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VCE Mathematical Methods ½
Transformations [2.4]
Workbook

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Outline:

Learning Objectives:

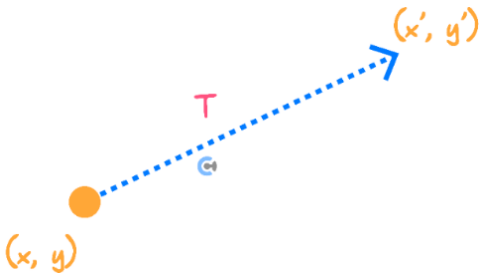
- ☐ **MM12 [2.4.1]** - Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations
- ☐ **MM12 [2.4.2]** - Find Transformed Functions
- ☐ **MM12 [2.4.3]** - Find Transformations From Transformed Function (Reverse Engineering)

Section A: Introduction to Transformations

Sub-Section: Image and Pre-Image

What do we call an original coordinate and a transformed coordinate?

Image and Pre-Image



- The original coordinate is called the pre-image.
- The transformed coordinate is called the image.

Pre-Image: (x, y)

Image: (x', y')

Question 1

It is known that $(1,4)$ transformed into $(3,5)$. State the value of x' and y'.

$x' = 3$

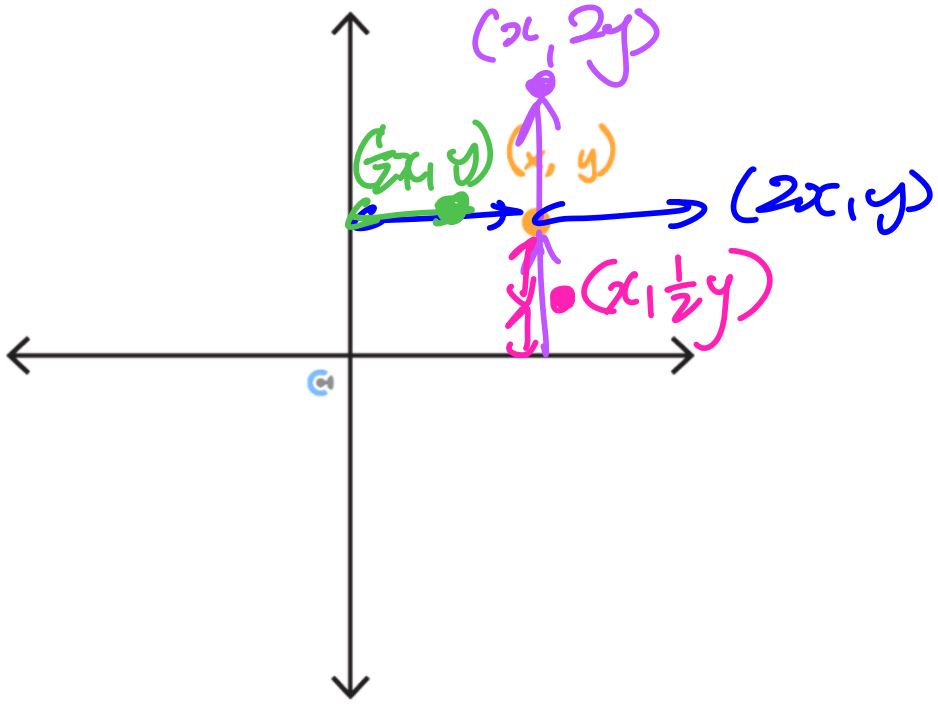
$y' = 5$

NOTE: The x' and y' notation will be used quite heavily!

Sub-Section: Dilation

Exploration: Dilation → stretch

- Consider the point below:



- Let's plot the coordinates:
  - $P1$: Dilation by a factor 2 from the x-axis.
  - $P2$: Dilation by a factor $\frac{1}{2}$ from the x-axis.
  - $P3$: Dilation by a factor 2 from the y-axis.
  - $P4$: Dilation by a factor $\frac{1}{2}$ from the y-axis.

Dilation

Dilation by a factor a from the x-axis: $y' = ay$

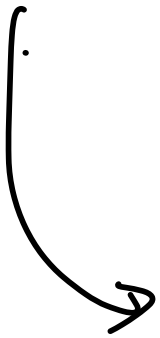
Dilation by a factor b from the y-axis: $x' = bx$

Question 2 Walkthrough.

Find the image (x', y') after applying the following transformations to (x, y).

