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VCE Mathematical Methods ½
Functions & Relations II [2.2]
Workbook

Outline:

Section A: Domain and Range

Sub-Section: Set Notation

Let's have a look at set notations!

Set Operators

- Intersection: "AND".

$$A \cap B = \textit{What values are in set } A \textit{ AND in set } B.$$

- Union: "OR".

$$A \cup B = \textit{What values are in set } A \textit{ OR in set } B.$$

- Set difference: "Except".

$$A \setminus B = \textit{What values are in set } A \textit{ except those also in set } B.$$

Space for Personal Notes

Question 1

For the sets given below, find:

$$A = \{0, 2, 3, 5, 6, 11\} \text{ and } B = \{0, 1, 2, 3, 5, 7, 9, 10\}$$

a. $A \cap B =$

$A = \{0, 2, 3, 5, 6, 11\}$ $B = \{0, 1, 2, 3, 5, 7, 9, 10\}$

$A \cap B = \{0, 2, 3, 5\}$

b. $A \cup B =$

$A = \{0, 2, 3, 5, 6, 11\}$ $B = \{0, 1, 2, 3, 5, 7, 9, 10\}$

$A \cup B = \{0, 1, 2, 3, 5, 6, 7, 9, 10, 11\}$

c. $A \backslash B =$

$A = \{0, 2, 3, 5, 6, 11\}$ $B = \{0, 1, 2, 3, 5, 7, 9, 10\}$

$A \backslash B = \{6, 11\}$

d. $B \backslash A =$

$A = \{0, 2, 3, 5, 6, 11\}$ $B = \{0, 1, 2, 3, 5, 7, 9, 10\}$

$B \backslash A = \{1, 7, 9, 10\}$

Sub-Section: Interval Notation

Now interval notation!

Interval Notation

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

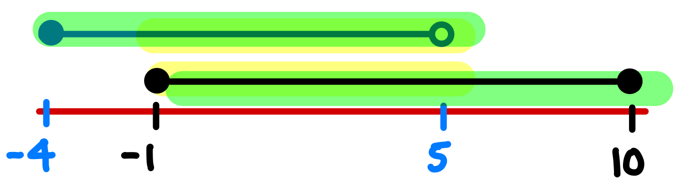
Question 2 Walkthrough.

Simplify the following set.

$$A = [-1, 10] \text{ and } B = [-4, 5)$$

a. Find $A \cap B$.

$= [-1, 5)$ //



b. Find $A \cup B$.

$= [-4, 10]$ //

NOTE: Use **number lines** to find the intersection and union of sets.

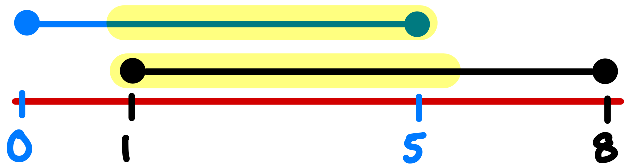
Now your turn!

Question 3

Find the following sets:

a. $[0,5] \cap [1,8]$

$= [1,5]$



b. $[-3,7] \cup \left(-11,\frac{1}{2}\right]$

$= (-11,7]$

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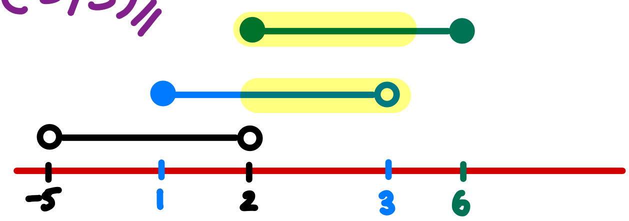
Question 4 Extension.

Find the following set.

$$[1,3) \cap [2,6] \cup (-5,2)$$

$[2,3) \cup (-5,2)$

$= (-5,3)$ //



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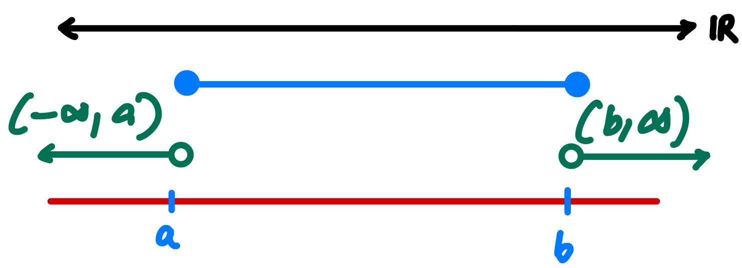
<u>Discussion:</u> What is $\mathbb{R}\backslash[a,b]$ equal to? Is it $(-\infty,a)\cup(b,\infty)$ or $(-\infty,a]\cup[b,\infty)$?



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Sub-Section: Maximal Domain

What is a maximal domain?

Maximal Domain

- The maximal domain is the longest possible domain for a rule without committing a mathematical warcime
- In Methods, we need to consider 3 important rules:

$$\sqrt{z}, \quad z \geq 0$$

$$\log(z), \quad z > 0$$

$$\frac{1}{z}, \quad z \neq 0$$

Head Tutor's Comment: Emphasise that this works WHATEVER THE z is.

NOTE: We will consider $\log$ in depth later throughout the year!

Space for Personal Notes

Question 5 Walkthrough.

Find the maximal domain of each of the following functions.

a. $f(x) = \underbrace{\sqrt{x-3}}_{\geq 0}$

$x-3 \geq 0$

$\therefore x \geq 3$

b. $h(x) = \log_2 \underbrace{(x+5)}_{>0}$

$x+5>0$

$\therefore x>-5$

Head Tutor's Comment: Emphasise the need for graphing when solving non $1:1$ inequalities.

c. $h(x) = \frac{1}{\underbrace{x-4}_{\neq 0}}$ $\Rightarrow$ $x-4 \neq 0$

$\therefore x \neq 4$

$\therefore x \in \mathbb{R} \setminus \{4\}$

Your turn!

Question 6

Find the maximal domain of the following functions.

a. $f(x) = \underbrace{\sqrt{-x-6}}_{\geq 0} - 5$

$-x-6 \geq 0$

$\therefore x \leq -6$

b. $h(x) = -\log_2 \underbrace{(x+10)}_{>0}$

$x+10>0$

$\therefore x > -10$

c. $\underbrace{\frac{1}{x^2-25}}_{\neq 0}$

$x^2-25 \neq 0$

$x^2 \neq 25$

$x \neq \pm 5$

$\therefore x \in \mathbb{R} \setminus \{5, -5\}$

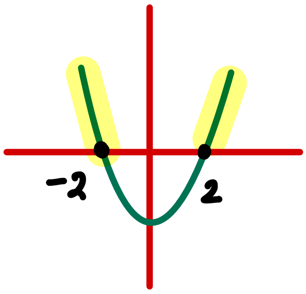
Now harder ones!

Question 7

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{\underline{x^2 - 4}} - 5$

≥ 0



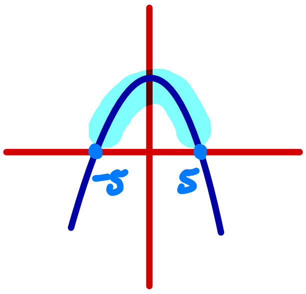
$x^2 - 4 \geq 0$

$x^2 \geq 4$

$\therefore x \geq 2$ or $x \leq -2$

b. $h(x) = -\log_2(\underline{25 - x^2})$

> 0



$25 - x^2 > 0$

$x^2 < 25$

$\therefore -5 < x < 5$

NOTE: Always sketch the function when solving inequalities for many to one functions.

