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VCE Mathematical Methods ½
Transformations [2.4]
Homework

Homework Outline:

Compulsory Questions	Pg 2 – Pg 19
Supplementary Questions	Pg 20 – Pg 42

Section A: Compulsory Questions

Sub-Section [2.4.1]: Applying x' and y' Notation to Find Transformed Points, Find Interpretation of Transformations and Altered Order of Transformations

Question 1

Consider the following transformations on the plane:

- S a dilation by a factor 2 from the x-axis.
- T a translation 3 units to the right and 2 units down.
- W a reflection in the x-axis.

Find the image, (x', y'), of the point (x, y) and the transformation:

a. S

b. T

c. S then T.

d. T then W then S.

Question 2

A transformation T is applied to points on the plane such that the image is given by $(x', y') = (2x + 4, -y + 2)$.

a. Describe T in words where dilations and reflections occur before translations.

b. Describe T in words where translations occur before reflections and dilations.

Question 3

The transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is described by the following sequence of transformations.

- A dilation by factor 2 from the x-axis.
- A dilation by factor 3 from the y-axis.
- A reflection in the x-axis.
- A translation 2 units left.
- A translation 6 units down.

a. Let (x', y') be the image of (x, y) under T. Find (x', y').

b. Describe in words, the transformations T, in the order of translations, reflections, and dilations.

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Sub-Section [2.4.2]: Find Transformed Functions

Question 4

a. Find the rule for the image of $f(x) = x^2$ under the transformations:

- A dilation by factor 2 from the x-axis.
- A translation 1 unit up.

b. Find the rule for the image of $f(x) = \sqrt{x}$ under the transformations:

- A dilation by factor 4 from the y-axis.
- A translation 1 unit down.

c. Find the rule for the image of $f(x) = \frac{1}{x}$ under the transformations:

- A dilation by factor 2 from the x-axis.
- A translation 1 unit up and 3 units to the left.

Question 5

a. Find the rule for the image of $f(x) = 2x^2 + 4$ under the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2,\ T(x, y) = (2x + 1, -y + 2)$.

b. Find the rule for the image of $f(x) = \frac{3}{x-3}$ under the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2,\ T(x, y) = (x + 1, -2y + 2)$.

c. Find the rule for the image of $f(x) = \sqrt{2x - 4} + 3$ under the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2, T(x, y) = (-2x + 1, 2y + 3)$.

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Question 6

a. Find the rule for the image of $f(x) = 2(x-1)^2 + 3$ under the transformations:

- A dilation by factor 2 from the x-axis.
- A translation 3 units to the left.
- A translation 1 unit up.
- A reflection in the x-axis.

b. Find the rule for the image of $f(x) = \frac{1}{x-1}$ under the transformations:

- A dilation by factor 4 from the x-axis.
- A dilation by factor $\frac{1}{2}$ from the y-axis.
- A reflection in the x-axis.
- A translation 2 units right.

c. Find the rule for the image of $f(x) = \sqrt{2x + 6} - 4$ under the transformations:

- A translation 1 unit up.
- A translation 4 units to the right.
- A reflection in the y-axis.
- A dilation by factor 2 from the x-axis.

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Sub-Section [2.4.3]: Find Transformations from Transformed Function

Question 7

a. Let $f(x) = x^2$ and $g(x) = 4x^2 + 1$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

b. Let $f(x) = \sqrt{x}$ and $g(x) = 2\sqrt{x+1} - 3$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

c. Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{3}{x+2}$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

Question 8

a. Let $f(x) = x^2$ and $g(x) = 4(x-2)^2 + 3$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

b. Let $f(x) = \sqrt{2x}$ and $g(x) = 2\sqrt{4x-2}$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

c. Let $f(x) = \frac{6}{x-1}$ and $g(x) = \frac{3}{x+2} + 1$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

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Question 9

a. Let $f(x) = 2\sqrt{x+1}$ and $g(x) = 5\sqrt{5-3x} + 4$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

b. Let $f(x) = 2(x-3)^2 + 2$ and $g(x) = x^2 + 4x + 7$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

c. Let $f(x) = \frac{3}{x^2} + 1$ and $g(x) = -\frac{6}{(2x-3)^2} + 4$.

Describe a sequence of transformations that maps the graph of f onto the graph of g.

