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VCE Mathematical Methods ½
Functions & Relations II [2.2]
Workbook

Outline:

Section A: Domain and Range

Sub-Section: Set Notation

Let's have a look at set notations!

Set Operators

- Intersection: "AND".

$$A \cap B = What\ values\ are\ in\ set\ A\ AND\ in\ set\ B.$$

- Union: "OR".

$$A \cup B = What\ values\ are\ in\ set\ A\ OR\ in\ set\ B.$$

- Set difference: "Except".

$$A \setminus B = What\ values\ are\ in\ set\ A\ except\ those\ also\ in\ set\ B.$$

Space for Personal Notes

Question 1

For the sets given below, find:

$$A = \{0, 2, 3, 5, 6, 11\} \text{ and } B = \{0, 1, 2, 3, 5, 7, 9, 10\}$$

a. $A \cap B =$

b. $A \cup B =$

c. $A \backslash B =$

d. $B \backslash A =$

Sub-Section: Interval Notation

Now interval notation!

Interval Notation

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Question 2 Walkthrough.

Simplify the following set.

$$A = [-1, 10] \text{ and } B = [-4, 5)$$

a. Find $A \cap B$.

b. Find $A \cup B$.

NOTE: Use **number lines** to find the intersection and union of sets.

Now your turn!

Question 3

Find the following sets:

a. $[0,5] \cap [1,8]$

b. $[-3,7] \cup \left(-11,\frac{1}{2}\right]$

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Question 4 Extension.

Find the following set.

$$[1,3) \cap [2,6] \cup (-5,2)$$

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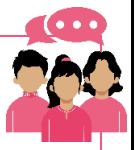

Discussion: What is $\mathbb{R}\backslash[a,b]$ equal to? Is it $(-\infty,a)\cup(b,\infty)$ or $(-\infty,a]\cup[b,\infty)$?

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Sub-Section: Maximal Domain

What is a maximal domain?

Maximal Domain

- The maximal domain is _____ domain for a rule without committing a _____.
- In Methods, we need to consider 3 important rules:

$\sqrt{z}$, z _____

$\log(z)$, z _____

$\frac{1}{z}$, z _____

NOTE: We will consider log in depth later throughout the year!

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Question 5 Walkthrough.

Find the maximal domain of each of the following functions.

a. $f(x) = \sqrt{x-3}$

b. $h(x) = \log_2(x+5)$

c. $h(x) = \frac{1}{x-4}$

Your turn!

Question 6

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{-x-6} - 5$

b. $h(x) = -\log_2(x + 10)$

c. $\frac{1}{x^2-25}$

Now harder ones!

Question 7

Find the maximal domain of the following functions.

a. $f(x) = \sqrt{x^2 - 4} - 5$

b. $h(x) = -\log_2(25 - x^2)$

NOTE: Always sketch the function when solving inequalities for many to one functions.

Calculator Commands

Mathematica

```
FunctionDomain[func, x]
```

TI-Nspire

Type up domain (or find it under the book button).

$$\text{domain}(\text{func}, x)$$

Casio Classpad

Sketch the function and analyse.

Question 8 Tech Active.

Find the maximal domain of the following function.

$$f(x) = \sqrt{x^2 - 9} + 1$$

Sub-Section: Range

Now the range!

Range

- The range is the possible values for the output of a function.

Question 9 Walkthrough.

Find the range of the following function:

$$f: [-4, 4] \backslash \{0\} \to \mathbb{R}, f(x) = \frac{1}{x}$$

TIP: Always **sketch** the function!

Question 10

Find the range of the following functions.

a. $f: [-4, 6) \to R, f(x) = x^2 - 16$

b. $f: [12, \infty) \to R, f(x) = 2\sqrt{x + 4} + 1$

Question 11 Extension.

Find the range of the following function.

$$f: [-2, 8) \to R, f(x) = \frac{1}{2}x^2 - 2x - 2$$

Sub-Section: Functional Notation

How do we represent a function?

Functional Notation

$$f: Domain \rightarrow Codomain, f(x) = Rule$$

- Codomain is simply all the values the function works within.
- Codomain is **not** the same as range.