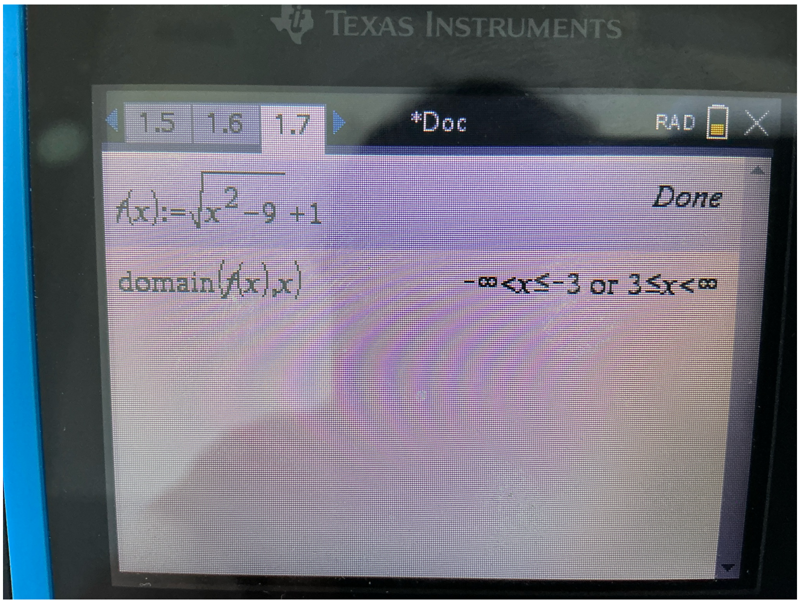
Calculator Commands

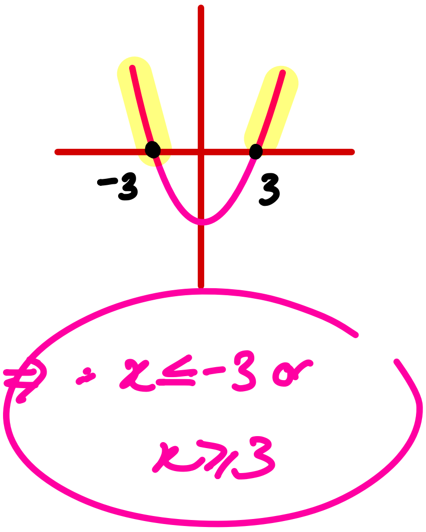
Mathematica	TI-Nspire	Casio Classpad
`FunctionDomain[func, x]`	Type up domain (or find it under the book button). $\text{domain}(\text{func}, x)$	Sketch the function and analyse.

Question 8 Tech Active.

Find the maximal domain of the following function.

$$f(x) = \sqrt{x^2 - 9} + 1$$





Sub-Section: Range

Now the range!

Range

- The range is the possible values for the output of a function.

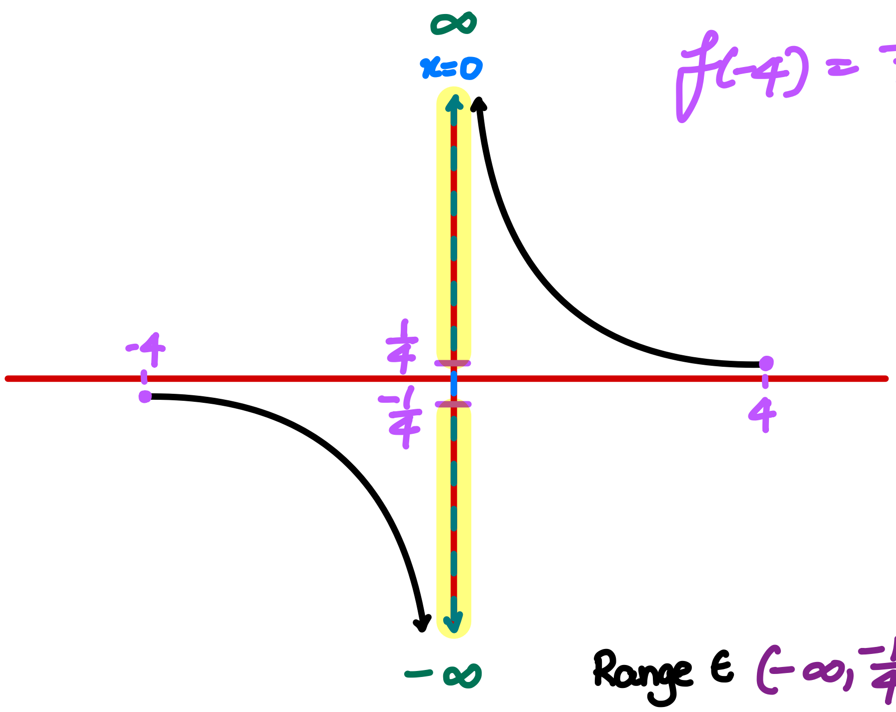
Question 9 Walkthrough.

Find the range of the following function:

$$f: [-4, 4] \backslash \{0\} \to \mathbb{R}, f(x) = \frac{1}{x}$$

$f(4) = \frac{1}{4}$

$f(-4) = -\frac{1}{4}$



Range $\in (-\infty, -\frac{1}{4}] \cup [\frac{1}{4}, \infty)$

or

$f(x) \in \mathbb{R} \backslash (-\frac{1}{4}, \frac{1}{4})$

TIP: Always **sketch** the function!

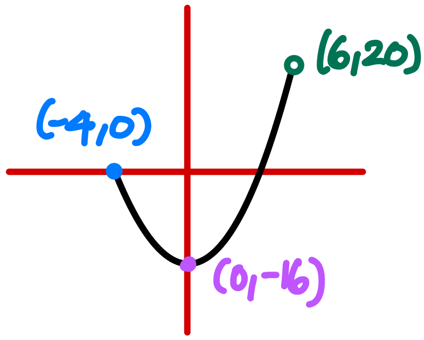
Question 10

Find the range of the following functions.

a. $f: [-4, 6) \rightarrow R, f(x) = x^2 - 16$

$f(-4) = 16 - 16 = 0$

$f(6) = 36 - 16 = 20$



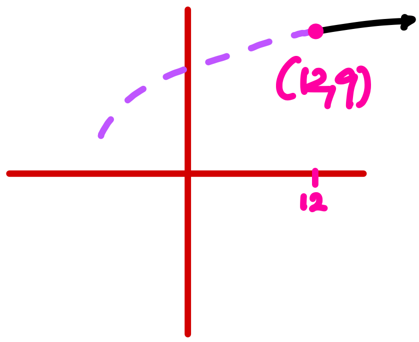
Range $\in [-16, 20)$

b. $f: [12, \infty) \rightarrow R, f(x) = 2\sqrt{x+4} + 1$

$f(12) = 2\sqrt{16} + 1$

$= 2 \cdot 4 + 1$

$= 9$



Range $\in [9, \infty)$

Question 11 Extension.

$(a+b)^2 = a^2 + 2ab + b^2$

Find the range of the following function.

$$f: [-2, 8) \rightarrow R, f(x) = \frac{1}{2}x^2 - 2x - 2$$

$f(x) = \frac{1}{2}(x^2 - 4x) - 2$

$= \frac{1}{2}((x-2)^2 - 4) - 2$

$= \frac{1}{2}(x-2)^2 - 4$

$f(-2) = \frac{1}{2}(4) + 4 - 2$

$= 2 + 2$

$= 4$

$f(8) = \frac{1}{2}(64) - 16 - 2$

$= 32 - 18$

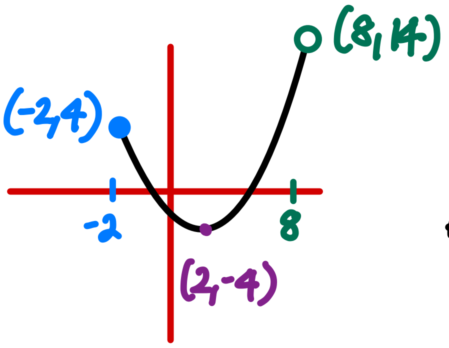
$= 14$

$\therefore x = \frac{-(-2)}{2(\frac{1}{2})} = 2$

$f(2) = \frac{1}{2}(4) - 4 - 2$

$= 2 - 6$

$= -4$



Range $\in [-4, 14)$

Sub-Section: Functional Notation

How do we represent a function?

Functional Notation

Dom x $\to$ Range y

$$f: Domain \to Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.
- Codomain is **not** the same as range.