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Sub-Section: The 'Final Boss'

Question 10

Consider the function $f(x) = x^2 - 4x + 7$ and the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2,\ T(x, y) = (3x - 6, -2y + 2)$.

a. Use words to describe the transformation T with:

i. Dilations and reflections before translations.

ii. Translations before reflections and dilations.

b. Write $f(x)$ in turning point form.

c. Find the rule for the image of $f(x)$ under T.

d. Determine a sequence of transformations that map $f(x)$ to $g(x) = 2x^2 - 16x + 28$.

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Section B: Supplementary Questions

Sub-Section [2.4.1]: Applying x' and y' Notation to Find Transformed Points, Find Interpretation of Transformations and Altered Order of Transformation

Question 11

Find the coordinates of the image point for the following:

a. The point $(2, 3)$ undergoes a dilation by a factor of 6 from the y-axis, a reflection in the x-axis, followed by a translation 1 unit up.

b. The point $(1,5)$ undergoes a translation 2 units left, a dilation by a factor of $\frac{1}{4}$ from the y-axis, a translation 3 units up, followed by a reflection in the x-axis.

c. The point $(-4,2)$ is dilated by a factor of 3 from the x-axis, translated 1 unit right, reflected in the x-axis, reflected in the y-axis, dilated by a factor of 2 from the y-axis, and then translated 5 units down.

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Question 12

Consider the sequence of transformations:

- A dilation by a factor of $\frac{1}{2}$ from the y-axis.
- A reflection in the x-axis.
- A dilation by a factor of 6 from the x-axis.
- A translation 4 units down.
- A translation 1 unit right.
- A translation 9 units up.

a. Rewrite the transformations in the order of a dilation, a translation, a dilation, a reflection, and then a translation.

b. Express the transformations as a sequence of two translations, followed by two dilations and a reflection.

c. Express the transformations in the order of a dilation, a translation, a dilation, a translation, and then a reflection.

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Question 13

The transformation T is defined as $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x, y) = (5 - 2x, 6y + 1)$.

a. Evaluate $T(-3,8)$.

b. Find the pre-image of $(7, -35)$ under the transformation T.

c. Express T as a sequence of two translations, two dilations, and a reflection.

d. Identify a sequence of transformations that maps the point $(-3,8)$ to the image of $(-3,8)$ under T and also maps the point $(1,-2)$ to the point $(23,-1)$.

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Question 14

a. Consider the transformation T described by:

- A translation 2 units left.
- A dilation by a factor of 3 from the x-axis.
- A dilation by a factor of $\frac{1}{4}$ from the y-axis.
- A reflection in the x-axis.
- A translation 1 unit up.
- A reflection in the line $y = x$.
- A translation 4 units right.

i. Apply T to the point $(5,2)$.

ii. Express T as a sequence of 2 dilations followed by 2 reflections, and then 2 translations.

b. Consider the transformation S described by:

- A dilation by a factor of 2 from the x-axis.
- A reflection in the y-axis.
- A dilation by a factor of $\frac{1}{3}$ from the y-axis.
- A reflection in the line $y = 4$.
- A translation 5 units down.
- A translation 1 unit right.

i. S can also be defined $S: \mathbb{R}^2 \to \mathbb{R}^2, S(x, y) = (ax + b, cy + d)$. Find the values of a, b, c, and d.

ii. Hence, evaluate $S(-2,4)$.

c. A point (x, y) undergoes the transformations T followed by S. Find the image point.

d. Given that the image point from **part c.**, is $(-4, 6)$, find the pre-image.

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Sub-Section [2.4.2]: Find Transformed Functions

Question 15

Find the resultant function when:

a. $y = x^2$ is dilated by a factor of 2 from the y-axis, reflected in the x-axis, translated 3 units up, and translated 1 unit left.

b. $y = \frac{1}{x}$ is reflected in the y-axis, translated 3 units up, dilated by a factor of 2 from the x-axis, dilated by a factor of $\frac{1}{4}$ from the y-axis, and translated 2 units right.

c. $y = \sqrt{x}$ is translated 3 units down, translated 5 units right, reflected in the y-axis, dilated by a factor of 3 from the x-axis, dilated by a factor of 2 from the y-axis, and reflected in the x-axis.

