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VCE Mathematical Methods ½

Functions & Relations Exam Skills [2.3]

Homework Solutions

Homework Outline:

Compulsory Questions	Pg 2 - Pg 28
Supplementary Questions	Pg 29 - Pg 59

Section A: Compulsory Questions

Sub-Section [2.3.1]: Restrict Domain Such that the Inverse Function Exists

Question 1

For each of the following functions, a domain restriction is given with an endpoint a or b. Determine the minimum value of a or maximum value of b such that the inverse function, f^{-1}, exists.

a. $f: [a, \infty) \to \mathbb{R}, f(x) = (x-3)^2 + 2$

$a = 3$

b. $f: (-\infty, b] \to \mathbb{R}, f(x) = -x^2 + 6x - 5$

$f(x) = -(x-3)^2 + 4$ and so $b = 3$

c. $f: [a, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 3$

$f(x) = (x-2)^2 - 1$ and so $a = 2$

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Question 2

All the functions in this question are trunci written in a non-standard form.

a. Consider the function:

$$f : (a, \infty) \to \mathbb{R}, \quad f(x) = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$

Find the minimum value of a such that $f(x)$ has an inverse.

$f(x) = 2 + \frac{3}{(x-1)^2}$. So $a = 1$

b. Consider the function:

$$g : (-\infty, a) \to \mathbb{R}, \quad g(x) = \frac{x^2 - 6x + 11}{x^2 - 6x + 9}$$

Find the maximum value of a such that $g(x)$ has an inverse.

$f(x) = 1 + \frac{2}{(x-3)^2}$. So $a = 3$

c. Consider the function:

$$h : (a, \infty) \to \mathbb{R}, \quad h(x) = \frac{2x^2 + 8x + 5}{x^2 + 4x + 4}$$

Find the minimum value of a such that $h(x)$ has an inverse.

$f(x) = 2 - \frac{3}{(x+2)^2}$. So $a = -2$

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Question 3

For each of the following semicircle functions, a domain restriction is given with an endpoint a.

Determine the minimum or maximum value of a such that the inverse function exists.

a. Consider the semicircle function:

$$f : [a, 3] \to \mathbb{R}, \qquad f(x) = \sqrt{9 - x^2}$$

Find the minimum value of a such that $f(x)$ has an inverse.

$r = 3$ so $a = 0$.

b. Consider the semicircle function:

$$g : [-2, a] \to \mathbb{R}, \qquad g(x) = \sqrt{12 + 4x - x^2} + 1$$

Find the maximum value of a such that $g(x)$ has an inverse.

$g(x) = \sqrt{16 - (x-2)^2} + 1$.
$r = 4$ so $a = -2 + 4 = 2$

c. Consider the semicircle function:

$$h : [a, 4] \to \mathbb{R}, \qquad h(x) = \sqrt{24 - 2x - x^2} + 3$$

Find the minimum value of a such that $h(x)$ has an inverse.

$h(x) = \sqrt{25 - (x+1)^2} + 3.$
$r = 5$ and so $a = 4 - 5 = -1$

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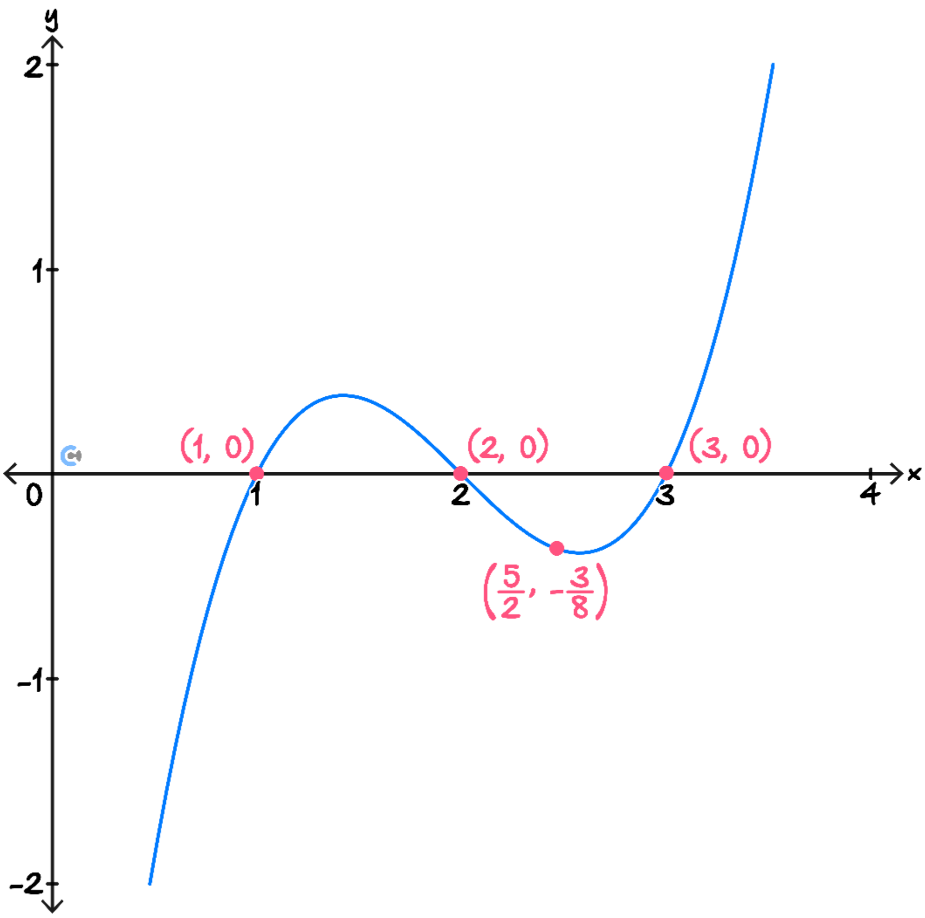
Sub-Section [2.3.2]: Figure out Possible Rule of a Graph

Question 4

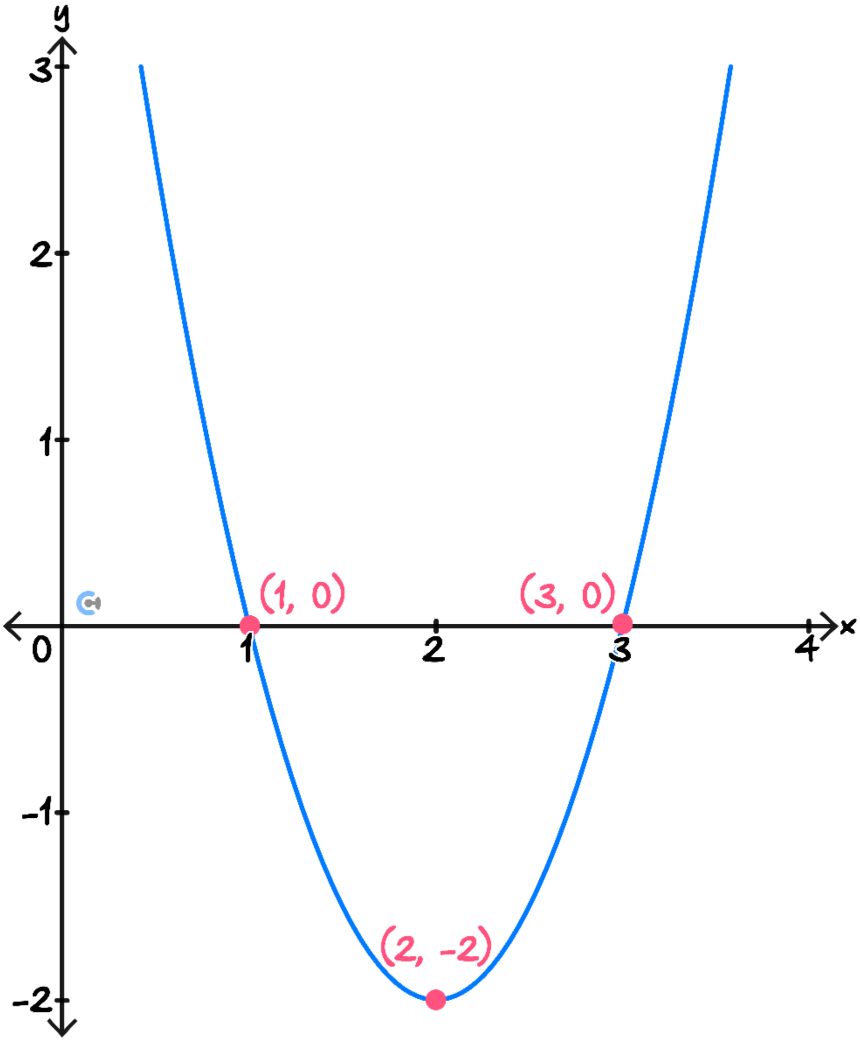
Determine a possible rule for the following graphs:

a.



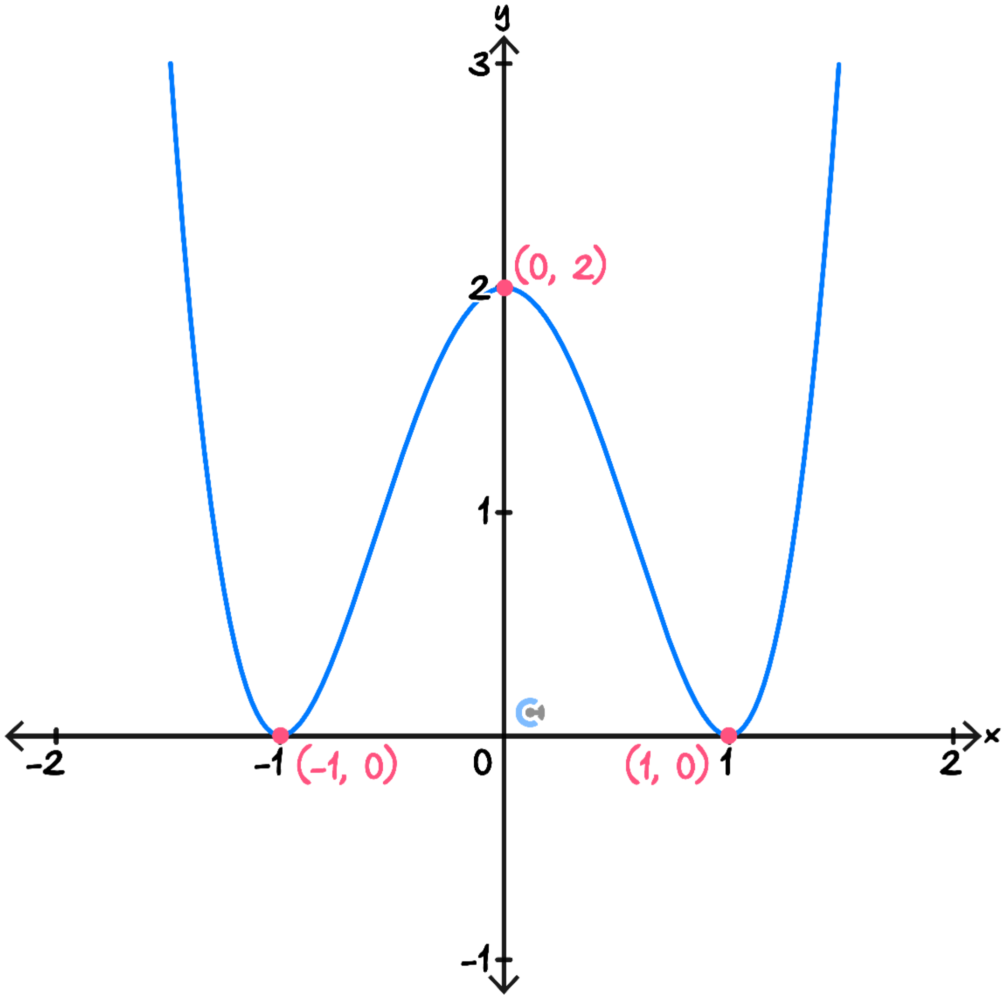
$$f(x) = (x-1)(x-2)(x-3)$$

b.



$$f(x) = 2(x-1)(x-3)$$

c.