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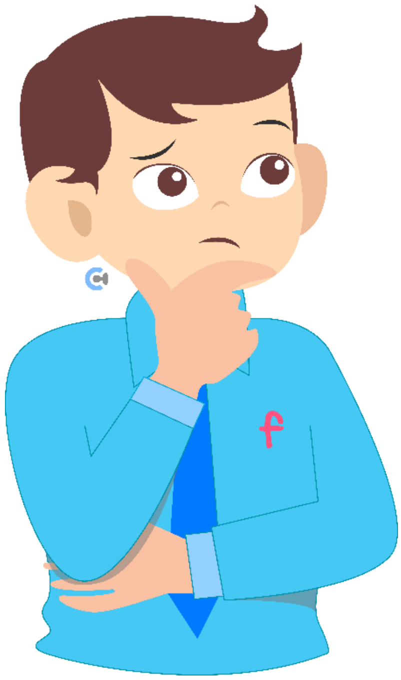
<u>Analogy</u>: Functional notation is a “business card” for functions.

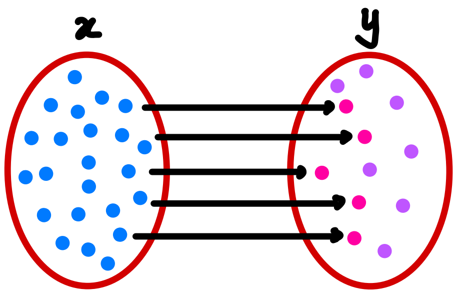
- A function f wants to make a business card for themself.



- They decide to put their name, working hours, company associated and their role.

$$\boldsymbol{Name\colon Working\ Hour \to Company, Role}$$

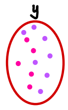
- Their name is simply f.
- Their working hours are their “domain”.
- Their company is the “CoDomain”.
- Their role is the rule!



$$\boldsymbol{f\colon Domain \to Codomain, f(x) = Rule}$$

• ⇒ Domain

• ⇒ Range



⇒ Codomain

- Now, does f have to make everything in their company?

No

- Hence, using this analogy, would his range (their output) be the same as the codomain (company)?

Range $\subseteq$ Codomain

Subset

“within”

Question 12

Consider the following function, written in functional notation:

$$f: [-1, 4] \to \mathbb{R}, f(x) = x + 5$$

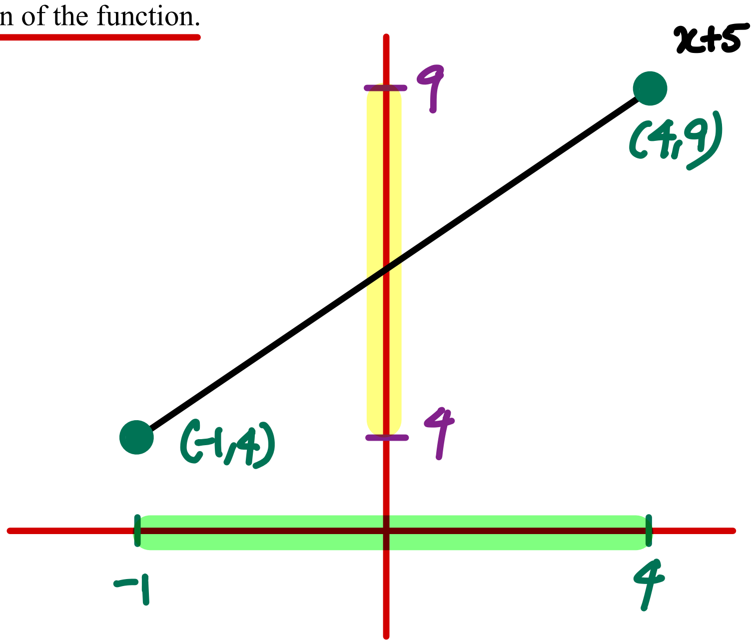
Identify the name, domain, range, and the equation of the function.

Name: f

Domain: $[-1, 4]$

Range: $[4, 9]$

Equation: $f(x) = x + 5$



Head Tutor's Comment: Don't spend too much time here just skim through quickly.

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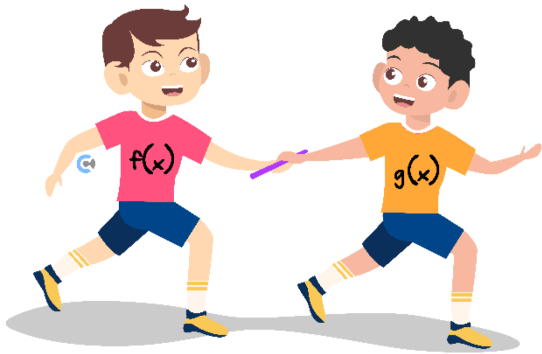
Section B: Hybrid (Piecewise) Functions

Analogy: Hybrid functions are like a relay race.

- Imagine the functions $f(x)$ and $g(x)$ participating in a relay race as part of the same team.



- $f(x)$ is running for $x < 4$ and $g(x)$ is running for $x \geq 4$.
- For $x = 5$ who do we look at?

$g(x)$

- This is how hybrid functions work!

Piecewise (Hybrid) Functions

- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x values for which the two functions are defined.
- The two domains do not have to join!

Space for Personal Notes

Question 13 Walkthrough.

Consider the hybrid function f.

$$f(x) = \begin{cases} x^2 - 5, & x \geq 0 \\ x + 4, & x < 0 \end{cases}$$

$x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$ ≈ 2.2

a. Find $f(-2)$.

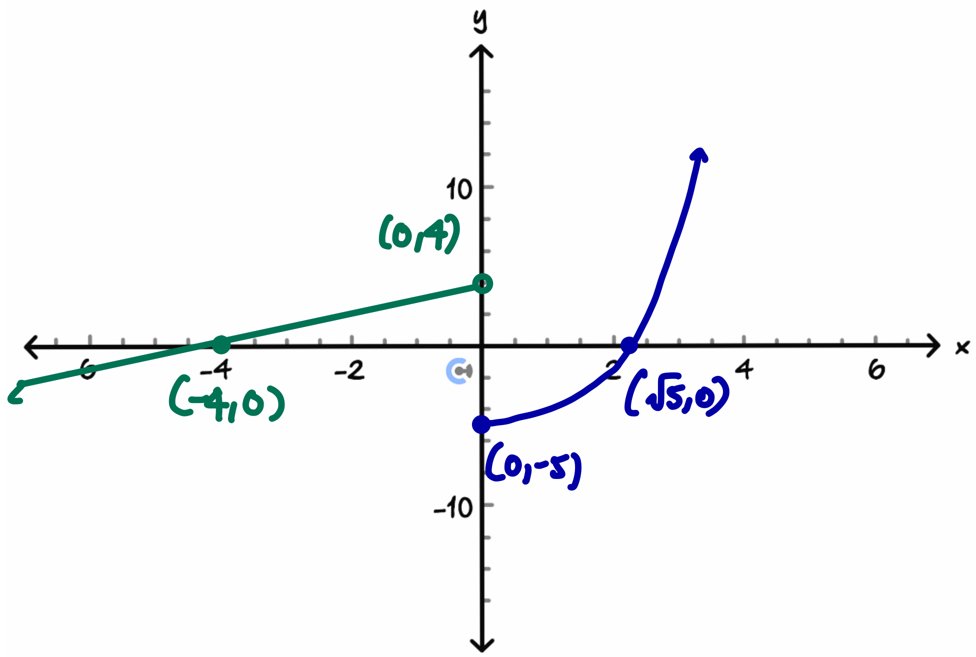
$f(-2) = 2$

b. Find $f(5)$.

$f(5) = 25 - 5 = 20$

c. Graph $y = f(x)$.

