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VCE Mathematical Methods ½
Functions & Relations Exam Skills [2.3]
Workbook

Outline:

Learning Objectives:

- ☐ MM12 [2.3.1] - Restrict domain such that the inverse function exists
- ☐ MM12 [2.3.2] - Figure out possible rule of a graph
- ☐ MM12 [2.3.3] - Solve number of solution problems graphically

Section A: Recap

If you were here last week skip to Section B - Warmup test.

Set Operators

- Intersection: "AND".

$$A \cap B = \textit{What values are in set } A \textit{ AND in set } B.$$

- Union: "OR".

$$A \cup B = \textit{What values are in set } A \textit{ OR in set } B.$$

- Set difference: "Except".

$$A \setminus B = \textit{What values are in set } A \textit{ except those also in set } B.$$

Space for Personal Notes

Question 1

For the sets given below, find:

$$A = \{0, 2, 3, 5, 6, 11\} \text{ and } B = \{0, 1, 2, 3, 5, 7, 9, 10\}$$

a. $A \cap B =$

$$\boxed{\{0, 2, 3, 5\}}$$

b. $A \cup B =$

$$\boxed{\{0, 1, 2, 3, 5, 6, 7, 9, 10, 11\}}$$

c. $A \backslash B =$

$$\boxed{\{6, 11\}}$$

d. $B \backslash A =$

$$\boxed{\{1, 7, 9, 10\}}$$

Interval Notation

- Parentheses (non-inclusive):

$$\boldsymbol{x \in (a, b) \Rightarrow a < x < b}$$

- Square brackets [inclusive]:

$$\boldsymbol{x \in [a, b] \Rightarrow a \leq x \leq b}$$

NOTE: Use **number lines** to find the intersection and union of sets.

Now, your turn!

Question 2

Find the following sets:

a. $[0, 5] \cap [1, 8]$

$$\boxed{[1,5]}$$

b. $[-3, 7] \cup \left(-11, \frac{1}{2}\right]$

$$\boxed{(-11,7]}$$

Maximal Domain

- The maximal domain is ___ the biggest possible ___ domain for a rule without committing a ___ mathematical crime ___
- In Methods, we need to consider 3 important rules:

$$\sqrt{z}, \quad z \ \boxed{\geq 0}$$

$$\log(z), \quad z \ \boxed{\geq 0}$$

$$\frac{1}{z}, \quad z \ \boxed{\geq 0}$$

Head Tutor's Comment: Emphasise that this works WHATEVER THE z is.

NOTE: We will consider $\log$ in depth later throughout the year!

Question 3

Find the maximal domain of the following functions:

a. $f(x) = \sqrt{-x-6} - 5$

$$\boxed{\begin{aligned} -x-6 &\geq 0 \\ -x &\geq 6 \\ x &\leq -6 \end{aligned}}$$

b. $h(x) = -\log_2(x+10)$

$$\boxed{\begin{aligned} x+10 &> 0 \\ x &> -10 \end{aligned}}$$

<u>Head Tutor's Comment</u>: Emphasise the need for graphing when solving non $1:1$ inequalities.

c. $\frac{1}{x^2-25}$

$$\boxed{\mathbb{R}\backslash\{-5, 5\}}$$

NOTE: Always sketch the function when solving inequalities for many-to-one functions.

Space for Personal Notes

Calculator Commands

- **Mathematica**

```
FunctionDomain[func, x]
```

- **TI-Nspire**
  - Type up domain (or find it under the book button).

$$\text{domain}(\text{func}, x)$$

- **Casio Classpad**
  - Sketch the function and analyse.

Question 4 Tech-Active.

Find the maximal domain of the following function:

$$f(x) = \sqrt{x^2 - 9} + 1$$

$$\boxed{x \leq -3 \text{ or } x \geq 3.}$$

Range

- The range is the possible values for the output of a function.

Space for Personal Notes

Question 5

Find the range of the following function:

$$f: [-4, 4] \backslash \{0\} \to \mathbb{R}, f(x) = \frac{1}{x}$$

<u>Head Tutor's Comment</u>: Sketch the function and find the range.

$$\boxed{\text{Range } \left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{1}{4}, \infty\right)}$$

TIP: Always **sketch** the function!

<u>Functional Notation</u>

$$f: Domain \to Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.
- Codomain is **not** the same as range.

Space for Personal Notes

Question 6

Consider the following function, written in functional notation:

$$f: [-1, 4] \to \mathbb{R}, f(x) = x + 5$$

Identify the name, domain, range, and the equation of the function.

- Name of the function is f.
- The domain of the function is $[-1, 4]$.
- **The range of the function is NOT NECESSARILY all real numbers. Instead, it's $[4, 9]$.**
- The equation of the function is $x + 5$.

Head Tutor's Comment: Don't spend too much time here just skim through quickly.

Definition

Piecewise (Hybrid) Functions

- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x-values for which the two functions are defined.
- The two domains do not have to join!

Space for Personal Notes

Question 7

Consider the hybrid function f.

$$f(x) = \begin{cases} x^2 - 5, & x \geq 0 \\ x + 4, & x < 0 \end{cases}$$

a. Find $f(-2)$.

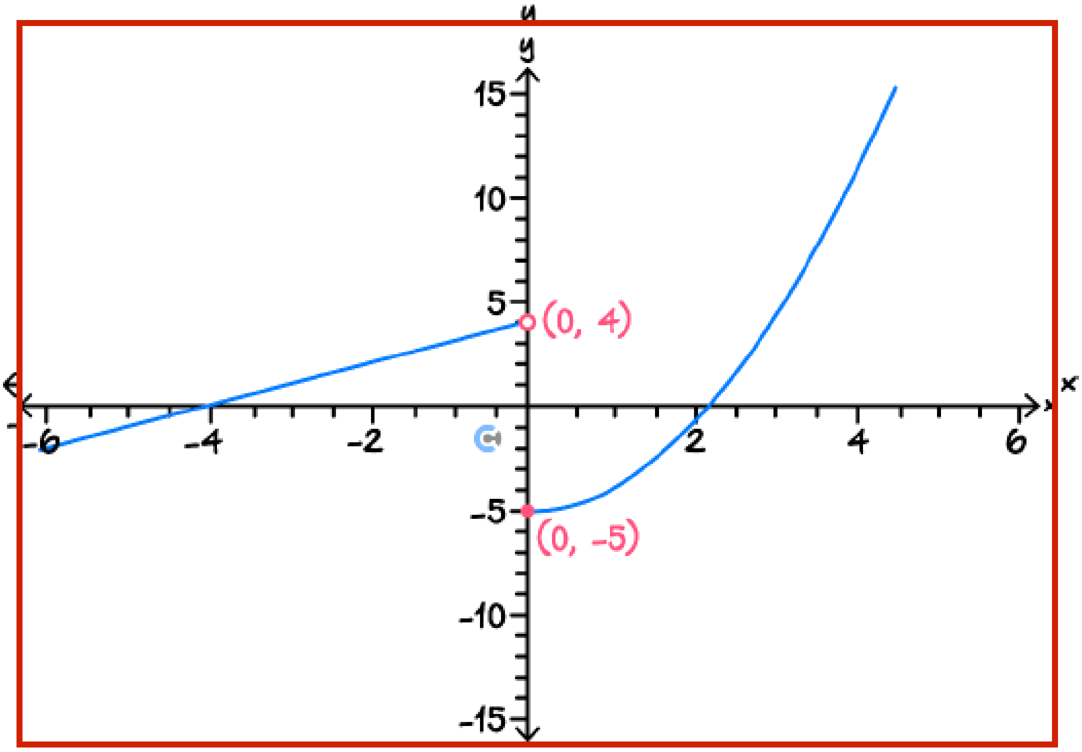
We sub into the linear, ANS: 2

b. Find $f(5)$.

We sub into the quadratic, ANS: 20

c. Graph $y = f(x)$.

OPEN CIRCLE AT $(0, 4)$.



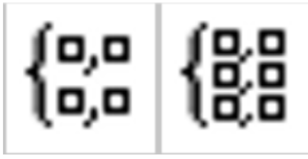
Defining Hybrid Functions on CAS

Mathematica

- "Esc PW" and Control Enter to create cells.

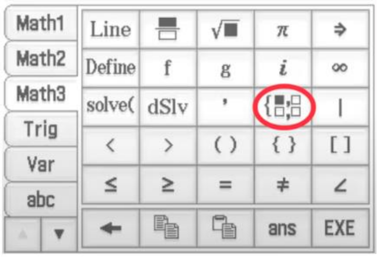
$$\begin{cases} \texttt{func1} & \texttt{dom1} \\ \texttt{func2} & \texttt{dom2} \end{cases}$$

TI-Nspire



$$\begin{cases} func1, dom1 \\ func2, dom2 \end{cases}$$

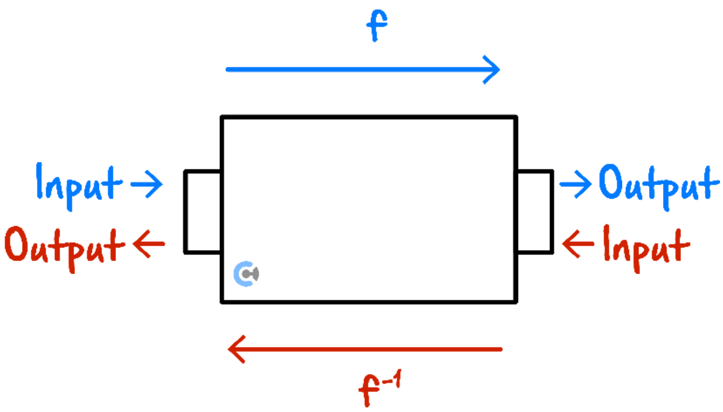
Casio Classpad



Inverse Relation

Head Tutor's Comment: Go through the basics quickly.

- **Definition**: Inverse is a relation which does the $\boxed{\text{opposite}}$.



Discussion: What would be the inverse of $f(x) = x + 2$?

$$\boxed{x - 2}$$

Question 8

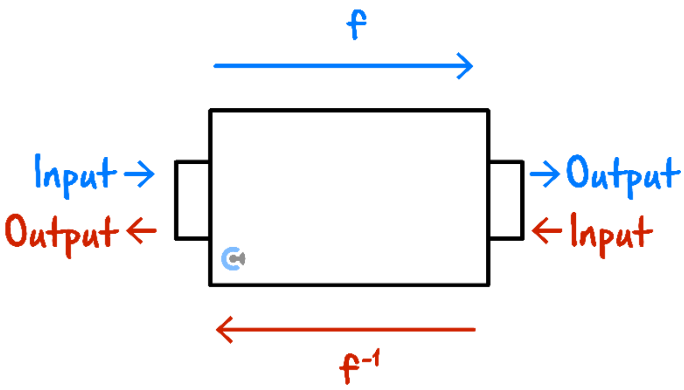
Find the inverse of $f(x) = 2x + 1$.

$$\boxed{f^{-1}(x) = \frac{x-1}{2}}$$

Head Tutor's Comment: Do NOT solve this by swapping x and y. Use the "opposite" idea only.

Solving for an Inverse Relation

- Swap x and y.



