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# VCE Mathematical Methods ½
## Functions & Relations Exam Skills [2.3]
## Homework

### Homework Outline:

| | |
|---|---|
| Compulsory Questions | Pg 2 - Pg 28 |
| Supplementary Questions | Pg 29 - Pg 59 |

## Section A: Compulsory Questions

### Sub-Section [2.3.1]: Restrict Domain Such that the Inverse Function Exists

**Question 1**

For each of the following functions, a domain restriction is given with an endpoint $a$ or $b$. Determine the minimum value of $a$ or maximum value of $b$ such that the inverse function, $f^{-1}$, exists.

**a.** $f: [a, \infty) \to \mathbb{R}, f(x) = (x-3)^2 + 2$

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**b.** $f : (-\infty, b] \to \mathbb{R}, f(x) = -x^2 + 6x - 5$

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**c.** $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 3$

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**Question 2**

All the functions in this question are trunci written in a non-standard form.

**a.** Consider the function:

$$f : (a, \infty) \to \mathbb{R}, \quad f(x) = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$

Find the minimum value of $a$ such that $f(x)$ has an inverse.

**b.** Consider the function:

$$g : (-\infty, a) \to \mathbb{R}, \quad g(x) = \frac{x^2 - 6x + 11}{x^2 - 6x + 9}$$

Find the maximum value of $a$ such that $g(x)$ has an inverse.

**c.** Consider the function:

$$h : (a, \infty) \to \mathbb{R}, \quad h(x) = \frac{2x^2 + 8x + 5}{x^2 + 4x + 4}$$

Find the minimum value of $a$ such that $h(x)$ has an inverse.

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**Question 3**

For each of the following semicircle functions, a domain restriction is given with an endpoint $a$.

Determine the minimum or maximum value of $a$ such that the inverse function exists.

**a.** Consider the semicircle function:

$$f : [a, 3] \to \mathbb{R}, \qquad f(x) = \sqrt{9 - x^2}$$

Find the minimum value of $a$ such that $f(x)$ has an inverse.

**b.** Consider the semicircle function:

$$g : [-2, a] \to \mathbb{R}, \qquad g(x) = \sqrt{12 + 4x - x^2} + 1$$

Find the maximum value of $a$ such that $g(x)$ has an inverse.

**c.** Consider the semicircle function:

$$h : [a, 4] \to \mathbb{R}, \qquad h(x) = \sqrt{24 - 2x - x^2} + 3$$

Find the minimum value of $a$ such that $h(x)$ has an inverse.

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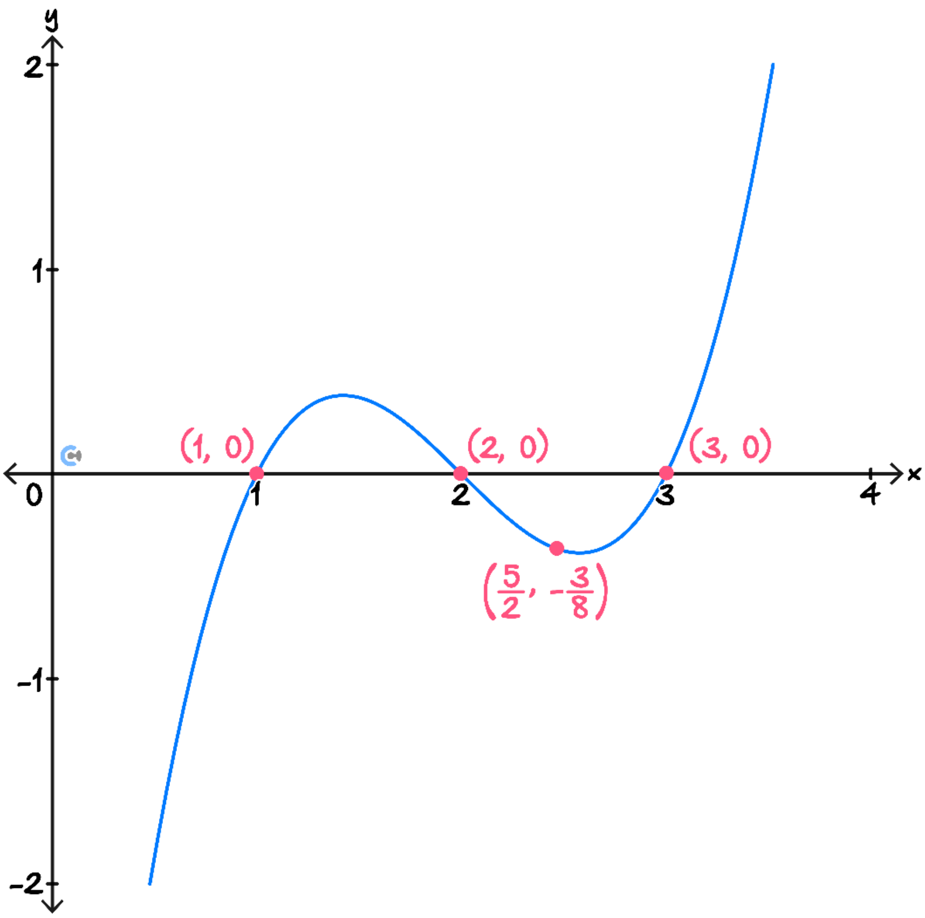
## Sub-Section [2.3.2]: Figure out Possible Rule of a Graph

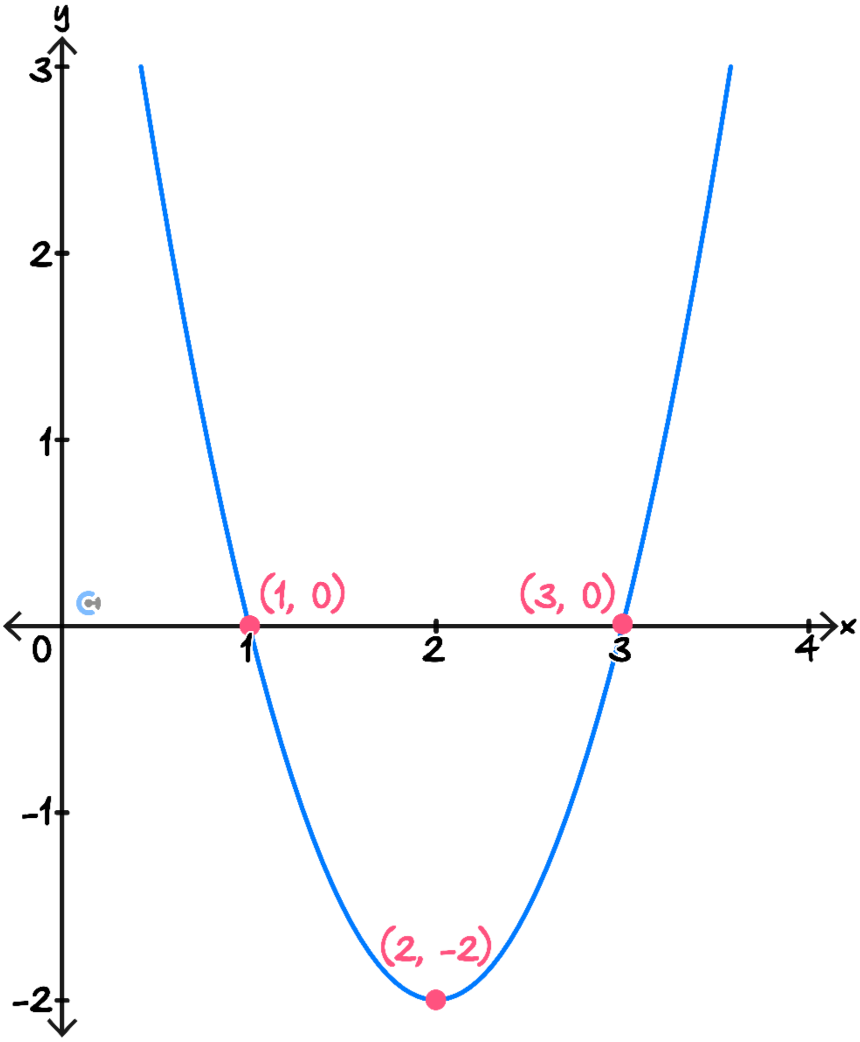
**Question 4**

Determine a possible rule for the following graphs:

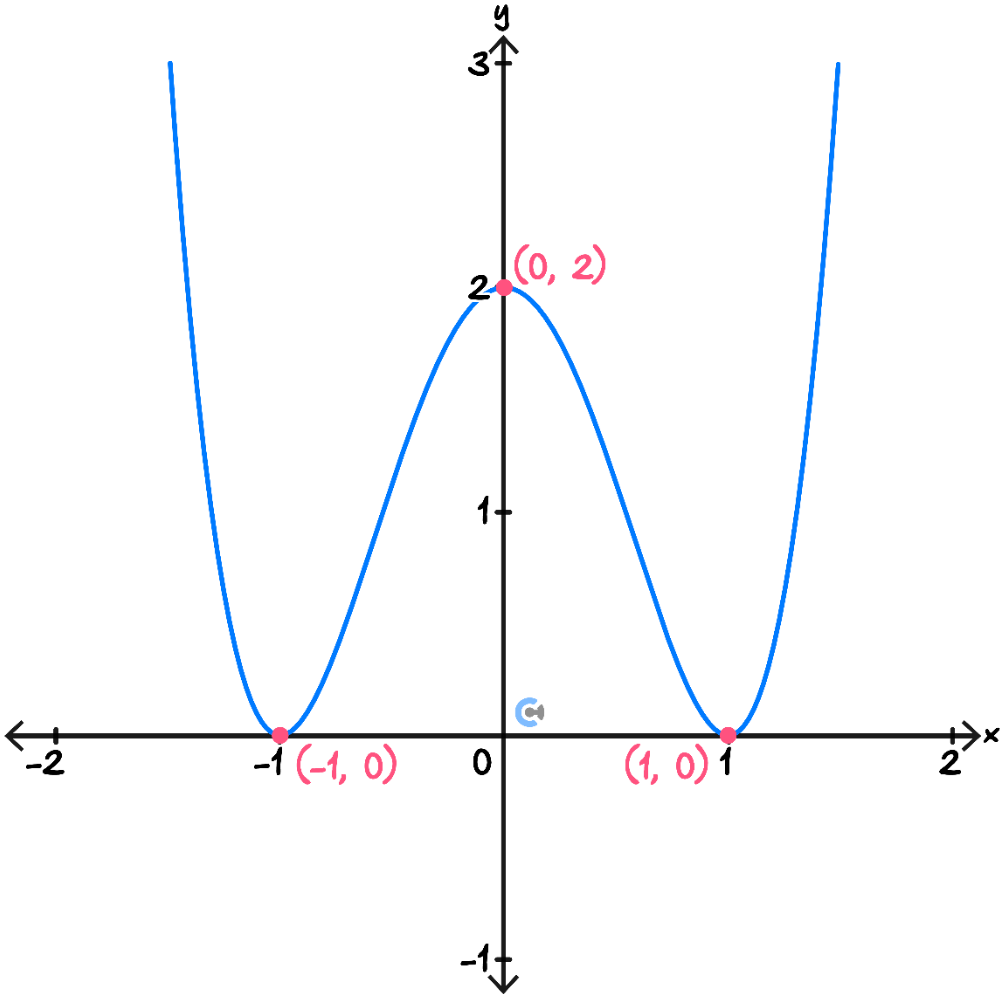
**a.**



**b.**



**c.**

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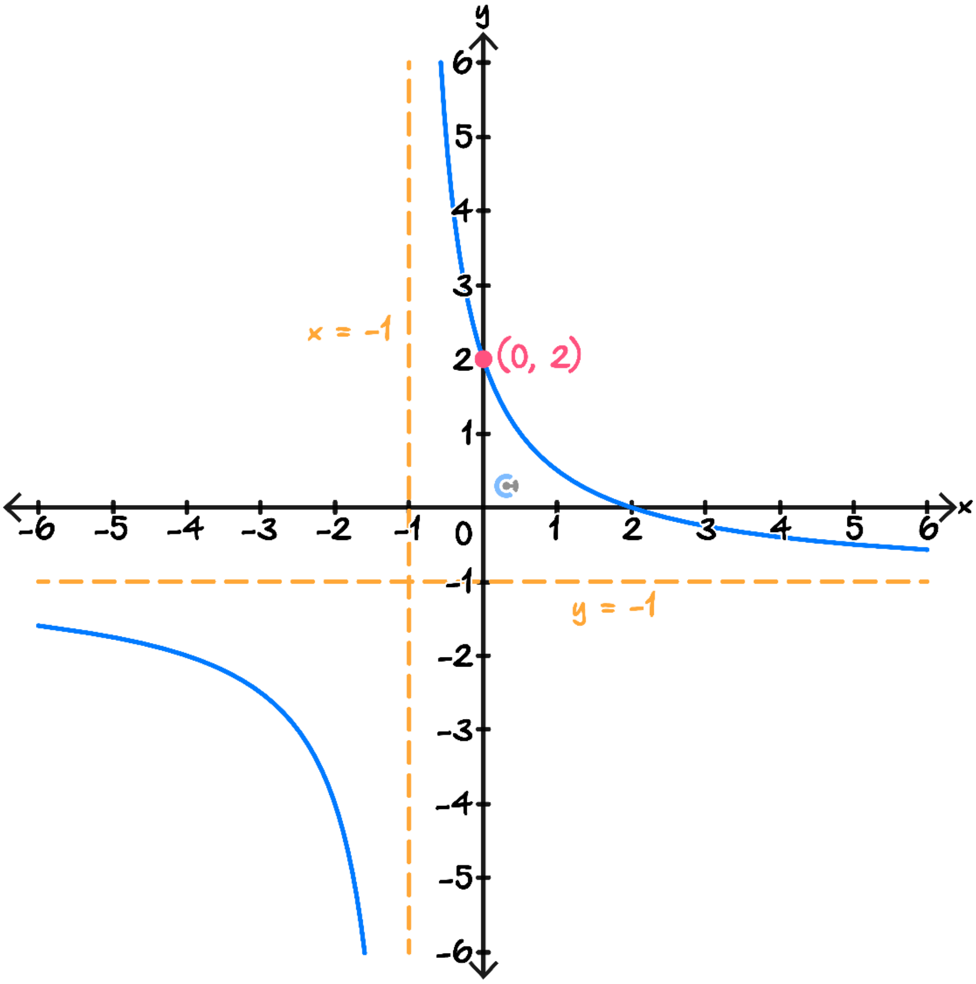
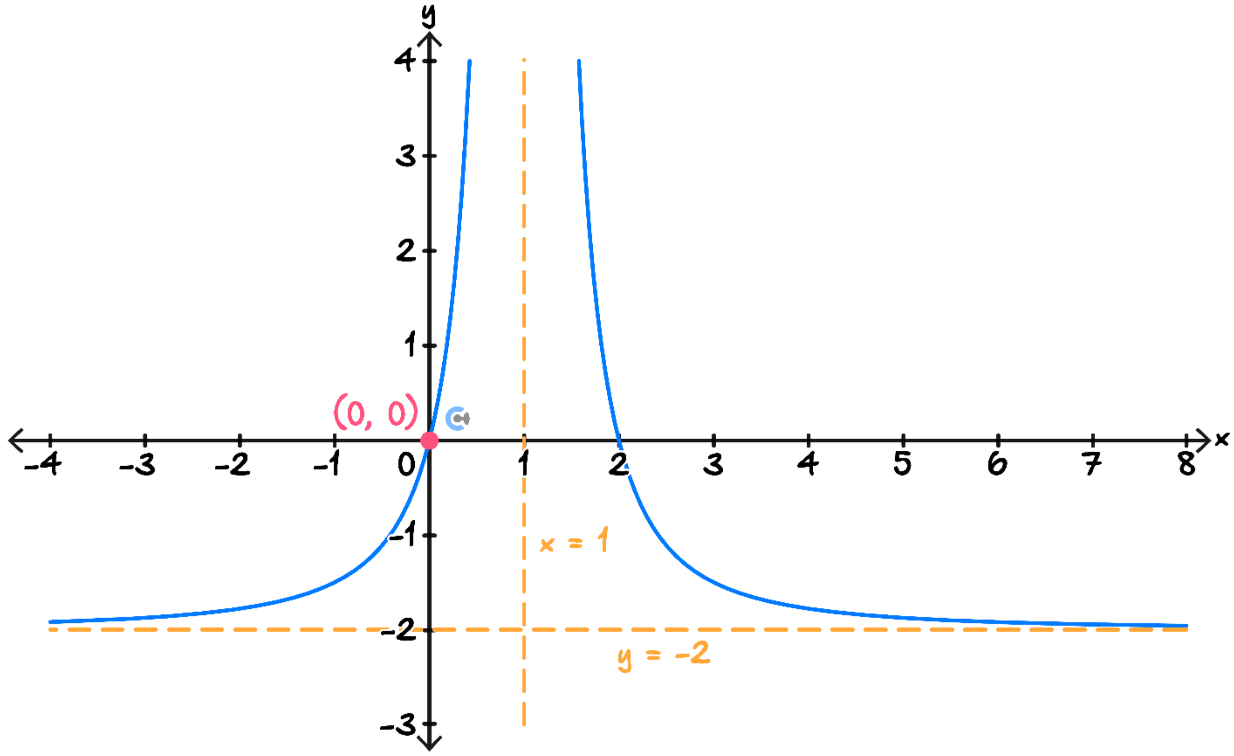
**Question 5**

Determine a possible rule for the graphs.

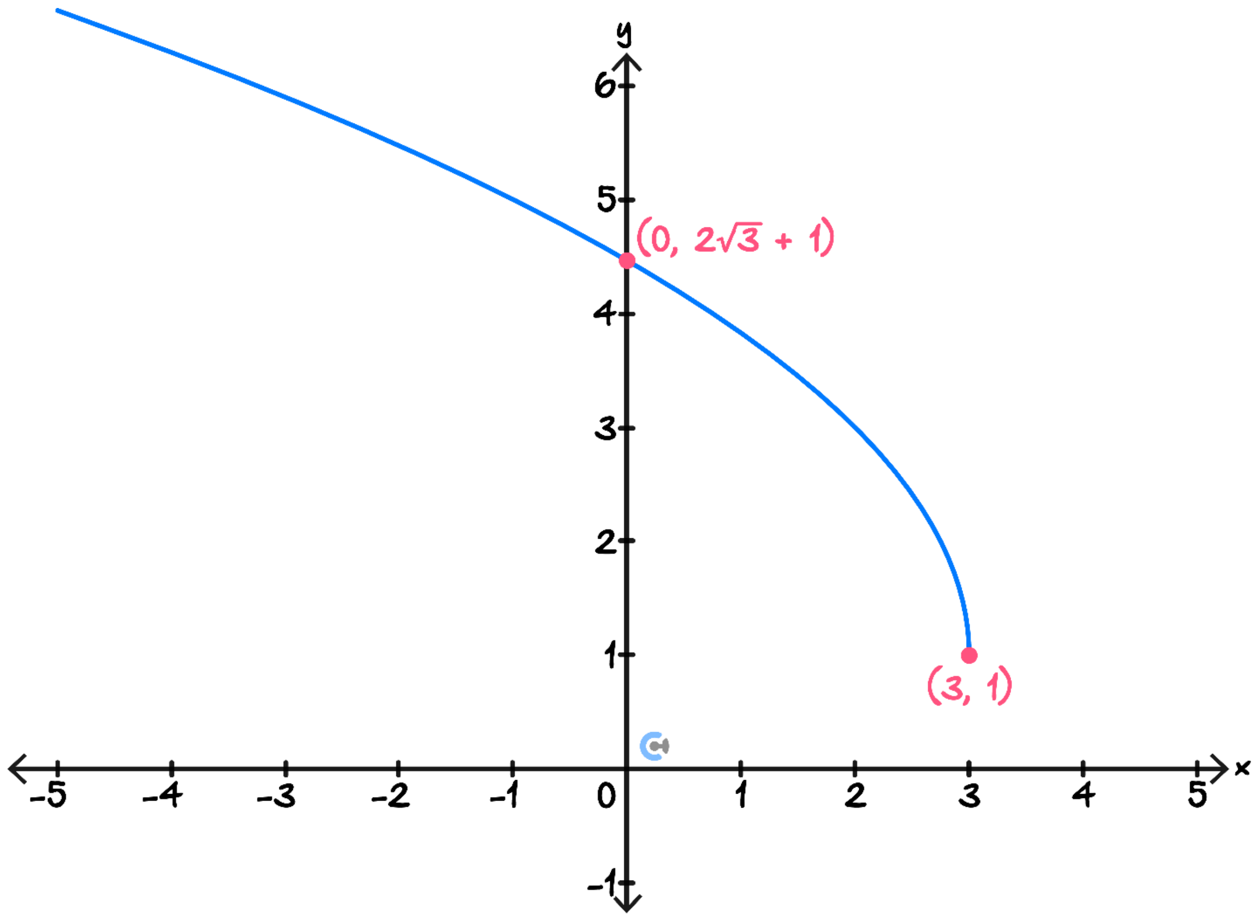
**a.**



**b.**



c.