OPEN CIRCLE AT $(0, 4)$



Question 14

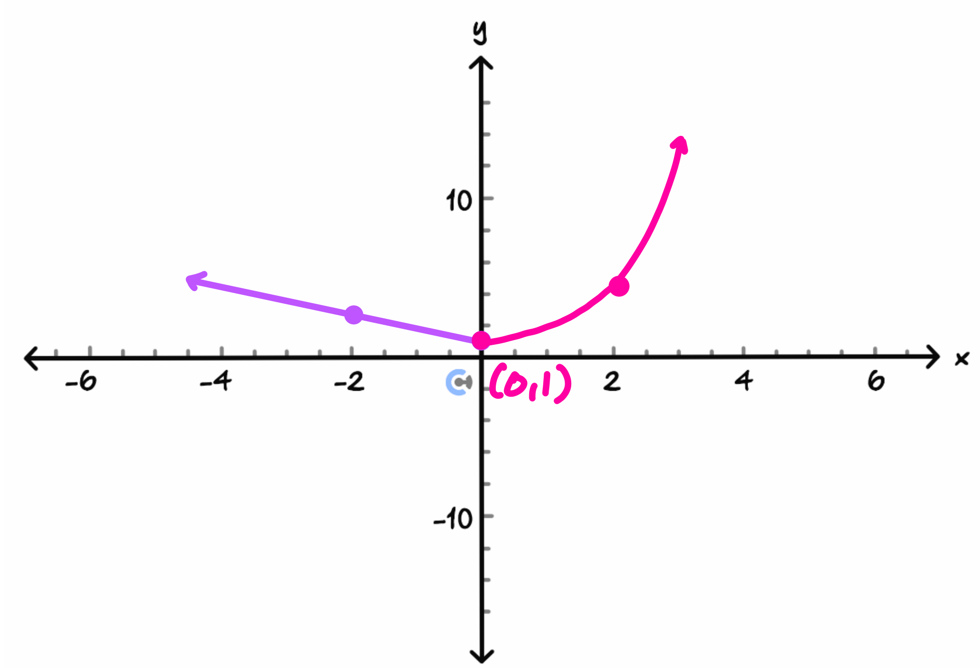
Consider the hybrid function g.

$f(2) = 5$

$f(-2) = 3$

$$g(x) = \begin{cases} x^2 + 1, & x \geq 0 \\ 1 - x, & x < 0 \end{cases}$$

a. Graph $y = g(x)$.



b. Find the range of $g(x)$.

Range $\in [1, \infty)$

Question 15 (CAS Active)

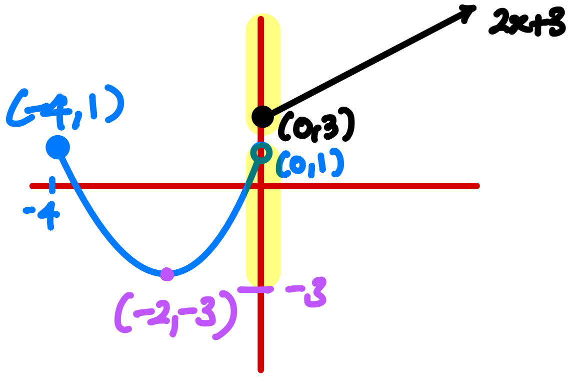
$g(-4) = 16 - 16 + 1$
$= 1$

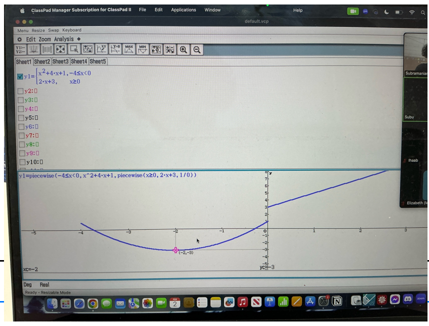
Consider the hybrid function g.

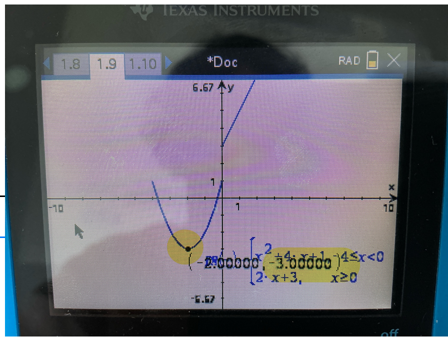
$$g(x) = \begin{cases} x^2 + 4x + 1, & -4 \leq x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$$

Find the range of $g(x)$.

Range $\in [-3, 1] \cup [3, \infty)$







Defining Hybrid Functions on CAS

Mathematica

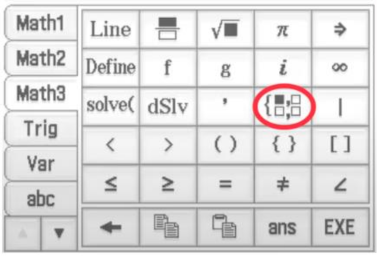
- "Esc PW" and Control Enter to create cells.

$$\begin{cases} \text{func1} & \text{dom1} \\ \text{func2} & \text{dom2} \end{cases}$$

TI-Nspire

$$\begin{cases} func\,1, dom\,1 \\ func\,2, dom\,2 \end{cases}$$

Casio Classpad



Question 16

Consider the hybrid function g.

$$g(x) = \begin{cases} x^3 + 6x - 5, & x < 1 \\ x + 4, & x \geq 1 \end{cases}$$

a. Evaluate $g(-2)$.

$g(-2) = -8 - 12 - 5$

$= -25$

b. Evaluate $g(3)$.

$g(3) = 7$

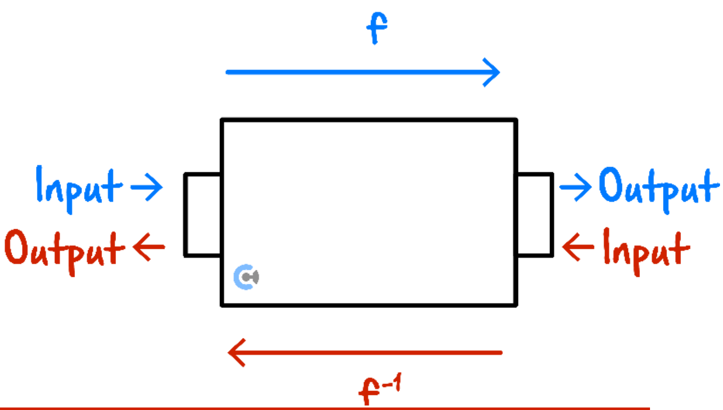
Section C: Inverse Functions

Sub-Section: Basics of Inverses

What does "inverse" mean?

Inverse Relation

- **Definition**: Inverse is a relation which does the opposite.



Head Tutor's Comment: Go through the basics quickly.

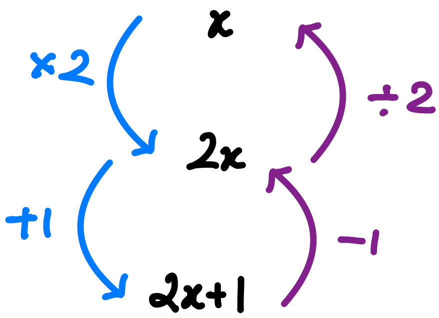
Discussion: What would be the inverse of $f(x) = x + 2$?

$f^{-1}(x) = x - 2$

Question 17

Find the inverse of $f(x) = 2x + 1$.

$\therefore f^{-1}(x) = \frac{x-1}{2}$



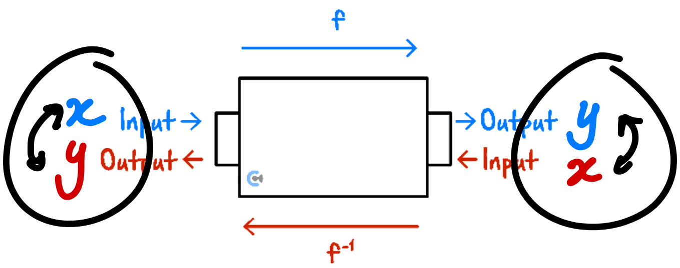
Head Tutor's Comment: Do NOT solve this by swapping x and y. Use the "opposite" idea only.

Sub-Section: Swapping x and y

Is there a better way of solving for an inverse relation?

Solving For an Inverse Relation

- Swap x and y.



Question 18

Find the inverse of $f(x) = 2x + 1$ by swapping x and y.