$$f(x) = 2(x-1)^2(x+1)^2$$

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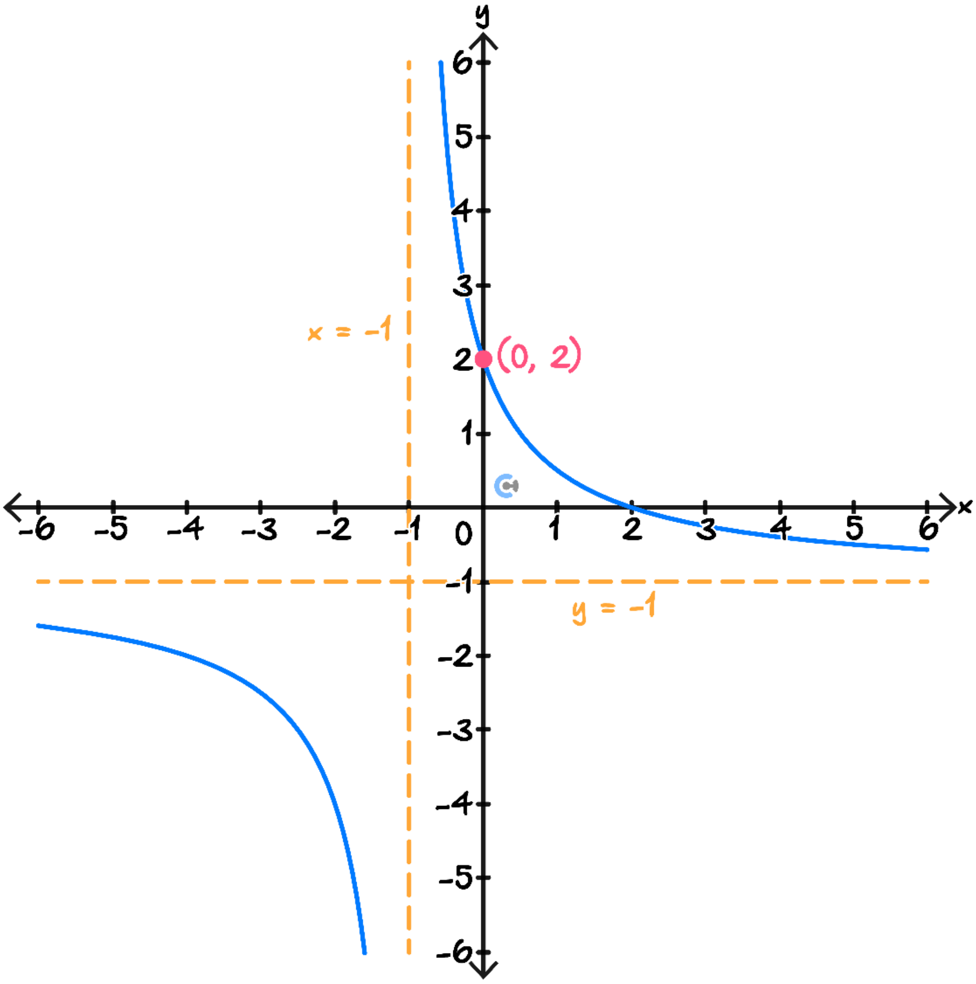
Question 5

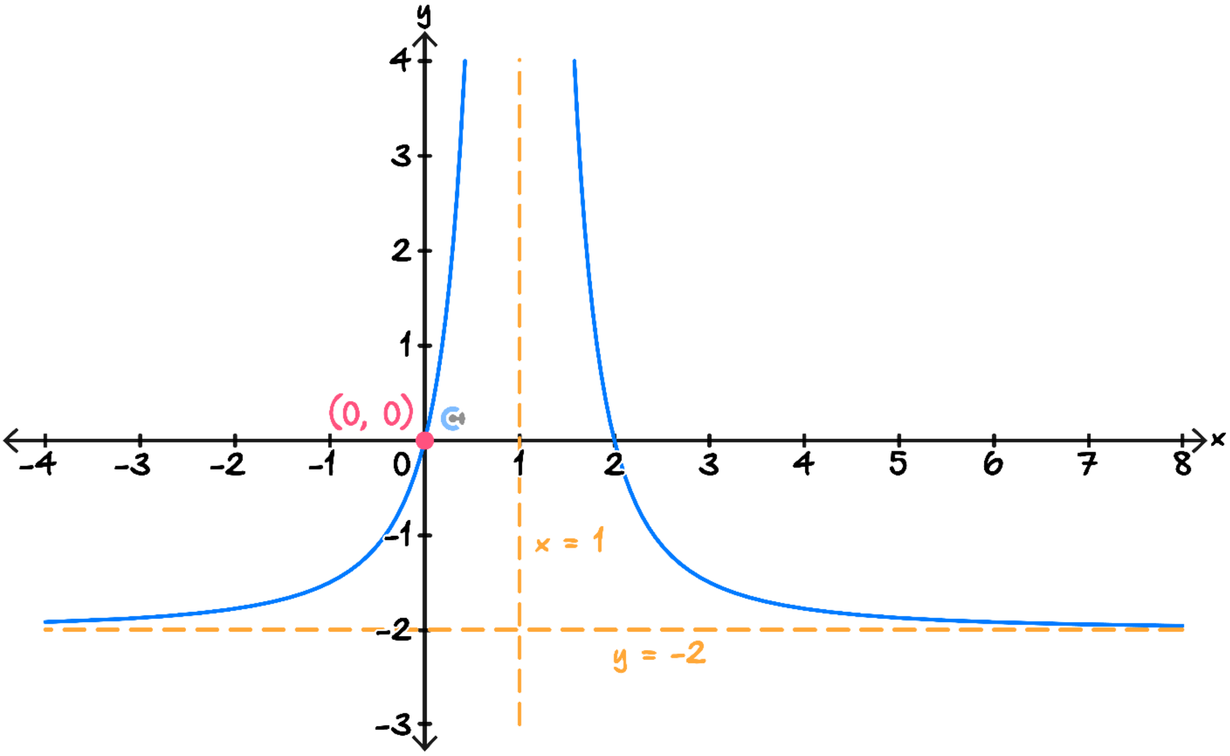
Determine a possible rule for the graphs.

a.



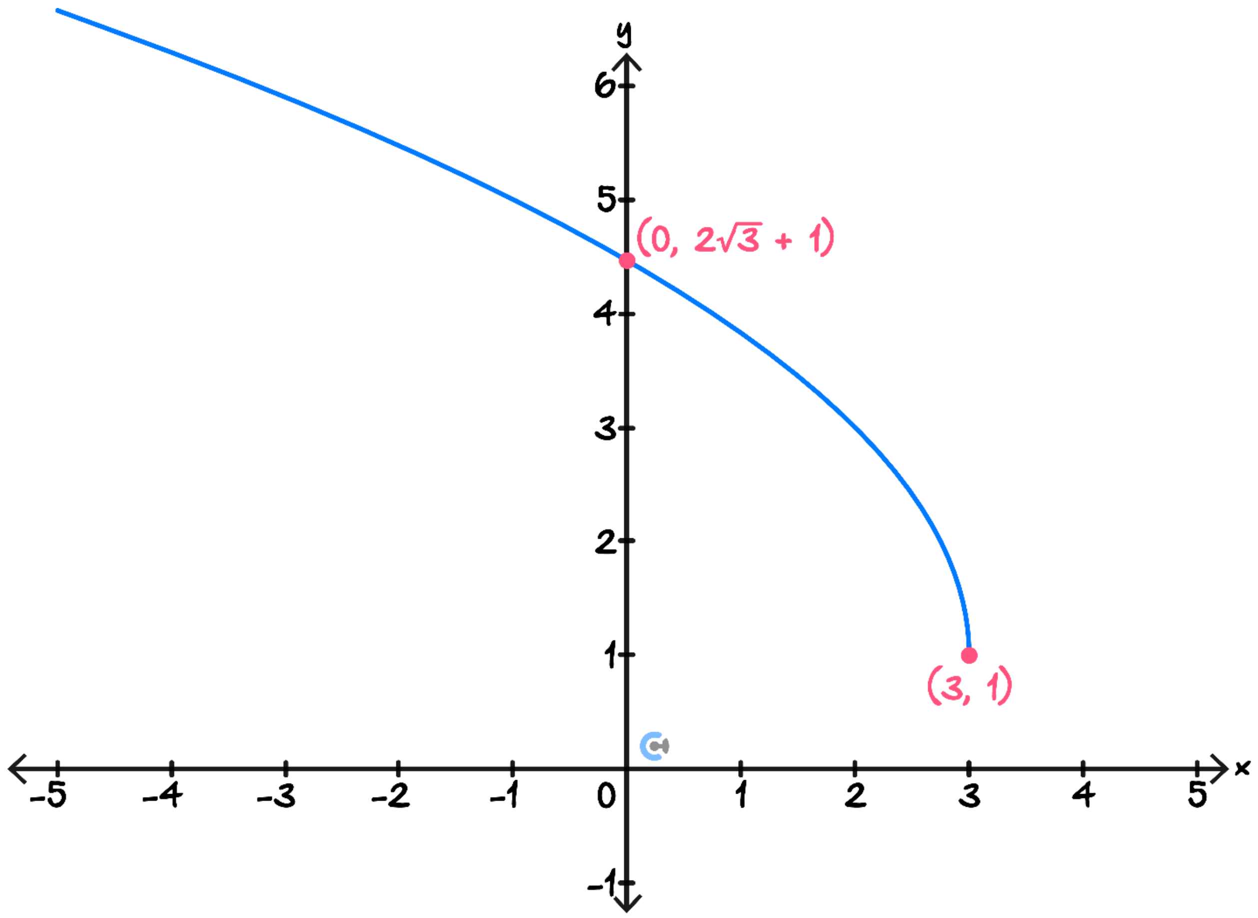
$$f(x) = \frac{3}{x+1} - 1$$

b.



$$f(x) = \frac{2}{(x-1)^2} - 2$$

c.



$$f(x) = 2\sqrt{3 - x} + 1$$

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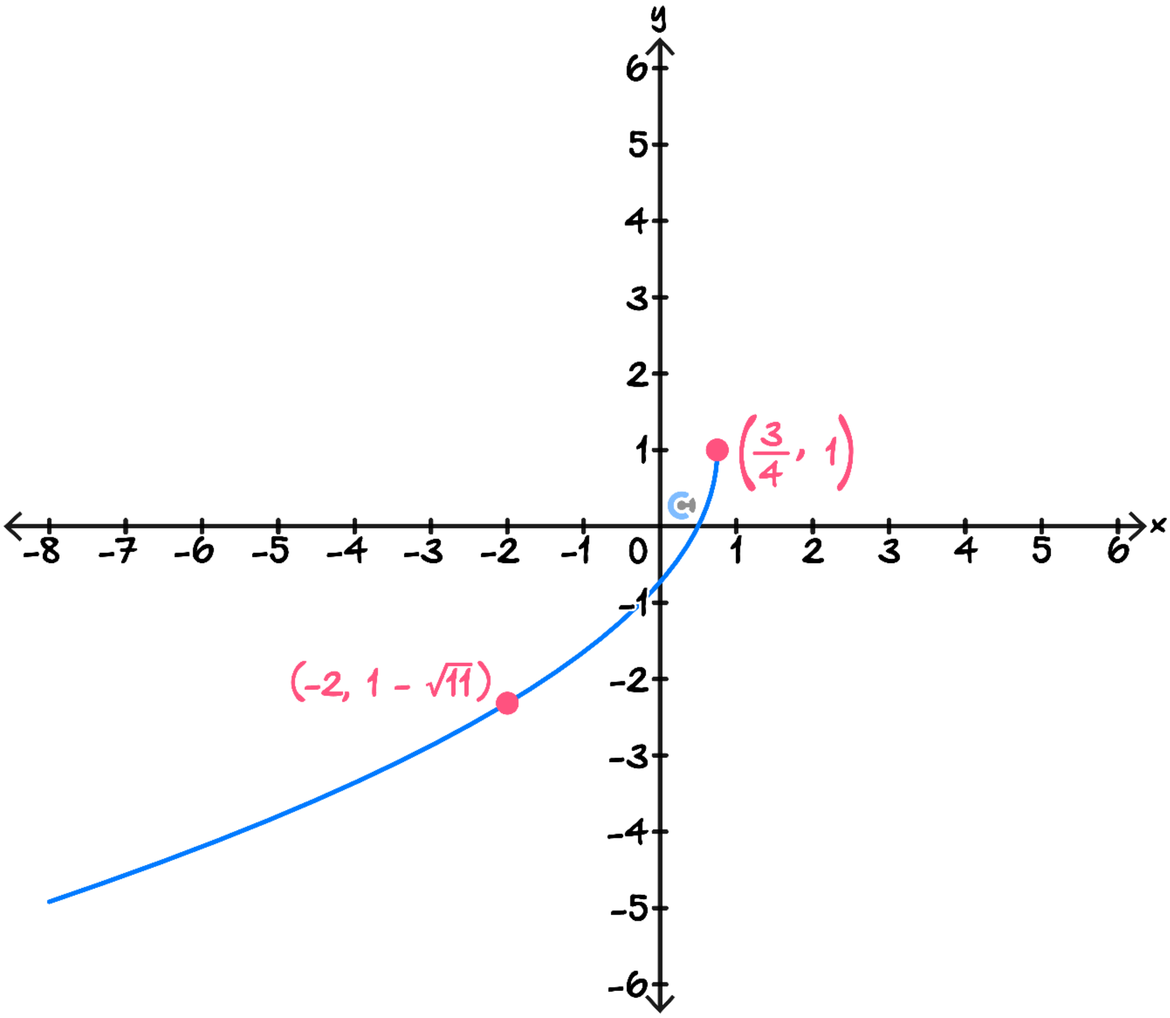
Question 6