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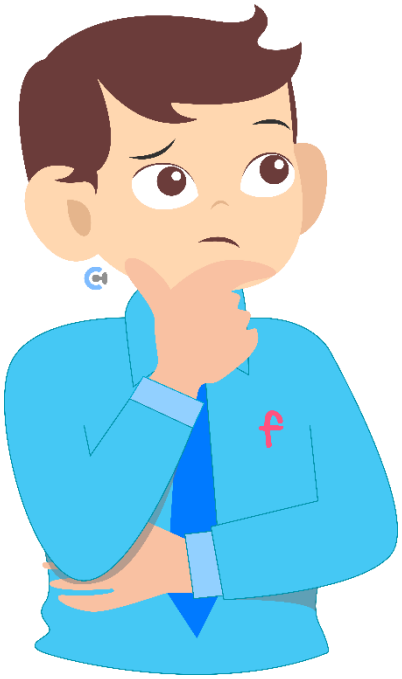
Analogy: Functional notation is a "business card" for functions.

- A function f wants to make a business card for themself.



- They decide to put their name, working hours, company associated and their role.

$$\boldsymbol{Name: Working\ Hour \rightarrow Company, Role}$$

- Their name is simply f.
- Their working hours are their "domain".
- Their company is the "CoDomain".
- Their role is the rule!

$$\boldsymbol{f: Domain \rightarrow Codomain, f(x) = Rule}$$

- Now, does f have to make everything in their company?

- Hence, using this analogy, would his range (their output) be the same as the codomain (company)?

Question 12

Consider the following function, written in functional notation:

$$f: [-1, 4] \to \mathbb{R}, f(x) = x + 5$$

Identify the name, domain, range, and the equation of the function.

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Section B: Hybrid (Piecewise) Functions

Analogy: Hybrid functions are like a relay race.

- Imagine the functions $f(x)$ and $g(x)$ participating in a relay race as part of the same team.

- $f(x)$ is running for $x < 4$ and $g(x)$ is running for $x \geq 4$.
- For $x = 5$ who do we look at?
- This is how hybrid functions work!

Piecewise (Hybrid) Functions

- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- $Domain_1$ and $Domain_2$ represent the x values for which the two functions are defined.
- The two domains do not have to join!

Space for Personal Notes

Question 13 Walkthrough.

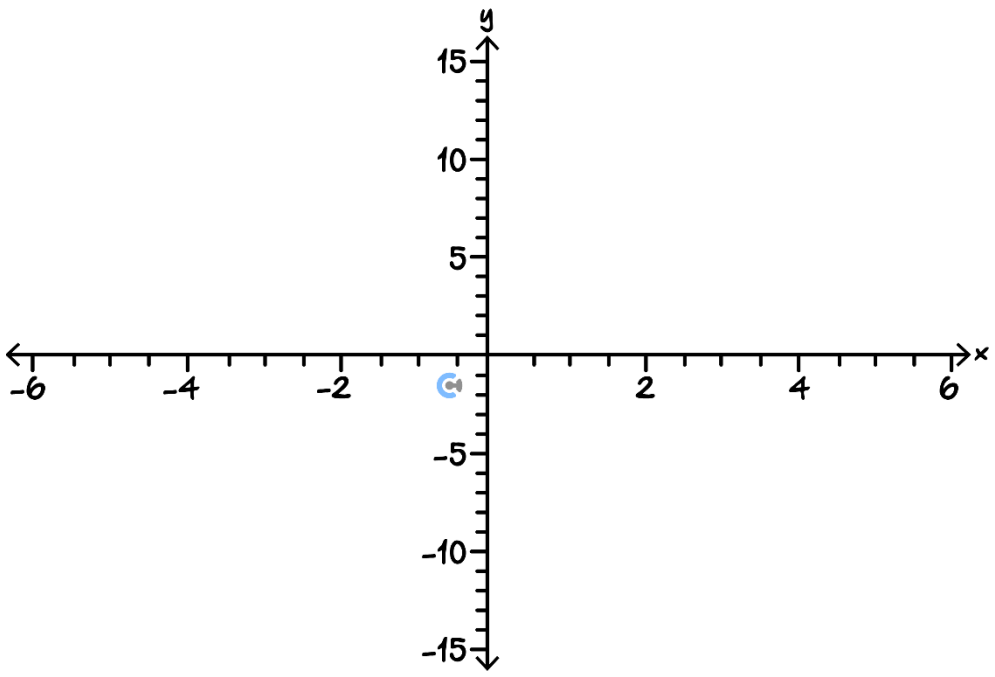
Consider the hybrid function f.