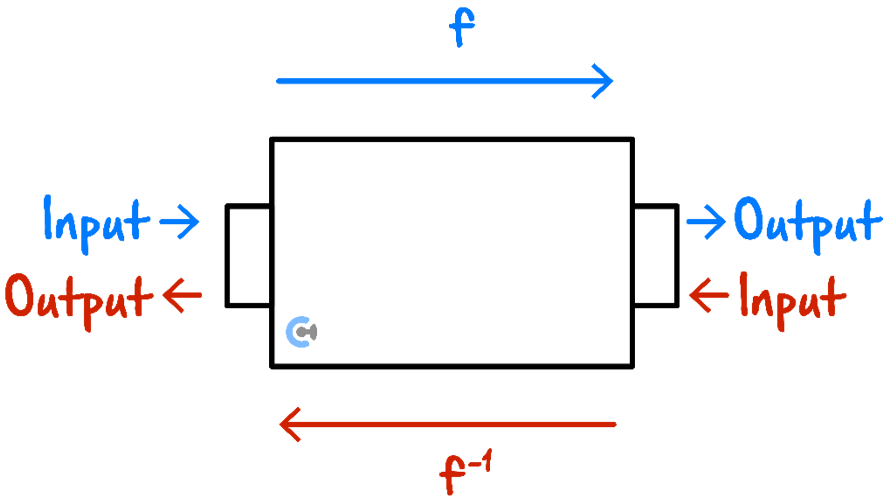
Question 9

Find the inverse of $f(x) = 2x + 1$ by swapping x and y.

$$\boxed{f^{-1}(x) = \frac{x-1}{2}}$$

NOTE: $f(x) = y$.

Domain and Range of Inverse Functions



$Dom\ f^{-1} =$ $\boxed{Ran\ f}$

$Ran\ f^{-1} =$ $\boxed{Dom\ f}$

Space for Personal Notes

Question 10

Consider the function $f: [0, 4] \to R, f(x) = 2x + 1$.

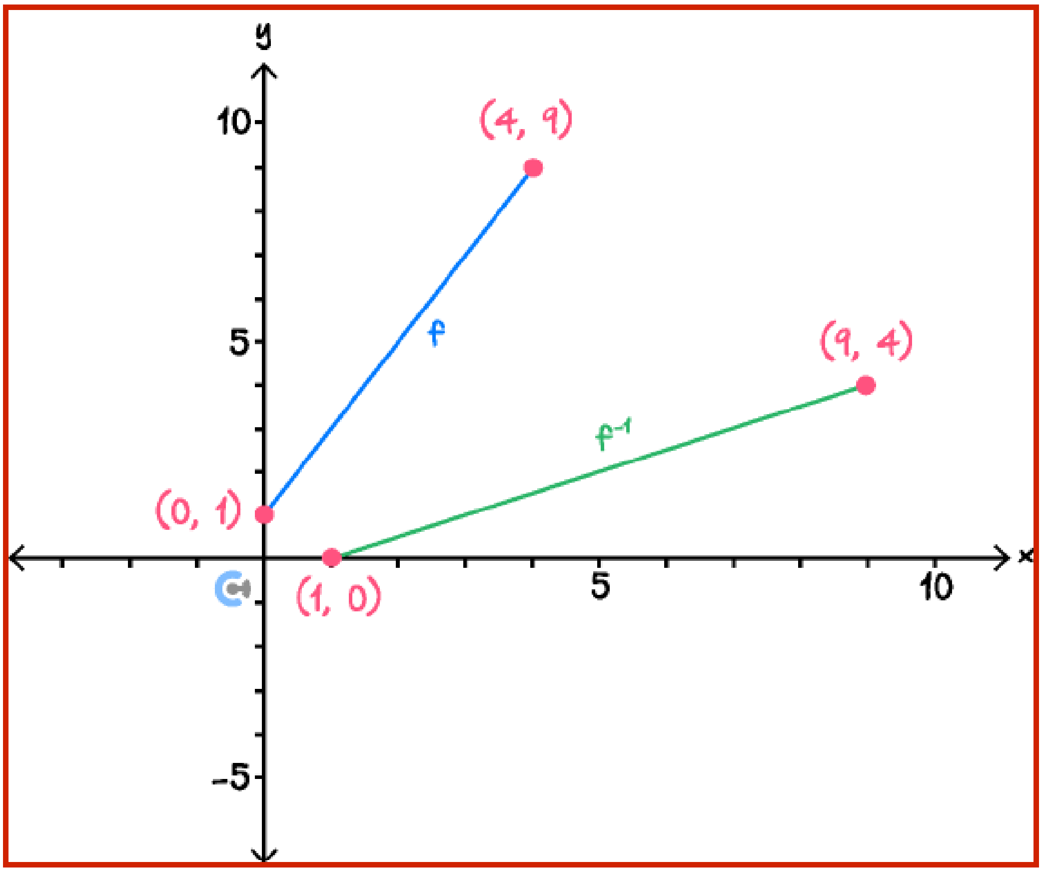
a. Find the rule for the inverse function.

$$\boxed{f^{-1}(x) = \frac{x-1}{2}}$$

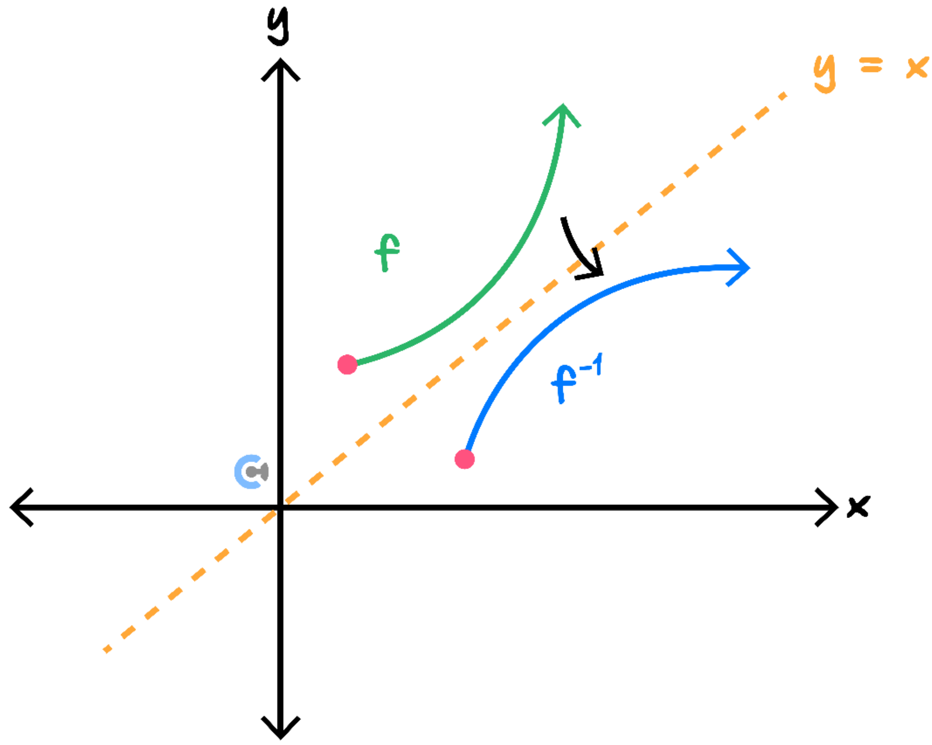
b. State the domain and range of the inverse function.

Range $= [0, 4]$
Domain $= [1, 9]$

c. Sketch the $f(x)$ and $f^{-1}(x)$ on the axis below.

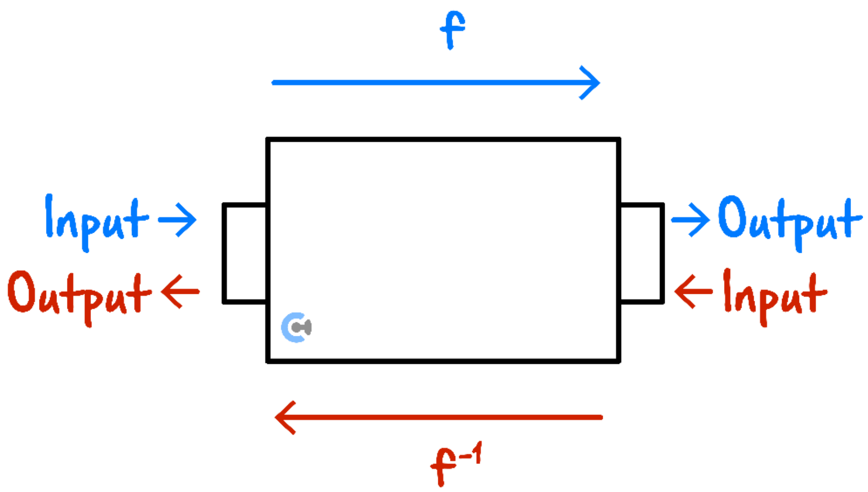


Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

Validity of Inverse Functions



- Requirement for Inverse Function:

f needs to be $\boxed{1:1}$

Question 11 (4 marks)

Consider the function $f: (-\infty, a] \to \mathbb{R}, f(x) = (x-2)^2 + 3$.

a. Find the largest possible value of a such that the inverse function f^{-1} exists.

$\boxed{a = 2}$

b. Find the domain and range of the inverse function. (2 marks)

Domain: $[3, \infty)$

Range: $(-\infty, 2]$

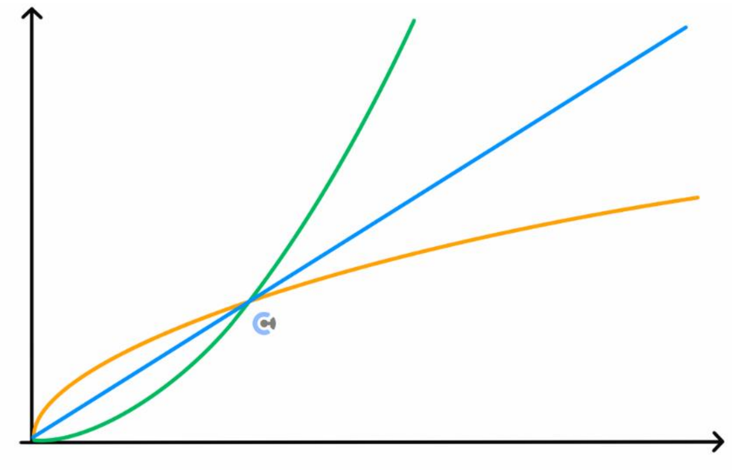
c. Find the rule for the inverse function. (2 marks)

$\boxed{f^{-1}(x) = 2 - \sqrt{x-3}}$

TIP: Always try sketching the function to find the domain such that an inverse function can exist!

NOTE: You will need to complete the square when finding the inverse of quadratic functions!

Intersection Between a Function and its Inverse



$$f(x) = x \text{ OR } f^{-1}(x) = x$$

Question 12

Find the intersection between $f: [0, \infty) \to R, f(x) = x^3$ and its inverse, without finding the inverse.

$$\boxed{x = 0 \text{ and } x = 1.}$$

Head Tutor's Comment: Emphasise that you cannot just cancel x's on either side of the equation. (For anything that could potentially be 0, we cannot cancel them.)

NOTE: We can always equate the function to x instead of the inverse function itself!

ALSO NOTE: This only works for an increasing function, however in VCAA, this is always the case. Something to note for SACS is that there could be intersections that are NOT on $y = x$.