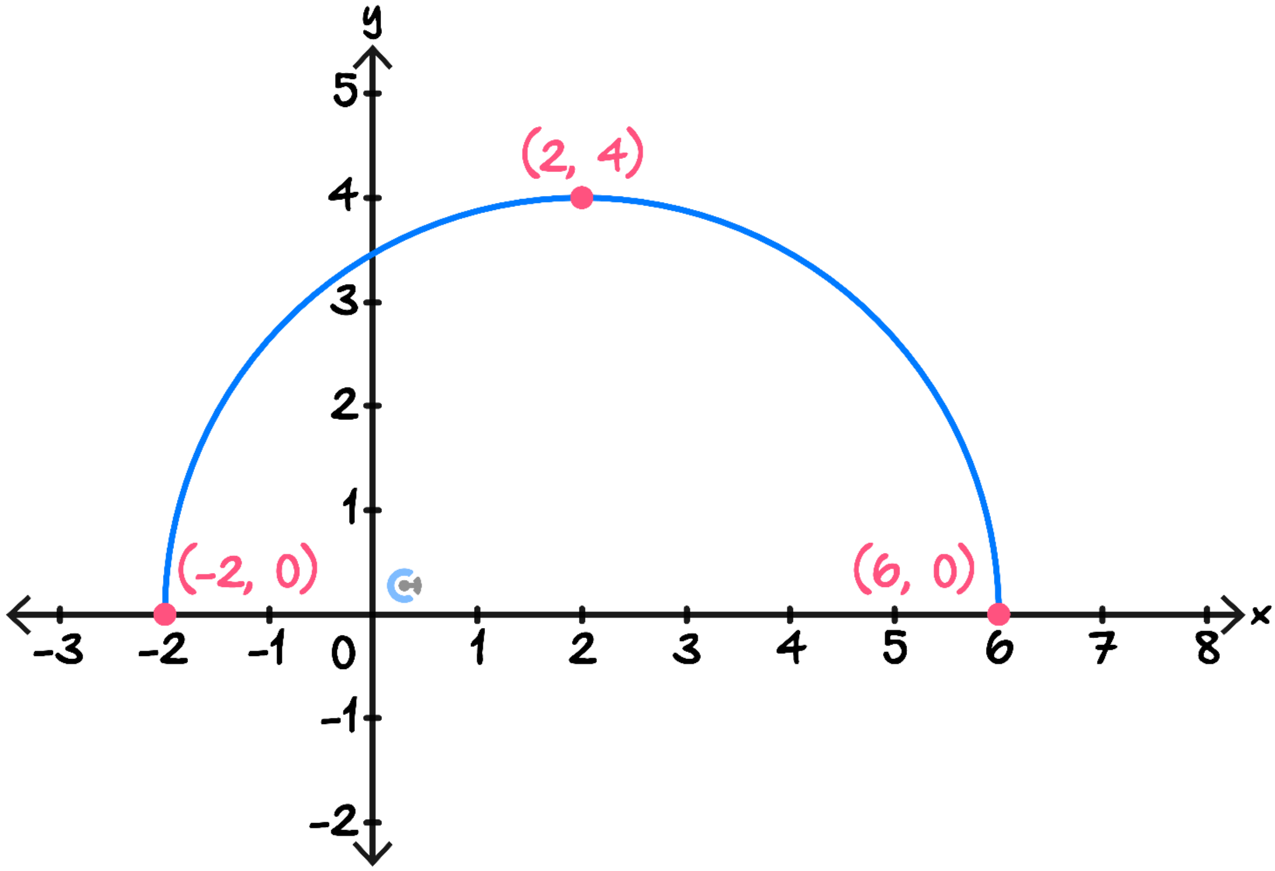
Determine a possible rule for the graphs.

a.



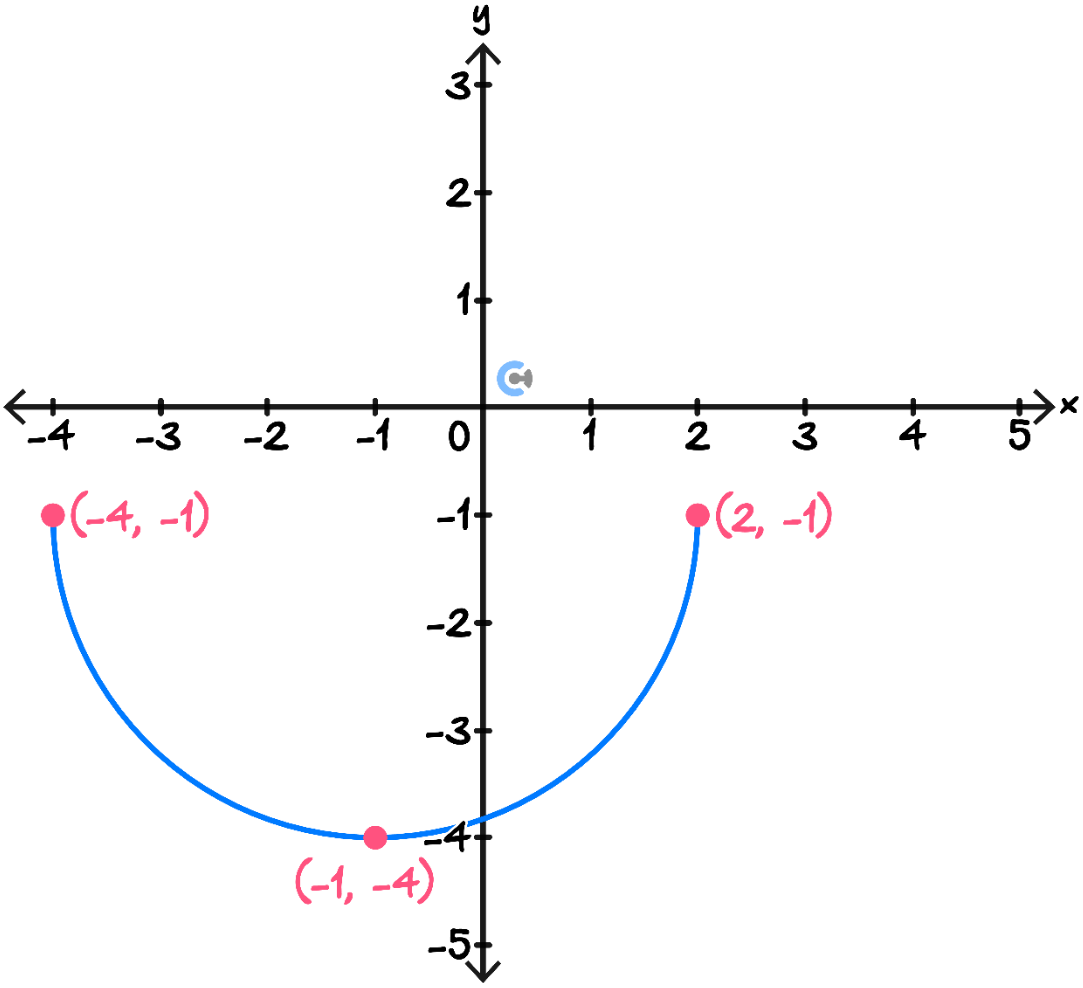
$$f(x) = -\sqrt{3-4x} + 1 = -2\sqrt{\frac{3}{4} - x} + 1$$

b.



$$f(x) = \sqrt{16 - (x-2)^2}$$

c.



$$f(x) = -\sqrt{9 - (x+1)^2} - 1$$

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Sub-Section [2.3.3]: Solve Number of Solution Problems Graphically

Question 7 Tech-Active.

Consider the function $f(x) = 2x^2 - 5x - 7$.

Determine the real values of k for which $f(x) = k$ has two solutions.

Complete the square/graph to see $k > -\frac{81}{8}$.

Question 8 Tech-Active.

Consider the function $f(x) = 2x^3 - 12x^2 + 18x + 4$.

Determine the real values of k for which $f(x) = k$ has three solutions.

Graphing the function we see turning points at $(1, 12)$ and $(3, 4)$ and so from the shape we conclude that $4 < k < 12$.

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Question 9 Tech-Active.

Consider the function $f(x) = 3x^4 - 12x^3 - 6x^2 + 36x + 4$.

Determine the real values of k for which $f(x) = k$ has two solutions.

Graph the function and see turning points at $(-1, 23), (1, 25)$ and $(3, -23)$. Looking at the shape we conclude that $k > 25$ or $k = -23$ for two solutions.

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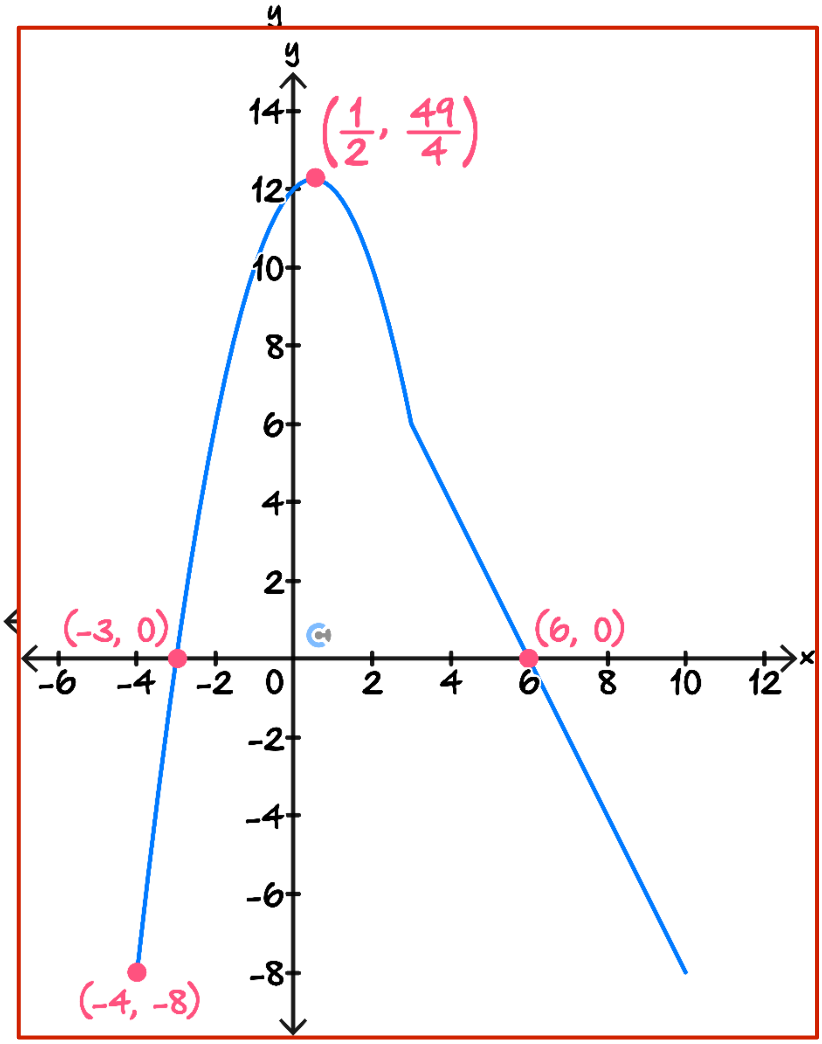
Sub-Section: Exam 1 Questions

Question 10

Consider the function:

$$f(x) = \begin{cases} -x^2 + x + 12, & -4 \leq x \leq 3 \\ 12 - 2x, & x > 3 \end{cases}$$

a. Sketch the graph of $y = f(x)$ on the axes below. Label all intercepts, endpoints, and turning points.



b. State the range of $f(x)$.

$$\left(-\infty, \frac{49}{4}\right]$$

c. Determine the range of values of k for which $f(x) = k$ has two solutions.

$$k \in \left[-4, \frac{49}{4}\right)$$

d. Justify whether or not $f^{-1}(x)$ is defined.

$f^{-1}(x)$ is not defined because $f(x)$ is a many to one function.

e. Find the maximal domain of the equation $y = \log_2(f(x))$.

We require $f(x) > 0$. Thus $x \in (-3, 6)$

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Question 11

Consider the function $f : D \to \mathbb{R}, f(x) = \frac{2x+1}{x-2}$.