$$f(x) = \begin{cases} x^2 - 5, & x \geq 0 \\ x + 4, & x < 0 \end{cases}$$

a. Find $f(-2)$.

b. Find $f(5)$.

c. Graph $y = f(x)$.

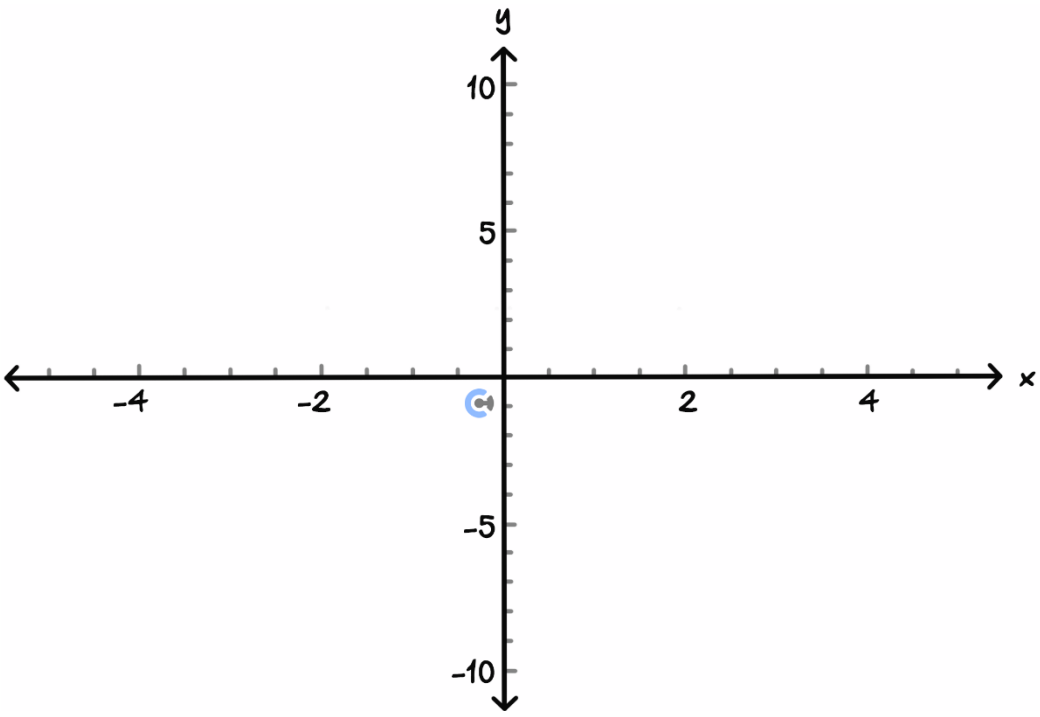


Question 14

Consider the hybrid function g.

$$g(x) = \begin{cases} x^2 + 1, & x \geq 0 \\ 1 - x, & x < 0 \end{cases}$$

a. Graph $y = g(x)$.



b. Find the range of $g(x)$.

Question 15

Consider the hybrid function g.

$$g(x) = \begin{cases} x^2 + 4x + 1, & -4 \leq x < 0 \\ 2x + 3, & x \geq 0 \end{cases}$$

Find the range of $g(x)$.