Dilation by factor 2 from the x-axis. $y' = 2y$

Dilation by factor $\frac{1}{3}$ from the y-axis. $x' = \frac{1}{3}x$



$(\frac{1}{3}x, 2y)$

Question 3

Find the image (x', y') after applying the following transformations to (x, y).

Dilation by factor $\frac{1}{2}$ from the x-axis. $y' = \frac{1}{2}y$

Dilation by factor 4 from the y-axis. $x' = 4x$

$(4x, \frac{1}{2}y)$

NOTE: We are applying the transformations on (x, y) not (x', y').

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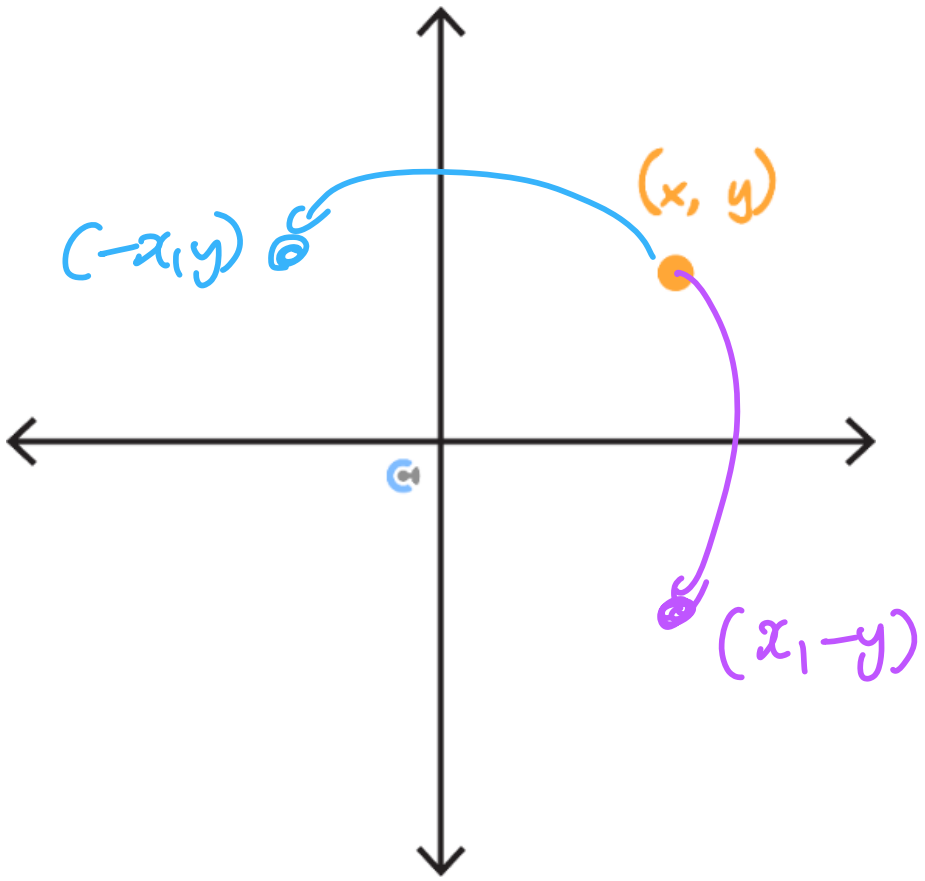
Sub-Section: Reflection

Exploration: Reflection

- Consider the point below:



- Let's plot the coordinates:
  - $P1$: Reflection in the x-axis.
  - $P2$: Reflection in the y-axis.