Question 16

Find the resultant function when:

a. $y = -2(x + 5)^2 + 1$ is dilated by a factor of $\frac{1}{3}$ from the x-axis, translated 4 units right, translated 1 unit down, reflected in the y-axis, and dilated by a factor of 2 from the y-axis.

b. $y = \frac{2}{(5-x)^2} + 7$ is reflected in the x-axis, translated 2 units up, dilated by a factor of 3 from the y-axis, reflected in the y-axis, translated 4 units right, and dilated by a factor of $\frac{1}{4}$ from the x-axis.

c. $y = 4 - 2(x+1)^3$ is translated 4 units right, dilated by a factor of 3 from the x-axis, reflected in the y-axis, translated 5 units up, reflected in the x-axis, and dilated by a factor of 2 from the y-axis.

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Question 17

Find the resultant function when:

a. $(x-2)^2+(y+5)^2=9$ is dilated by a factor of 3 from the y-axis, reflected in the x-axis, translated 4 units up, translated 1 unit left, and dilated by a factor of 3 from the x-axis.

b. $y=2x^2+3x-6$ is reflected in the y-axis, dilated by a factor of 4 from the x-axis, translated 5 units down, translated 1 unit right, dilated by a factor of $\frac{1}{2}$ from the y-axis.

c. $x = -\sqrt{-y^2 + 6y + 15} + 4$ is translated 2 units down, dilated by a factor of 3 from the x-axis, reflected in the y-axis, translated 5 units left, dilated by a factor of $\frac{1}{4}$ from the y-axis, translated 5 units up, reflected in the y-axis, and dilated by a factor of 2 from the x-axis.

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Question 18

a. When the graph $y = 6 - 2(x + 1)^2$ undergoes the transformation T, described as:

- A translation 4 units right.
- A dilation by a factor of 2 from the y-axis.
- A translation 4 units down.
- A reflection in the y-axis.
- A dilation by a factor of 3 from the x-axis.

It is mapped onto an equation $y = a(x - h)^2 + k$, where $a, h, k \in \mathbb{R}$.

Find the values of a, h, and k.

b. The graph $y = 6 - 2(x + 1)^2$ can also be mapped to the same equation from **part a.**, by a sequence of 2 dilations, a reflection, and 2 translations. Describe this sequence of transformations.

c. Find the pre-image, that when undergoes the transformation T, results in the equation $y = 6 - 2(x + 1)^2$.

d. The graph of $y = 6 - 2(x + 1)^2$ undergoes the transformation T, followed by a dilation by a factor of 2 from the x-axis, a reflection in the line $x = 6$, a reflection in the line $y = x$, and a translation 1 unit up. Find the image equation.

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Sub-Section [2.4.3]: Find Transformations from Transformed Function (Reverse Engineering)

Question 19

Find the sequence of transformations that map:

a. $y = x^2$ to $y = -3(x+1)^2 + 7$.

b. $y = \frac{1}{x}$ to $y = \frac{3}{5-2x} + 6$.

c. $y = \sqrt{x}$ to $y = 1 - \frac{\sqrt{4-3x}}{2}$.

Question 20

Find the sequence of transformations that map:

a. $y = 4(x+8)^3 - 5$ to $y = 5 - 2(6x-1)^3$.

b. $y = 3\sqrt{16-(x+1)^2} + 5$ to $y = 1 - 2\sqrt{16-(3x+5)^2}$.

c. $y = \frac{3}{(4-2x)^2} + 7$ to $y = -\frac{6}{(x+1)^2} + 5$.

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Question 21

a. The function $y = -2(3(x-1))^4 + 5$ undergoes a sequence of 2 transformations, a reflection, and 2 dilations to become the graph $y = 6(2-x)^4 - 1$.

b. The transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (ax+b, y+c)$ maps the equation $y = 11 + 5(x+3)^2$ onto the equation $y = 20(x-6)^2 + 9$. Find the values of a, b, and c.

c. The graph $y = \frac{\sqrt{6x-4}}{3} + 2$ is mapped onto $y = 5 - 2\sqrt{-1-x}$ by a sequence of 2 dilations and 2 reflections, followed by a translation.

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Question 22

Find the sequence of transformations that map:

a. $y = x^2 - 4x + 6$ onto $y = -2x^2 + 10x - 7$.

b. $y = 2\sqrt{(x+4)^2 + 1} - 5$ onto $y = 3 - \sqrt{(2x-6)^2 + 9}$.

c. $f : [-4, \infty) \to \mathbb{R}, f(x) = -x^2 - 8x + 9$ onto $g : (-\infty, 5] \to \mathbb{R}, g(x) = 2x^2 - 20x + 13$.

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