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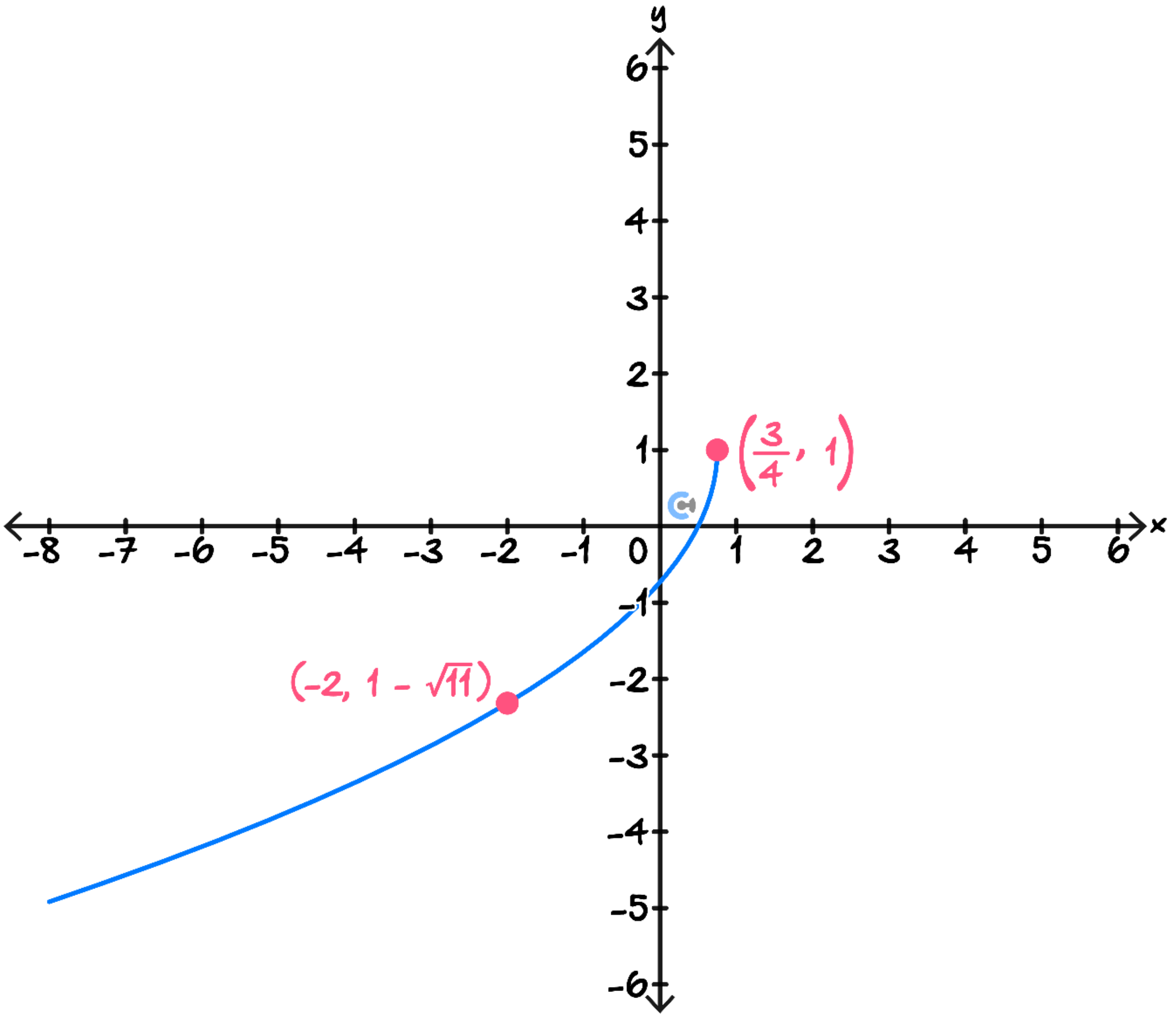
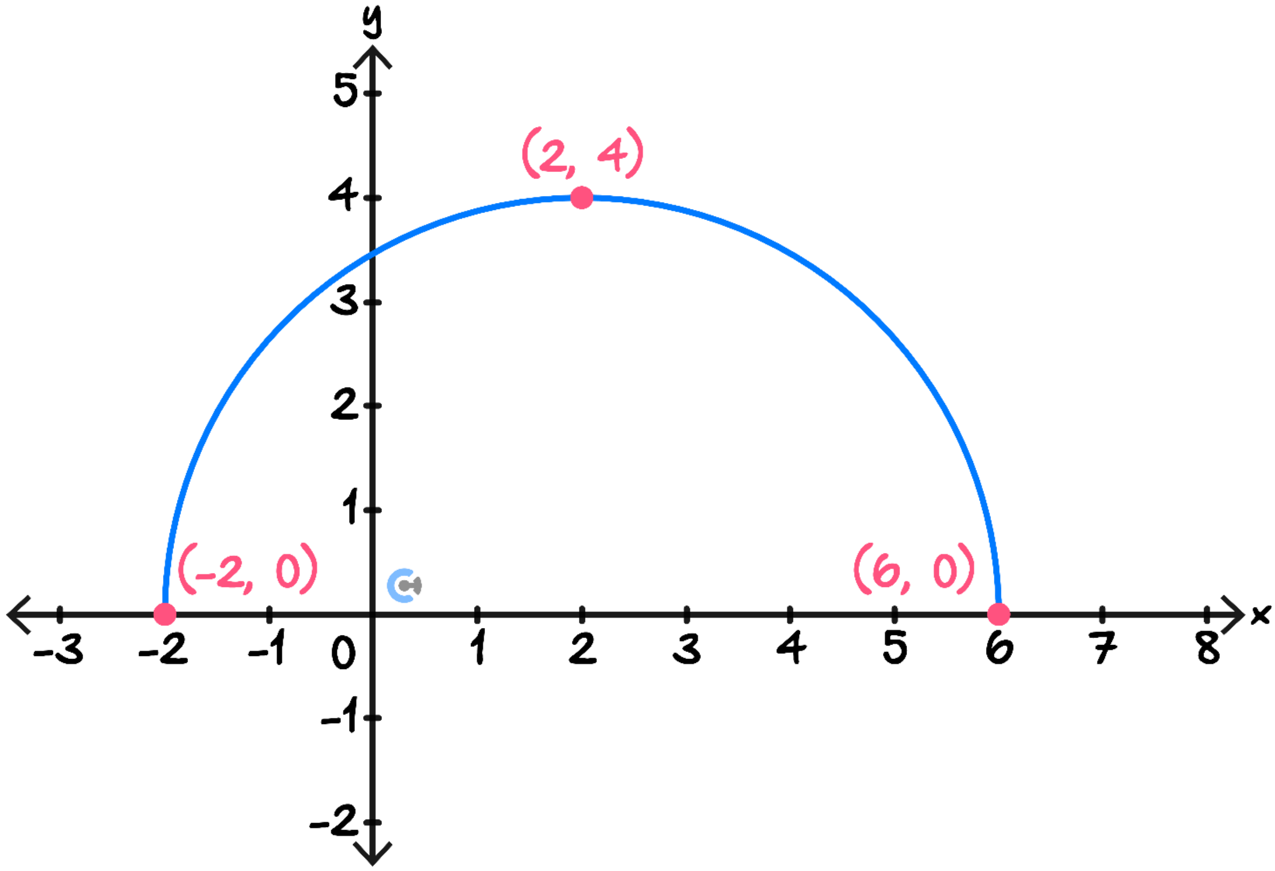
**Question 6**

Determine a possible rule for the graphs.

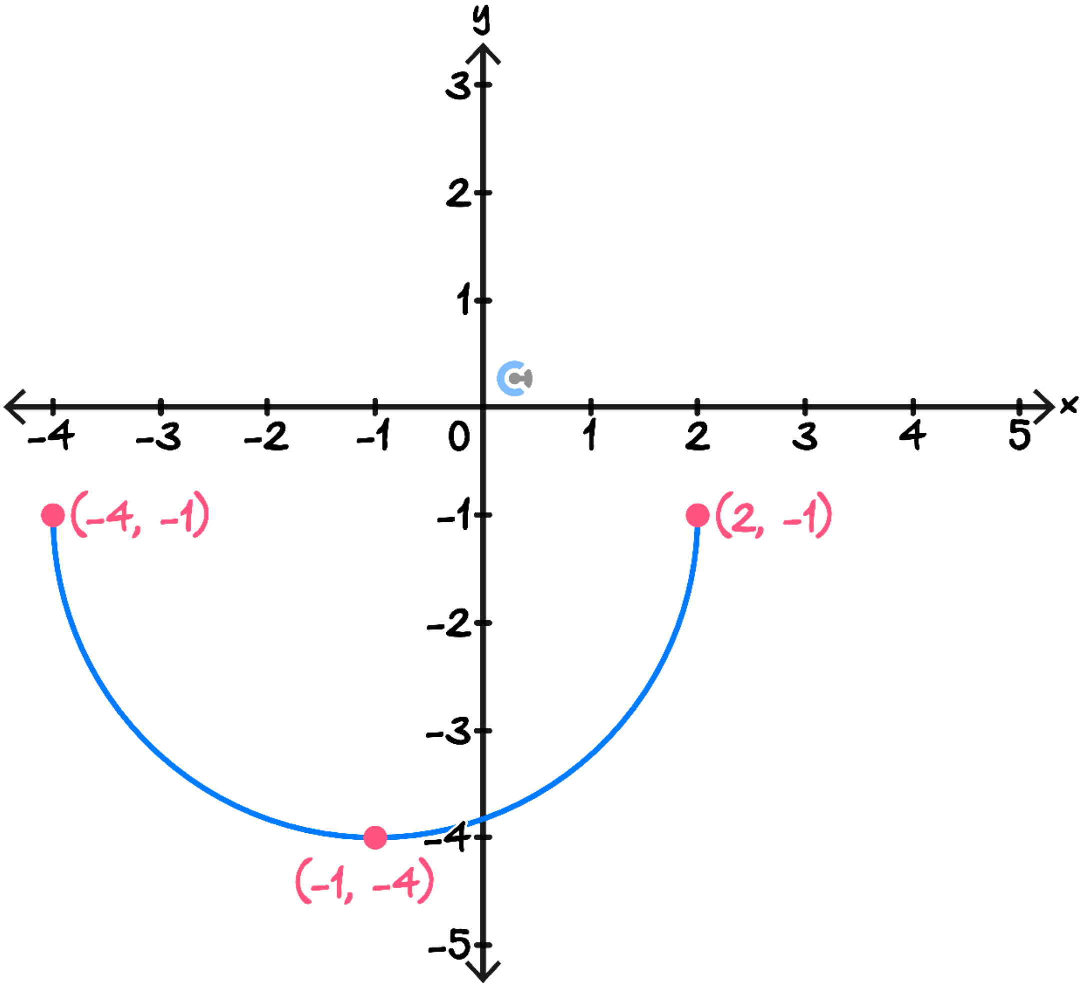
**a.**



**b.**



**c.**

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## Sub-Section [2.3.3]: Solve Number of Solution Problems Graphically

**Question 7 Tech-Active.**

Consider the function $f(x) = 2x^2 - 5x - 7$.

Determine the real values of $k$ for which $f(x) = k$ has two solutions.

**Question 8 Tech-Active.**

Consider the function $f(x) = 2x^3 - 12x^2 + 18x + 4$.

Determine the real values of $k$ for which $f(x) = k$ has three solutions.

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**Question 9 Tech-Active.**

Consider the function $f(x) = 3x^4 - 12x^3 - 6x^2 + 36x + 4$.

Determine the real values of $k$ for which $f(x) = k$ has two solutions.

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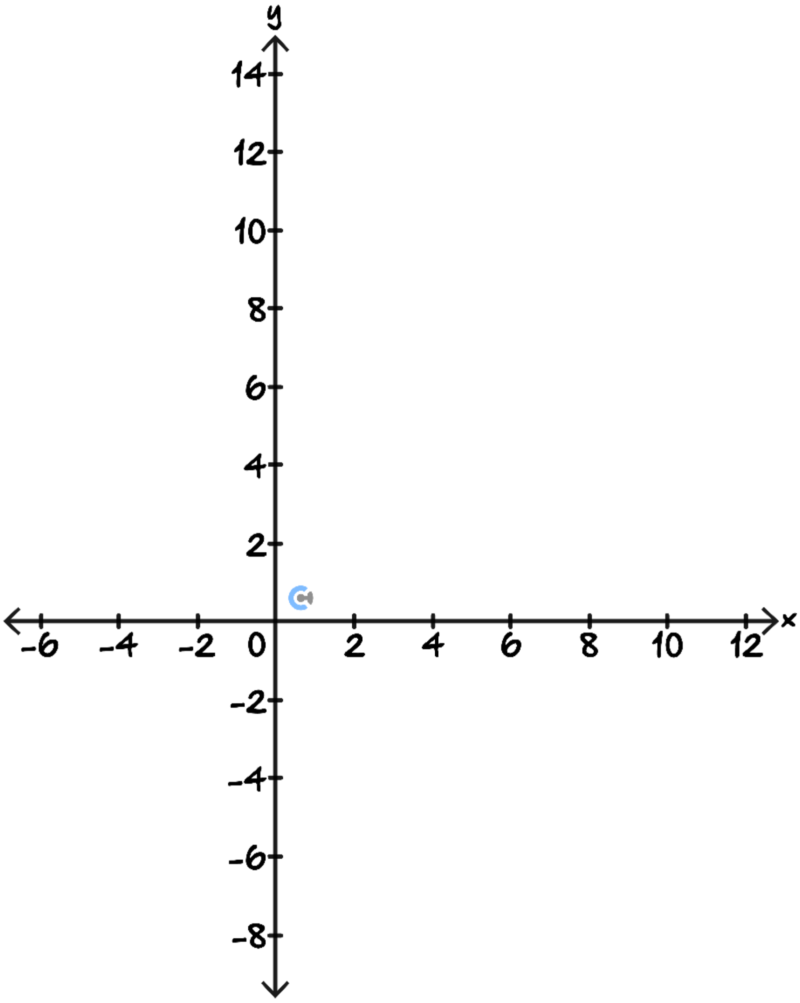
## Sub-Section: Exam 1 Questions

**Question 10**

Consider the function:

$$f(x) = \begin{cases} -x^2 + x + 12, & -4 \leq x \leq 3 \\ 12 - 2x, & x > 3 \end{cases}$$

**a.** Sketch the graph of $y = f(x)$ on the axes below. Label all intercepts, endpoints, and turning points.



**b.** State the range of $f(x)$.

**c.** Determine the range of values of $k$ for which $f(x) = k$ has two solutions.

**d.** Justify whether or not $f^{-1}(x)$ is defined.

**e.** Find the maximal domain of the equation $y = \log_2\big(f(x)\big)$.

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**Question 11**

Consider the function $f : D \to \mathbb{R}, f(x) = \frac{2x+1}{x-2}$.