Reflection

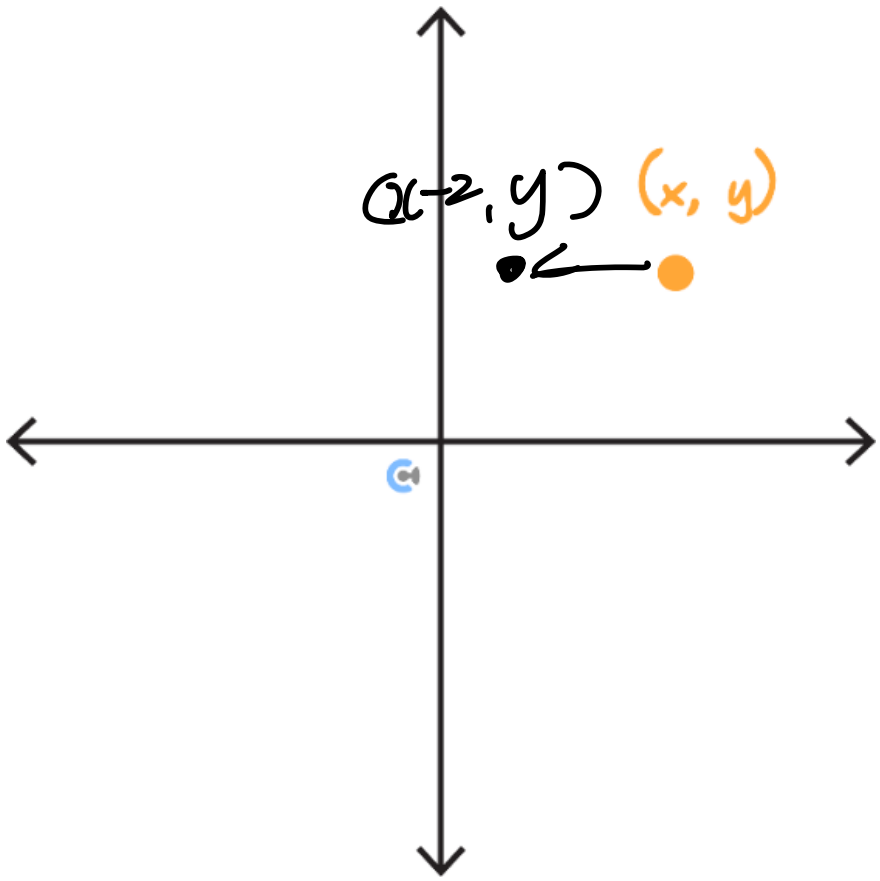
Reflection in the x-axis: $y' = -y$

Reflection in the y-axis: $x' = -x$

Sub-Section: Translation

Exploration: Translation → move

- Consider the point below:



- Let's plot the coordinates (ignore the scale):
  - $P1$: Translation by 2 units in the negative direction of the x-axis. → left
  - $P2$: Translation by 3 units in the negative direction of the y-axis. → down

Translation

Translation by c units in the positive direction of the x-axis: $x' = x + c$

Translation by d units in the positive direction of the y-axis: $y' = y + d$

Question 4

Find the image (x', y') after applying the following transformations to (x, y).

Translation by 3 units in the positive direction of the x-axis. $x' = x + 3$

Translation by 2 units in the negative direction of the y-axis. $y' = y - 2$

$(x+3, y-2)$

Key Takeaways

- ☑ The transformed point is called the image and is denoted by (x', y').
- ☑ The dilation factor is multiplied by the original coordinates.
- ☑ Reflection makes the original coordinates the negative of their original values.
- ☑ Translation adds a unit to the original coordinates.

Section B: Transformation of Points

DRT

Sub-Section: Basic Transformation of Points

Let's try to apply all types of transformations to a point!

Question 5 Walkthrough.

Find the image (x', y') after applying the following transformations to (x, y).

y x

Dilation by a factor 2 from the x-axis. $y' = 2y$

Dilation by a factor 4 from the y-axis. $x' = 4x$

Reflection in the x-axis. $y' = -2y$

Translation by 2 units in the negative direction of the x-axis. $x' = 4x - 2$

Translation by 3 units in the positive direction of the y-axis. $y' = -2y + 3$

$(4x - 2, -2y + 3)$

Question 6

Find the image (x', y') after applying the following transformations to (x, y).

Translation by 4 units in the positive direction of the x-axis. $x' = x + 4$

Translation by 3 units in the negative direction of the y-axis. $y' = y - 3$

Dilation by a factor of $\frac{1}{5}$ from the x-axis. $y' = \frac{1}{5}(y - 3)$

Dilation by a factor of 2 from the y-axis. $x' = 2(x + 4)$

Reflection in the x-axis. $y' = -\frac{1}{5}(y - 3)$

$$\left(2(x+4),\ -\frac{1}{5}(y-3)\right)$$

NOTE: Order Matters.

Question 7 Extension.

Find the image (x', y') after applying the following transformations to (x, y).

Translation by a units in the negative direction of the x-axis.

Translation by b units in the positive direction of the y-axis.

Dilation by a factor c from the x-axis.