Section B: Warmup Test (19 Marks)

INSTRUCTION: 19 Marks. 15 Minutes Writing.

Question 13 (2 marks)

$4 + [-\infty, 0] \Rightarrow [-\infty, 4]$

$4 - \sqrt{\ }$

State the implied domain and range for $y = 4 - \underbrace{\sqrt{3 - 2x}}_{\geq 0}$.

$\Rightarrow 3 - 2x \geq 0$

$3 \geq 2x$

$x \leq \frac{3}{2}$

Dom : $x \in (-\infty, \frac{3}{2}]$ //

Ran : $y \in (-\infty, 4]$ //

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Question 14 (6 marks)

Let $f(x) = \frac{x+2}{x-2}$.

y-int:

$y = \frac{2}{-2} = -1$

x-int:

$0 = 1 + \frac{4}{x-2}$

$\frac{4}{x-2} = -1$

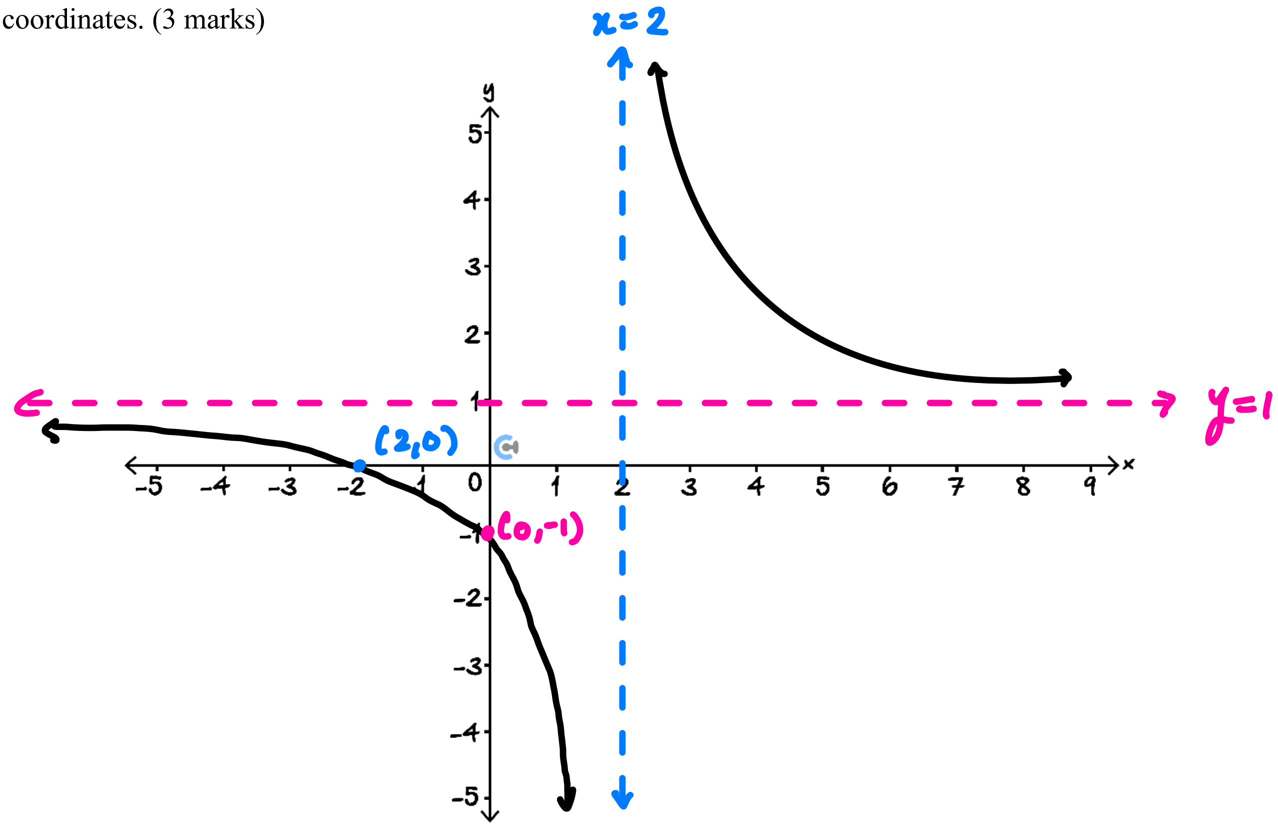
$x - 2 = -4$

$\therefore x = -2$

a. Write $f(x)$ in the form $a + \frac{b}{x-2}$ for integers a and b. (1 mark)

$f(x) = \frac{x-2+4}{x-2} = 1 + \frac{4}{x-2}$ //

b. Sketch the graph of $y = f(x)$ on the axes below. Label asymptotes with equations and axes intercepts with coordinates. (3 marks)



c. Find the rule and domain of f^{-1}, the inverse function of f. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = 1 + \frac{4}{y-2}$

$x - 1 = \frac{4}{y-2}$

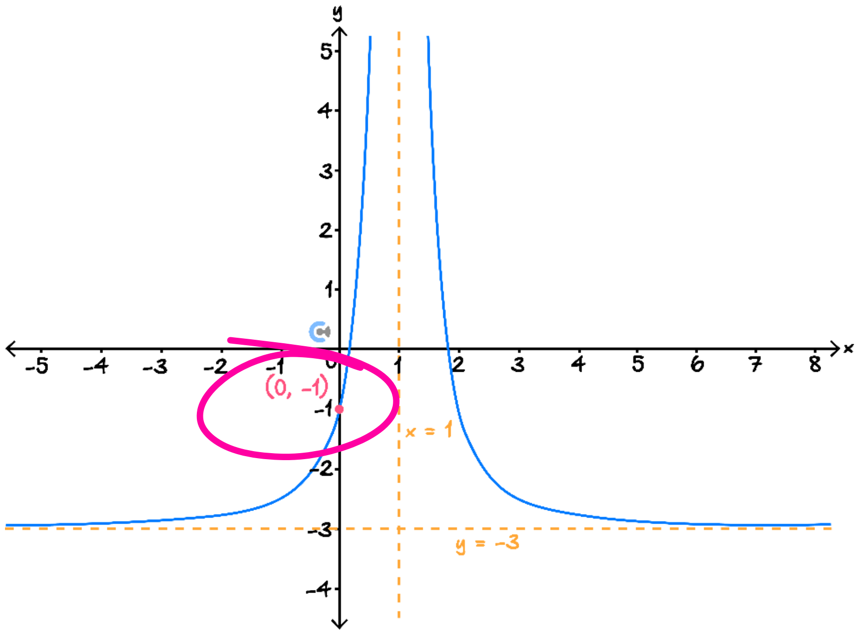
$y - 2 = \frac{4}{x-1}$

$y = \frac{4}{x-1} + 2 \Rightarrow \therefore f^{-1}(x) = \frac{4}{x-1} + 2$

Dom f^{-1} = Ran f = $\mathbb{R} \setminus \{1\}$

Question 15 (2 marks)

Find the rule of the following truncus:



$\therefore h = 1, k = -3$

$\therefore y = \frac{a}{(x-1)^2} - 3$

Sub $(0,-1)$:

$-1 = \frac{a}{(0-1)^2} - 3$

Space for Personal Notes

$\therefore 2 = a \rightarrow \therefore y = \frac{2}{(x-1)^2} - 3$

Question 16 (2 marks)

Find the centre and radius of the cirele given by the equation $x^2 - 6x + y^2 - 4y + 4 = 0$.

$(x-3)^2 - 9 + (y-2)^2 = 0$

$\therefore (x-3)^2 + (y-2)^2 = 9$

Centre: $(3,2)$ //

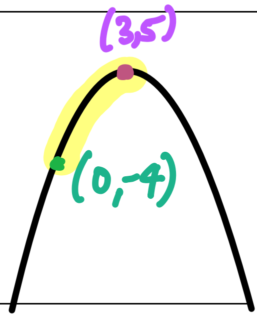
radius $= \sqrt{9} = 3$ //

Space for Personal Notes

Question 17 (7 marks)

TP: $(3,5)$

Consider the function $f:[0,a] \to R, f(x) = -(x-3)^2 + 5$.



a. Find the largest value of a such that the inverse function f^{-1} exists. (1 mark)

f must be 1:1

$\therefore a = 3$

b. State the domain and range of the inverse of f. (1 mark)

$\text{Dom } f^{-1} = \text{Ran } f = [-4,5]$

$\text{Ran } f^{-1} = \text{Dom } f = [0,3]$

c. Determine the equation of the inverse function f^{-1}. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = -(y-3)^2 + 5$

$x - 5 = -(y-3)^2$

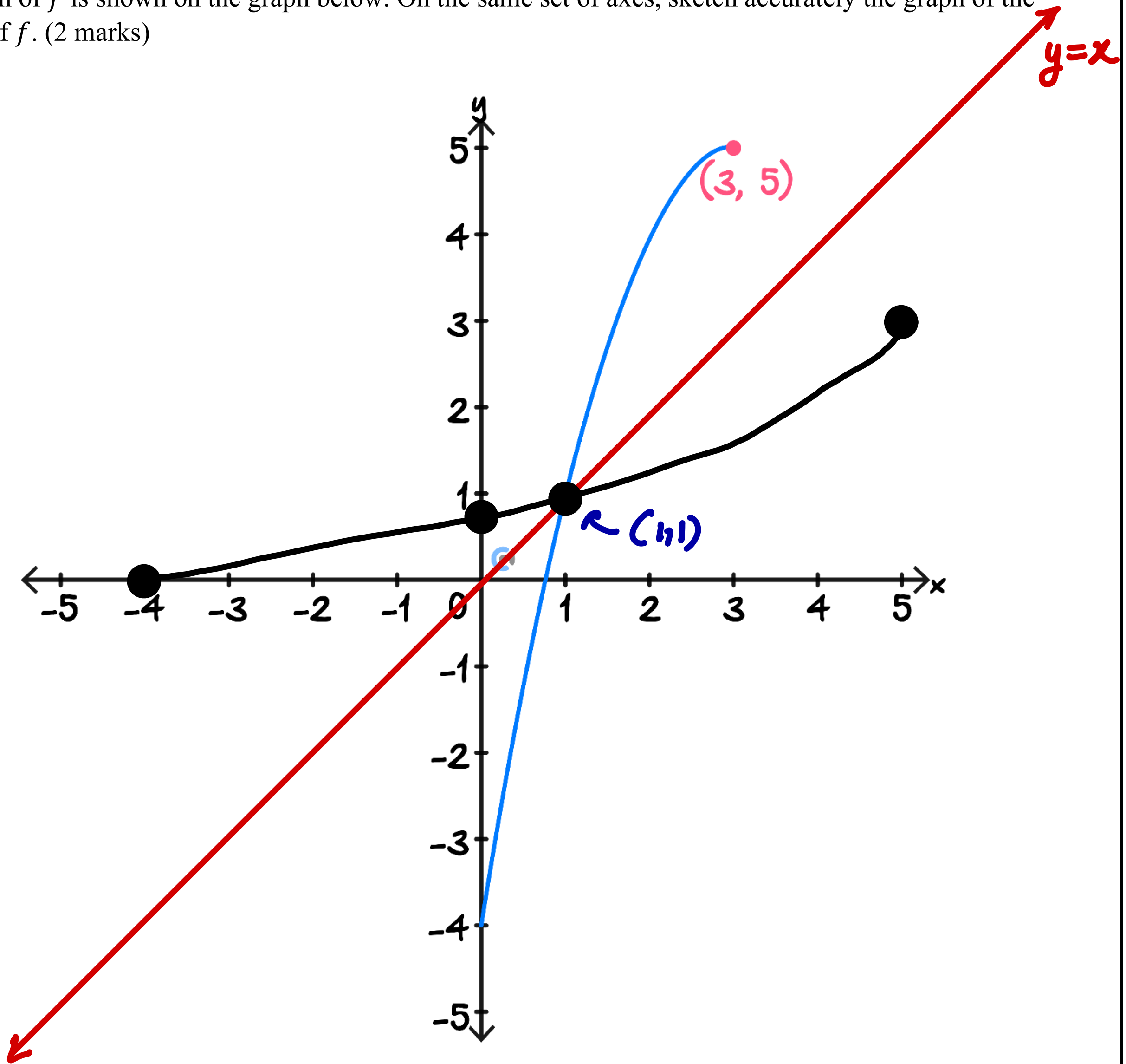
$(y-3)^2 = 5 - x$

$y - 3 = \pm\sqrt{5-x}$

$y = 3 \pm \sqrt{5-x}$

$\therefore f^{-1}(x) = 3 - \sqrt{5-x}$

d. The graph of f is shown on the graph below. On the same set of axes, sketch accurately the graph of the inverse of f. (2 marks)



e. Find an intersection point between f and f^{-1}. (1 mark)