**a.** State the maximal domain $D$ of $f$.

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**b.** Express $f$ in the form $a + \frac{b}{x-2}$ and state the values of $a$ and $b$.

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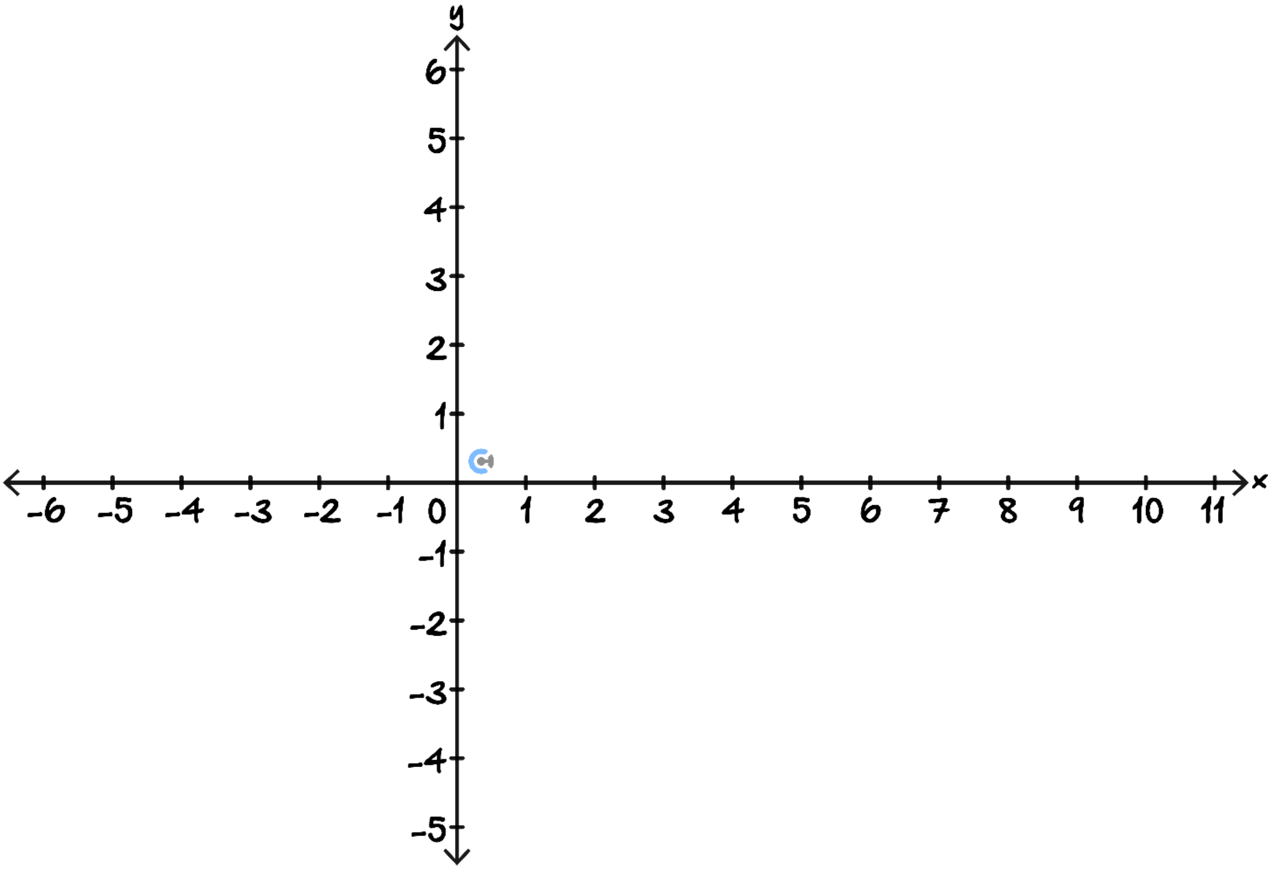
**c.** Sketch the graph of $y = f(x)$ on the axes below. Label asymptotes with their equations and axes intercepts with the coordinates.



**d.** Find the values of $x$ for which $f(x) \geq 1$.

**e.**

**i.** Determine the rule for $f^{-1}(x)$, the inverse of $f(x)$.

**ii.** Hence, determine the $x$-value for all points of intersection between $f(x)$ and $f^{-1}(x)$.

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**Question 12**

Consider the function $f : [1, \infty) \to \mathbb{R}, f(x) = x^2 - 2x + k$.

**a.** Define the inverse function of $f$.

**b.** Determine the values of $k$ for which $f$ and $f^{-1}$ do not intersect each other.

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## Sub-Section: Exam 2 Questions

**Question 13**

The maximal domain of the function $f(x) = \frac{1}{\sqrt{x^2-x-6}}$ is:

**A.** $x \in (0, \infty)$

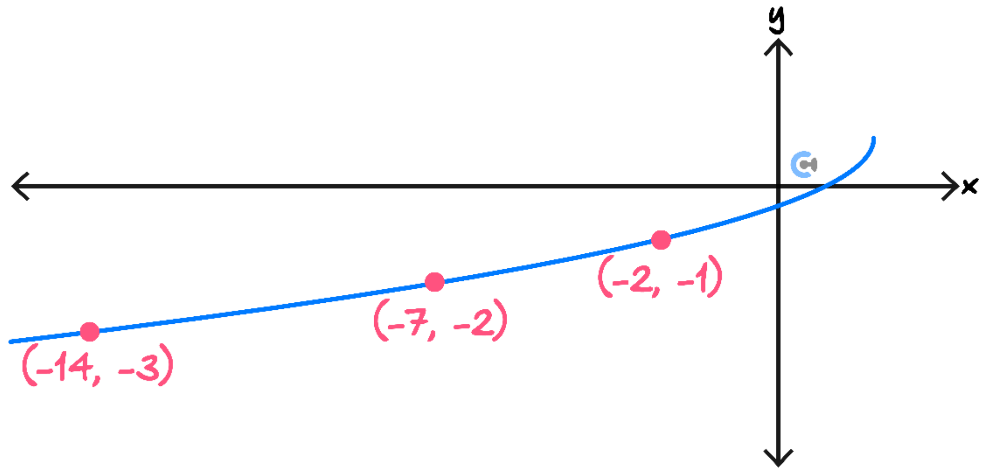
**B.** $x \in (-2, 3)$

**C.** $x \in (-\infty, 2] \cup [3, \infty)$

**D.** $x \in \mathbb{R} \backslash [-2, 3]$

**Question 14**

The most likely rule for the following graph is:




**A.** $\sqrt{3-x}+1$

**B.** $-\sqrt{2-x}+1$

**C.** $3\sqrt{x+1}-1$

**D.** $-3\sqrt{2-x}+1$

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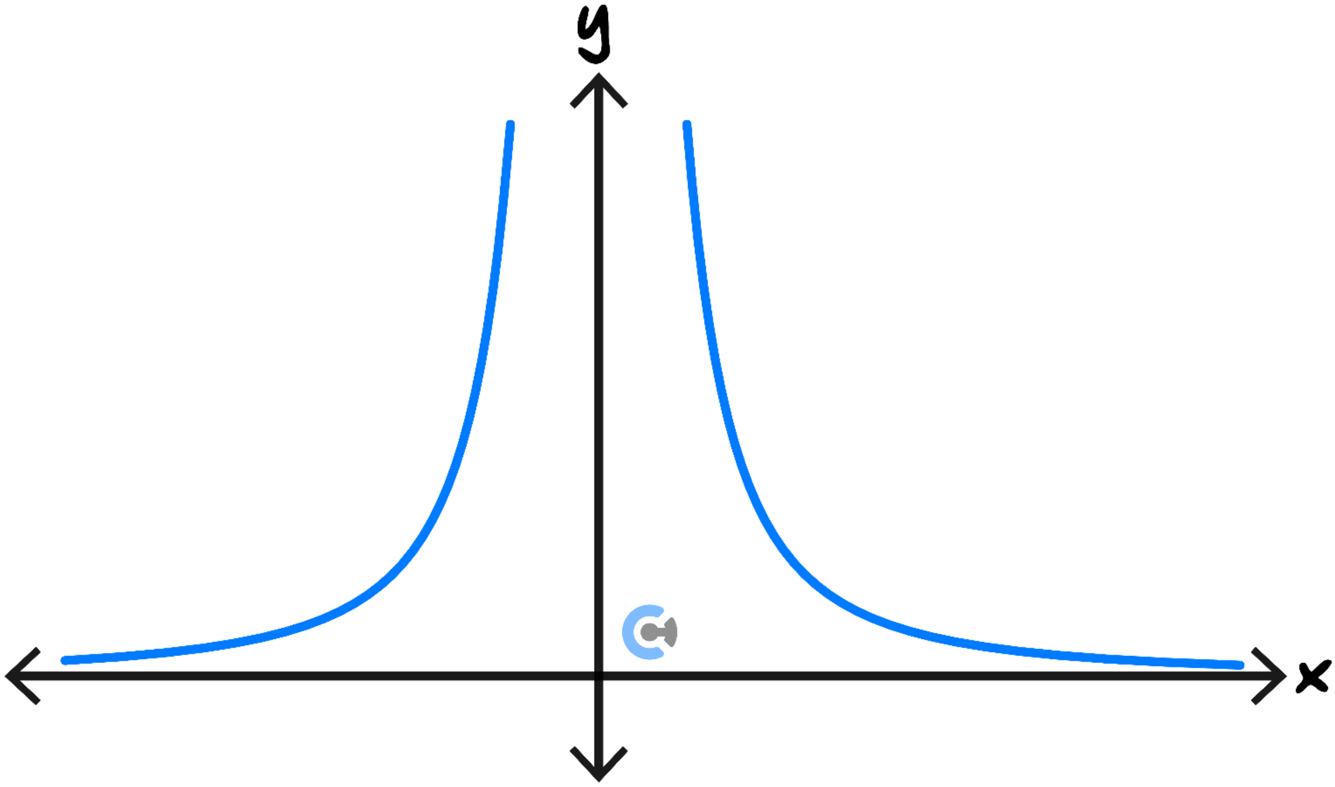
**Question 15**

The function $g$ with graph shown below is best described as:




**A.** One-to-one.

**B.** One-to-many.

**C.** Many-to-one.

**D.** Many-to-many.

**Question 16**

The line with equation $4y + 3x = 25$ intersects the circle $x^2 + y^2 = 25$ exactly once at the point $P(3, 4)$. The equation for the radius of the circle that passes through $P$ is:

**A.** $3y - 4x = 0$

**B.** $3y + 4x = 25$

**C.** $3y + 4x = 0$

**D.** $3y - 4x = 25$

**Question 17**

Consider the function $f(x) = 12x^5 + 90x^4 + 140x^3 - 180x^2 - 480x - 200$.

The equation $f(x) = k$ will have three solutions for:

**A.** $k \in (-618, -24) \cup (38, 632)$

**B.** $k \in [-618, -24] \cup [38, 632]$

**C.** $k \in (-24, 38)$

**D.** $k \in [24, 38]$

**Question 18**

The function $f$ is defined as $f : [a, a+2] \to \mathbb{R}, f(x) = x^2 - 4x - 8$.