Dilation by a factor $\frac{3}{d}$ from the y-axis.

Reflection in the x-axis.

$$(x', y') = \left(\frac{3}{d}(x - a), -c(y + b)\right)$$

Space for Personal Notes

Sub-Section: The Order of Transformations

Discussion: From the previous question, what happens when the translation is applied first?

* DRT → Expanded

TPD → Factorised

What is the order of transformations the same as?

The Order of Transformation

DRT → expanded

Order = BODMAS Order

DRT

Question 8 Walkthrough.

Consider the point (x, y) which was transformed into a point $(3x + 6, y)$ by the transformation T.

Isha

~~Jennifer~~ thinks the transformation was:

"Translation 6 units in the positive direction of the x-axis and dilation by a factor of 3 from the y-axis."

Meanwhile, David thinks the transformation was:

"Dilation by a factor of 3 from the y-axis and translation 6 units in the positive direction of the x-axis."

Who is correct? And why?

$x' = 3x$ $\quad$ $x' = 3x + 6$

David!

Question 9

Consider the point (x, y) was transformed into a point $(2(x-5), y)$ by the transformation T.

Mary thinks the transformation was:

"Translation 5 units in the negative direction of the x-axis and dilation by a factor of 2 from the y-axis."

Meanwhile, Sam thinks the transformation was:

"Dilation by a factor of 2 from the y-axis and translation 5 units in the negative direction of the x-axis."

Who is correct? And why?

Question 10 Extension.

Consider the point (x, y) was transformed into a point $(2ax + 6a, y)$ by the transformation T.

Jennifer thinks the transformation was:

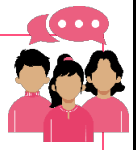
"A translation by 3 units in the positive direction of the x-axis, followed by a dilation by a factor $2a$ from the y-axis."

Meanwhile, David thinks the transformation was:

"A dilation by a factor $2a$ from the y-axis, followed by a translation by $3a$ units in the positive direction of the x-axis."

Who is correct? And why?

Jennifer is correct.
$2a(x + 3) = 2ax + 6a$



Discussion: If the order is the same as the **BODMAS** order, how do we change the order of transformations?

DRT factorise or expand TRD

Question 11 Walkthrough.

DRT $y' = \frac{1}{2}y + 3$

The series of transformations, "a dilation by a factor $\frac{1}{2}$ from the x-axis and a translation by 3 units up" yields the same result as the series of transformations, "a translation by a units up and a dilation by a factor b from the x-axis." Find the values of a and b.

TRD $y' = b(y+a)$

$$\frac{1}{2}y + 3 = b(y+a)$$

$$\frac{1}{2}y + 3 = by + ab$$

$b = \frac{1}{2}$

$ab = 3$

$\frac{a}{2} = 3$

$a = 6$

$a = 6,\ b = \frac{1}{2}$

Question 12

$x' = -4x - 8$

The series of transformations, "a dilation by a factor 4 from the y-axis, a reflection in the y-axis and a translation by 8 units left" yields the same result as the series of transformations, "a translation by c units right, a reflection in the y-axis and a dilation by a factor d from the y-axis." Find the values of c and d.

$x' = -d(x+c)$

$-4x-8 = -d(x+c)$

$-4(x+2) = -d(x+c)$

$d = 4$

$c = 2$

Question 13 Extension.

The series of transformations, "a dilation by a factor 2 from the y-axis, a reflection in the y-axis, a dilation by a factor 2 from the x-axis, a translation by 4 units left and a translation by 6 units down", yields the same result as the series of transformations, "a translation by c units right, a reflection in the y-axis, a dilation by a factor d from the y-axis, a translation k units down, and a dilation by a factor m from the x-axis." Find the values of c, d, k and m.

$$(-2x' - 4, 2y' - 6) = (-2(x' + 2), 2(y' - 3))$$

Therefore, $c = 2,\ d = 2,\ k = 3,\ m = 2$

NOTE: Dilation factors don't change!

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Sub-Section: Interpreting the Transformation of Points

Active Recall: Order of Transformation

DRT (NO) Order = BODMAS Order

Question 14 Walkthrough.