Let $y = f(x)$:

Swap x & y:

$x = 2y + 1$

$\therefore y = \frac{x-1}{2}$

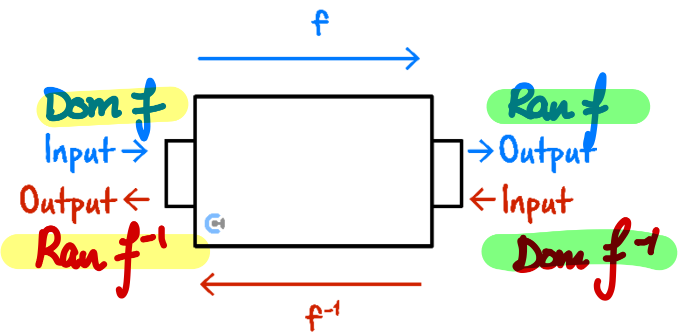
$\therefore f^{-1}(x) = \frac{x-1}{2}$

NOTE: $f(x) = y$.

Discussion: Hence, what would happen to the domain and range of the function when we find its inverse?

Swap Dom & Ran as well

Domain and Range of Inverse Functions



$Dom\ f^{-1} =$ Ran f

$Ran\ f^{-1} =$ Dom f

Question 19 Walkthrough.

$\to x+2 \geq 0 \Rightarrow x \geq -2$

Consider the function $f(x) = \underbrace{\sqrt{x+2}}_{\geq 0} - 1$ defined for its maximal domain.

a. Find the rule for the inverse function.

Let $y = f(x)$:

Swap x & y:

$x = \sqrt{y+2} - 1$

$x + 1 = \sqrt{y+2}$

$y + 2 = (x+1)^2$

$\therefore y = (x+1)^2 - 2$

$\therefore f^{-1}(x) = (x+1)^2 - 2$

b. State the domain and range of inverse function.

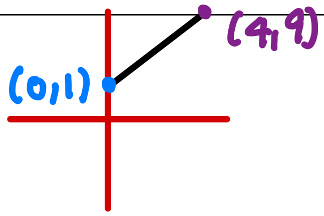
$\sqrt{x} - 1$

$[0, \infty) - 1 \Rightarrow [-1, \infty)$

Dom f^{-1} = Ran f = $[-1, \infty)$

Ran f^{-1} = Dom f = $[-2, \infty)$

Question 20



Consider the function $f: [0, 4] \to R, f(x) = 2x + 1$.

a. Find the rule for the inverse function.

Let $y = f(x)$:

Swap x & y:

$x = 2y + 1$

$\therefore y = \frac{x-1}{2}$

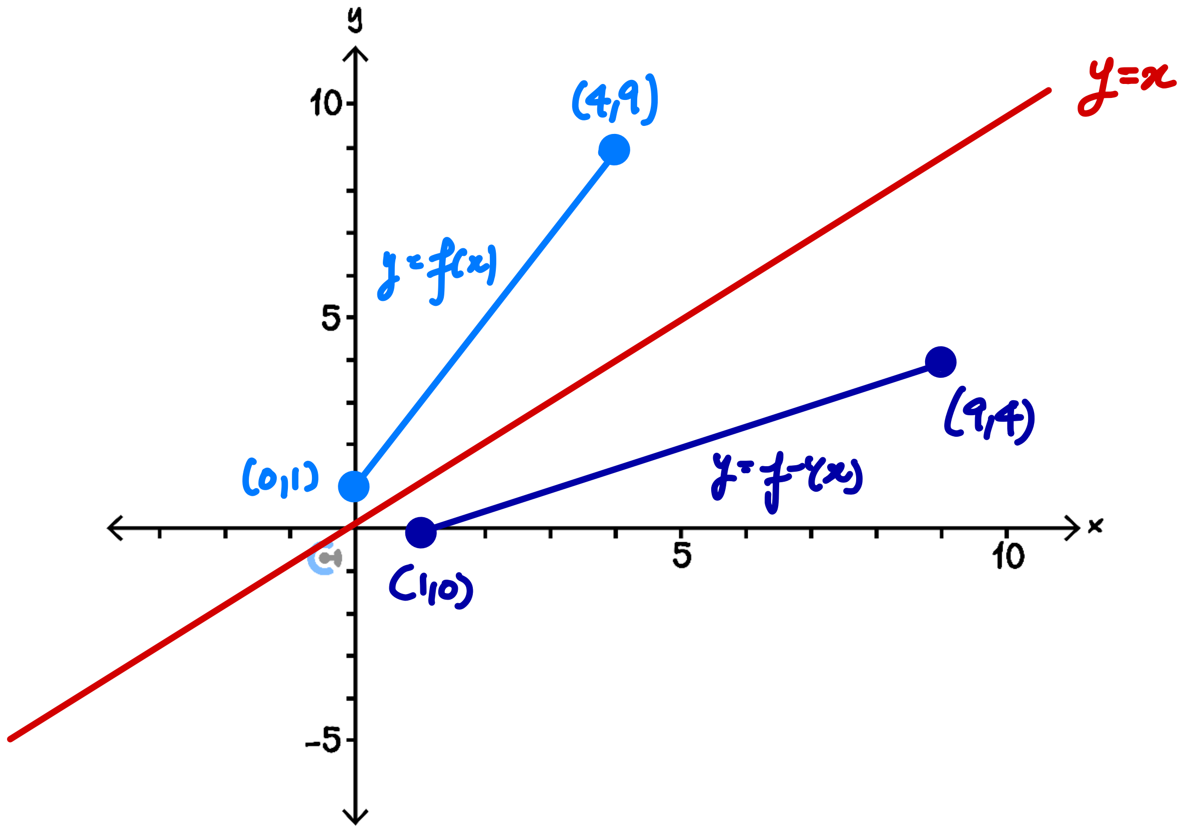
$\therefore f^{-1}(x) = \frac{x-1}{2}$

b. State the domain and range of inverse function.

Dom f^{-1} = Ran f = $[1, 9]$

Ran f^{-1} = Dom f = $[0, 4]$

c. Sketch the $f(x)$ and $f^{-1}(x)$ on the axis below.



Question 21 Extension.

Consider the function $f: (-\infty, 2] \to R, f(x) = \frac{1}{2}x^2 - 2x + 4$.

$\therefore f(x) = \frac{1}{2}(x^2 - 4x) + 4$

$= \frac{1}{2}((x-2)^2 - 4) + 4$

$= \frac{1}{2}(x-2)^2 + 2$

↳ TP: $(2,2)$

a. Find the rule for the inverse function.

Let $y = f(x)$:

Swap x & y:

$x = \frac{1}{2}(y-2)^2 + 2$

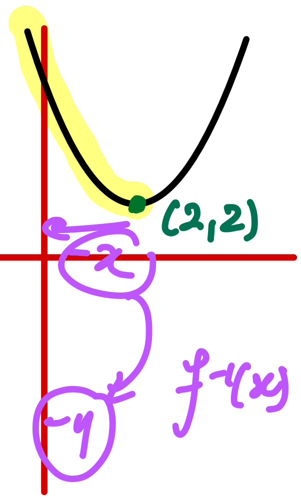
$x - 2 = \frac{1}{2}(y-2)^2$

$2x - 4 = (y-2)^2$

$\therefore y - 2 = \pm\sqrt{2x-4}$

$y = 2 \pm \sqrt{2x-4}$

$\therefore f^{-1}(x) = 2 - \sqrt{2x-4}$



b. State the domain and range of inverse function.

$\text{Dom } f^{-1} = \text{Ran } f = [2, \infty)$

$\text{Ran } f^{-1} = \text{Dom } f = (-\infty, 2]$

Discussion: In the previous question, which line were the two inverses symmetrical to?