**a.** Find the turning point of $f(x)$.

**b.** Find the values of $a$ such that:

**i.** The range of $f(x)$ is $[-8, 4]$.

**ii.** The inverse function $f^{-1}$ exists.

**iii.** $\sqrt{f(x)}$ does not exist.

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**Question 19**

The line with equation $y = mx$ intersects the circle with centre $(4,0)$ and radius 2 exactly once at the point $P(x, y)$.

(**Note:** A line that intersects a circle exactly once is called a line that is tangent to the circle.)

**a.** Find the equation of the circle.

**b.** Show that the $x$-coordinate of the point $P$ satisfies the equation:

$$(1 + m^2)x^2 - 8x + 12 = 0$$

**c.** Use the discriminant to find the possible values of $m$.

**d.** Hence, find the two possible sets of coordinates for $P$.

**e.** Find the distance of $P$ from the origin.

**f.** Find the acute angle that the two lines tangent to the circle make at the origin.

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## Section B: Supplementary Questions

### Sub-Section [2.3.1]: Restrict Domain Such that the Inverse Function Exists

**Question 20**

For each of the following functions, a domain restriction is given with an endpoint $a$ or $b$. Determine the minimum value of $a$ or maximum value of $b$ such that the inverse function, $f^{-1}$, exists.

**a.** $f: (-\infty, b] \to \mathbb{R}, f(x) = (x+1)^2 - 3$

**b.** $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 4x + 7$

**c.** $f : [a, \infty) \to \mathbb{R}, f(x) = -x^2 + 8x - 11$

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**Question 21**

All the functions in this question are trunci written in a non-standard form.

**a.** Consider the function:

$$f : (-\infty, a) \to \mathbb{R}, \quad f(x) = \frac{11 + 12x + 3x^2}{x^2 + 4x + 4}$$

Find the maximum value of $a$ such that $f(x)$ has an inverse.

**b.** Consider the function:

$$g : (a, \infty) \to \mathbb{R}, \quad g(x) = \frac{x^2 + 8x + 18}{x^2 + 8x + 16}$$

Find the minimum value of $a$ such that $g(x)$ has an inverse.

**c.** Consider the function:

$$h : (a, \infty) \to \mathbb{R}, \quad h(x) = \frac{3x^2 + 6x - 2}{x^2 + 2x + 1}$$

Find the minimum value of $a$ such that $h(x)$ has an inverse.

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**Question 22**

For each of the following semicircle functions, a domain restriction is given with an endpoint $a$.

Determine the minimum or maximum value of $a$ such that the inverse function exists.

**a.** Consider the semicircle function:

$$f : [-4, a] \to \mathbb{R}, f(x) = \sqrt{4 - (x + 2)^2}$$

Find the minimum value of $a$ such that $f(x)$ has an inverse.

**b.** Consider the semicircle function:

$$g : [a, 4] \to \mathbb{R}, \qquad g(x) = 2 - \sqrt{8 + 2x - x^2}$$

Find the maximum value of $a$ such that $g(x)$ has an inverse.

**c.** Consider the semicircle function:

$$h : [-5, a] \to \mathbb{R}, \qquad h(x) = \sqrt{20 - 16x - 4x^2} + 1$$

Find the maximum value of $a$ such that $h(x)$ has an inverse.

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**Question 23**

Consider the function:

$$f : [a, \infty) \to \mathbb{R}, \qquad f(x) = \frac{2x^2 + 8x + 11}{5 + 4x + x^2}$$

Find the maximum value of $a$ such that $f(x)$ has an inverse.

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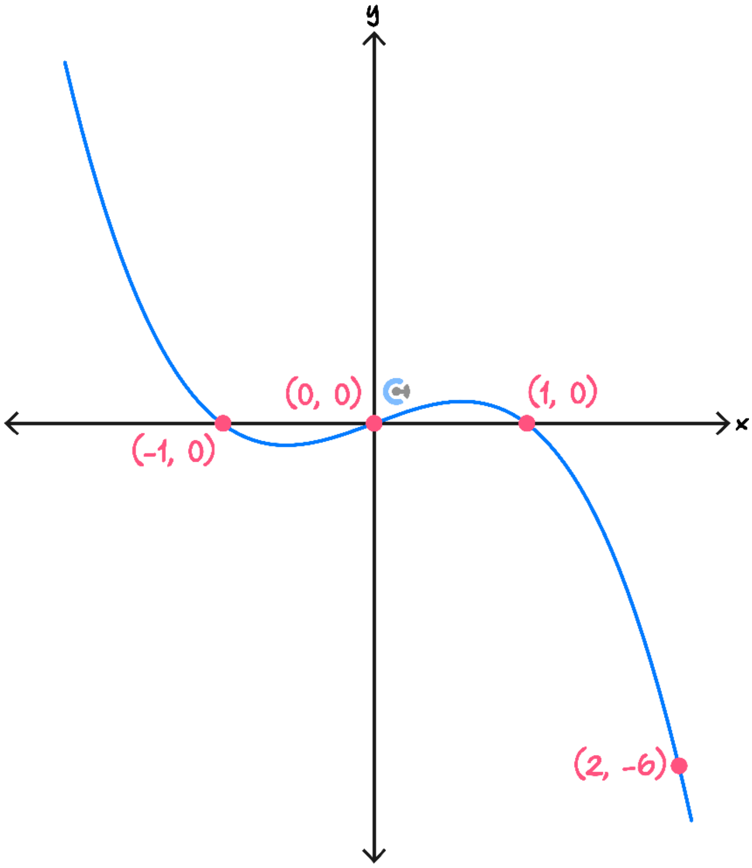
## Sub-Section [2.3.2]: Figure Out Possible Rule of a Graph

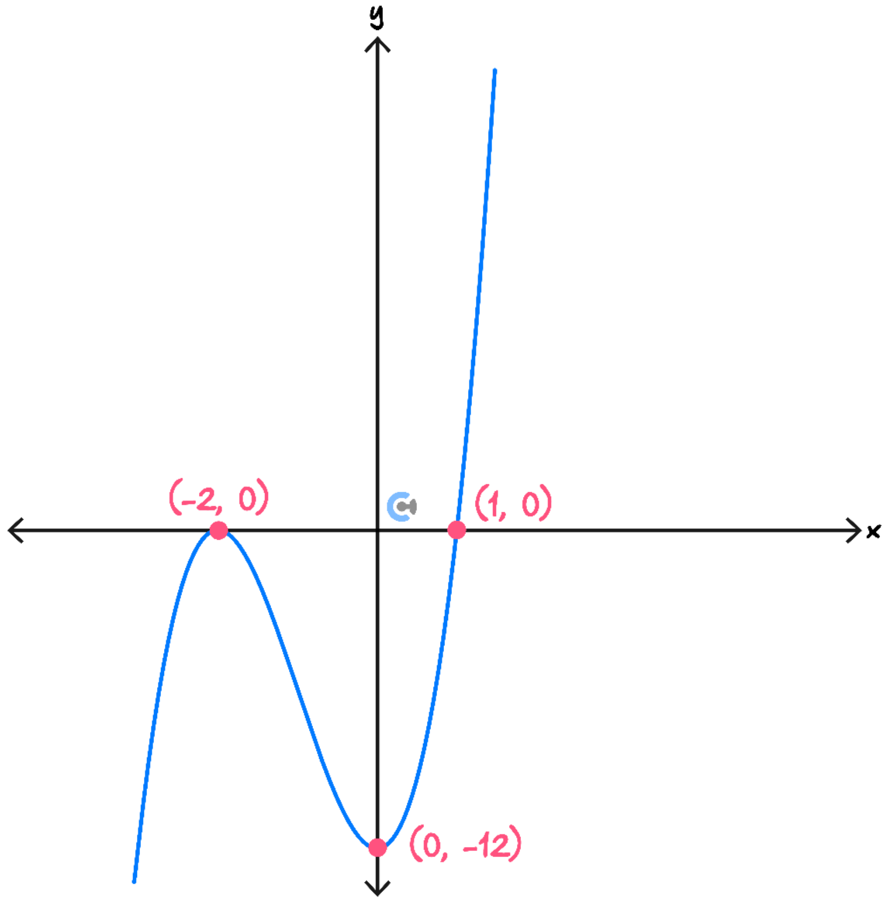
**Question 24**

Determine a possible rule for the following graphs:

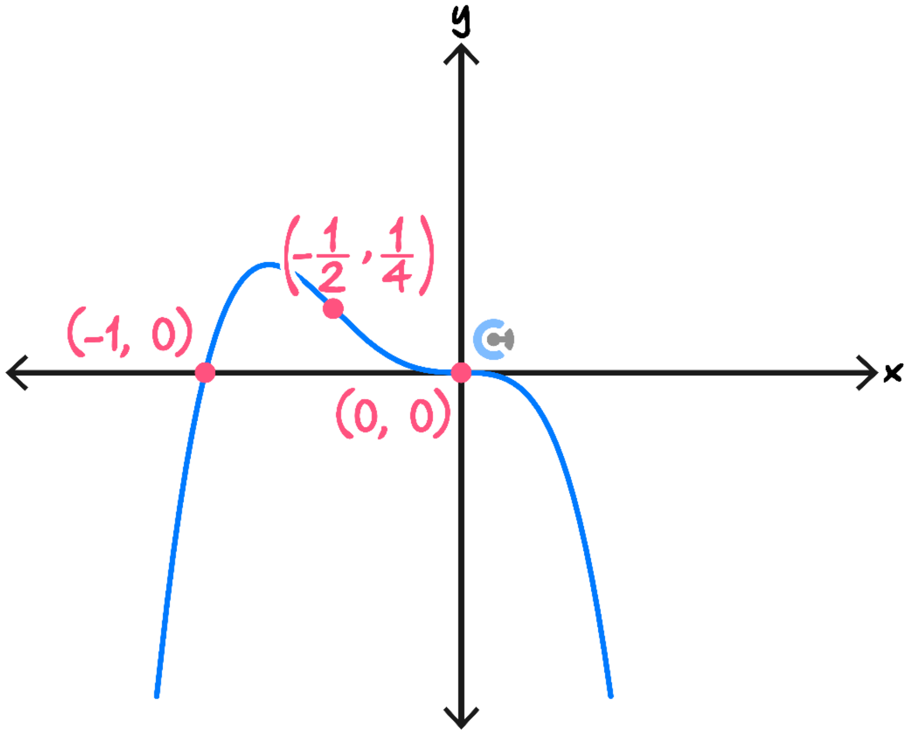
**a.**



**b.**



**c.**

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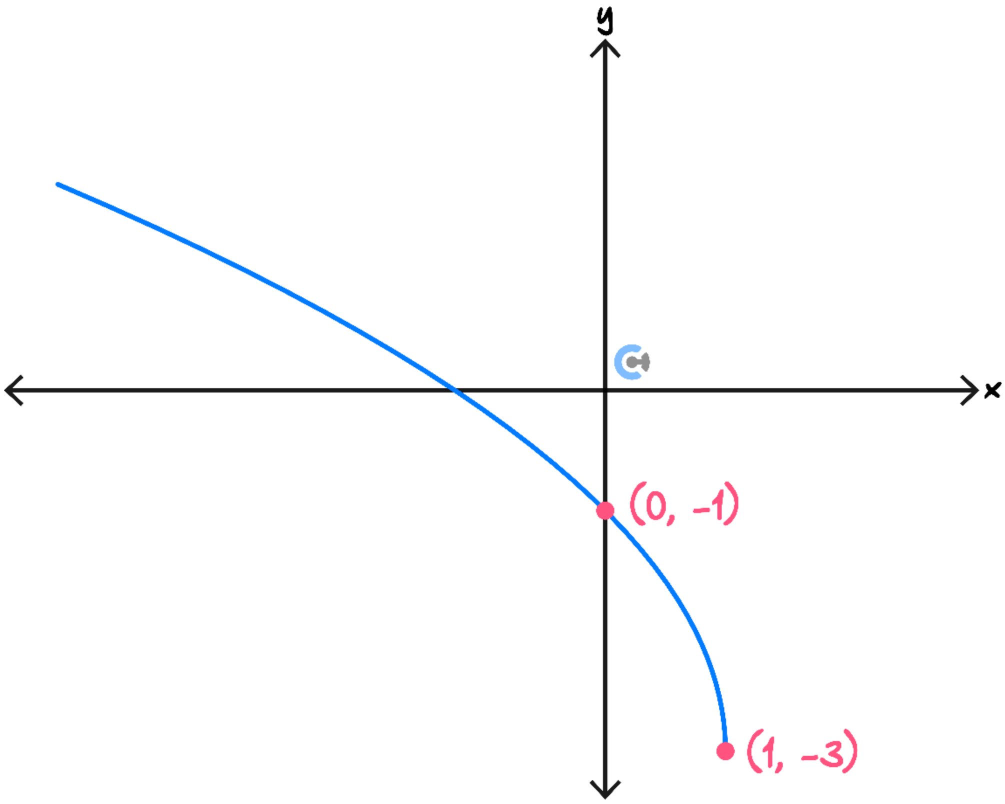
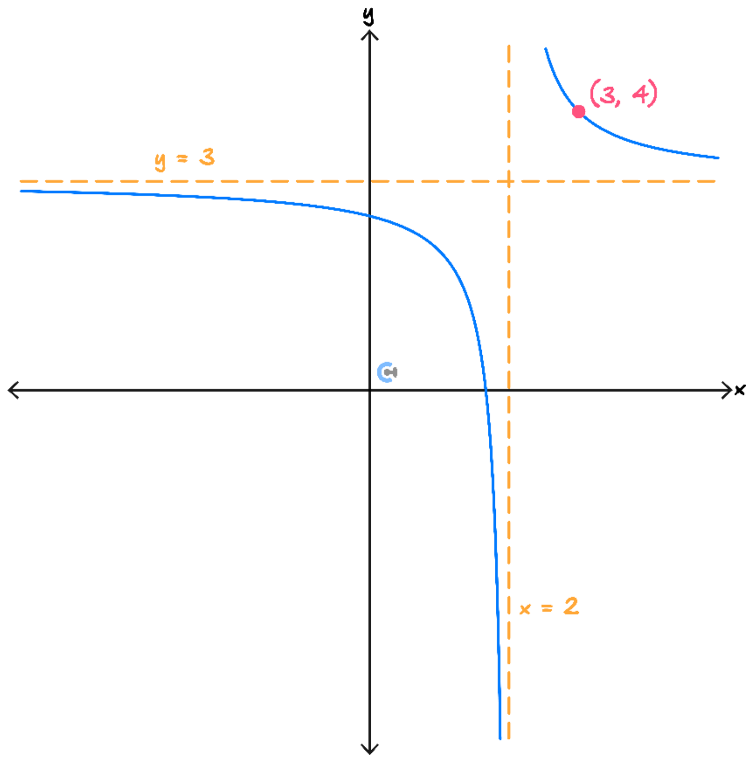
**Question 25**

Determine a possible rule for the graphs.

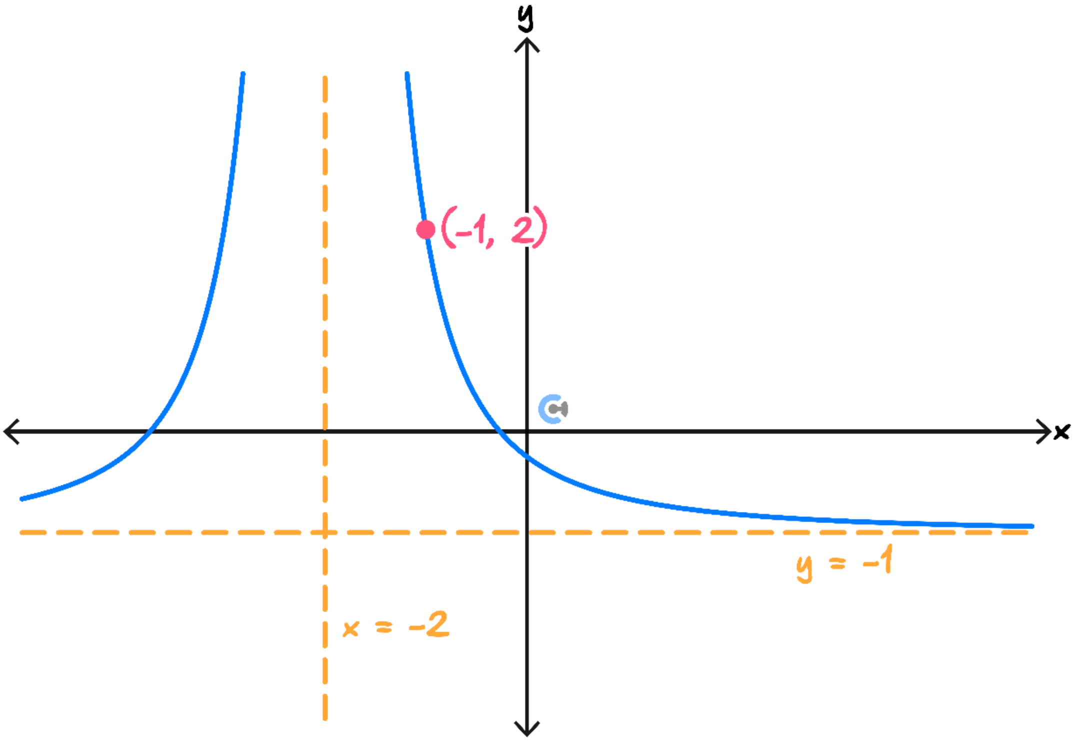
**a.**



**b.**



**c.**

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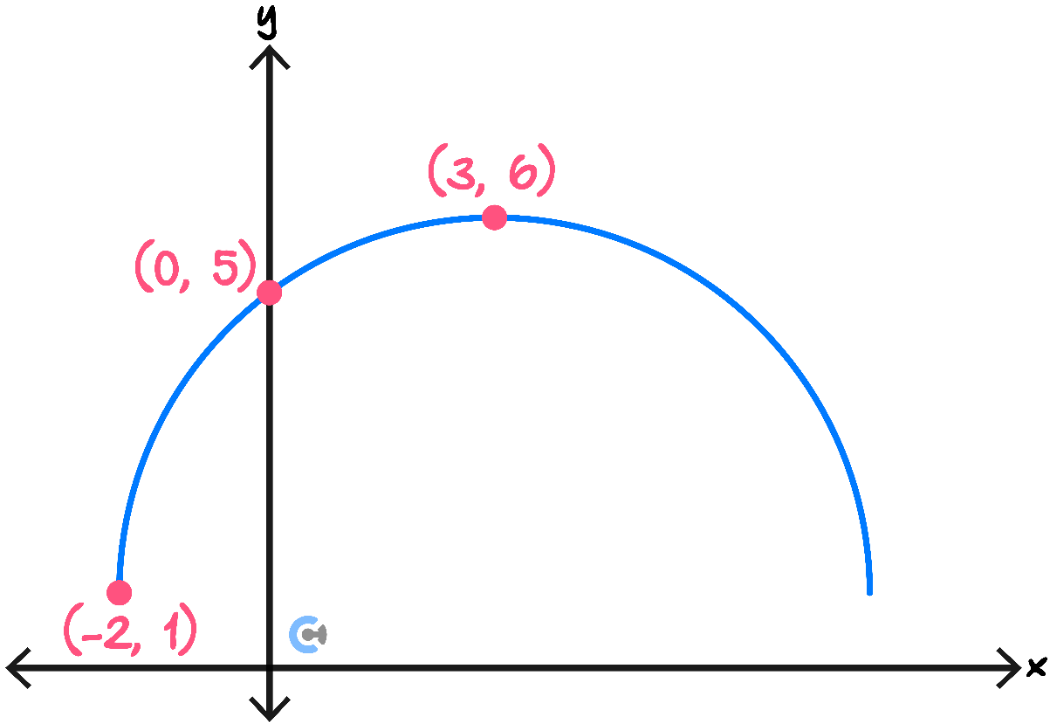
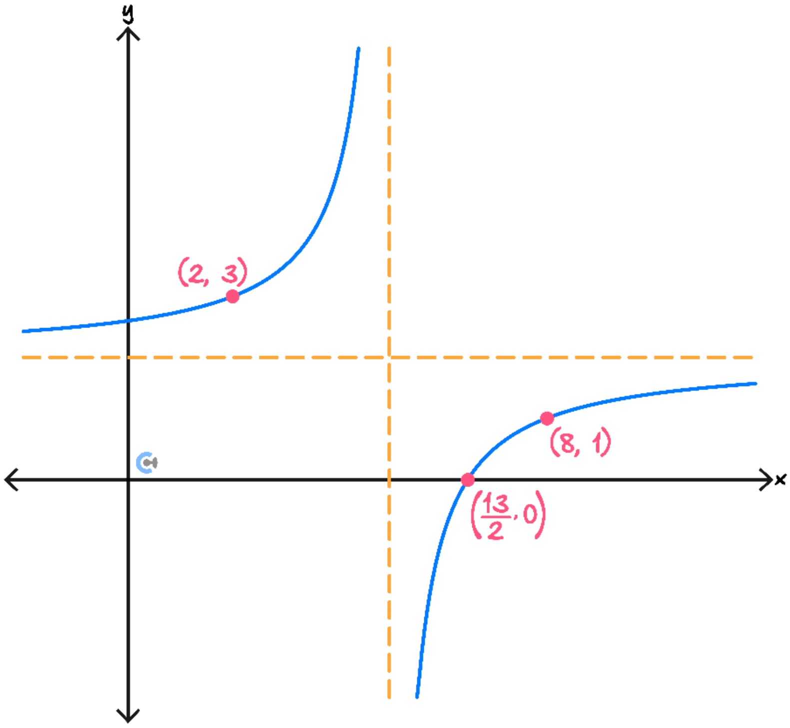
**Question 26**

Determine a possible rule for the graphs.