Consider the transformation which maps:

$$x' = 2x + 4 \quad = 2(x+2)$$

↳ expanded $$y' = -3(y - 1) \quad = -3y + 3$$

a. State the transformation in DRT (Dilation, Reflection, Translation) order.

x. Dilation by factor of 2 from the y-axis
y. Dilation by factor of 3 from the x-axis
y. Reflection in x-axis
x. Translate 4 right
y. Translate 3 up

b. State the transformation in the translation first order. TRD → factorised

x. T 2 right
y. T 1 down
y. R in x
x. D 2 from y
y. D 3 from x

NOTE: Expanding or factorising changes the order of transformation.

Question 15

Consider the transformation which maps:

$$x' = 3x + 6 \quad = 3(x+2)$$
$$y' = -2(y + 2) \quad = -2y - 4$$

a. State the transformation in DRT (Dilation, Reflection, Translation) order.

x D factor 3 from y
y D factor 2 from x
y R in x
x T 6 right
y T 4 down

b. State the transformation in the translation first order.

x T 2 right
y T 2 up
y R in x
x D factor 3 from y-axis
y D factor 2 from x

Space for Personal Notes

Discussion: Could the order of x and y transformations change?

Yes → x & y independent

Key Takeaways

- ☑ Transformations should be interpreted when x' and y' are isolated.
- ☑ The order of transformation follows the BODMAS order.
- ☑ To change the order of transformations, we either factorise or expand.

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Section C: Transformation of Functions

Sub-Section: Applying Transformations to Functions

Let's now work with Functions!

Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:

1. Transform the points.
2. Make x and y the subjects.
3. Substitute them into the function.

Question 16 Walkthrough.

Apply the transformations given below to $y = x^2$.

Reflect in the y-axis. $x' = -x$

Translate 1 unit to the right. $x' = -x + 1$

Dilate by a factor of 2 from the y-axis. $x' = 2(-x+1)$

$y' = y$

② $\frac{x'}{2} = -x + 1$

$x = -\frac{x'}{2} + 1$

③ $y = x^2$

$y' = \left(-\frac{1}{2}x' + 1\right)^2$

Image: $y = \left(-\frac{1}{2}x + 1\right)^2$

Your turn!

Active Recall: Transformation of Functions

- The aim is to get rid of the old variables, x and y, and have the new variables, x' and y', instead.

$$y = f(x) \rightarrow y' = f(x')$$

- Steps:

1. Transform the point.
2. Make x and y the Subject.
3. Sub them into the function.

Space for Personal Notes

Question 17

Apply the following transformations to the functions given:

a. $f(x) = x^2$

Step 1 transform point

Dilation by factor 3 from the x-axis. $y' = 3y$

Reflect in the y-axis. $x' = -x$

Translate 3 units to the left. $x' = -x - 3$

Dilate by a factor of 5 from the y-axis. $x' = 5(-x-3)$

Step 2 Rearrange for x, y

$y = \frac{1}{3}y'$

$\frac{x'}{5} = -x - 3$

$\Rightarrow x = -\frac{x'}{5} - 3$

Step 3 sub in

$\frac{1}{3}y' = \left(-\frac{x'}{5} - 3\right)^2$

$y = 3\left(-\frac{x}{5} - 3\right)^2$

$f(x) = 3\left(-\frac{x}{5} - 3\right)^2$

$= 3\left(-\left(\frac{x}{5} + 3\right)\right)^2$

$= 3\left(\frac{x}{5} + 3\right)^2$

b. $f(x) = \sqrt{x}$

Dilate by a factor of $\frac{1}{4}$ from the y-axis. $x' = \frac{1}{4}x$ ①

Dilate by a factor of 3 from the x-axis. $y' = 3y$

Translate 4 units to the left. $x' = \frac{1}{4}x - 4$

Translate 1 unit up. $y' = 3y + 1$

Reflect in the y-axis. $x' = -\left(\frac{1}{4}x - 4\right)$

② $y = \frac{y' - 1}{3}$

Sub $x = 4(-x' + 4)$

$\frac{y'-1}{3} = \sqrt{4(-x'+4)}$

$\frac{y'-1}{3} = 2\sqrt{-x'+4}$

$y' = 6\sqrt{-x'+4} + 1$

$f(x) = 6\sqrt{-x+4} + 1$

Question 18 Extension.

Apply the following transformations to $y = 2^x$.

Translation by 2 units to the right.

Reflection in the y-axis.

Dilation by a factor 3 from the y-axis.

Translation by 3 units up.

A dilation by a factor 2 from the x-axis.