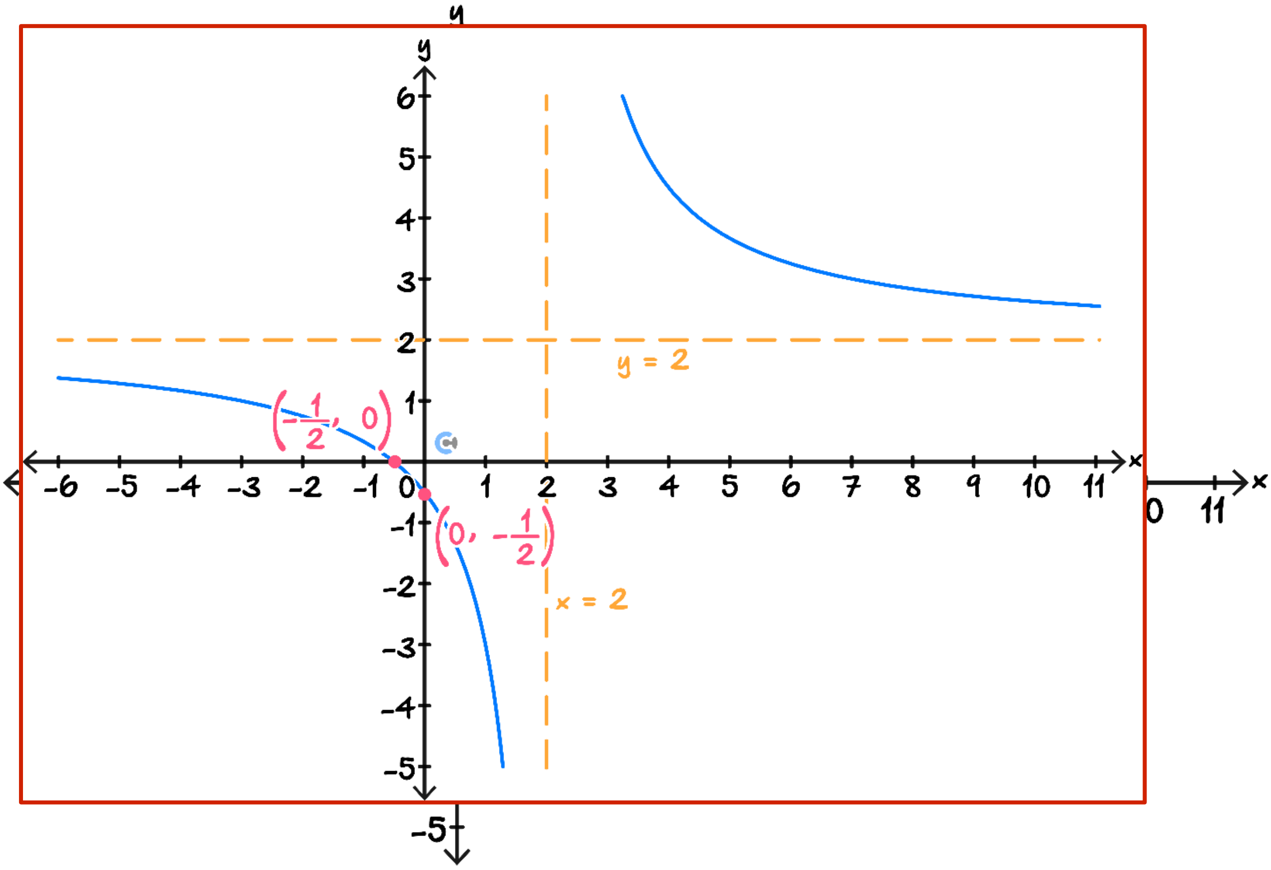
a. State the maximal domain D of f.

$D = \mathbb{R} \setminus \{2\}$.

b. Express f in the form $a + \frac{b}{x-2}$ and state the values of a and b.

$f(x) = \frac{2(x-2)+5}{x-2} = 2 + \frac{5}{x-2}$.
$a = 2$ and $b = 5$.

c. Sketch the graph of $y = f(x)$ on the axes below. Label asymptotes with their equations and axes intercepts with the coordinates.



d. Find the values of x for which $f(x) \geq 1$.

$x \in (-\infty, -3] \cup (2, \infty)$

e.

i. Determine the rule for $f^{-1}(x)$, the inverse of $f(x)$.

Let $y = f^{-1}(x)$, then

$$x = 2 + \frac{5}{y-2} \implies y - 2 = \frac{5}{x-2} \implies y = \frac{5}{x-2} + 2$$

so $f^{-1}(x) = 2 + \dfrac{5}{x-2}$.

ii. Hence, determine the x-value for all points of intersection between $f(x)$ and $f^{-1}(x)$.

f is its own inverse. So intersection for all $x \in \mathbb{R} \setminus \{2\}$.

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Question 12

Consider the function $f : [1, \infty) \to \mathbb{R}, f(x) = x^2 - 2x + k$.

a. Define the inverse function of f.

$f(x) = (x-1)^2 + k - 1$. Swap x and y.

$$x = (y-1)^2 + k - 1 \implies y - 1 = \pm\sqrt{x + 1 - k}$$

ran $f^{-1} = \text{dom } f = [1, \infty)$. So

$$f^{-1} : [k-1, \infty) \to \mathbb{R},\ f^{-1}(x) = \sqrt{x + 1 - k} + 1$$

b. Determine the values of k for which f and f^{-1} do not intersect each other.

Consider solving $x^2 - 2x + k = x \implies x^2 - 3x + k = 0$. Now consider the discriminant.

$$\Delta = 9 - 4k < 0 \implies k > \frac{9}{4}$$

No intersection if $k > \frac{9}{4}$.

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Sub-Section: Exam 2 Questions

Question 13

The maximal domain of the function $f(x) = \frac{1}{\sqrt{x^2-x-6}}$ is:

A. $x \in (0, \infty)$

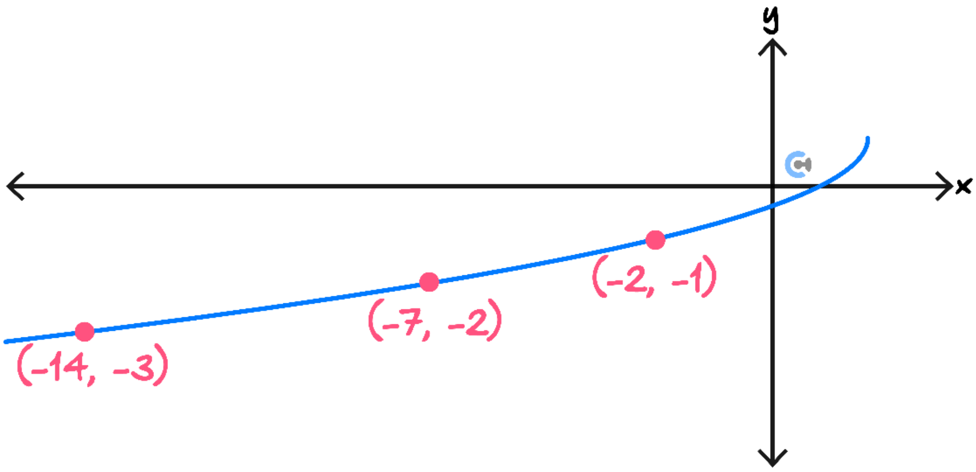
B. $x \in (-2, 3)$

C. $x \in (-\infty, 2] \cup [3, \infty)$

D. $x \in \mathbb{R}\backslash[-2, 3]$

Question 14

The most likely rule for the following graph is:



A. $\sqrt{3-x}+1$

B. $-\sqrt{2-x}+1$

C. $3\sqrt{x+1}-1$

D. $-3\sqrt{2-x}+1$

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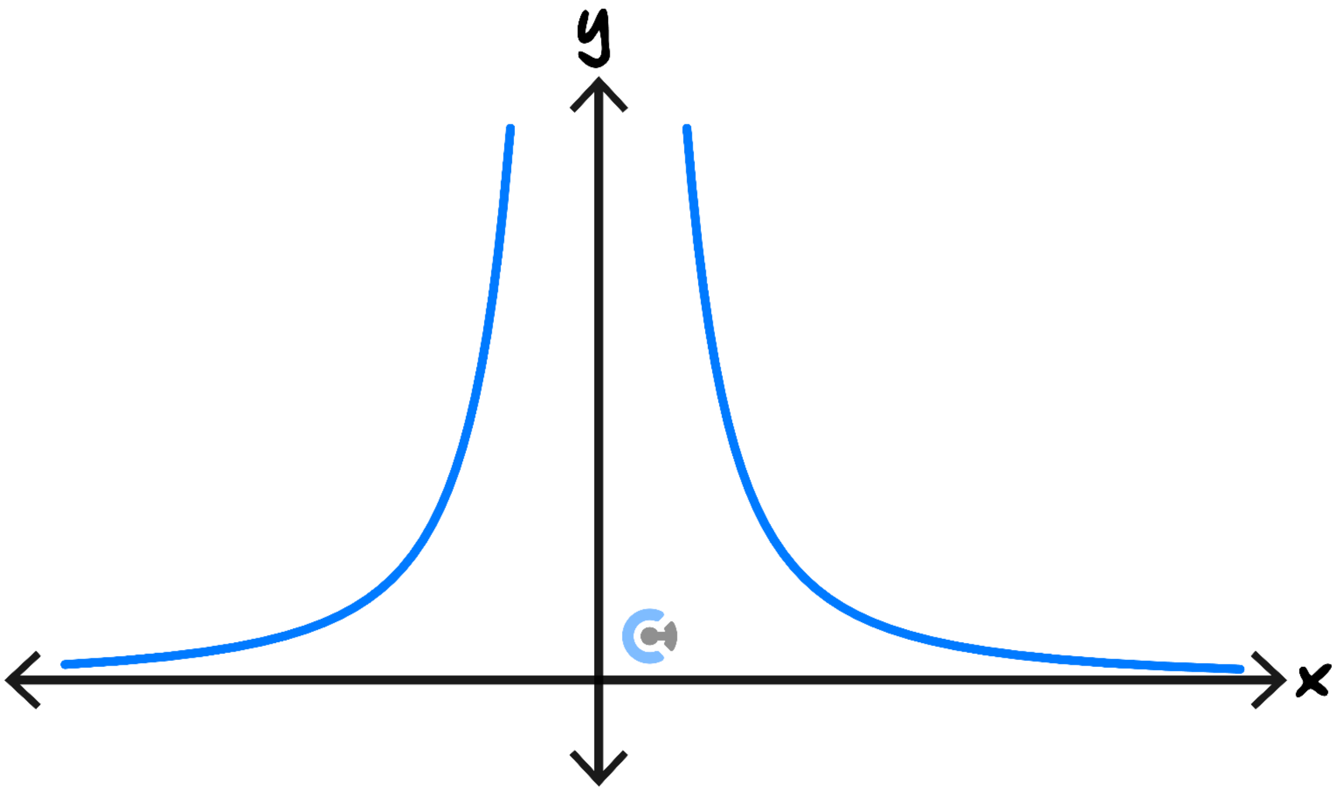
Question 15

The function g with graph shown below is best described as:



A. One-to-one.

B. One-to-many.

C. Many-to-one.

D. Many-to-many.

Question 16

The line with equation $4y + 3x = 25$ intersects the circle $x^2 + y^2 = 25$ exactly once at the point $P(3, 4)$. The equation for the radius of the circle that passes through P is:

A. $3y - 4x = 0$

B. $3y + 4x = 25$

C. $3y + 4x = 0$

D. $3y - 4x = 25$

Question 17

Consider the function $f(x) = 12x^5 + 90x^4 + 140x^3 - 180x^2 - 480x - 200$.

The equation $f(x) = k$ will have three solutions for:

A. $k \in (-618, -24) \cup (38, 632)$

B. $k \in [-618, -24] \cup [38, 632]$

C. $k \in (-24, 38)$

D. $k \in [24, 38]$

Question 18

The function f is defined as $f : [a, a+2] \to \mathbb{R}, f(x) = x^2 - 4x - 8$.

a. Find the turning point of $f(x)$.

$f(x) = (x-2)^2 - 12$.
Thus turning point is (2, -12).

b. Find the values of a such that:

i. The range of $f(x)$ is $[-8, 4]$.

First consider $f(a) = -8 \implies a = 0, 4$.
$f(0+2) = -12 \neq 4$ so reject $a = 0$.
$f(4+2) = 4$, therefore accept $a = 4$.
Now consider $f(a) = 4 \implies a = -2, 6$.
$f(-2+2) = -8$ therefore accept $a = -2$.
$f(6+2) = 24 \neq -8$, therefore reject $a = 6$.
Conclude that $a = -2$ or $a = 4$.

ii. The inverse function f^{-1} exists.

We need f to be a one to one function.
Consider when the endpoints are on the turning point.
$a = 0$ or $a = 2$.
Therefore f^{-1} exists for $x \in (-\infty, 0] \cup [2, \infty)$

iii. $\sqrt{f(x)}$ does not exist.

Does not exist when $f(x) < 0$ this occurs when $2 - 2\sqrt{3} < x < 2 + 2\sqrt{3}$.
Hence for $a \in (2 - 2\sqrt{3}, 2\sqrt{3})$

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Question 19

The line with equation $y = mx$ intersects the circle with centre $(4,0)$ and radius 2 exactly once at the point $P(x, y)$.

(**Note:** A line that intersects a circle exactly once is called a line that is tangent to the circle.)

a. Find the equation of the circle.

$(x - 4)^2 + y^2 = 4$

b. Show that the x-coordinate of the point P satisfies the equation:

$$(1 + m^2)x^2 - 8x + 12 = 0$$

Sub in $y = mx$ into the circle equation.

$$(x-4)^2 + (mx)^2 = 4$$
$$x^2 - 8x + 16 + m^2x^2 = 4$$
$$(1+m^2)x^2 - 8x + 12 = 0$$

c. Use the discriminant to find the possible values of m.

We want $(1+m^2)x^2 - 8x + 12 = 0$ to have only one solution. Therefore

$$\Delta = 64 - 48(1+m^2) = 0 \implies m = \pm\frac{1}{\sqrt{3}}$$

d. Hence, find the two possible sets of coordinates for P.