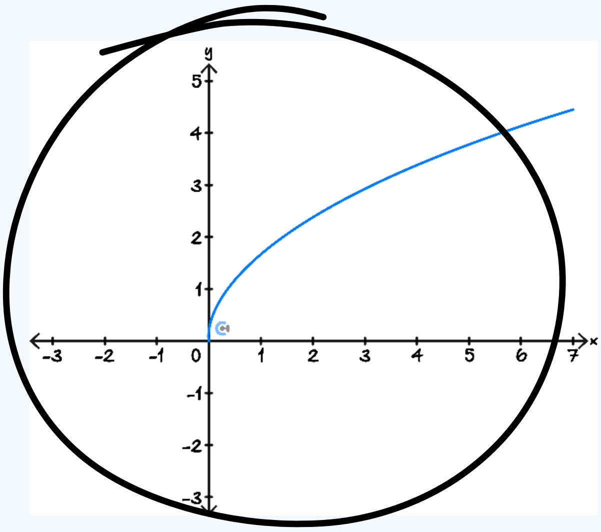
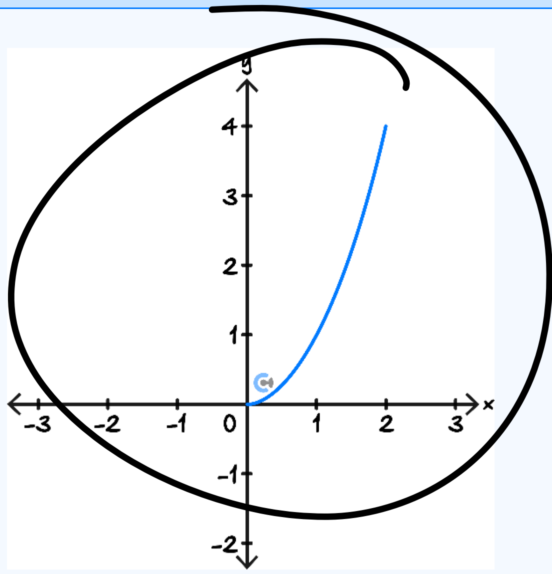
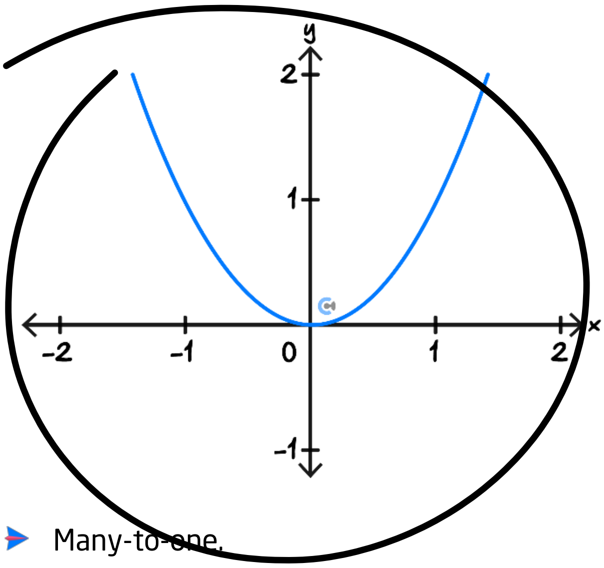
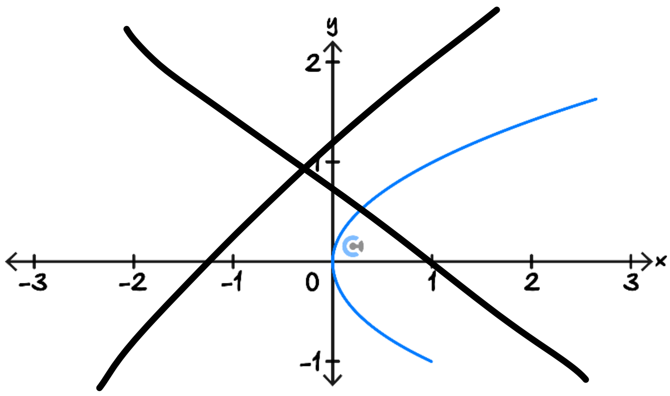
IP: (1,1)

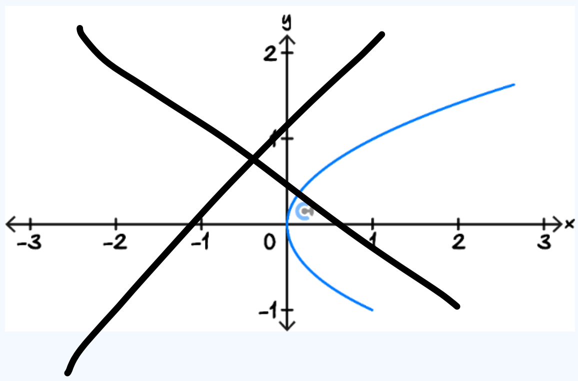
Section C: Exam Skills

Sub-Section: Restrict Domain Such That the Inverse Function Exists

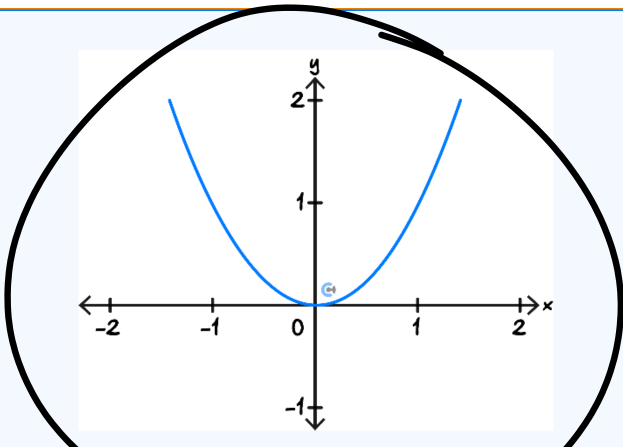
Exploration: How does taking the inverse affect a function/relation's type?

- The graphs of f and f^{-1} are pictured below. Fill in the type of function for the f^{-1} column.

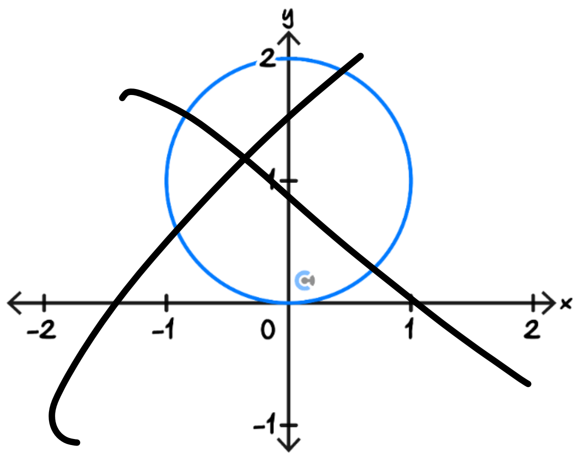
$f(x)$	$f^{-1}(x)$
 One-to-one.	 1:1
 Many-to-one.	 1:Many



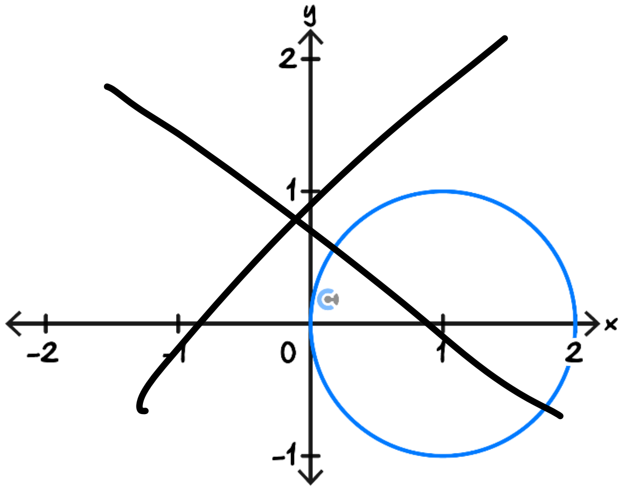
- One-to-many.



- Many : 1.



- Many-to-many.



- Many : Many.

- You may notice that swapping 'x' and 'y' means both sides of the 'to' are swapped around!
- Circle the functions on the left-hand column.
- Circle the functions on the right-hand column.
- Hence, which type of function remains a function when it is inversed?

1:1

<u>**Restricting a Function Such That Its Inverse Function Exists**</u>

Definition

- To restrict a function such that its inverse exists, we restrict the domain such that it is **one-to-one**.

Question 18 Walkthrough.

Consider the function $f: (-\infty, a] \to R, f(x) = x^2 - 4x + 9$.

Find the maximum value of a such that f^{-1} exists.

$\therefore a = \frac{-(-4)}{2(1)} = 2$ //

TIP: Find the turning point!

<u>Active Recall</u>

- For the inverse function to exist, the function must be 1:1.

Question 19

Consider the function $f: [a, \infty) \to R, f(x) = x^2 + 10x + 4$.

Find the minimum value of a such that f^{-1} exists.

$\therefore a = \frac{-10}{2} = -5$ //

Question 20 (1 mark) **Extension.**

Which of the following has an inverse which is a function?