A reflection in the x-axis.

$$y = -2^{-1-\frac{x}{3}} - 6$$

OR

$$y = -2\left(2^{-2-\frac{x}{3}} + 3\right)$$

Sub-Section: Finding the Applied Transformations

Now let's go backwards!

Reverse Engineering

- **Steps:**

1. Add the dashes (') back to the transformed function.
2. Make $f(\ )$ the subject.
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make x' and y' the subjects and interpret the transformations.

Question 19 ~~Walkthrough:~~

Find the transformations required for $y = x^2$ to be transformed to $y' = 3\left(\frac{x'+3}{2}\right)^2 + 5$.

$\frac{y'-5}{3} = \left(\frac{x'+3}{2}\right)^2$

$x = \frac{x'+3}{2}$ $y = \frac{y'-5}{3}$

$x' = 2x - 3$ $y' = 3y + 5$

x. D factor 2 from y

y. D factor 3 from x

x. T 3 left

y. T 5 up

Your turn!

Active Recall: Steps for reverse engineering

- Steps:

1. Add the dashes (') back to the _____ (Image (transformed function))
2. Make $f(\ )$ the _____ (subject / main operation ⇒ isolate x & y).
3. Equate the LHS of the original and transformed functions to the RHS of the original and transformed functions.
4. Make _____ (x', y') the subjects and interpret the transformations.

(↓ read)

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Question 20

State a series of transformations (in order) that allow $f(x)$ to be transformed into $g(x)$.

a. $f(x) = 2(x+1)^2 + 3$ and $g(x) = 6(x-4)^2 - 3$.

walk-through

$y = 2(x+1)^2 + 3$ $\quad y' = 6(x'-4)^2 - 3$

$\frac{y-3}{2} = (x+1)^2$ $\quad \frac{y'+3}{6} = (x'-4)^2$

$\frac{y-3}{2} = \frac{y'+3}{6}$ $\quad (x+1)^2 = (x'-4)^2$

Rearrange for x' y'

$y'+3 = 3(y-3)$ $\quad x+1 = x'-4$

$y' = 3y - 12$ $\quad x' = x + 5$

y D factor 3 from x

x T 5 right

y T 12 down

b. $f(x) = 3(x-1)^2$ and $g(x) = \frac{1}{2}(2x+3)^2 + 1$.

$\frac{y}{3} = (x-1)^2$ $\quad 2(y'-1) = (2x'+3)^2$

$\frac{y}{3} = 2(y'-1)$ $\quad x - 1 = 2x' + 3$

$\frac{y}{6} = y' - 1$ $\quad x' = \frac{x-4}{2}$

$y' = \frac{y}{6} + 1$ $\quad x' = \frac{1}{2}x - 2$

x D $\frac{1}{2}$ from y

y D $\frac{1}{6}$ from x

x T 2 left

y T 1 up

Question 21 Extension.

Find a sequence of transformations required for $y = 2(x-3)^2 + 4$ to be transformed to $y = -x^2 - 4x - 9$.

Complete the square for second function.

$y = -(x+2)^2 - 5$

Dilation by factor $\frac{1}{2}$ from the x-axis.

Reflection in the x-axis.

Translation 3 units down.

Translation 5 units to the left.

Key Takeaways

- ☑ We transform the coordinates first, then transform the function.
- ☑ To transform the function, replace its old variables with the new ones.
- ☑ To find the transformations, simply equate LHS with RHS after separating the transformations of x and y.

Contour Checklist

☐ Learning Objective: [2.4.1] – Applying x' and y' Notation to Find Transformed Points, Find the Interpretation of Transformations and Altered Order of Transformations

Key Takeaways

- ☐ The transformed point is called the image and is denoted by (x', y').
- ☐ The dilation factor is multiply to the original coordinate.
- ☐ Reflection makes the original coordinates the negative of their original values.
- ☐ Translation add a unit to the original coordinate.
- ☐ Transformations should be interpreted when x', y' are isolated.
- ☐ The order of transformation follows the NO BODMAS order.
- ☐ To change the order of transformations, we either factorise / expand.

☐ Learning Objective: [2.4.2] – Find Transformed Functions

Key Takeaways

- ☐ To transform the function, replace its x, y with the new one.

Learning Objective: [2.4.3] – Find Transformations From Transformed Function (Reverse Engineering)

Key Takeaways

- To find the transformations, simply equate the _____ after separating the transformations of x and y.

x-region
y-region

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