Sub in $y = \pm\frac{1}{\sqrt{3}}x$ into the equation of the circle to find the points of intersection.
When $m = \frac{1}{\sqrt{3}}$ intersection at $(3, \sqrt{3})$.
When $m = -\frac{1}{\sqrt{3}}$ intersection at $(3, -\sqrt{3})$.

e. Find the distance of P from the origin.

$$d = \sqrt{9+3} = 2\sqrt{3}$$

f. Find the acute angle that the two lines tangent to the circle make at the origin.

The line $y = \frac{1}{\sqrt{3}}x$ makes an angle of 30° with the positive x-axis. By symmetry the angle made by the two tangents at the origin is 60°.

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Section B: Supplementary Questions

Sub-Section [2.3.1]: Restrict Domain Such that the Inverse Function Exists

Question 20

For each of the following functions, a domain restriction is given with an endpoint a or b. Determine the minimum value of a or maximum value of b such that the inverse function, f^{-1}, exists.

a. $f: (-\infty, b] \to \mathbb{R}, f(x) = (x+1)^2 - 3$

$a = -1$

b. $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 7$

$f(x) = (x-2)^2 + 3$ and so $a = 2$

c. $f : [a, \infty) \to \mathbb{R}, f(x) = -x^2 + 8x - 11$

$f(x) = 5 - (x-4)^2$ and so $a = 4$

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Question 21

All the functions in this question are trunci written in a non-standard form.

a. Consider the function:

$$f : (-\infty, a) \to \mathbb{R}, \quad f(x) = \frac{11 + 12x + 3x^2}{x^2 + 4x + 4}$$

Find the maximum value of a such that $f(x)$ has an inverse.

$f(x) = 3 - \frac{1}{(x+2)^2}$. So $a = -2$

b. Consider the function:

$$g : (a, \infty) \to \mathbb{R}, \quad g(x) = \frac{x^2 + 8x + 18}{x^2 + 8x + 16}$$

Find the minimum value of a such that $g(x)$ has an inverse.

$f(x) = 1 + \frac{2}{(x+4)^2}$. So $a = -4$

c. Consider the function:

$$h : (a, \infty) \to \mathbb{R}, \quad h(x) = \frac{3x^2 + 6x - 2}{x^2 + 2x + 1}$$

Find the minimum value of a such that $h(x)$ has an inverse.

$f(x) = -3 + \frac{5}{(x+1)^2}$. So $a = -1$

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Question 22

For each of the following semicircle functions, a domain restriction is given with an endpoint a.

Determine the minimum or maximum value of a such that the inverse function exists.

a. Consider the semicircle function:

$$f : [-4, a] \to \mathbb{R}, f(x) = \sqrt{4 - (x+2)^2}$$

Find the minimum value of a such that $f(x)$ has an inverse.

$r = 4$ so $a = -2.$

b. Consider the semicircle function:

$$g : [a, 4] \to \mathbb{R}, \qquad g(x) = 2 - \sqrt{8 + 2x - x^2}$$

Find the maximum value of a such that $g(x)$ has an inverse.

$g(x) = 2 - \sqrt{9 - (x-1)^2}.$

$r = 3$ so $a = 4 - 3 = 1$

c. Consider the semicircle function:

$$h : [-5, a] \to \mathbb{R}, \qquad h(x) = \sqrt{20 - 16x - 4x^2} + 1$$

Find the maximum value of a such that $h(x)$ has an inverse.

$h(x) = 2\sqrt{9 - (x+2)^2} + 1.$
So $a = -2$

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Question 23

Consider the function:

$$f : [a, \infty) \to \mathbb{R}, \qquad f(x) = \frac{2x^2 + 8x + 11}{5 + 4x + x^2}$$

Find the maximum value of a such that $f(x)$ has an inverse.

$f(x) = 2 + \dfrac{1}{(x+2)^2 + 1}$, so $a = -2$.

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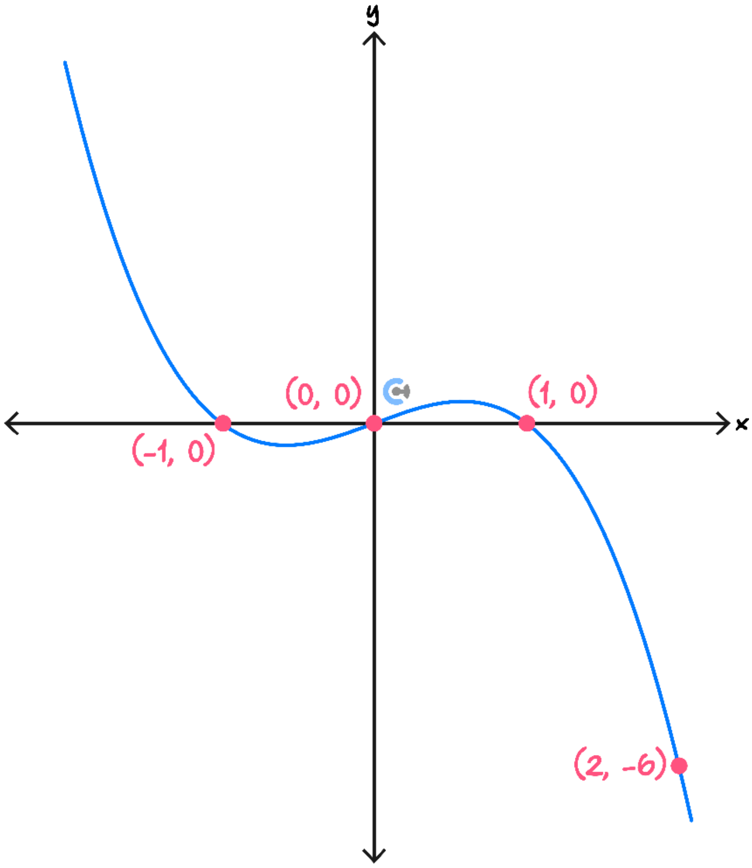
Sub-Section [2.3.2]: Figure Out Possible Rule of a Graph

Question 24

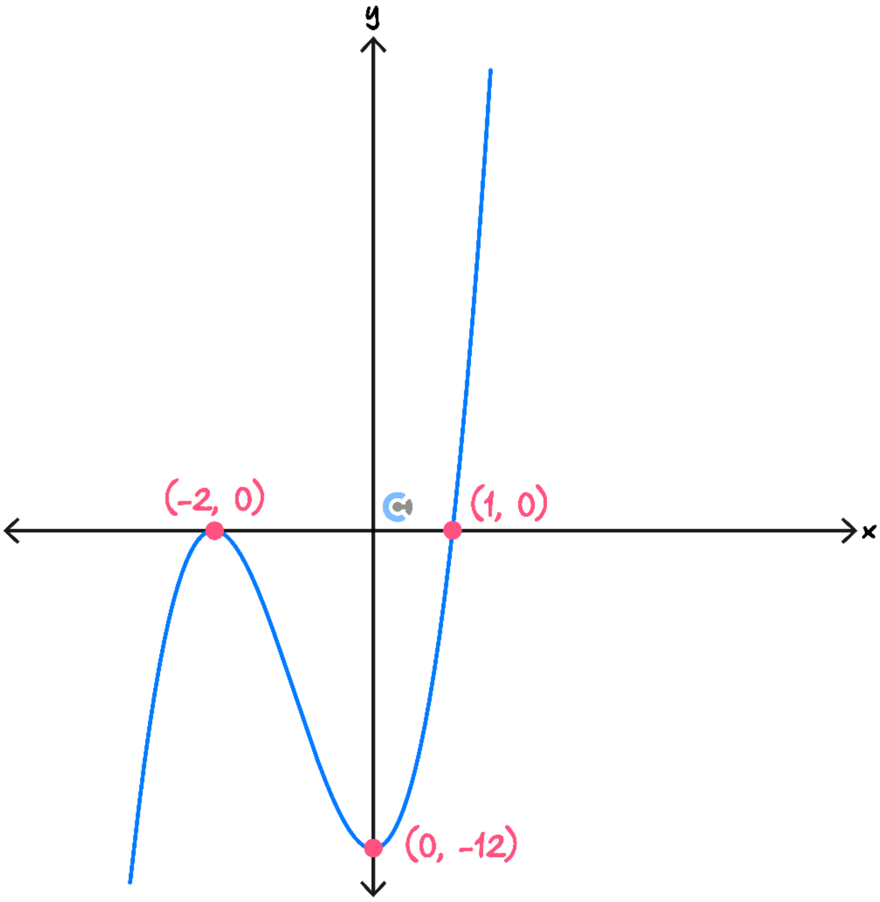
Determine a possible rule for the following graphs:

a.



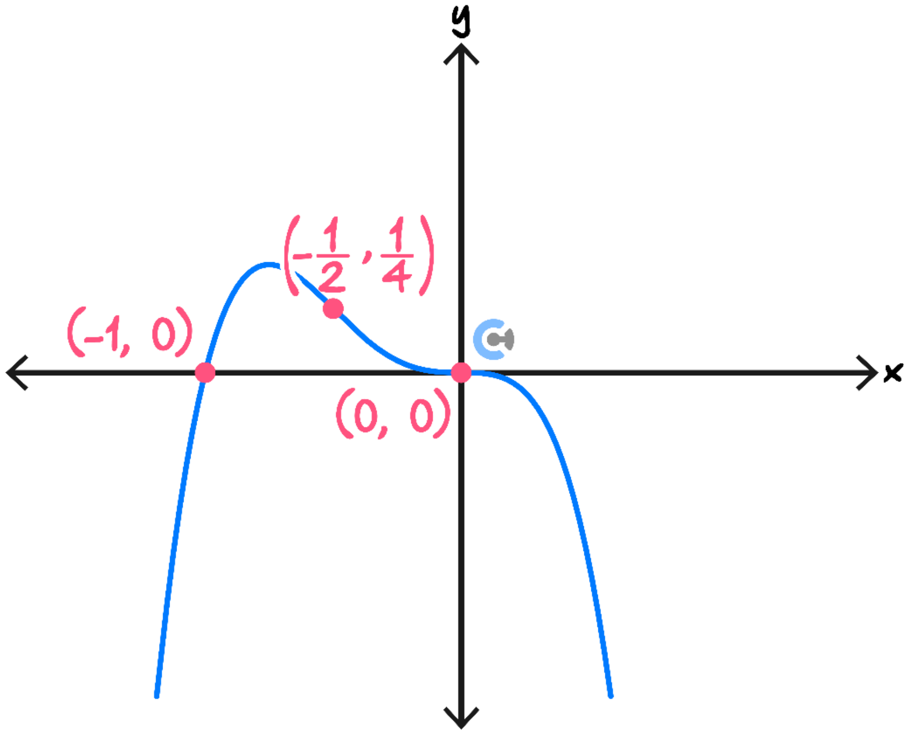
$$f(x) = -x(x-1)(x+1)$$

b.



$$f(x) = 3(x+2)^2(x-1)$$

c.



$$f(x) = -4x^3(x+1)$$

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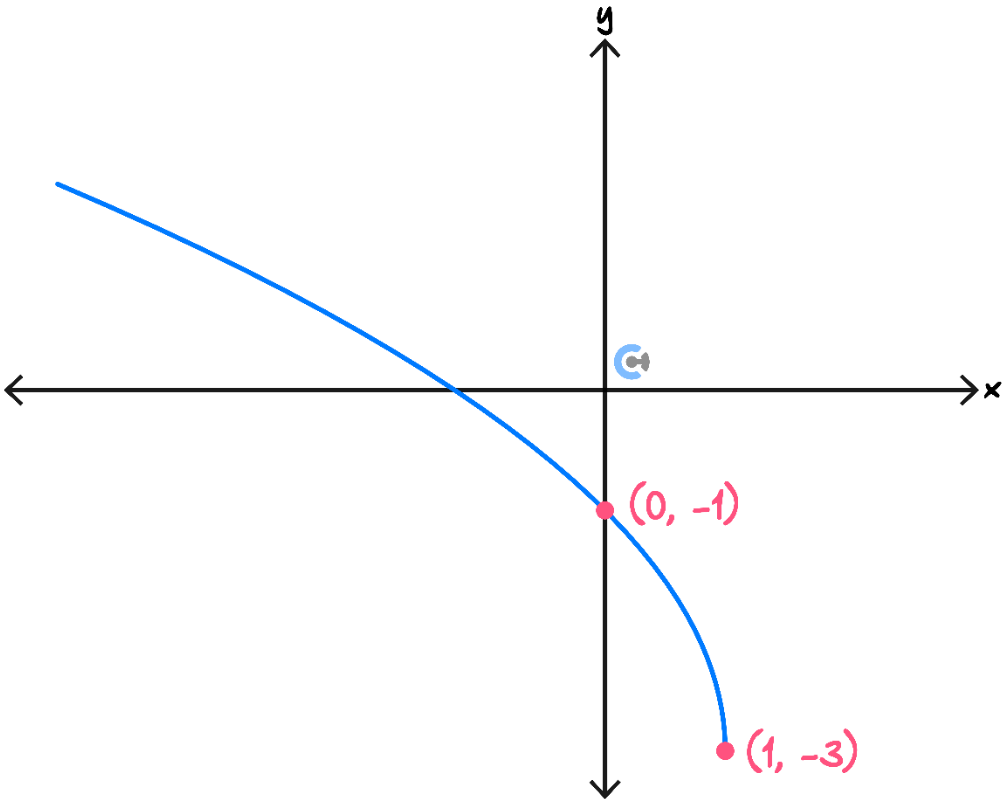
Question 25