1:1

A. $y = 2x^2 + 1$

B. $x^2 + y^2 = 9$

C. $y = \frac{3}{2x-5} + 2$

D. $y = \sqrt{9 - x^2}$

Space for Personal Notes

Sub-Section: Figure Out Possible Rule of a Graph

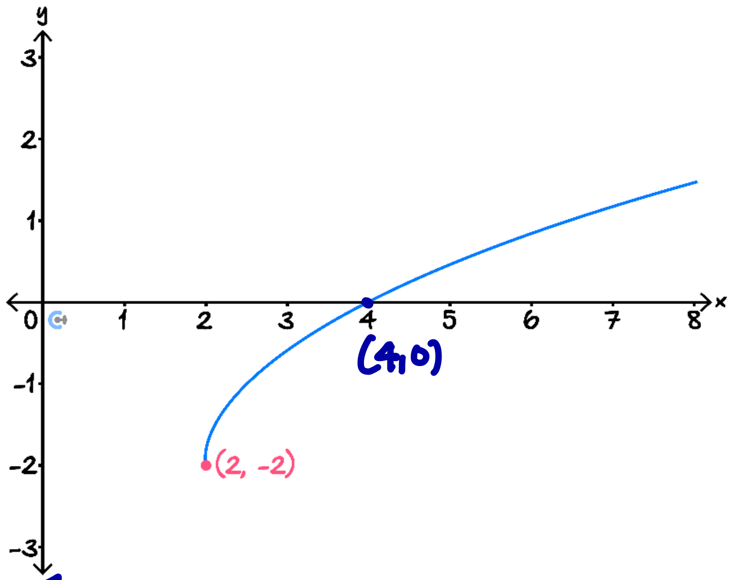
We've gone from rule to graph. How do we go from having the graph to getting the rule?

Figuring Out the Possible Rule of a Function Through 'Characteristics'

- Identify the 'characteristic(s)' that changes between each option (asymptote, x-intercepts, y-intercepts etc).
- Use elimination based on the rule to get the right option.

Question 21 (1 mark) **Walkthrough.**

The equation that best represents the graph below is:



A. $y = \sqrt{x-2} + 2$ 4 $\neq 0$

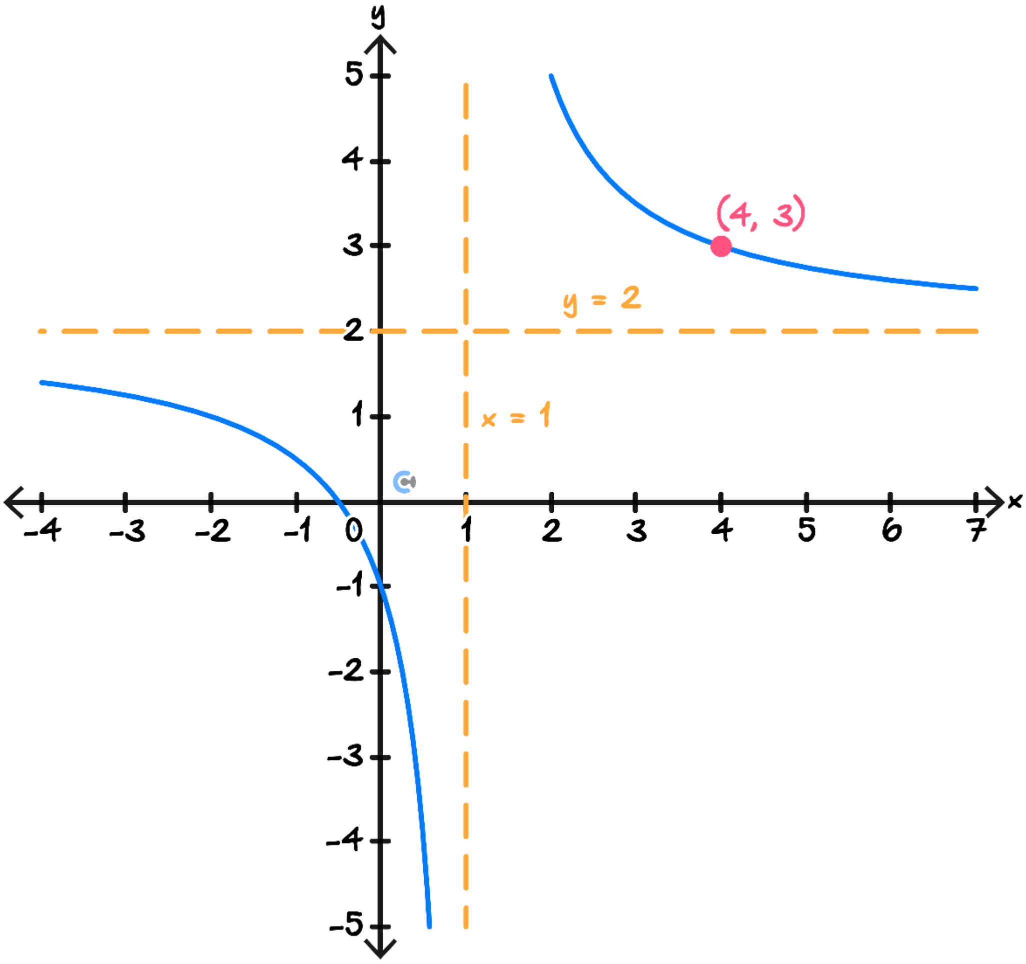
B. $y = \sqrt{2-4x} - 2$ 4 $\neq 0$

C. ~~$y = \frac{2}{2x-4} + 2$~~

D. $y = \sqrt{2x-4} - 2$ 4 $= 0$

Question 22 (1 mark)

The equation that best represents the graph below is:



A. $y = \frac{2}{(x-1)^2} + 2$

B. $y = \frac{1}{x-1} + 2$ 4 ≠ 3

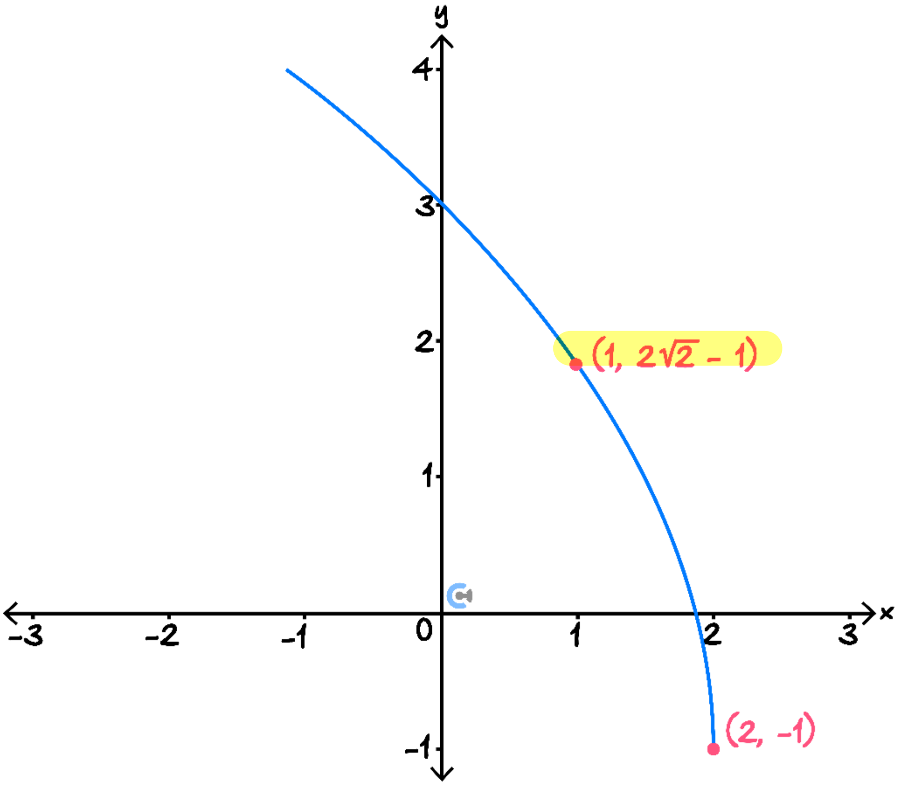
C. $y = \frac{2}{x-1} + 2$ 4 ≠ 3

D. $y = \frac{3}{x-1} + 2$ 4 = 3

Space for Personal Notes

Question 23 (1 mark) **Extension.**

The equation that best represents the graph below is:



A. $y = -\sqrt{2-2x} + 2$

B. $y = 3\sqrt{2-x} - 1$

C. $y = 2\sqrt{4-2x} - 1$

D. $y = \sqrt{6-2x} - 1$

Space for Personal Notes

Section D: Exam 1 (16 Marks)

INSTRUCTION: 16 Marks. 20 Minutes Writing.

Question 24 (9 marks)

Consider the function $f(x) = \sqrt{\underbrace{x^2 + 4x - 12}_{\geq 0}}$.

a. Find the maximal domain of $f(x)$. (2 marks)

$x^2 + 4x - 12 \geq 0$

$(x+6)(x-2) \geq 0$

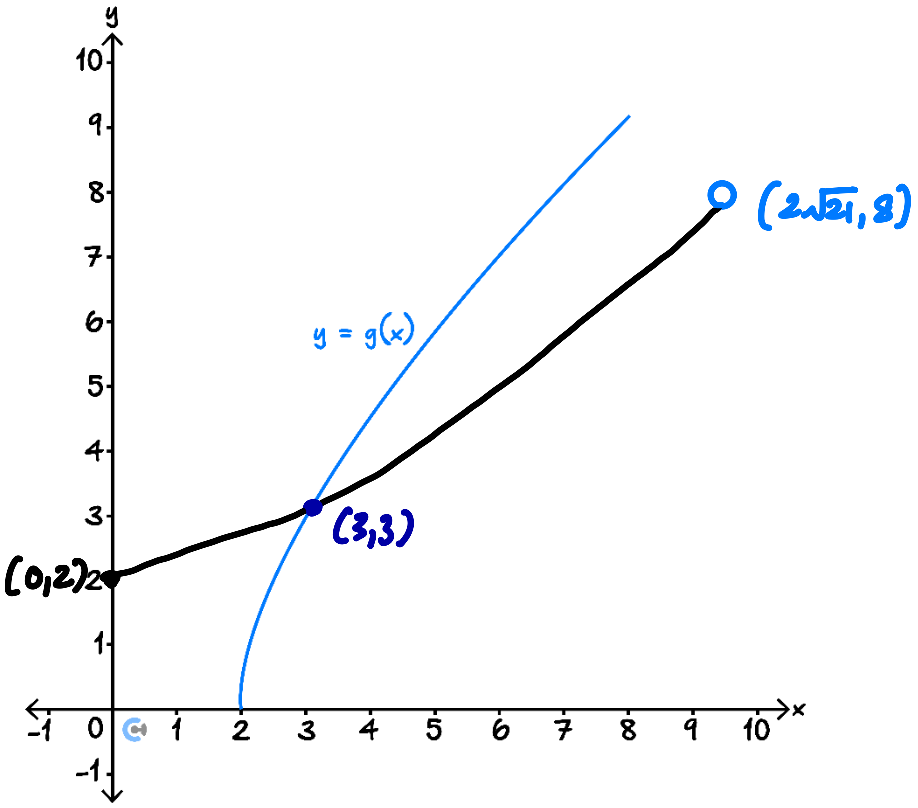
$\therefore x \leq -6$ or $x \geq 2$

$(8, 2\sqrt{21})$

$f(8) = \sqrt{64+32-12}$

$= \sqrt{84} = 2\sqrt{21}$

The graph of $g(x)$ is sketched below, where $g: [2,8) \to \mathbb{R}, g(x) = f(x)$.



b. Find the equation of $g^{-1}(x)$. (2 marks)

Let $y = g(x)$:

Swap x & y:

$x = \sqrt{y^2+4y-12}$

$x^2 = y^2+4y-12$

$(y+2)^2 - 16 = x^2$

$(y+2)^2 = x^2+16$

$y+2 = \pm\sqrt{x^2+16}$

$y = -2 \pm \sqrt{x^2+16}$

$\therefore g^{-1}(x) = \sqrt{x^2+16} - 2$

c. Find the point of intersection between $g(x)$ and $g^{-1}(x)$. (2 marks)

Let $g(x) = x$:

$\sqrt{x^2+4x-12} = x$

$\cancel{x^2}+4x-12 = \cancel{x^2}$

$4x-12=0$

$4x=12$

$\therefore x=3$

$\therefore y=x=3$

$\therefore$ IP: $(3,3)$

d. Sketch the graph of $g^{-1}(x)$ on the axes above. Label all endpoints and any points of intersection with $g(x)$. (3 marks)

Space for Personal Notes

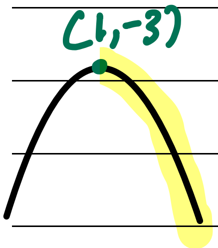
Question 25 (7 marks)

The function $f(x)$ is defined as $f: [a, \infty) \to \mathbb{R}, f(x) = -x^2 + 2x - 4$.