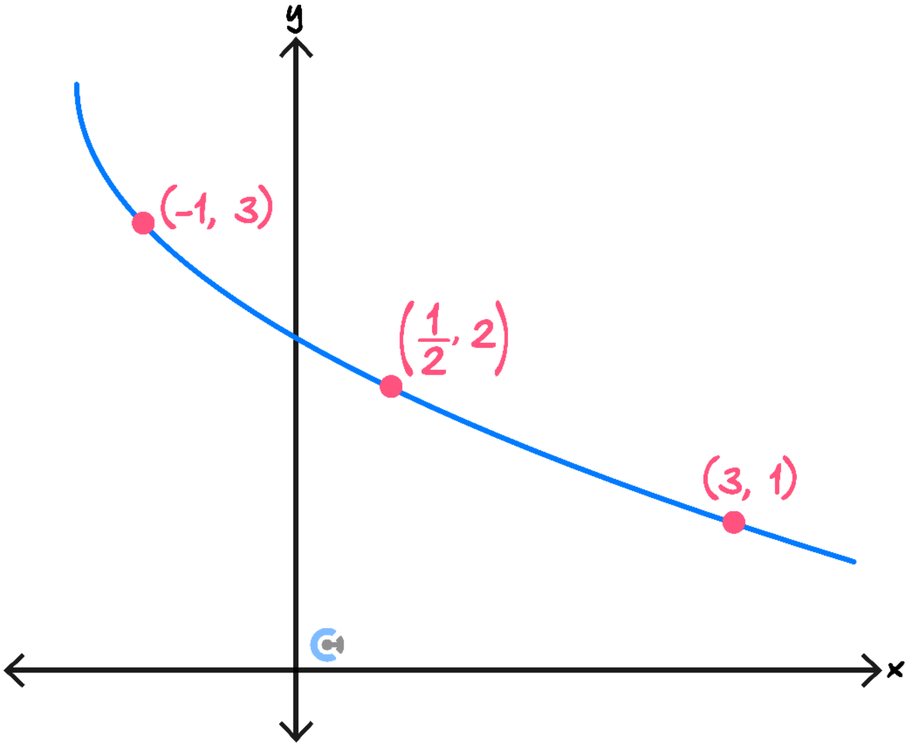
**a.**



**b.**



**c.**

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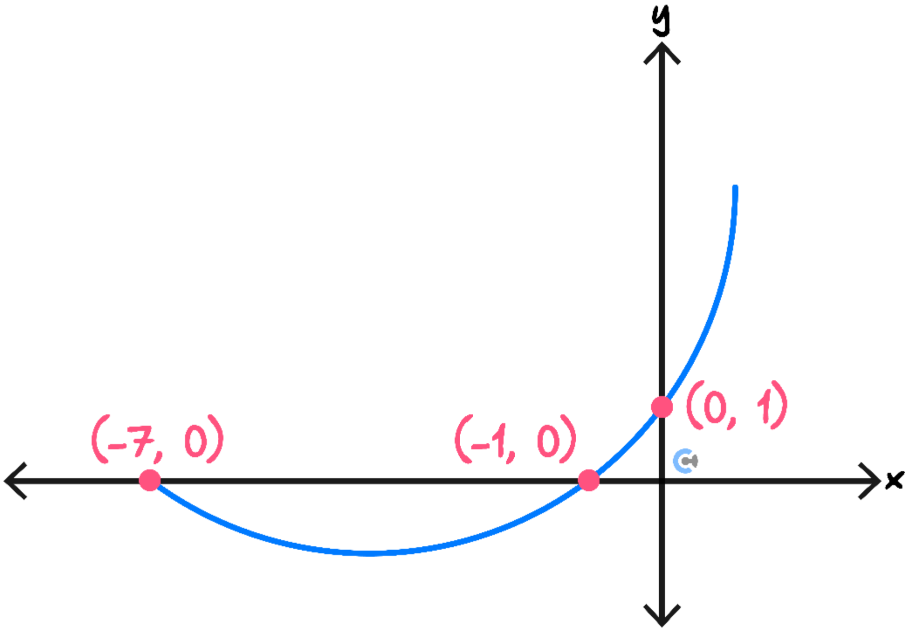
**Question 27**

Determine a possible function for this graph.



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## Sub-Section [2.3.3]: Solve Number of Solution Problems Graphically

**Question 28 Tech-Active.**

Consider the function $f(x) = 4x^2 - 4x + 5$.

Determine the real values of $k$ for which $f(x) = k$ has two solutions.

**Question 29 Tech-Active.**

Consider the function $f(x) = x^3 + 3x^2 - 9x + 2$.

Determine the real values of $k$ for which $f(x) = k$ has three:

**a.** Two solutions.

**b.** Three solutions.

**Question 30 Tech-Active.**

Consider the function $f(x) = x^4 - 8x^3 + 6x^2 + 40x - 14$.

Determine the real values of $k$ for which $f(x) = k$ has :

**a.** Three solutions.

**b.** Two solutions.

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**Question 31**

Consider the function $f(x) = 3x^3 + k$.

Determine the real value of $k$ for which $f(x) = f^{-1}(x)$ has three solutions.

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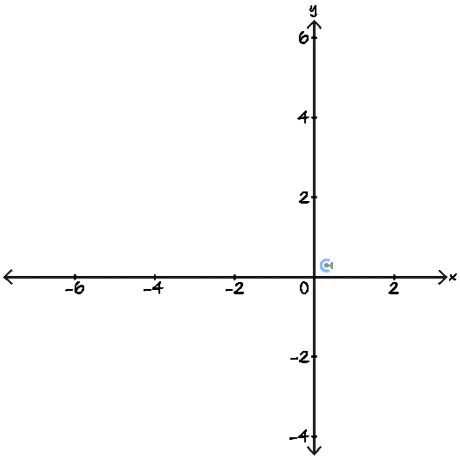
## Sub-Section: Exam 1 Questions

**Question 32**

Let $f(x) = \frac{2x+6}{x+4}$ be defined on its maximal domain.

**a.** Sketch the graph of $f(x)$ on the axes below. Labelling all asymptotes with their equations and axial intercepts with their coordinates.




**b.** State the domain and range of $f^{-1}$.

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**c.** Find the values of $x$ for which $f(x) > 1$.

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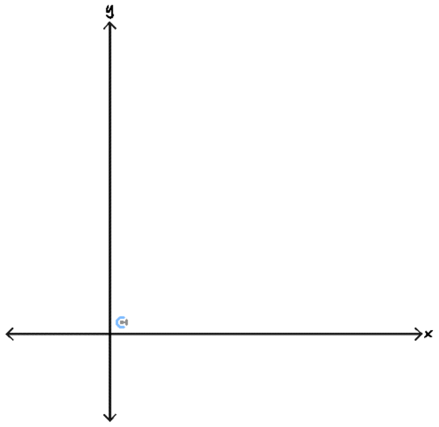
**Question 33**

Consider the function $f : \mathbb{R}\backslash\{h\} \to \mathbb{R}, \ f(x) = \frac{a}{(x-h)^2} + k$.

The graph of $f$ is drawn below.




**a.** Show that $a = -2, h = 2$, and $k = 5$.

**b.** Find the maximal domain of $g(x) = \sqrt{4 - (f(x) - 1)^2}$.

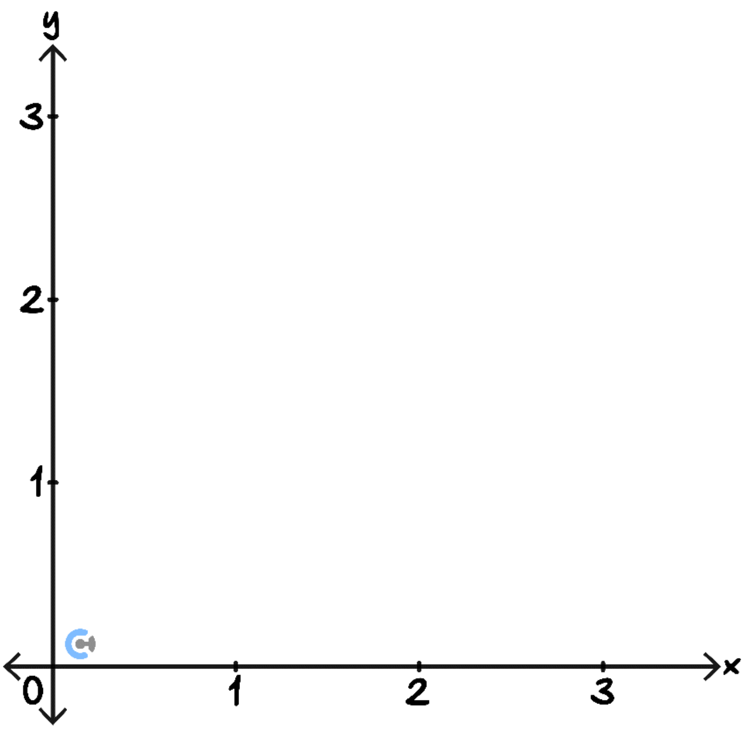
**Question 34**

Consider the function $f : [a, \infty) \to \mathbb{R}, f(x) = x^2 - 2x + 2$.

**a.** Find the smallest value of $a$ for which the inverse function of $f$, $f^{-1}$ exists.

**b.** State the domain and range of $f^{-1}$.

**c.** The graph of $y = f(x)$ is drawn on the axis below, sketch the graph of $y = f^{-1}(x)$ on the same axis, labelling points of intersection with their coordinates.



**d.** Let $g: [1, \infty) \to \mathbb{R}, g(x) = (x - 1)^2 + k$.

**i.** Find the values of $k$ for which $g(x) = g^{-1}(x)$ has no solutions.

**ii.** Find the values of $k$ for which $g(x) = g^{-1}(x)$ has two solutions.

**iii.** Find the values of $k$ for which $g(x) = g^{-1}(x)$ has one solution.

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## Sub-Section: Exam 2 Questions

**Question 35**

The maximal domain of the function $f$ is $(-2, \infty)$.

A possible rule for $f$ is:

**A.** $f(x) = \log_2(x - 2)$

**B.** $f(x) = \sqrt{2 - x}$

**C.** $f(x) = \frac{1}{x+2}$

**D.** $f(x) = \frac{1}{\sqrt{x+2}}$

**Question 36**

Consider the function $f : (a, b] \to \mathbb{R}, f(x) = \frac{1}{x-1}$ where $a < b < 1$.

The range of $f$ is:

**A.** $\left(\frac{1}{a-1}, \frac{1}{b-1}\right]$

**B.** $\left[\frac{1}{b-1}, \frac{1}{a-1}\right)$

**C.** $(b, a]$

**D.** $\left[\frac{1}{a-1}, \frac{1}{b-1}\right)$

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**Question 37**

Consider the function $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$.

The equation $f(x) = k$ will have three solutions for:

**A.** $k > -3$

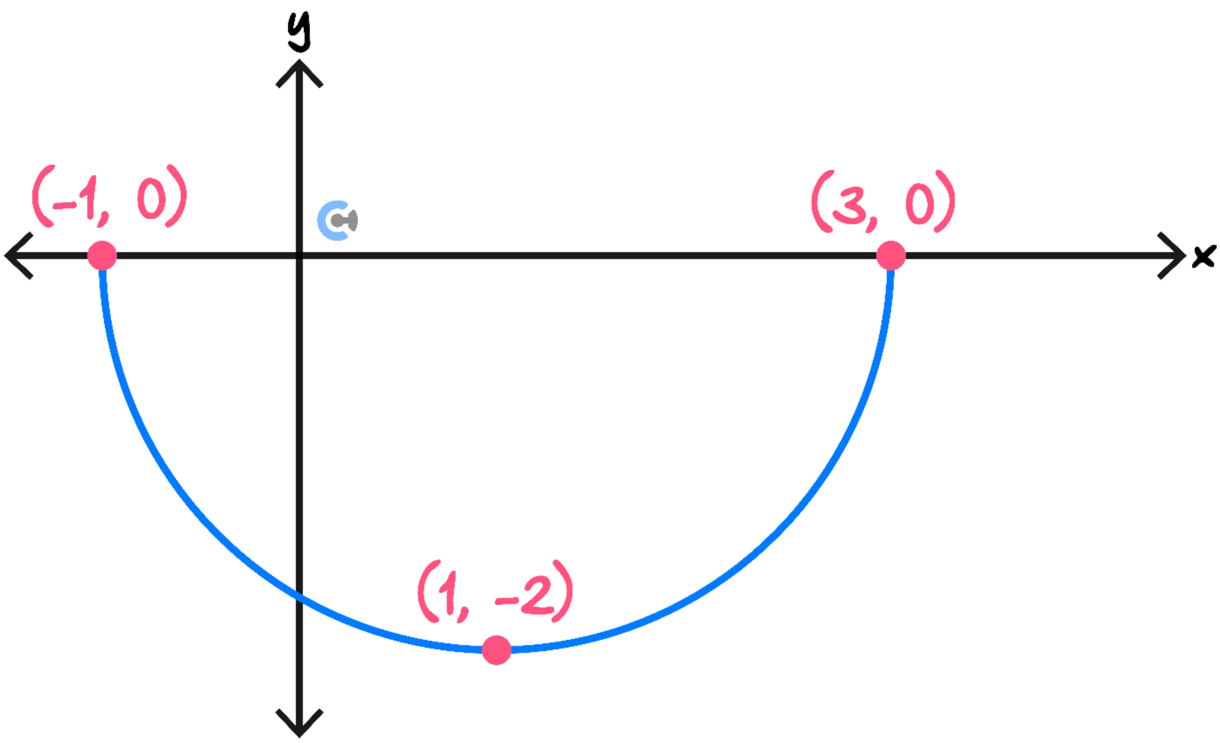
**B.** $k < 2$

**C.** $-3 < k < 2$

**D.** $k = -3$ or $k = 2$

**Question 38**

The equation that best represents the graph below is:




**A.** $y = -\sqrt{3 + 2x - x^2}$

**B.** $y = -\sqrt{3 - 2x - x^2}$

**C.** $y = \sqrt{4 - (x - 1)^2}$

**D.** $(x + 1)^2 + y^2 = 4$

**Question 39**

Consider the function $f: [-20, a] \to \mathbb{R}, f(x) = 2x^2 - 12x + 5$.