$y = x$

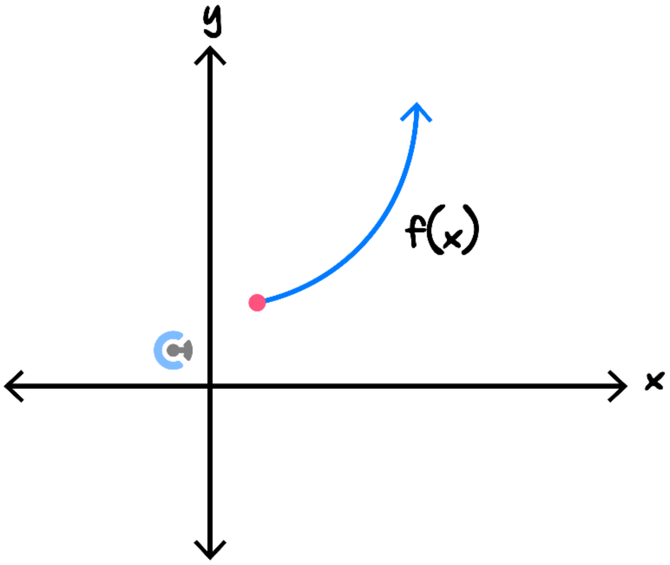
"mirror line"

Sub-Section: Symmetry Around $y = x$

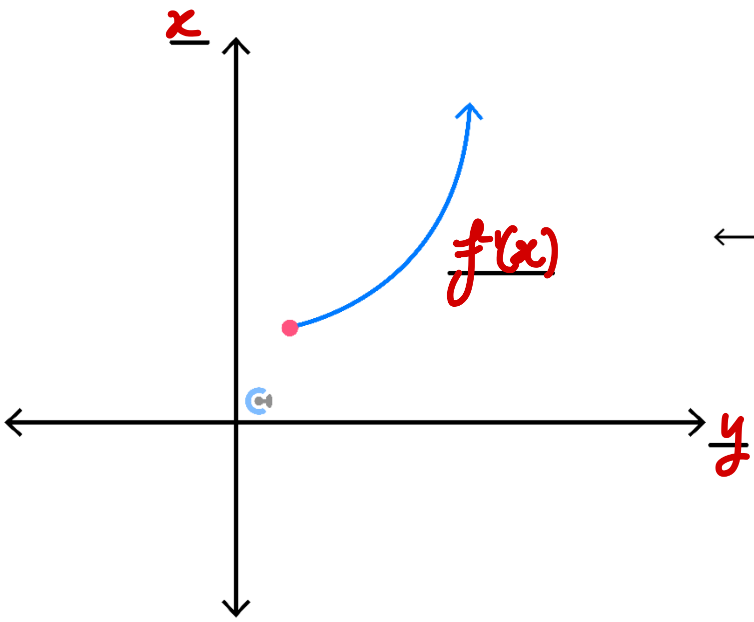
Why does this happen?

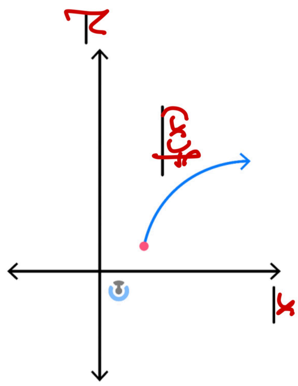
Exploration: Symmetry around $y = x$.

- Consider the following function:

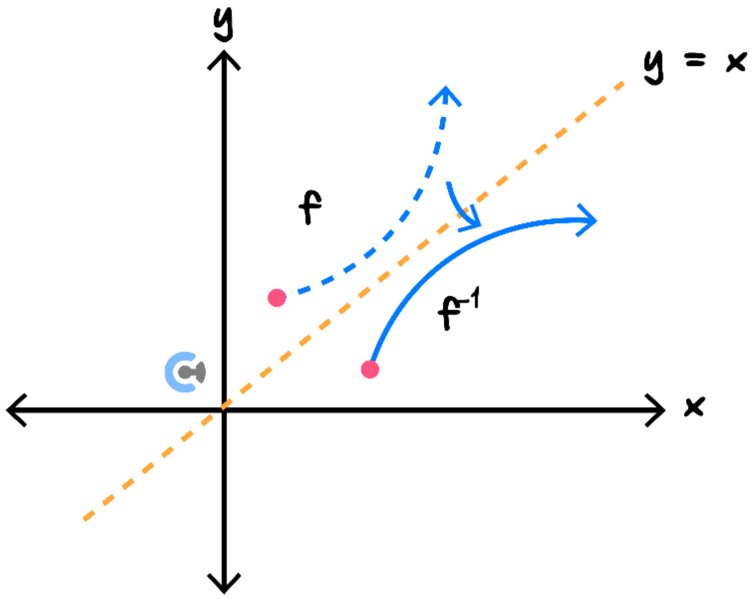


- What happens if you swap the x and y-axis on the label on our graph?

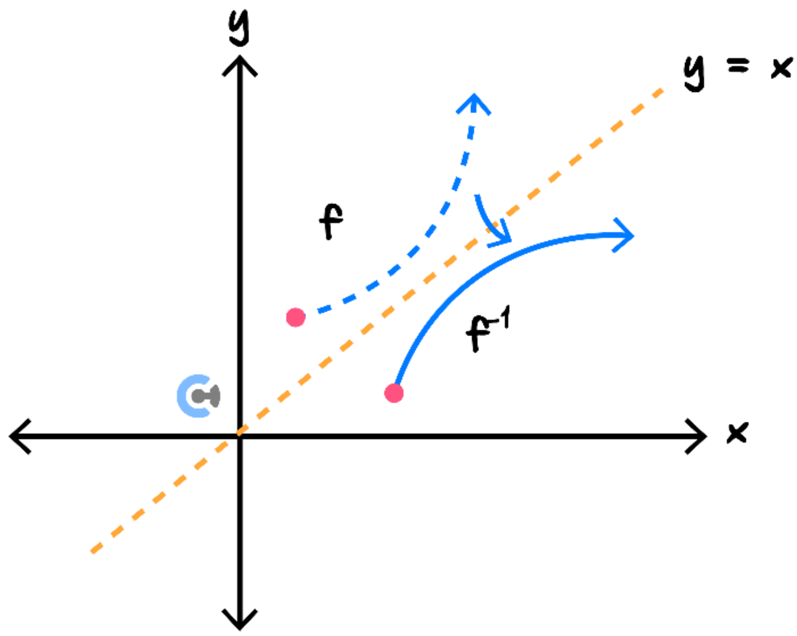




- Wait...do we want the x-axis to be the vertical one? [Yes/No]
- How should we reflect the graph so that the x and y-axis becomes horizontal and vertical again?



Symmetry of Inverse Functions



- Inverse functions are always symmetrical around $y = x$.

Sub-Section: Validity of Inverse Function

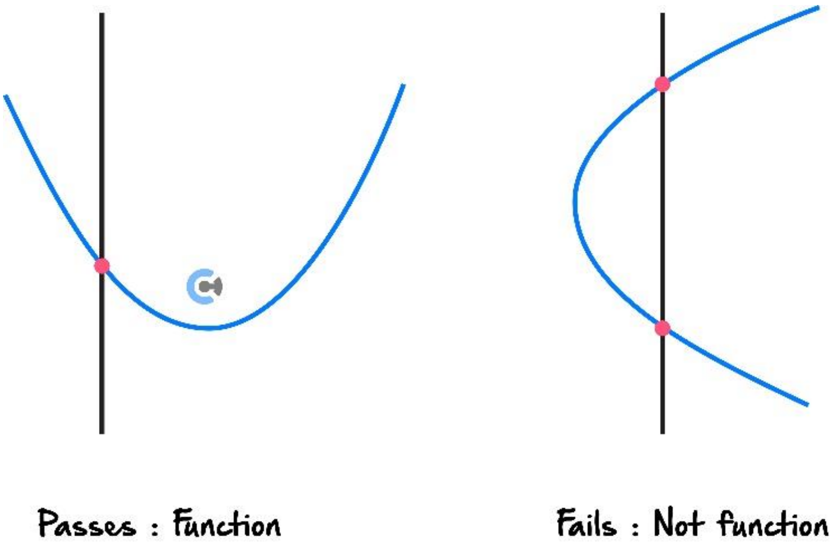
Does an inverse function always exist?

Discussion: If you find an inverse, can you guarantee that it is always a function? Hence, is it always an inverse function?