$a^2 + 2ab + b^2 = (a+b)^2$, $\frac{2ab}{2}$

a. Find the minimum value of a such that $f^{-1}(x)$ exists. (2 marks)

$f(x) = -(x^2 - 2x) - 4$



$= -((x-1)^2 - 1) - 4$

$= -(x-1)^2 - 3$

TP: $(1,-3)$

$\therefore a = 1$

b. Define the function $f^{-1}(x)$. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = -(y-1)^2 - 3$

$(y-1)^2 = -x - 3$

$y - 1 = \pm\sqrt{-x-3}$

$y = 1 \pm \sqrt{-x-3}$

$\therefore f^{-1}(x) = 1 + \sqrt{-x-3}$

Dom f^{-1} = Ran f $= (-\infty, -3]$

$f^{-1}: (-\infty, -3] \to \mathbb{R}, f^{-1}(x) = 1 + \sqrt{-x-3}$

c. State the range of $f^{-1}(x)$. (1 mark)

Ran f^{-1} = Dom f $= [1, \infty)$

d. Find the number of intersections between $f^{-1}(x)$ and $f(x)$. (2 marks)

Let $f(x) = x$:

$-x^2+2x-4 = x$

$x^2 - x + 4 = 0$

$\therefore \Delta = (-1)^2 - 4(1)(4)$

$= 1 - 16$

$= -15$

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$\therefore \Delta < 0 \Rightarrow$ No solutions $\Rightarrow$ 0 intersection

for $g(x) = g^{-1}(x)$ //

Section E: Tech-Active Exam Skills (4 Marks)

Sub-Section: Find the Rule From the Graph with CAS

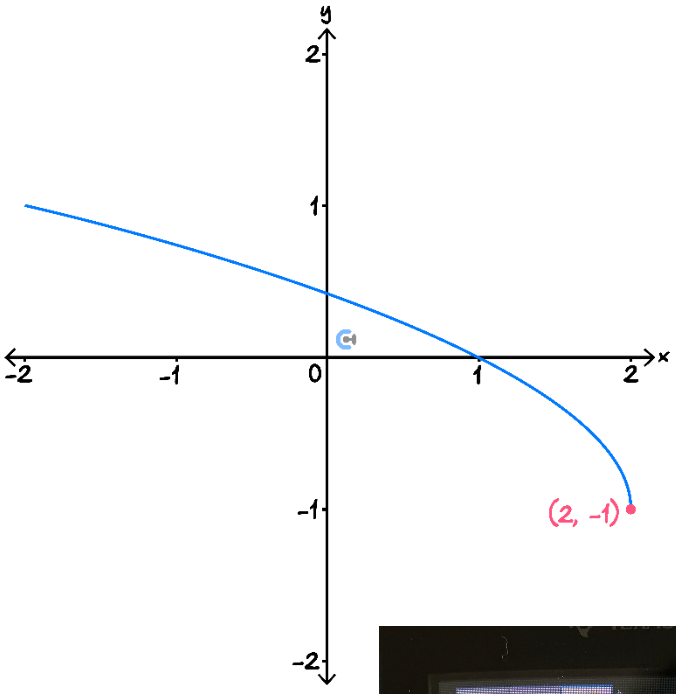
Definition

The 'Easy' Way to Find the Rule From the Graph

1. Graph each option.
2. Set the scale of the CAS to the **same** scale as the axes in the question.
3. See which option matches.

Question 26 (1 mark) **Walkthrough.**

The equation that best represents the graph below is:

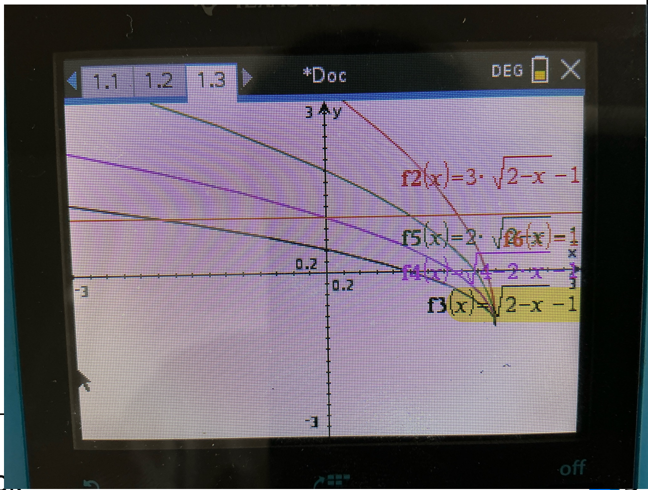


A. $y = 3\sqrt{2-x} - 1$

B. $y = \sqrt{2-x} - 1$

C. $y = \sqrt{4-2x} - 1$

D. $y = 2\sqrt{2-x} - 1$

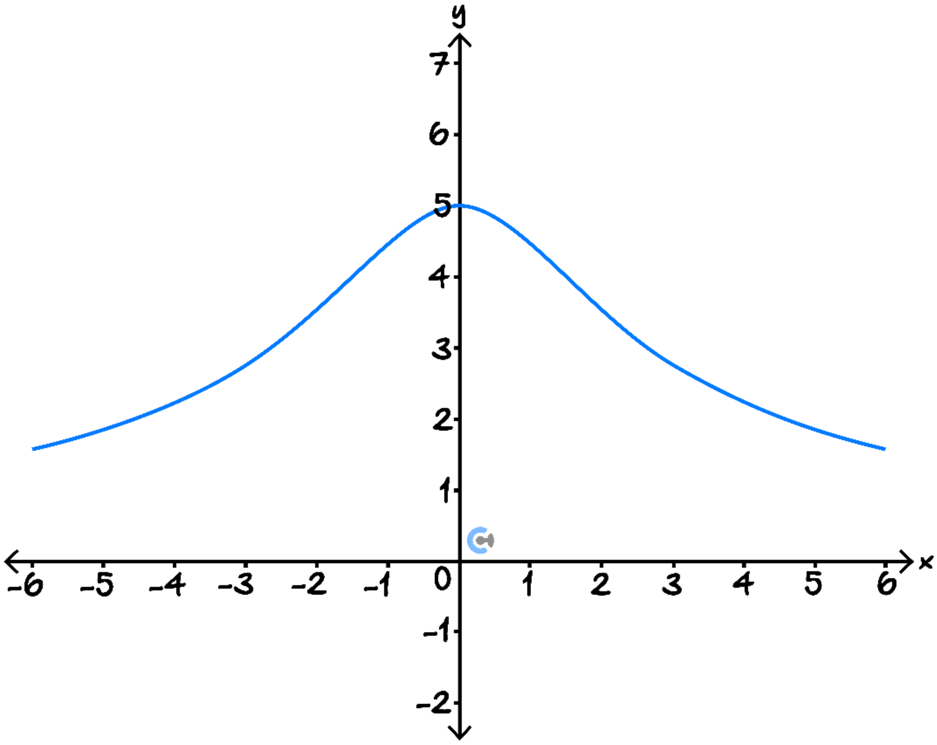


Active Recall: Finding the rule from CAS by graphing options:

- Make sure the CAS is set to the right scale.

Question 27 (1 mark) **Walkthrough.**

The equation that best represents the graph below is:

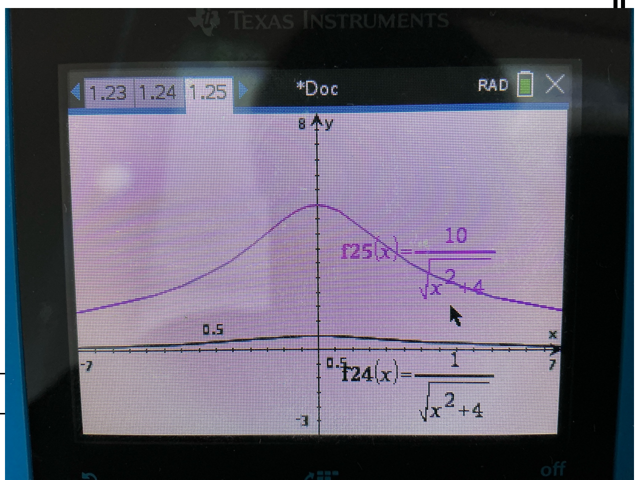


A. $y = \frac{1}{\sqrt{x^2+4}}$

B. $y = \frac{10}{\sqrt{x^2+4}}$

C. $y = \frac{5}{\sqrt{x^2+4}}$

D. $y = \frac{5}{\sqrt{x^2-4}}$



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Sub-Section: Solve Number of Solutions Problems Graphically

Calculator Commands: Using Sliders/Manipulate on CAS

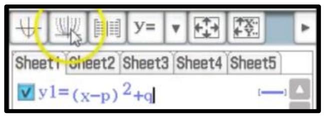
Mathematica

```
Manipulate[Plot[function, {x, xmin, xmax}],
    {unknown, lowerbound, upperbound}]
```

NOTE: The function **must** be typed out instead of using its saved name.

TI-Nspire

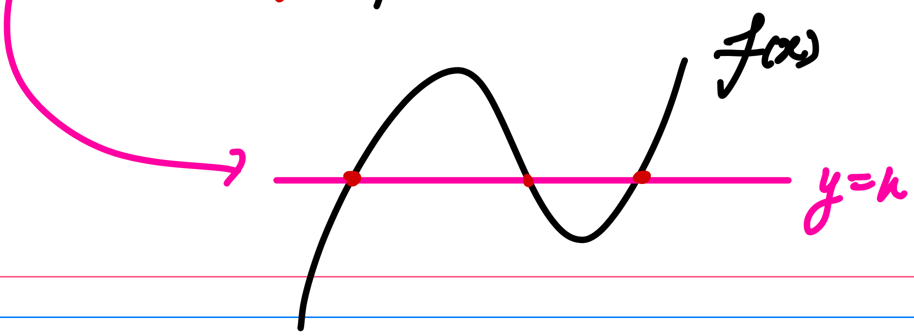
Casio Classpad



Discussion: How can we visualise the solutions of $f(x) = k$?

intersections

$\underbrace{y=f(x)}$ & $\underbrace{y=k}$



Solutions for $f(x) = k$

1. Graph $y = f(x)$ and $y = k$.
2. Slide $y = k$ between the intervals in the options.
3. Count the number of intersections until an interval matches.