The smallest value of $a$ for which the inverse function of $f$, $f^{-1}$ exists is:

**A.** $a = 6$

**B.** $a = -6$

**C.** $a = 3$

**D.** $a = -3$

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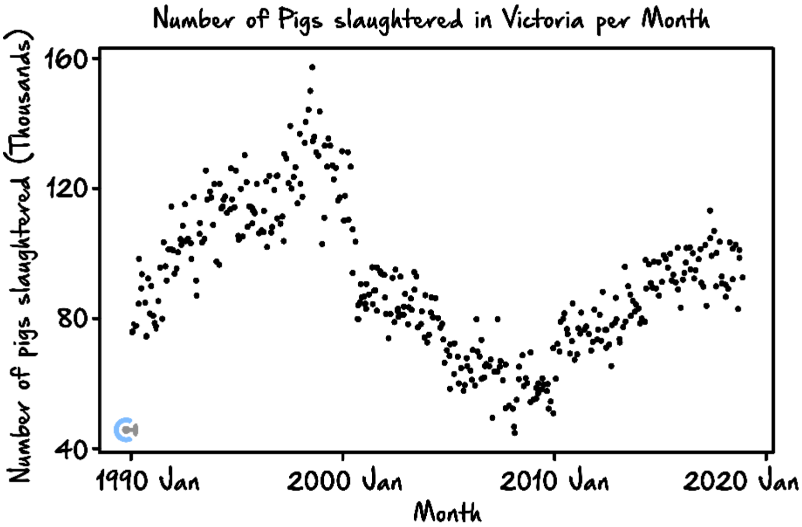
**Question 40**

The points shown on the chart below represent the number of pigs slaughtered in Australia, from 1990 to 2018.




We can attempt to model $y$, the number of pigs slaughtered in thousands, as a function of time.

Specifically, the variable $t$ which represents the month when the pigs were slaughtered, where $t = 1$ corresponds to January 1990, $t = 2$ corresponds to February 1990 and so on.

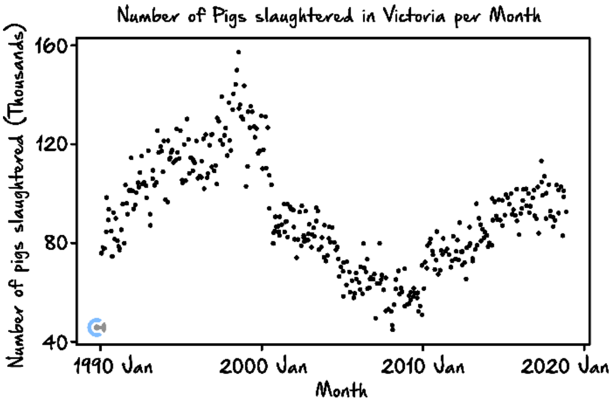
Our first attempt is setting $y = f(t)$, where $f : [1, \infty) \to \mathbb{R}, f(t) = \frac{a}{1000}t^3 + \frac{b}{100}t^2 + \frac{c}{10}t + d$ ,is a cubic polynomial.

We can do this by ensuring $f$ reflects some suitable points.

**a.** If we want $f(1) = 76$, $f(103) = 157.1$, $f(219) = 44.8$, and $f(348) = 92.3$, find the values of $a, b, c, d$ correct to 3 decimal places.

**b.** Plot the graph of $f$ over the interval $[1, 348]$ on the axis below, labelling the four points mentioned in **part a.i** with their coordinates.

You can use the fact that $t = 103$ corresponds to July, 1998, $t = 219$ corresponds to March 2008 and $t = 348$ corresponds to December 2018.




**c.**

**i.** According to this model, what is the earliest month after 2018 for which the number of pigs slaughtered will be greater than 157100?

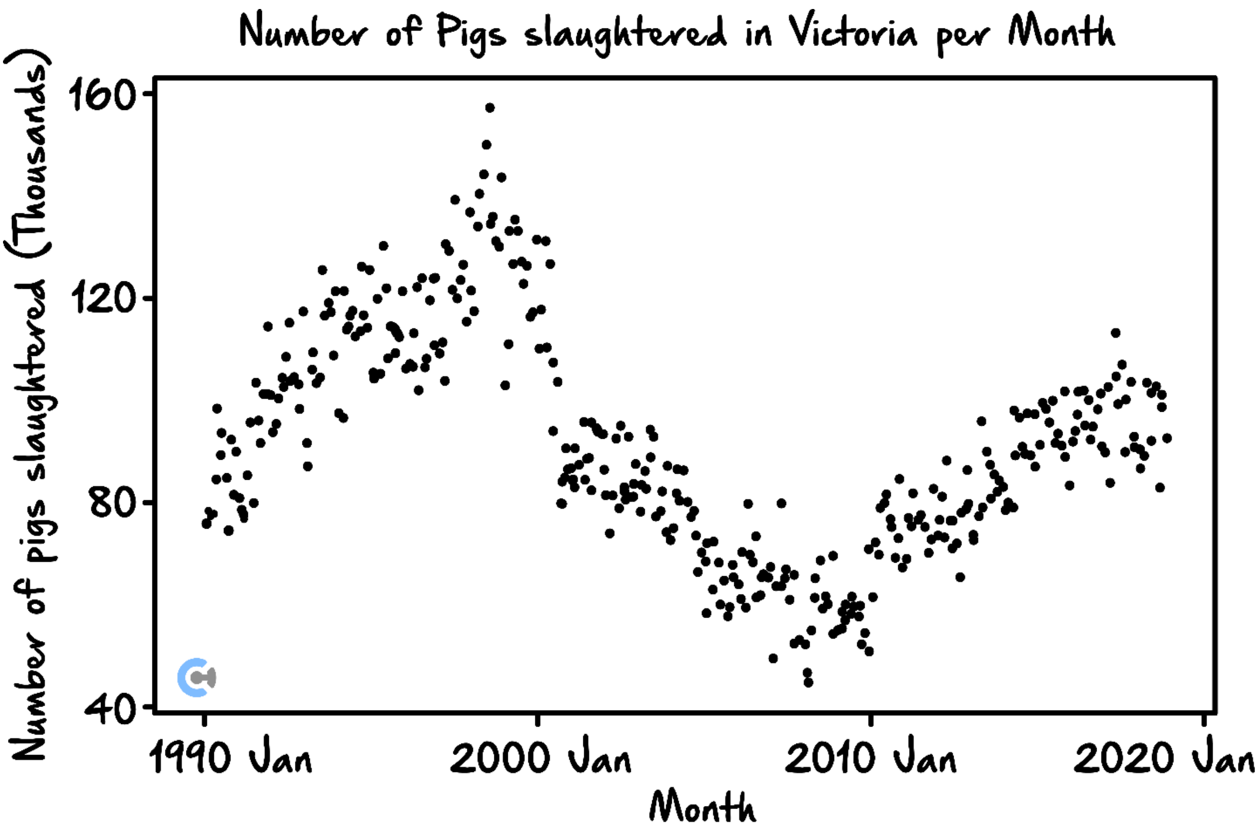
**ii.** For what values of $k$, does $f(t) = k$ have two solutions? Give your answer to 2 decimal places.

**iii.** Does our cubic model accurately reflect the minimum and maximum number of pigs slaughtered from 1990 to 2018?

**d.** An alternative model can be $y = g(t)$, where $g(t) = \sqrt{at^3 + bt^2 + ct + d}$.

**i.** Explain why the restrictions $g(1) = 76$, $g(103) = 157.1$, $g(219) = 44.8$, and $g(348) = 92.3$ are unusable.

**ii.** Sketch the graph of $y = g(t)$ over the interval $[1, 348]$ on the axis below if $g(1) = 76$, $g(103) = 157.1$, $g(262) = 70.2$ and $g(348) = 92.3$. Label endpoints and turning points with their coordinates correct to 2 decimal places.



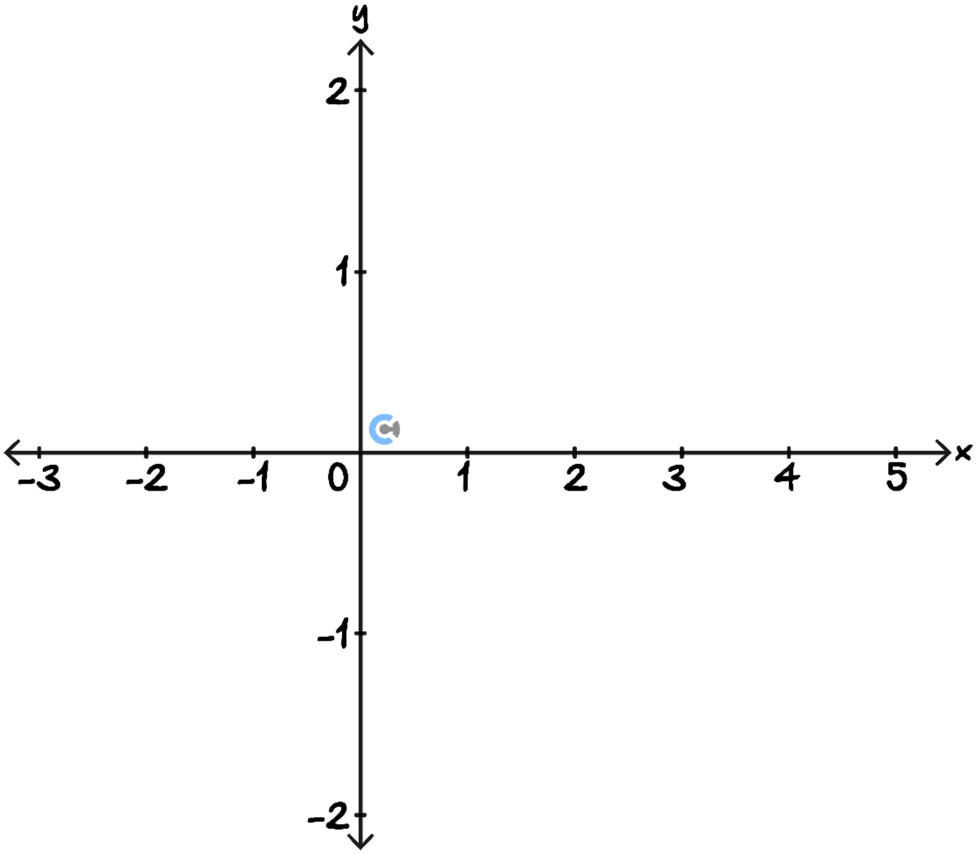

Space for Personal Notes

**Question 41**

Consider the function $f(x) = \frac{5-8x+2x^2}{x^2-4x+4}$.

**a.** State the maximum domain and range of $f$.

**b.** Sketch the graph of $f$ on the axis below, labelling asymptotes with their equations and axes intercepts with their coordinates.



**c.** Consider the function $g(x) = 3x^4 + 8x^2 - 6x^2 - 24x + k$.

**i.** State the turning points of $g(x)$ in terms of $k$.

**ii.** For what values of $k$, does the equation $g(x) = 2$ have exactly two solutions?

**iii.** For what values of $k$, does the equation $g(x + a) = f(x)$ never have a solution for any value of $a$?

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