0, 60)$

Question 45

Consider the function $f(x) = 4\sqrt{3x+7} + 2$.

Apply the following transformations to $f(x)$:

1. Dilation by a factor of $\frac{1}{2}$ from the x-axis.

2. Translated 3 units in the positive direction of the y-axis.

3. Reflection in the x-axis.

4. Translated 2 units in the negative direction of the x-axis.

5. Dilated by a factor of 2 from the y-axis.

$$f(x) \to 2\sqrt{3x+7} + 1 \to 2\sqrt{3x+7} + 4 \to -2\sqrt{3x+7} - 4$$

$$\to -2\sqrt{3(x+2)+7} - 4 \to -2\sqrt{3\left(\frac{1}{2}x+2\right)+7} - 4$$

Therefore, the image is $y = -2\sqrt{3\left(\frac{1}{2}x+2\right)+7} - 4 = -2\sqrt{\frac{3}{2}x + 13} - 4$

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Sub-Section: Exam 2 Questions

Question 46

The graph of the function f passes through the point $(2, -3)$.

If $h(x) = 3f(x - 2)$, then the graph of the function h must pass through the point:

A. $(4, -9)$

B. $(0, -9)$

C. $(4, -1)$

D. $(0, -1)$

Question 47

The graph of the function $f: \mathbb{R} \to \mathbb{R}, f(x) = 3^x - 1$, is reflected in the y-axis and then translated 2 units to the left and then 3 units up.

Which one of the following is the rule of the transformed graph?

A. $y = \left(\frac{1}{3}\right)^{x+2} + 2$

B. $y = \frac{1}{3} \times 3^{x+2} + 3$

C. $y = 3^{-x} + 3$

D. $y = 3^{-x+2} + 3$

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Question 48

The graph of the function g is obtained from the transformed graph of the function:

$$f: [-2,6] \to \mathbb{R}, f(x) = 3x^2 + 5x - 2$$

which undergoes a dilation of factor 2 from the y-axis, followed by a dilation of factor $\frac{1}{4}$ from the x-axis, followed by a reflection in the x-axis, and finally followed by a translation of 6 units in the positive direction of the y-axis. The domain and range of g are respectively:

A. $[-4, 12]$ and $[-12, 4]$

B. $[-4, 12]$ and $\left[-28, \frac{337}{48}\right]$

C. $[-12, 4]$ and $\left[-\frac{239}{48}, 40\right]$

D. $[-4, 12]$ and $\left[-40, \frac{239}{48}\right]$

Question 49

The image of the function $f(x) = x^4$ is $y = -40(x + 2)^4$. The transformations that could have been applied are:

A. Reflection in the x-axis, then translation in the positive direction of the x-axis by 2 units, followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

B. Reflection in the x-axis, then translation in the negative direction of the x-axis by 2 units, followed by a dilation from the x-axis by a factor of 5 and a dilation by factor 2 from the y-axis.

C. Reflection in the x-axis, then a dilation from the x-axis by a factor of 2, followed by a translation in the positive direction of the x-axis by 2 units, and finally a dilation from the y-axis by a factor of 2.

D. Reflection in the x-axis, then a dilation from the y-axis by a factor of $\frac{1}{2}$, followed by a translation in the negative direction of the x-axis by 2 units, and finally a dilation from the x-axis by a factor of $\frac{5}{2}$.

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Question 50

Consider the function $f: (-3,1) \to \mathbb{R}, f(x) = (x+3)(x+2)(3x-3)$.

a. State the range of f, correct to 3 decimal places.

$\text{ran}(f) = (-18.194, 2.638)$

b. The following sequence of transformations, T, map the graph of f onto the graph of g.

- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 2 units up and 1 unit left, followed by,
- A reflection in the x-axis.

i. State the rule of g.

$g(x) = -\left(f\left(2(x+1)\right) + 2\right) = -62 - 174x - 120x^2 - 24x^3$

ii. State the domain of g.

Apply the transformation $x \to \frac{1}{2}x - 1$ onto the interval $(-3, 1)$ to get the domain of g.
Thus, the domain of g is $\left(-\frac{5}{2}, -\frac{1}{2}\right)$.

iii. State the range of g correct to 3 decimal places.

Apply the transformation $y \to -(y+2)$ onto the interval $(-18.194, 2.638)$ to get the domain of g.
Thus the range of g is $(-4.638, 16.194)$

iv. Find the image of the point $(1, 0)$ under T.

$$\boxed{\left(-\frac{1}{2}, -2\right)}$$

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