Question 28 (1 mark) **Walkthrough.**

Find the values of k such that $\underbrace{x^3 - 6x^2 + 9x - 5}_{y = f(x)} = k$ has three solutions:

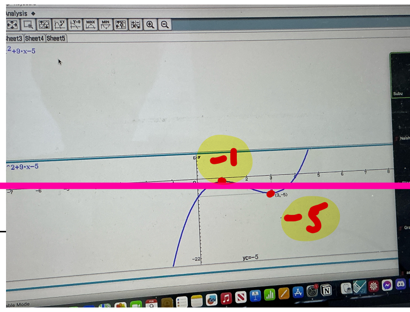
$\&$ $y = k$ 3 intersections

A. $k > -1$

B. $-5 < k < -1$

C. $-5 < k < 1$

D. $k = 1$



Question 29 (1 mark)

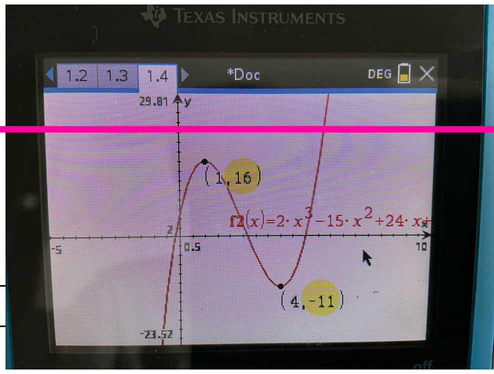
Find the values of k such that $2x^3 - 15x^2 + 24x + 5 = k$ has one solution:

A. $k < 20$ or $k > 2$.

B. $-11 < k < 16$

C. $k = 16$

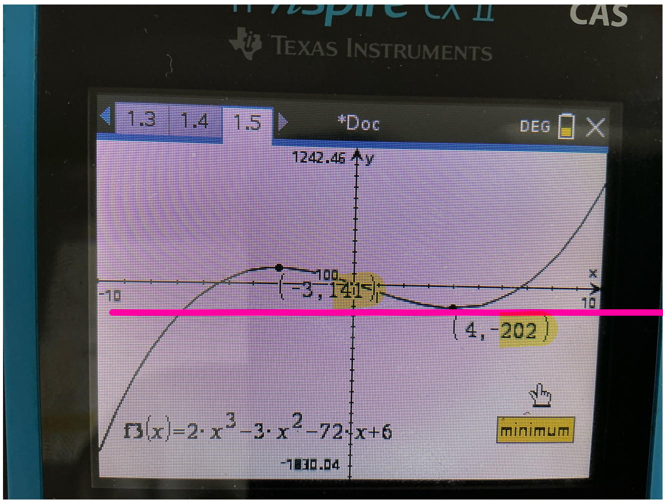
D. $k < -11$ or $k > 16$.



Question 30 Extension.

Find the values of k such that $2x^3 - 3x^2 - 72x + 6 = k$ has two solutions.

$\Rightarrow \therefore k = 141$ or -202

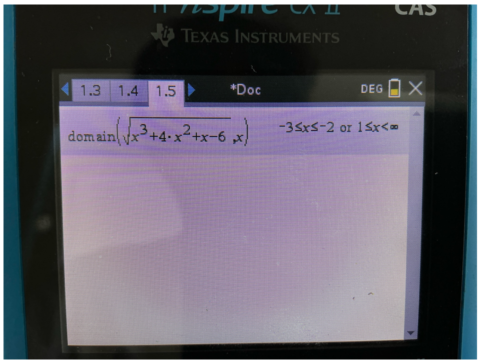


Section F: Exam 2 (16 Marks)

INSTRUCTION: 16 Marks. 20 Minutes Writing.

Question 31 (1 mark)

The maximal domain of the function $f(x) = \sqrt{x^3 + 4x^2 + x - 6}$ is:



A. $x \in (-\infty, -3] \cup [-2, 1]$

B. $x \in [-3, \infty) \backslash (-2, 1)$

Not inclusive

C. $x \in [-3, -2] \cup [1, \infty]$

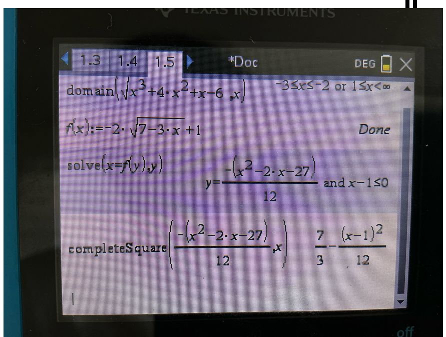
D. $x \geq -3$ or $x \geq -2$ or $x \geq 1$.

Question 32 (1 mark)

$y = -2\sqrt{} + 1$

$(-\infty, 0] + 1 = (-\infty, 1]$

The inverse of the equation $f(x) = -2\sqrt{7 - 3x} + 1$ is:



A. $f^{-1}: \left(-\infty, \frac{7}{3}\right] \to \mathbb{R}, f^{-1}(x) = \frac{1}{2}\left(\frac{1}{3}x - 7\right)^2 - 1$

B. $f^{-1}: [1, \infty) \to \mathbb{R}, f^{-1}(x) = \frac{1}{12}(-x^2 + 2x + 27)$

C. $f^{-1}: \mathbb{R} \to \mathbb{R}, f^{-1}(x) = -\frac{1}{3}\left(\frac{1}{2} - \frac{x}{2}\right)^2 + \frac{7}{3}$

D. $f^{-1}: (-\infty, 1] \to \mathbb{R}, f^{-1}(x) = -\frac{1}{12}(x - 1)^2 + \frac{7}{3}$

Question 33 (1 mark)

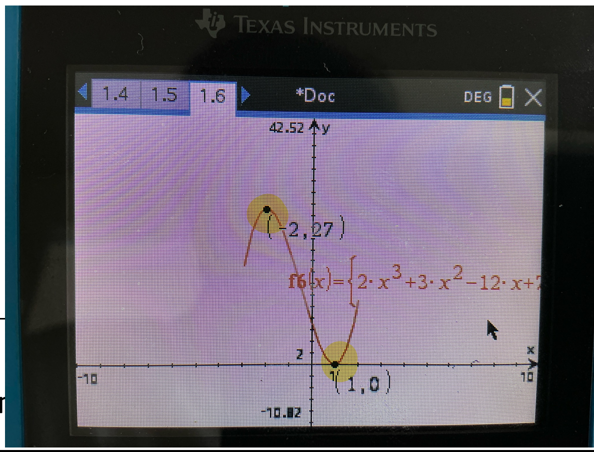
The range of the function $f: [-3,2) \to \mathbb{R}, f(x) = 2x^3 + 3x^2 - 12x + 7$:

A. $[0, 27]$

B. $[11, 16)$

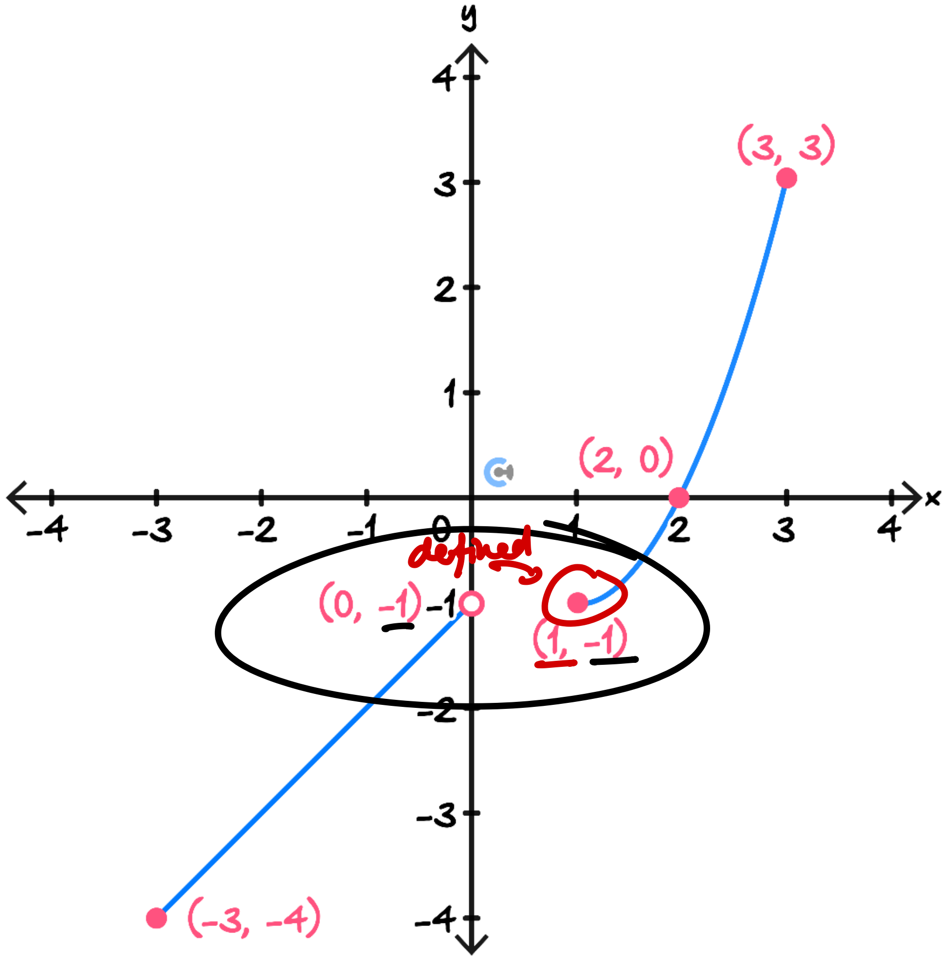
C. $(6, 11]$

D. $[0, 27)$



Question 34 (1 mark)

The graph $f(x)$ is sketched on the axes below.