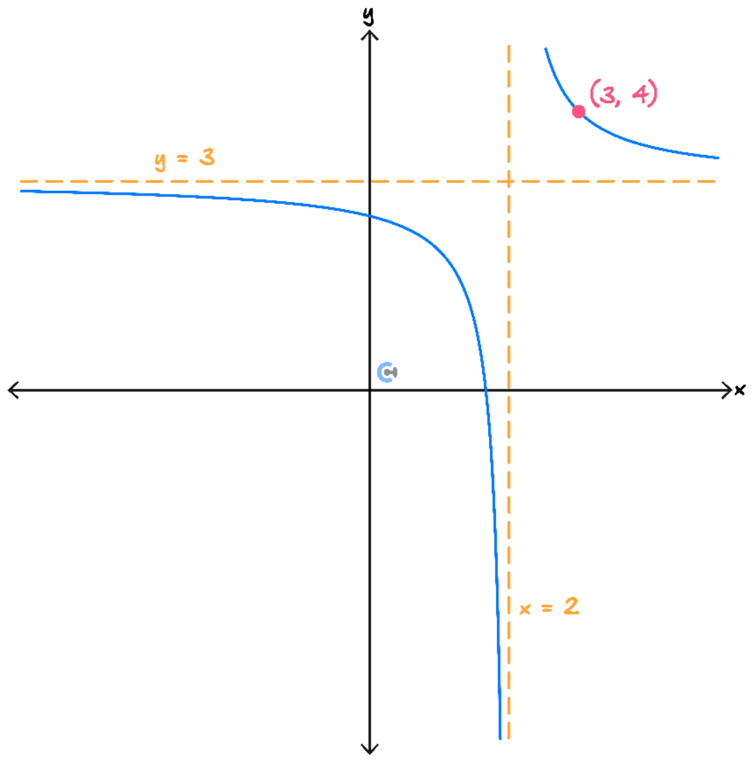
Determine a possible rule for the graphs.

a.



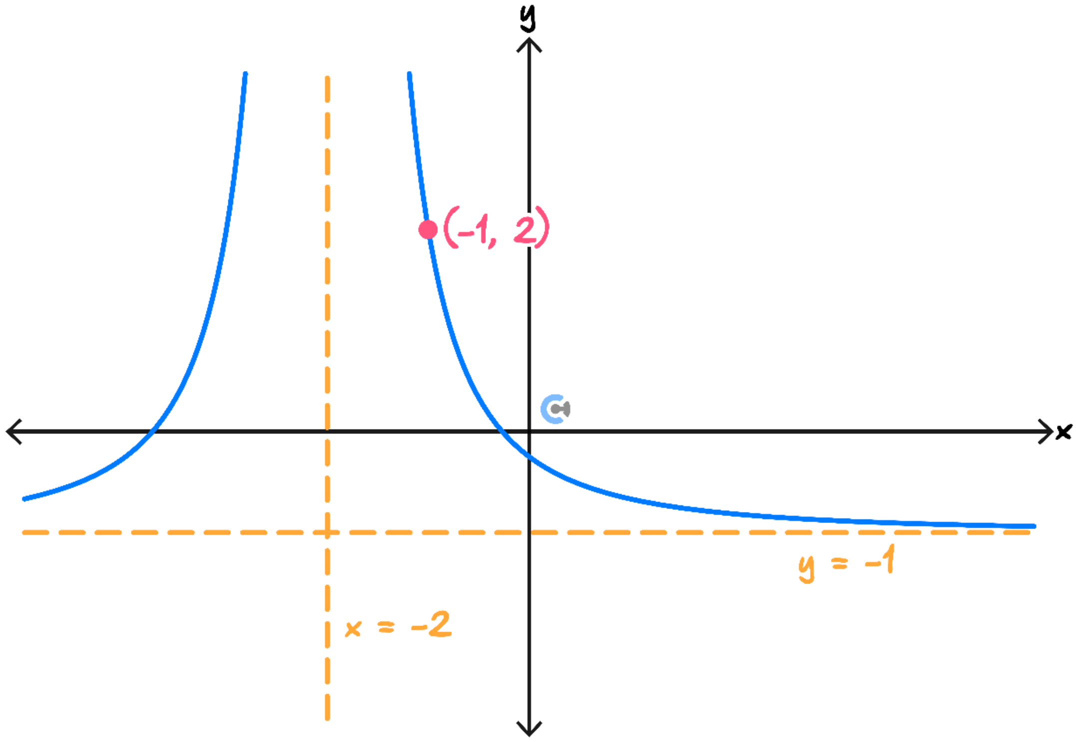
$$f(x) = 2\sqrt{1-x} - 3$$

b.



$$f(x) = 3 - \frac{1}{2 - x}$$

c.



$$f(x) = \frac{3}{(x+2)^2} - 1$$

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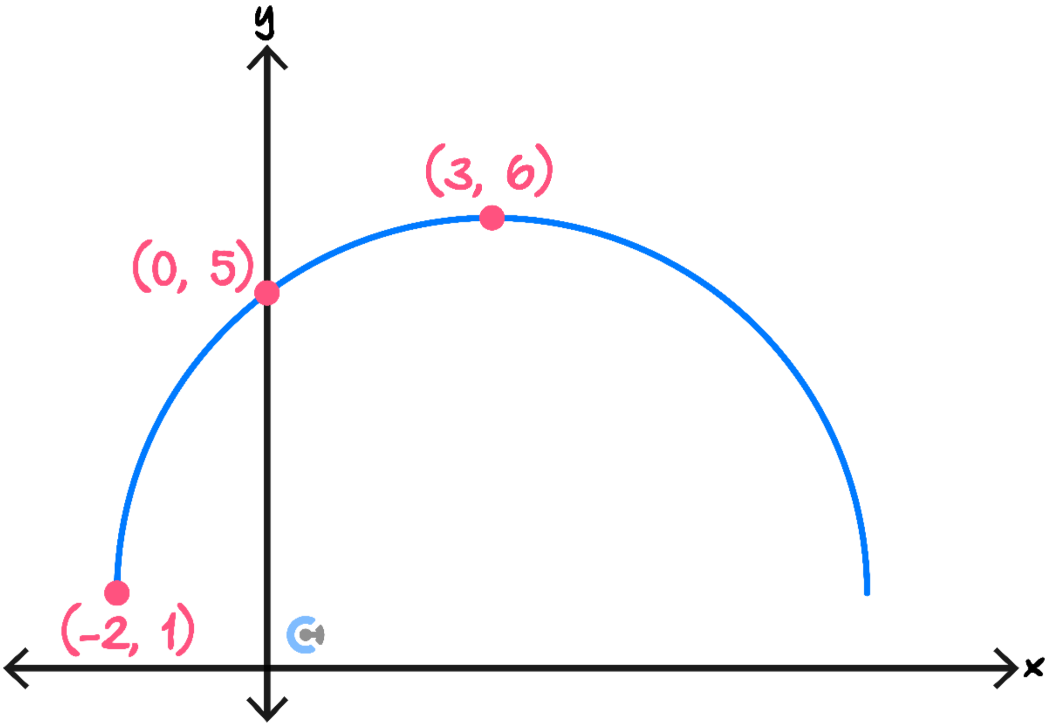
Question 26

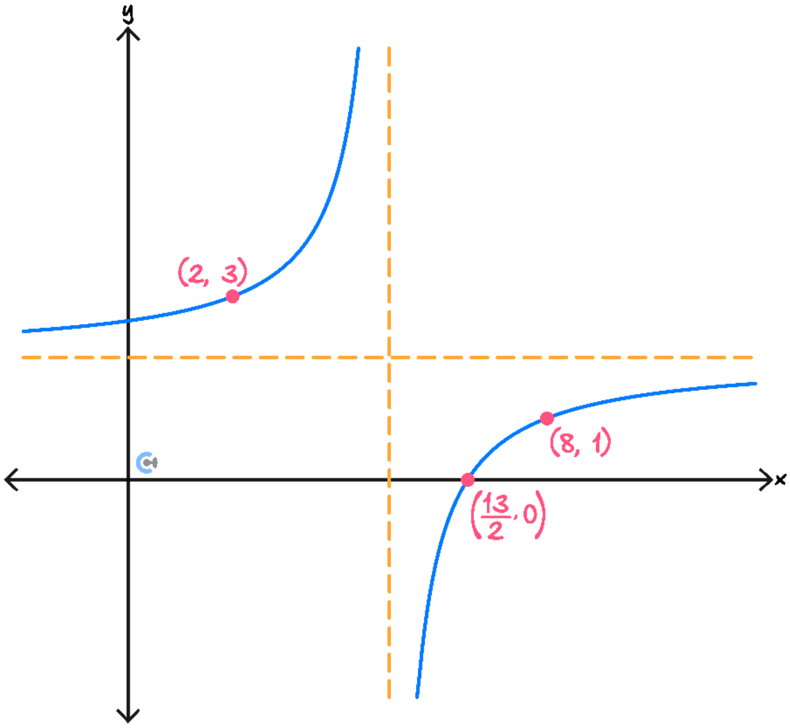
Determine a possible rule for the graphs.

a.



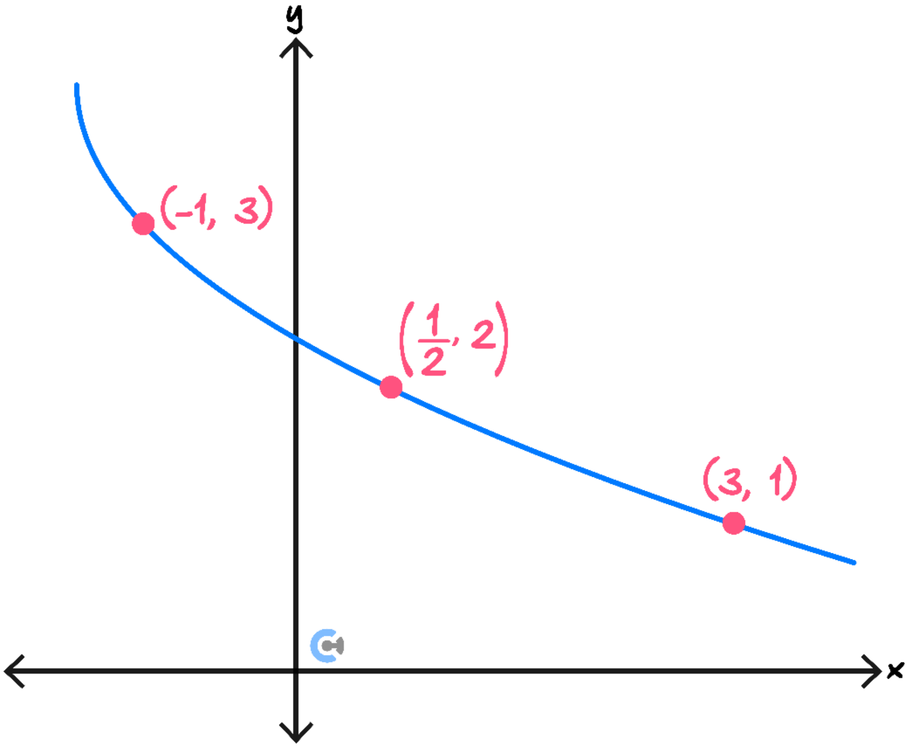
$$f(x) = \sqrt{25 - (x-3)^2} + 1$$

b.



$$f(x) = 2 - \frac{3}{x-5}$$

c.



$$f(x) = 4 - \sqrt{3 + 2x}$$

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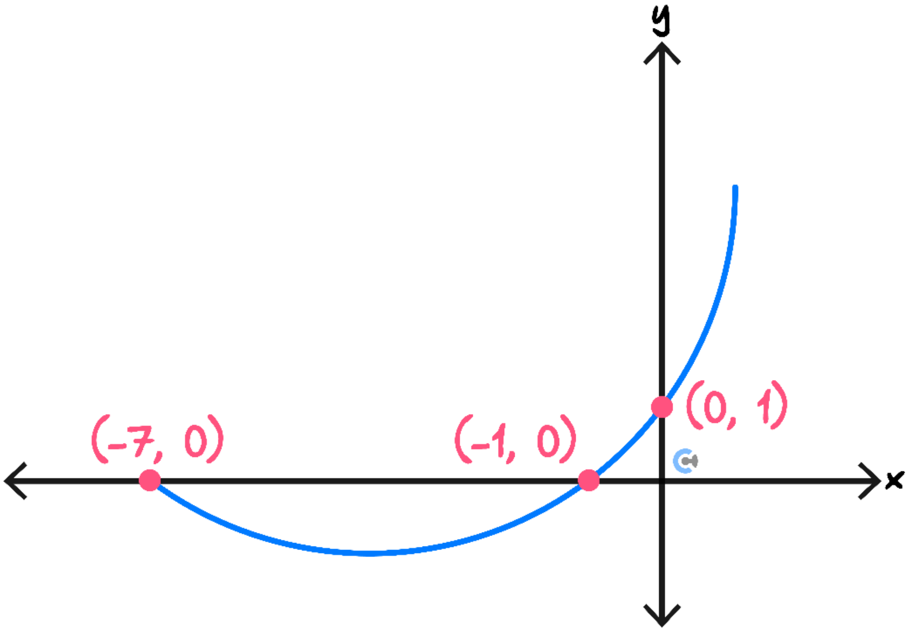
Question 27

Determine a possible function for this graph.



$$f : [-7, 1] \to \mathbb{R}, f(x) = 4 - \sqrt{25 - (x + 4)^2}$$

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Sub-Section [2.3.3]: Solve Number of Solution Problems Graphically

Question 28 Tech-Active.