ALWAYS inverse relation

NOT ALWAYS inverse function

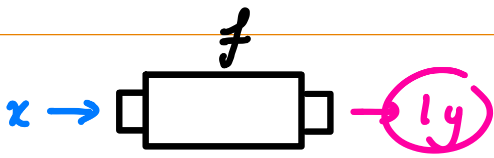
REMINDER: Functions



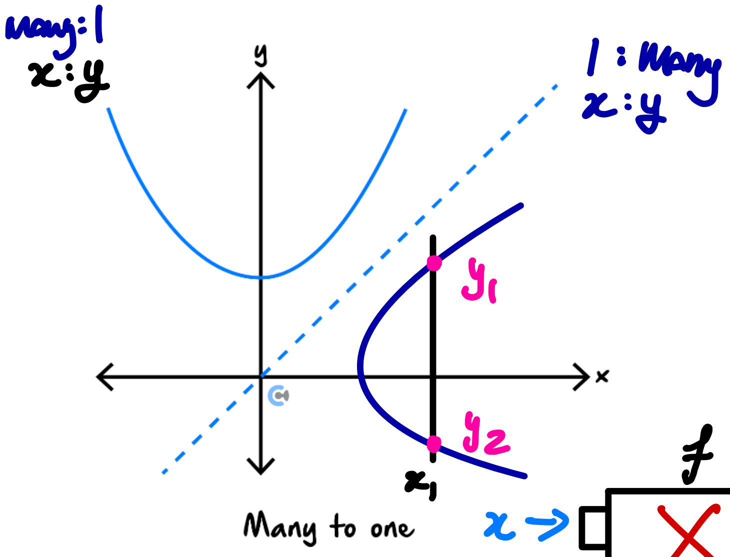
- Functions pass a vertical line test.

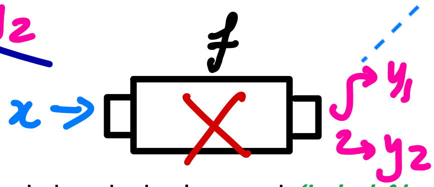
x Many : 1 y
1 : 1

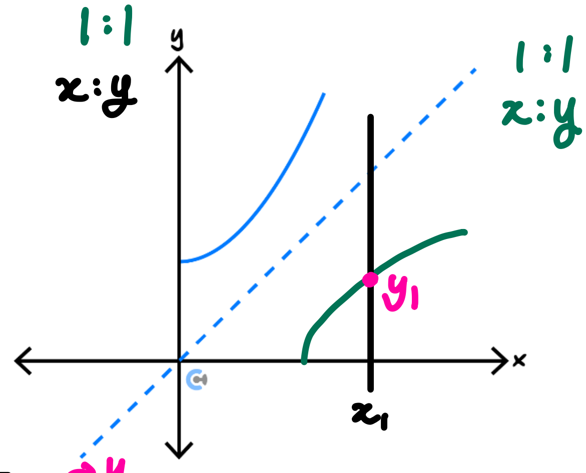
Exploration: Validity of inverse functions.

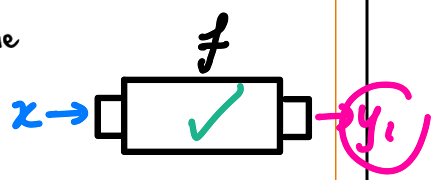


- Consider the many to one and one to one functions.







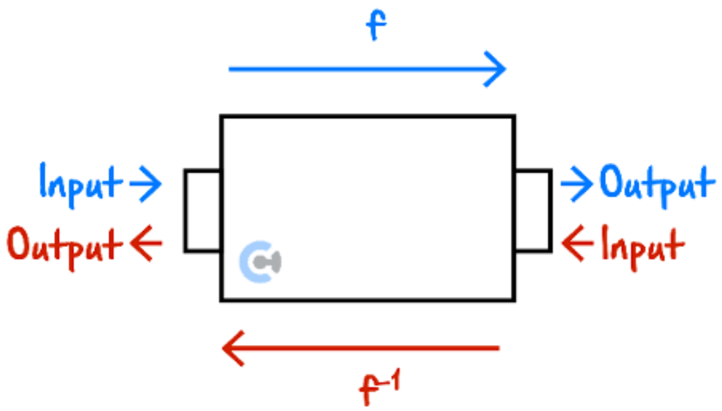


- Reflect them around $y = x$ and sketch the inverse! *(Label Above)*
- Which inverse is a function? (Passes through a vertical line test?)

  [neither] / [left] / [right] / [both]

- For an inverse **function** to exist, what must the original function be? [many to one] / [one to one]

Validity of Inverse Functions



- **Requirement for Inverse Function:**

f *needs to be* 1 : 1

Question 22 Walkthrough.

$\rightarrow$ TP: $(2,3)$

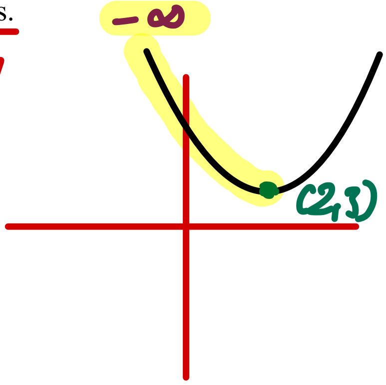
$-\infty, 2$

Consider the function $f: (-\infty, a] \to \mathbb{R}, f(x) = (x-2)^2 + 3$.

a. Find the largest possible value of a such that the inverse function f^{-1} exists.

f must be 1:1

$\therefore a = 2$



b. Find the domain and range of the inverse function. (2 marks)

$\text{Dom } f^{-1} = \text{Ran } f = [3, \infty)$

$\text{Ran } f^{-1} = \text{Dom } f = (-\infty, 2]$

c. Find the rule for the inverse function. (2 marks)

Let $y = f(x)$:

Swap x & y:

$x = (y-2)^2 + 3$

$x - 3 = (y-2)^2$

$y - 2 = \pm\sqrt{x-3}$

$y = 2 \pm \sqrt{x-3}$

$\therefore f^{-1}(x) = 2 - \sqrt{x-3}$

TIP: Always try sketching the function to find the domain such that an inverse function can exist!

NOTE: You will need to complete the square when finding the inverse of quadratic functions!

Space for Personal Notes

Your turn!

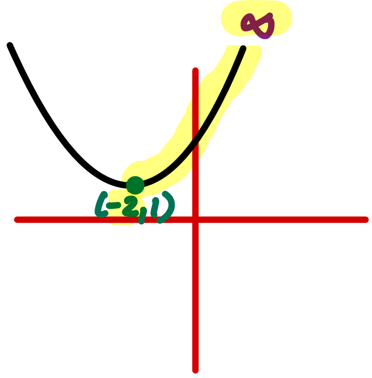
Question 23

Consider the function $g: [b, \infty) \to \mathbb{R}, g(x) = (x+2)^2 + 1$. $\rightarrow$ TP: $(-2, 1)$

a. Find the smallest possible value of b such that the inverse function g^{-1} exists.

g must be 1:1

$\therefore b = -2$



b. Find the domain and range of the inverse function. (2 marks)

Dom g^{-1} = Ran g = $[1, \infty)$

Ran g^{-1} = Dom g = $[-2, \infty)$

c. Find the rule for the inverse function. (2 marks)

Let $y = g(x)$:

Swap x & y:

$x = (y+2)^2 + 1$

$x - 1 = (y+2)^2$

$y + 2 = \pm\sqrt{x-1}$

$y = -2 \pm \sqrt{x-1}$

$\therefore g^{-1}(x) = -2 + \sqrt{x-1}$

Space for Personal Notes

Question 24 Extension.