The value of $f^{-1}(-1)$ is:

$f(x) = y$

$f^{-1}(y) = x$

A. -2

B. 1

C. 0

D. Undefined

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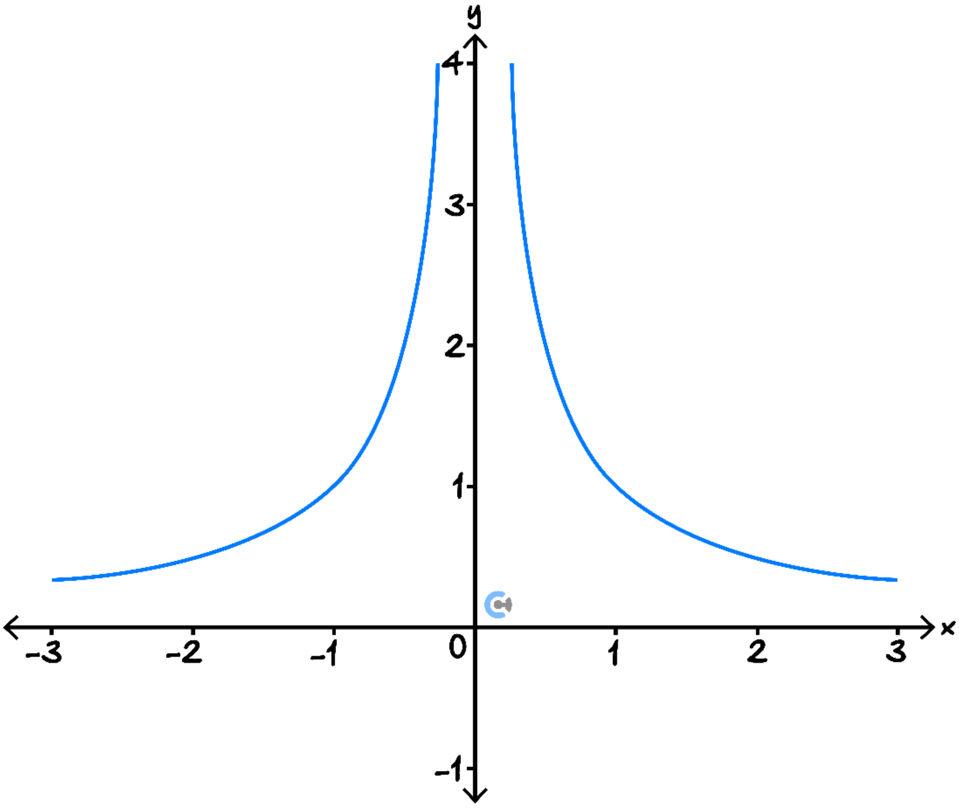
Question 35 (1 mark)

The graph $f(x)$ is sketched on the axes below.



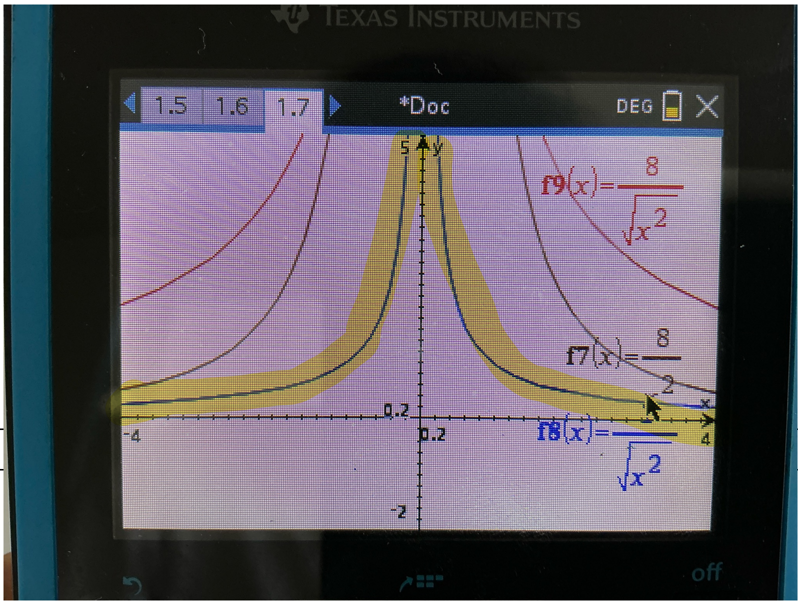
The rule for $f(x)$ is most likely:

A. $f(x) = \frac{8}{x^2}$

B. $f(x) = \frac{1}{\sqrt{x^2}}$

C. $f(x) = \frac{8}{\sqrt{x^2}}$

D. $f(x) = \frac{100}{x^2}$



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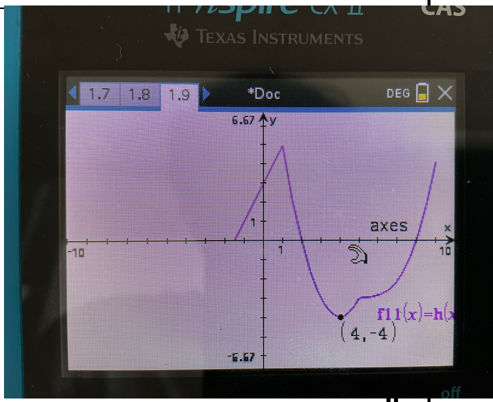
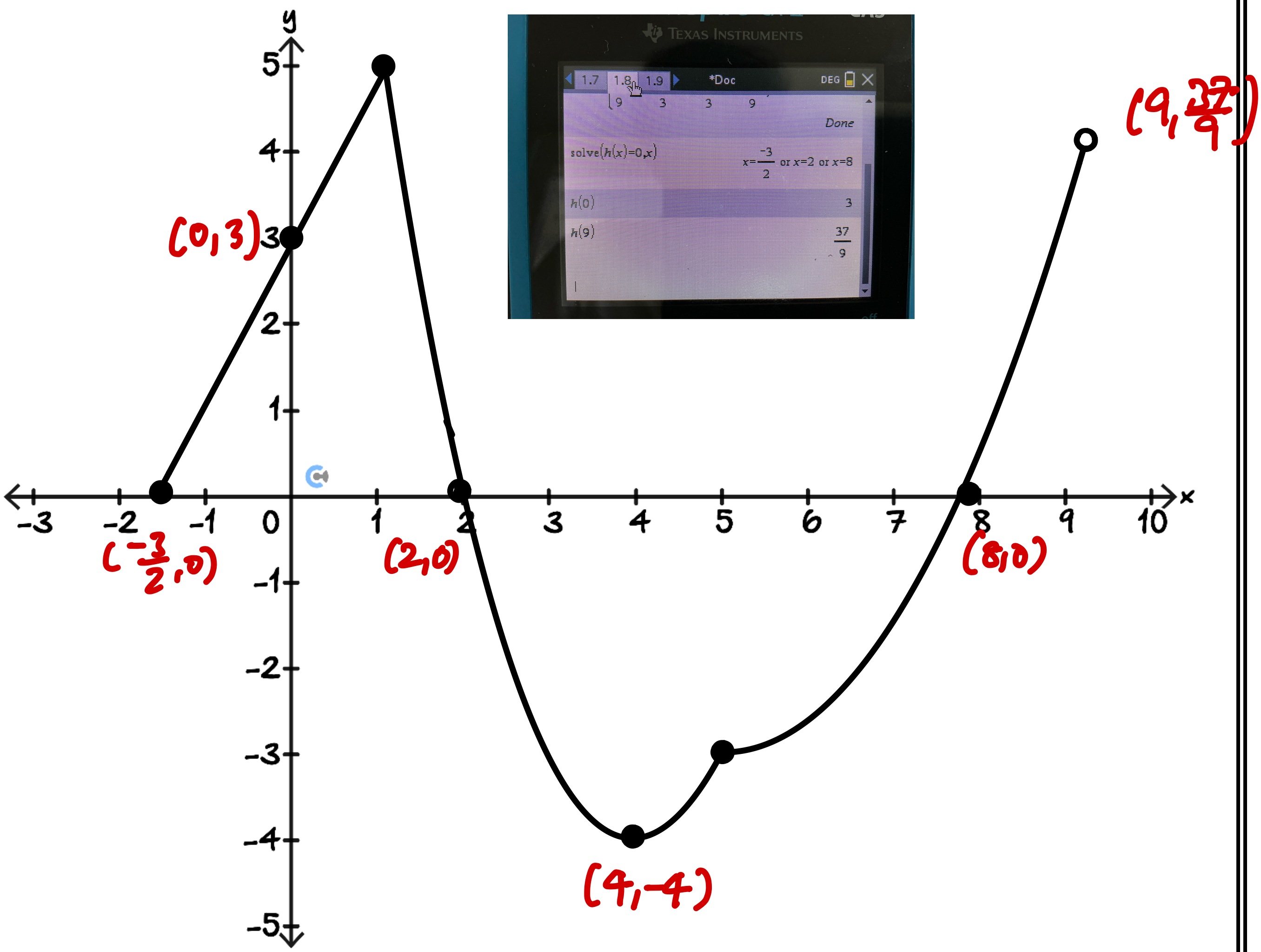
Question 36 (11 marks)



The height of a rollercoaster, in metres, is modelled by the function:

$$h(x) = \begin{cases} 2x + 3, & -\frac{3}{2} \le x \le 1 \\ x^2 - 8x + 12, & 1 < x \le 5 \\ \frac{1}{9}x^3 - \frac{5}{3}x^2 + \frac{25}{3}x - \frac{152}{9}, & 5 < x < 9 \end{cases}$$

a. Sketch the graph of $h(x)$ on the axes below. Label all endpoints, intercepts and turning points. (4 marks)

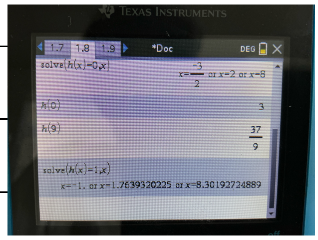


b. State the maximum height of the rollercoaster and the x-value at which this occurs. (2 marks)

Max height = 5m

c. For what values of x, correct to 3 decimal places, is the rollercoaster 1 m off the ground? (2 marks)

$\because h(x) = 1$ $\quad y=1$



$\therefore x = -1$ or 1.764 or 8.302

The "Fun Factor" of the rollercoaster is dependent on the height of the rollercoaster and can be calculated using the equation $f = 4h^{\frac{5}{2}} + 12h + 1$, where h is the height of the rollercoaster and f is the Fun Factor of the rollercoaster. $f(h)$

d. Find the values of x for which the Fun Factor is defined. (2 marks)

$f(h) = 4h^{\frac{5}{2}} + 12h + 1$

$h^{\frac{5}{2}} = h^{\frac{4}{2}} \cdot h^{\frac{1}{2}}$

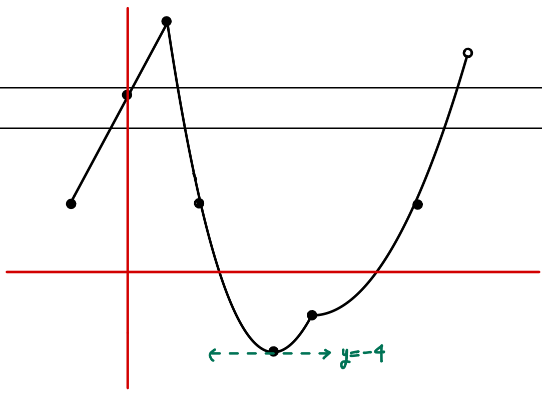
$= h^2 \cdot \sqrt{h}$

$= 4h^2\sqrt{h} + 12h + 1$

$\therefore x \in \left[-\frac{3}{2}, 2\right] \cup [8, 9)$

e. It is decided to raise the entire rollercoaster upwards on stilts. Find the minimum height of the stilts required to ensure the Fun Factor can be calculated at any point on the rollercoaster. (1 mark)

$\therefore$ min height $= 4m$



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Contour Checklist

☐ Learning Objective: [2.3.1] – Restrict Domain Such That the Inverse Function Exists

Key Takeaways

- ☐ A function must be 1:1 for the inverse function to exist.

☐ Learning Objective: [2.3.2] – Figure Out Possible Rule of a Graph

Key Takeaways

- ☐ If the question is Tech-Active, make the scale of your graph the same as the question.
- ☐ Get to the correct answer through elimination.

☐ Learning Objective: [2.3.3] – Solve Number of Solution Problems Graphically

Key Takeaways

- ☐ Solutions to $f(x) = k$ are the intersection points between $y = f(x)$ and $y = k$.

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