Consider the function $f(x) = 4x^2 - 4x + 5$.

Determine the real values of k for which $f(x) = k$ has two solutions.

Complete the square / graph to see that $k > 4$

Question 29 Tech-Active.

Consider the function $f(x) = x^3 + 3x^2 - 9x + 2$.

Determine the real values of k for which $f(x) = k$ has three:

a. Two solutions.

Graphing the function we see turning points at $(-3, 29)$ and $(1, -3)$, and so from the shape we conclude that $k = -3, 29$.

b. Three solutions.

$-3 < k < 29$

Question 30 Tech-Active.

Consider the function $f(x) = x^4 - 8x^3 + 6x^2 + 40x - 14$.

Determine the real values of k for which $f(x) = k$ has :

a. Three solutions.

Graphing the function we see turning points at $(-1, -39), (5, -39)$ and $(2, 42)$, and so from the shape we conclude that $k = 42$.

b. Two solutions.

$k = -39$ or $k > 42$.

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Question 31

Consider the function $f(x) = 3x^3 + k$.

Determine the real value of k for which $f(x) = f^{-1}(x)$ has three solutions.

A solution to $f(x) = f^{-1}(x)$ will lie on the line $y = x$.
Hence we just need to solve $f(x) = x \implies 3x^3 - x = -k$.
From the graph of $3x^3 - x$ we see that the equation $3x^3 - x = -k$ has 3 solutions if $-\frac{2}{9} < k < \frac{2}{9}$.
Hence $f(x) = f^{-1}(x)$ has three solutions if $-\frac{2}{9} < k < \frac{2}{9}$.

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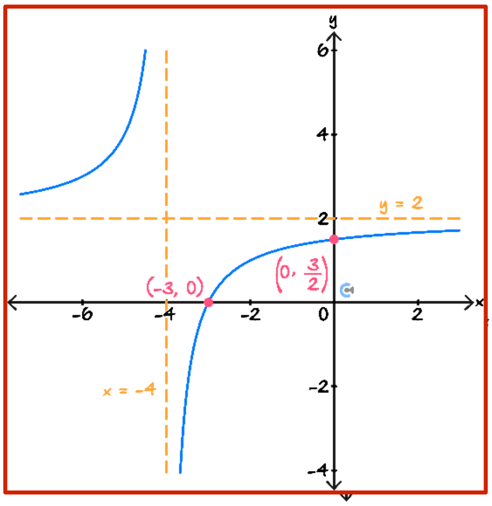
Sub-Section: Exam 1 Questions

Question 32

Let $f(x) = \frac{2x+6}{x+4}$ be defined on its maximal domain.

a. Sketch the graph of $f(x)$ on the axes below. Labelling all asymptotes with their equations and axial intercepts with their coordinates.



b. State the domain and range of f^{-1}.

$\text{Dom} = \mathbb{R}\backslash\{2\}$
$\text{Range} = \mathbb{R}\backslash\{-4\}$

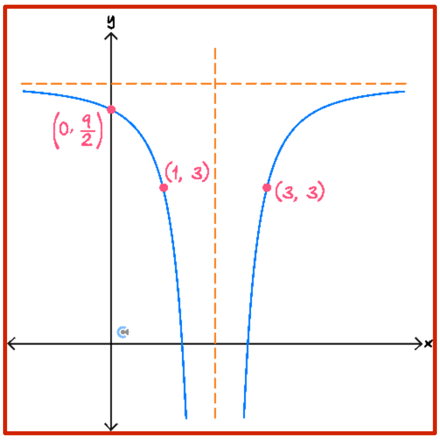
c. Find the values of x for which $f(x) > 1$.

$f(x) = 2 \implies x = -2.$
Hence $x > -2$ or $x < -4.$

Question 33

Consider the function $f : \mathbb{R}\backslash\{h\} \to \mathbb{R},\ \ f(x) = \frac{a}{(x-h)^2} + k$.

The graph of f is drawn below.



a. Show that $a = -2, h = 2$, and $k = 5$.

The graph of f must be symmetrical on the horizontal asymptote $x = h$.
Thus h must be the average of 1 and 3 $\implies h = 2$.
Substituting the point $(1, 3)$ into the equation $y = f(x)$ yields, $3 = a + k$.
Substituting the point $\left(0, \frac{9}{2}\right)$ into the equation $y = f(x)$ yields, $9 = \frac{1}{2}a + 2k$.
Solving simultaneously yields $a = -2$ and $k = 5$.

b. Find the maximal domain of $g(x) = \sqrt{4 - (f(x) - 1)^2}$.

As the maximal domain of $\sqrt{4 - (x-1)^2}$ is $[-1, 3]$, we require $f(x) \leq 3$ and $f(x) \geq -1$.
From the graph we see that $f(x) = 3 \implies x = 1, 3$.
If $f(x) = -1$ we see that $\frac{2}{(x-2)^2} = 6 \implies x - 2 = \pm\frac{1}{\sqrt{3}}$.
From the graph of f, we see that $f(x) \in [-1, 3]$ if $x \in \left[1, 2 - \frac{1}{\sqrt{3}}\right] \cup \left[2 + \frac{1}{\sqrt{3}}, 3\right]$.
Hence the maximal domain of $g(x)$ is $\left[1, 2 - \frac{1}{\sqrt{3}}\right] \cup \left[2 + \frac{1}{\sqrt{3}}, 3\right]$.

Question 34

Consider the function $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 2x + 2$.

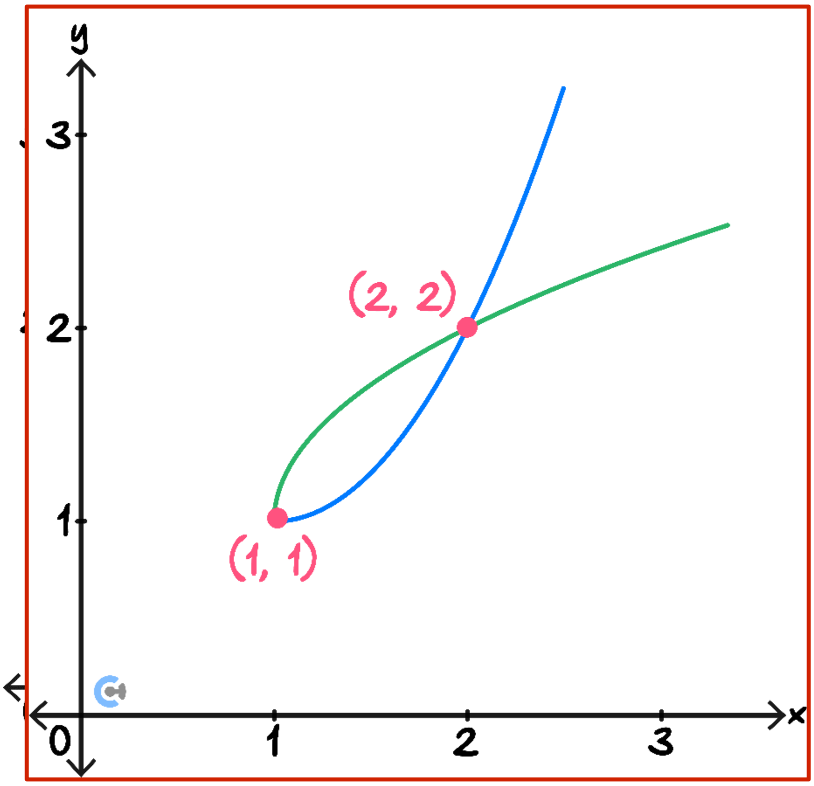
a. Find the smallest value of a for which the inverse function of f, f^{-1} exists.

$f(x) = (x-1)^2 + 1$, hence $a = 1$.

b. State the domain and range of f^{-1}.

$\text{Dom} = [1, \infty)$
$\text{Range} = [1, \infty)$

c. The graph of $y = f(x)$ is drawn on the axis below, sketch the graph of $y = f^{-1}(x)$ on the same axis, labelling points of intersection with their coordinates.



d. Let $g: [1, \infty) \to \mathbb{R}, g(x) = (x-1)^2 + k$.

i. Find the values of k for which $g(x) = g^{-1}(x)$ has no solutions.

Observe that solving $g(x) = g^{-1}(x)$ is equivalent to solving $g(x) = x \implies (x-1)^2 - x = x^2 - 3x + 1 = -k$.
By completing the square of $x^2 - 3x + 1 = \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$, we see that $x^2 - 3x + 1 = -k$ has no solutions if $-k < \frac{5}{4} \implies k > \frac{5}{4}$.

ii. Find the values of k for which $g(x) = g^{-1}(x)$ has two solutions.

$1 < k < \frac{5}{4}$.

iii. Find the values of k for which $g(x) = g^{-1}(x)$ has one solution.

$k = \frac{5}{4}$ or $k < 1$.

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Sub-Section: Exam 2 Questions

Question 35

The maximal domain of the function f is $(-2, \infty)$.

A possible rule for f is:

A. $f(x) = \log_2(x-2)$

B. $f(x) = \sqrt{2-x}$

C. $f(x) = \frac{1}{x+2}$

D. $f(x) = \frac{1}{\sqrt{x+2}}$

Question 36

Consider the function $f : (a, b] \to \mathbb{R}, f(x) = \frac{1}{x-1}$ where $a < b < 1$.

The range of f is:

A. $\left(\frac{1}{a-1}, \frac{1}{b-1}\right]$

B. $\left[\frac{1}{b-1}, \frac{1}{a-1}\right)$

C. $(b, a]$

D. $\left[\frac{1}{a-1}, \frac{1}{b-1}\right)$

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Question 37

Consider the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$.

The equation $f(x) = k$ will have three solutions for:

A. $k > -3$

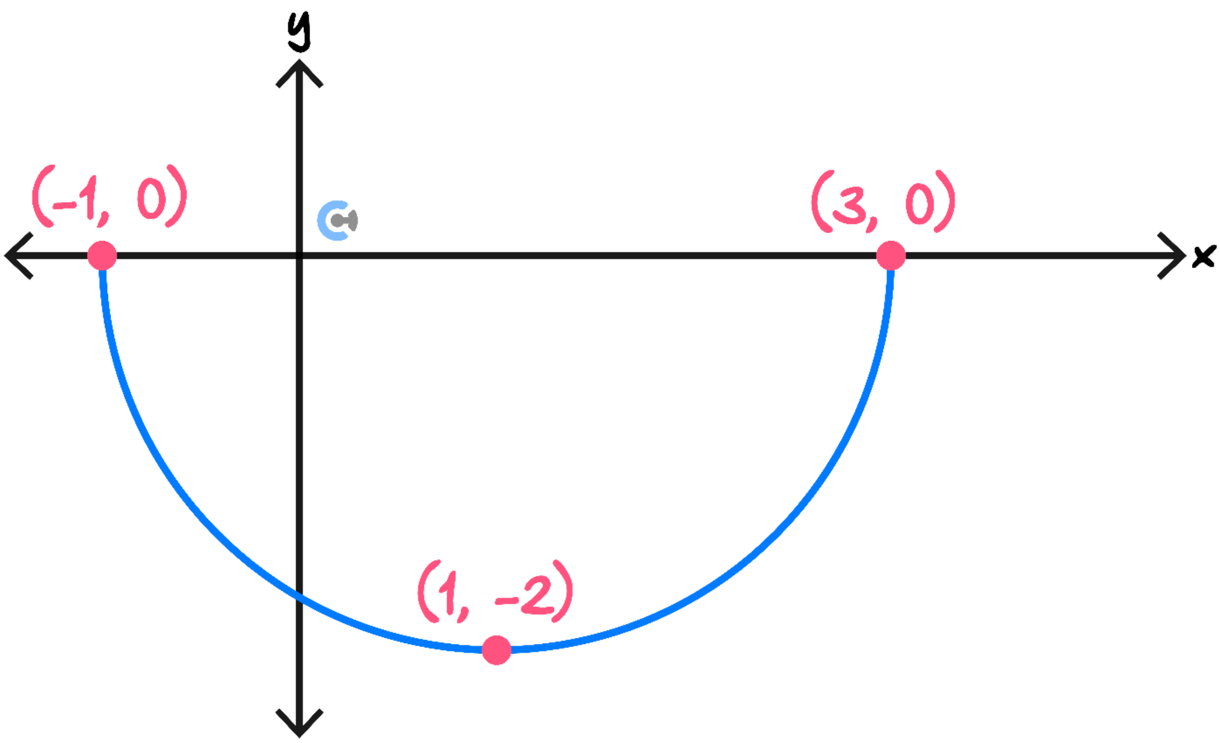
B. $k < 2$

C. $-3 < k < 2$

D. $k = -3$ or $k = 2$

Question 38

The equation that best represents the graph below is:



A. $y = -\sqrt{3 + 2x - x^2}$

B. $y = -\sqrt{3 - 2x - x^2}$

C. $y = \sqrt{4 - (x - 1)^2}$

D. $(x + 1)^2 + y^2 = 4$

Question 39

Consider the function $f: [-20, a] \rightarrow \mathbb{R}, f(x) = 2x^2 - 12x + 5$.