$f(x) = -(x^2-4x)-3$

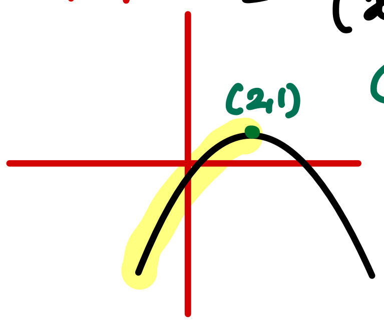
$= -((x-2)^2-4)-3$

$= -(x-2)^2+1$

Consider the function $g: (-\infty, b] \to \mathbb{R}, g(x) = -x^2 + 4x - 3$.

a. Find the largest possible value of b such that the inverse function g^{-1} exists.

g must be 1:1



↳ TP: $(2,1)$

$\therefore b = 2$

b. Find the domain and range of the inverse function. (2 marks)

Dom g^{-1} = Ran g = $(-\infty, 1]$

Ran g^{-1} = Dom g = $(-\infty, 2]$

c. Find the rule for the inverse function. (2 marks)

Let $y = g(x)$:

Swap x & y:

$x = -(y-2)^2+1$

$(y-2)^2 = 1-x$

$y - 2 = \pm\sqrt{1-x}$

$y = 2 \pm \sqrt{1-x}$

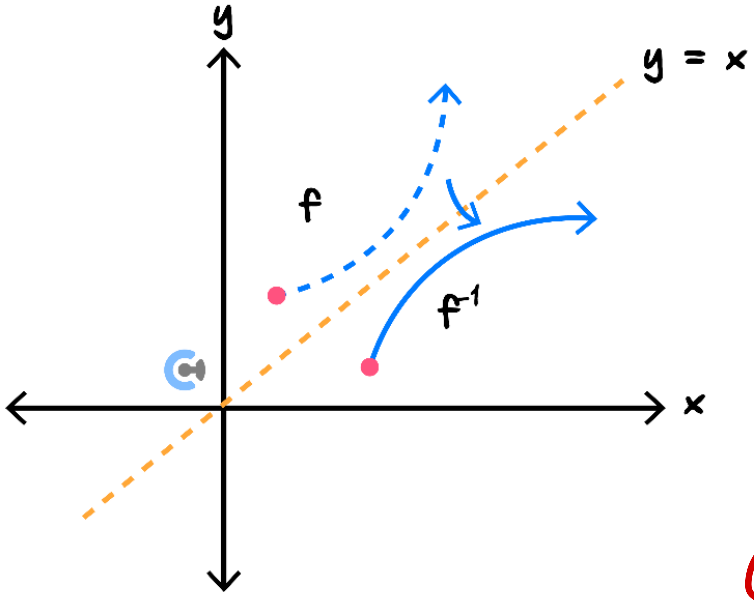
$\therefore g^{-1}(x) = 2 - \sqrt{1-x}$

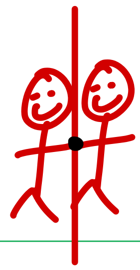
Space for Personal Notes

Sub-Section: Intersection Between Inverses

Where do inverses meet?

Active Recall: Symmetry around $y = x$.





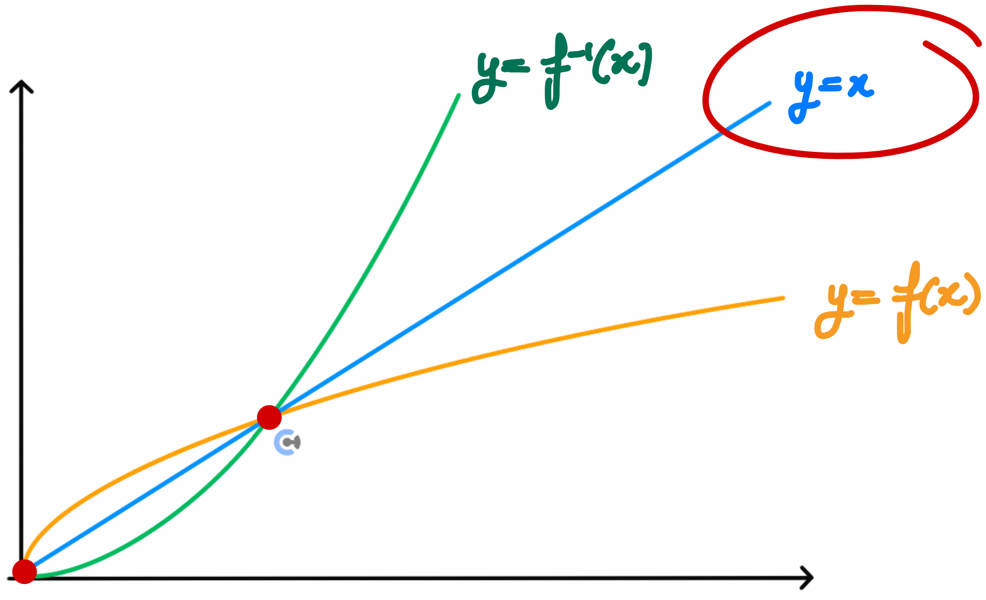
- Inverse functions are always symmetrical around $y = x$.

Discussion: Where could a function and its inverse meet?

∴ on the mirror line $y=x$.

Exploration: Intersections between a function and its inverse.

- Consider a function and its inverse below.



- Note the symmetry around $y = x$ for inverses!
- Circle the point where the two functions intersect.
- Where does this point also lie?

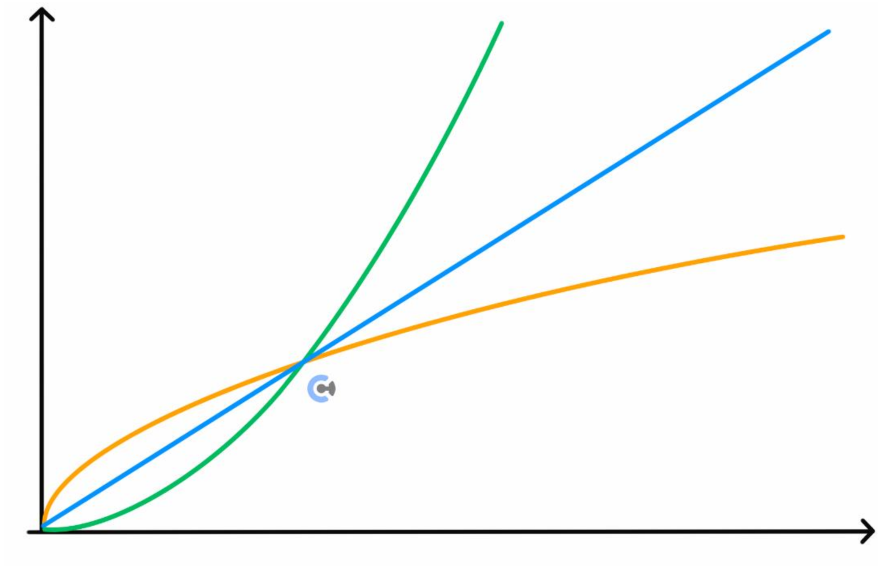
↪ $y = x$

Discussion: Hence, instead of solving $f(x) = f^{-1}(x)$, what can we solve instead of finding the point where a function and its inverse intersect?

$y = f(x)$ & $y = x$

↪ let $f(x) = x$

Intersection Between a Function and Its Inverse



$f(x) = x$ OR $f^{-1}(x) = x$

Head Tutor's Comment: Emphasise that you cannot just cancel x's on either side of the equation. (For anything that could potentially be 0, we cannot cancel them).

Question 25

(x,y)

Find the intersection between $f: [0, \infty) \to R, f(x) = x^3$ and its inverse, without finding the inverse.

Let $f(x) = x$:

$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x+1)(x-1) = 0$

$\therefore x = 0, 1, -1 \rightarrow$ reject as $x \geq 0$

$\therefore$ IPs: $(0,0)$ & $(1,1)$

NOTE: We can always equate the function to x instead of the inverse function itself!

ALSO NOTE: This only works for an increasing function, however in VCAA, this is always the case. Something to note for SACS is that there COULD be intersections that are NOT on $y = x$.

Contour Checklist

☐ Learning Objective: [2.2.1] – Find Domain and Range of Functions

Key Takeaways

Interval Notation:

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Maximal Domain:

- Inside of a $\log$ must be >0.
- Inside of a root must be ≥ 0.
- Denominator $\neq 0$.

Learning Objective: [2.2.2] – Sketch and Find the Domain and Range of Hybrid Functions

Key Takeaways

Piecewise (Hybrid) Functions:

- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- When we have an x intercept for one graph, sum graph intersects the other graph.
- $Domain_1$ and $Domain_2$ represent the x values for which the two functions are defined.
- The two domains do not have to join!

Learning Objective: [2.2.3] – Find the Rule, Domain, Range, and Intersections Between Inverse Functions

Key Takeaways

- f needs to be 1:1 for f^{-1} to exist.
- Domain of the inverse function equals to range of the original and vice versa.
- Symmetrical around $y=x$.
- For intersections of inverses, we can equate the function to x.

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VCE Mathematical Methods ½

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