The smallest value of a for which the inverse function of f, f^{-1} exists is:

A. $a = 6$

B. $a = -6$

C. $a = 3$

D. $a = -3$

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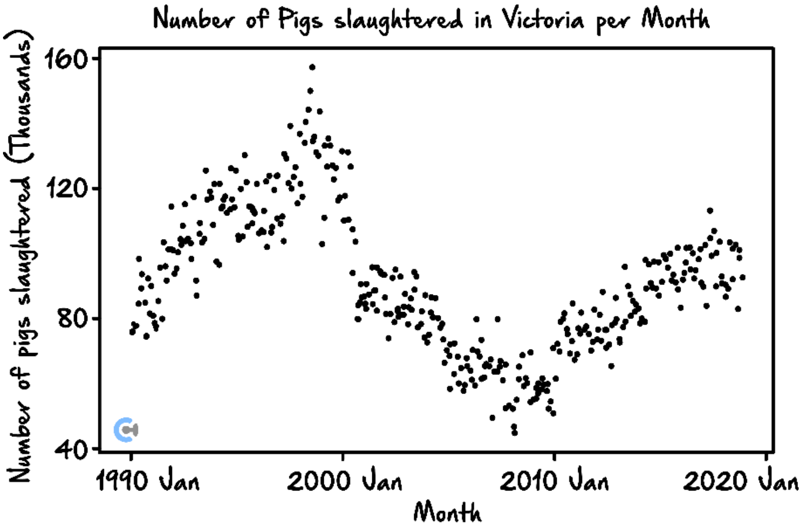
Question 40

The points shown on the chart below represent the number of pigs slaughtered in Australia, from 1990 to 2018.



We can attempt to model y, the number of pigs slaughtered in thousands, as a function of time.

Specifically, the variable t which represents the month when the pigs were slaughtered, where $t = 1$ corresponds to January 1990, $t = 2$ corresponds to February 1990 and so on.

Our first attempt is setting $y = f(t)$, where $f : [1, \infty) \to \mathbb{R}, f(t) = \frac{a}{1000}t^3 + \frac{b}{100}t^2 + \frac{c}{10}t + d$,is a cubic polynomial.

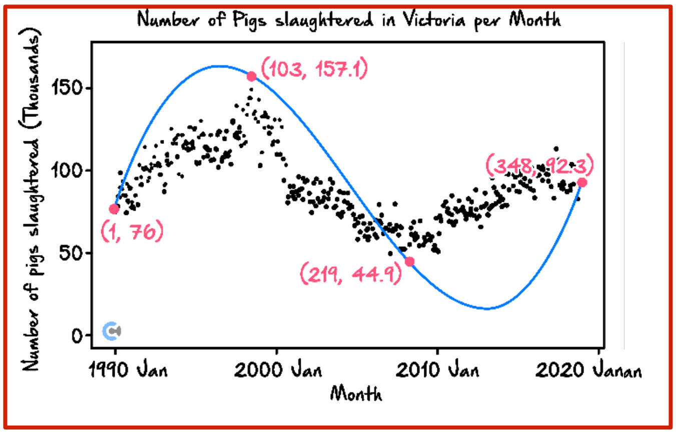
We can do this by ensuring f reflects some suitable points.

a. If we want $f(1) = 76$, $f(103) = 157.1$, $f(219) = 44.8$, and $f(348) = 92.3$, find the values of a, b, c, d correct to 3 decimal places.

$$a = 0.039, b = -2.069, c = 25.292, d = 73.491$$

b. Plot the graph of f over the interval $[1, 348]$ on the axis below, labelling the four points mentioned in **part a.i** with their coordinates.

You can use the fact that $t = 103$ corresponds to July, 1998, $t = 219$ corresponds to March 2008 and $t = 348$ corresponds to December 2018.



c.

i. According to this model, what is the earliest month after 2018 for which the number of pigs slaughtered will be greater than 157100?

Solve $f(t) = 157.1$ our desired solution is 371.2, round up to 372. This corresponds to December 2020.

ii. For what values of k, does $f(t) = k$ have two solutions? Give your answer to 2 decimal places.

$15.68 < k < 76$ or $k = 163.39$

iii. Does our cubic model accurately reflect the minimum and maximum number of pigs slaughtered from 1990 to 2018?

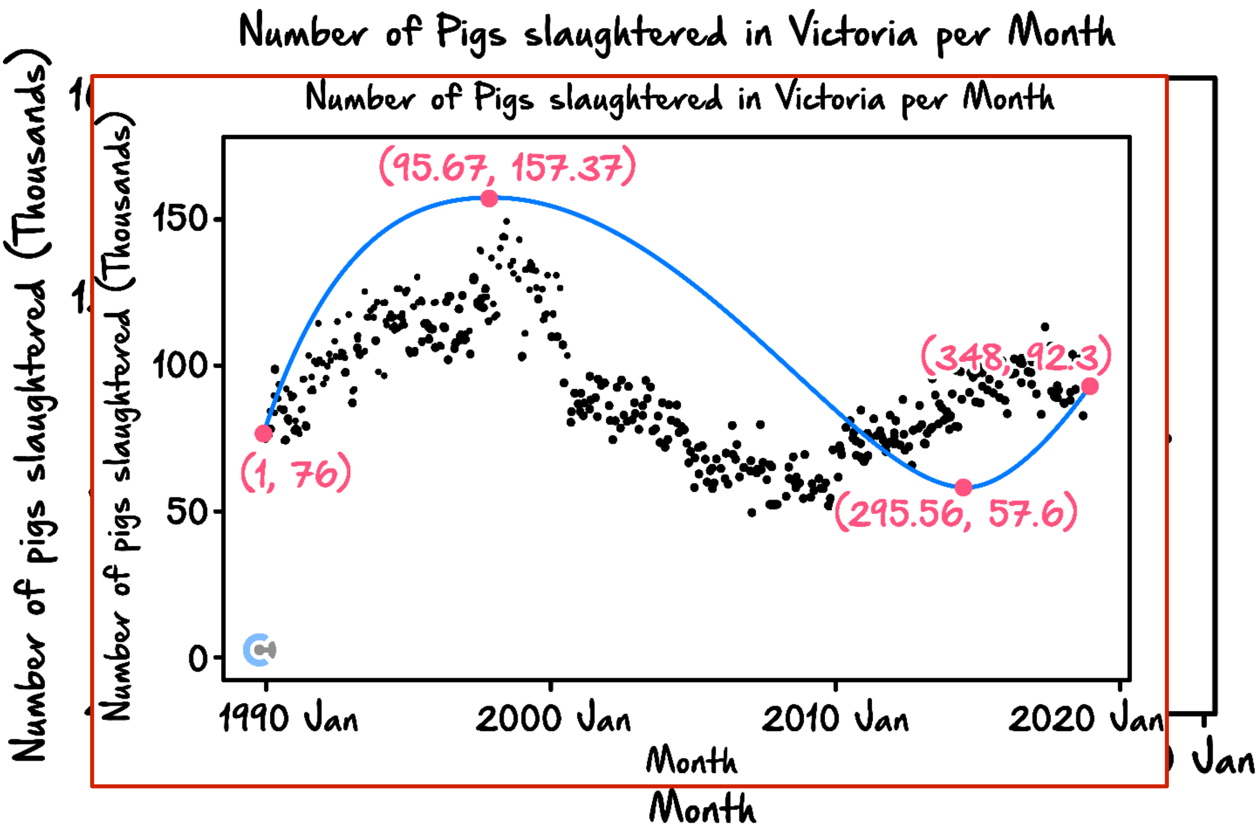
No, it suggest a maximum slaughter of $163, 3900$ pigs and a minimum slaughter of $15, 6800$ pigs, numbers that are too big and too small respectively.

d. An alternative model can be $y = g(t)$, where $g(t) = \sqrt{at^3 + bt^2 + ct + d}$.

i. Explain why the restrictions $g(1) = 76$, $g(103) = 157.1$, $g(219) = 44.8$, and $g(348) = 92.3$ are unusable.

If we solve for a, b, c and d with these restrictions we get $g(t) = \sqrt{0.00792687t^3 - 4.30716t^2 + 548.361t + 5231.94}$.
However $0.00792687t^3 - 4.30716t^2 + 548.361t + 5231.94 < 0$ for some $t \in [1, 348]$ meaning $g(t)$ is not defined for all $t \in [1, 348]$, making this model unworkable.

ii. Sketch the graph of $y = g(t)$ over the interval $[1, 348]$ on the axis below if $g(1) = 76$, $g(103) = 157.1$, $g(262) = 70.2$ and $g(348) = 92.3$. Label endpoints and turning points with their coordinates correct to 2 decimal places.



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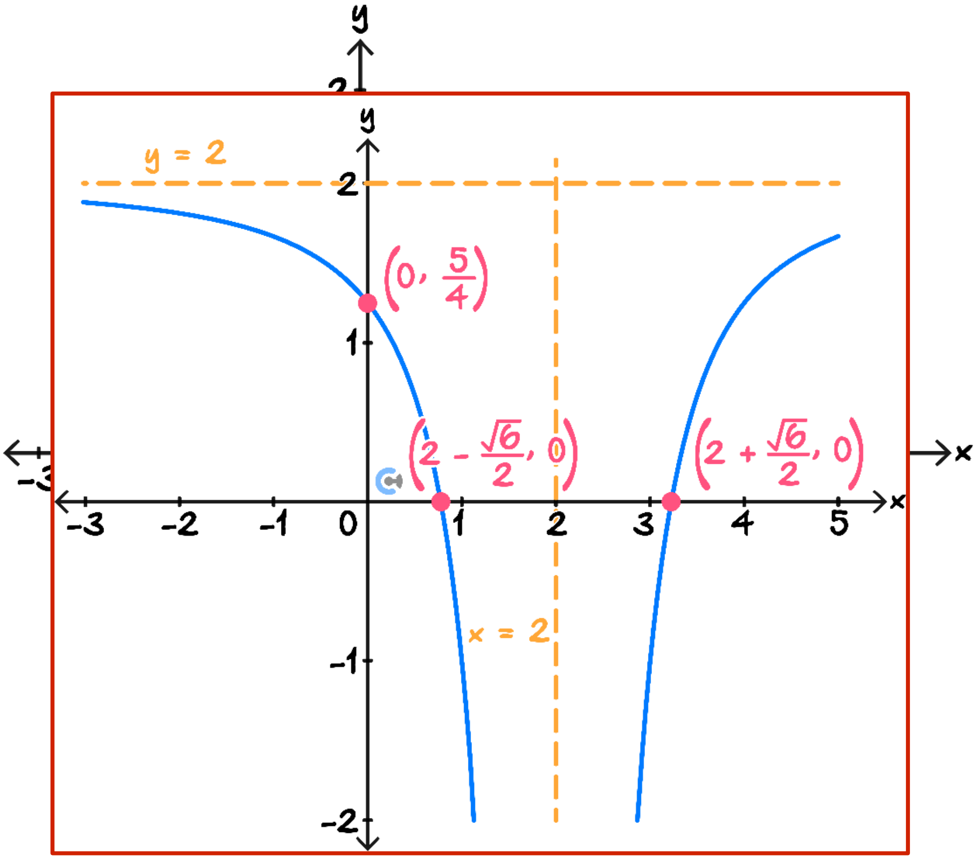
Question 41

Consider the function $f(x) = \frac{5-8x+2x^2}{x^2-4x+4}$.

a. State the maximum domain and range of f.

$f(x) = 2 - \frac{3}{(x-2)^2}$, hence the domain of f is $\mathbb{R}\backslash\{2\}$ and the range is $(-\infty, 2)$.

b. Sketch the graph of f on the axis below, labelling asymptotes with their equations and axes intercepts with their coordinates.



c. Consider the function $g(x) = 3x^4 + 8x^2 - 6x^2 - 24x + k$.

i. State the turning points of $g(x)$ in terms of k.

$(1, k - 19), (-2, 8 + k)$ and $(-1, 13 + k)$

ii. For what values of k, does the equation $g(x) = 2$ have exactly two solutions?

$-6 < k < 18$ or $k < -11$

iii. For what values of k, does the equation $g(x + a) = f(x)$ never have a solution for any value of a?

$k \geq 21$

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