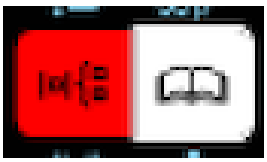
Defining Hybrid Functions on CAS

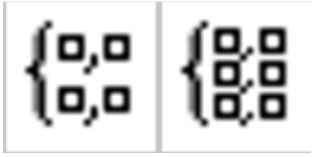
Mathematica

- "Esc PW" and Control Enter to create cells.

```
{ func1  dom1
  func2  dom2
```

TI-Nspire





$$\begin{cases} func\,1, dom\,1 \\ func\,2, dom\,2 \end{cases}$$

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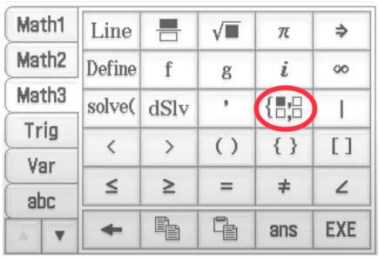


Question 16

Consider the hybrid function g.

$$g(x) = \begin{cases} x^3 + 6x - 5, & x < 1 \\ x + 4, & x \geq 1 \end{cases}$$

a. Evaluate $g(-2)$.

b. Evaluate $g(3)$.

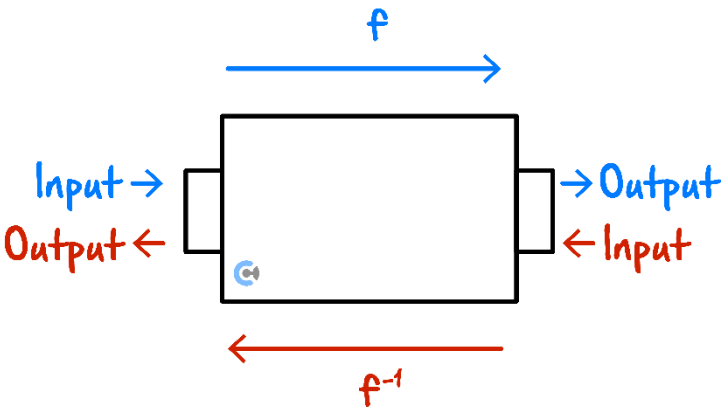
Section C: Inverse Functions

Sub-Section: Basics of Inverses

What does "inverse" mean?

Inverse Relation

- **Definition**: Inverse is a relation which does the _____.



Discussion: What would be the inverse of $f(x) = x + 2$?

Question 17

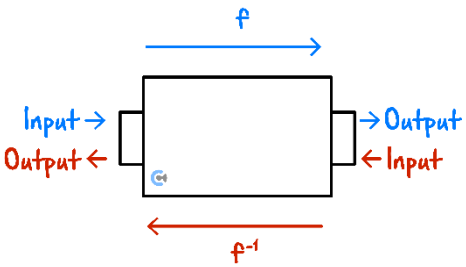
Find the inverse of $f(x) = 2x + 1$.

Sub-Section: Swapping x and y

Is there a better way of solving for an inverse relation?

Solving For an Inverse Relation

- Swap x and y.



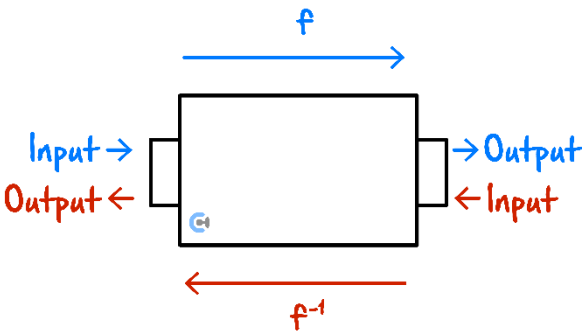
Question 18

Find the inverse of $f(x) = 2x + 1$ by swapping x and y.

NOTE: $f(x) = y$.

Discussion: Hence, what would happen to the domain and range of the function when we find its inverse?

Domain and Range of Inverse Functions



$$Dom\ f^{-1} = \_\_\_\_\_$$

$$Ran\ f^{-1} = \_\_\_\_\_$$

Question 19 Walkthrough.

Consider the function $f(x) = \sqrt{x+2} - 1$ defined for its maximal domain.

a. Find the rule for the inverse function.

b. State the domain and range of inverse function.

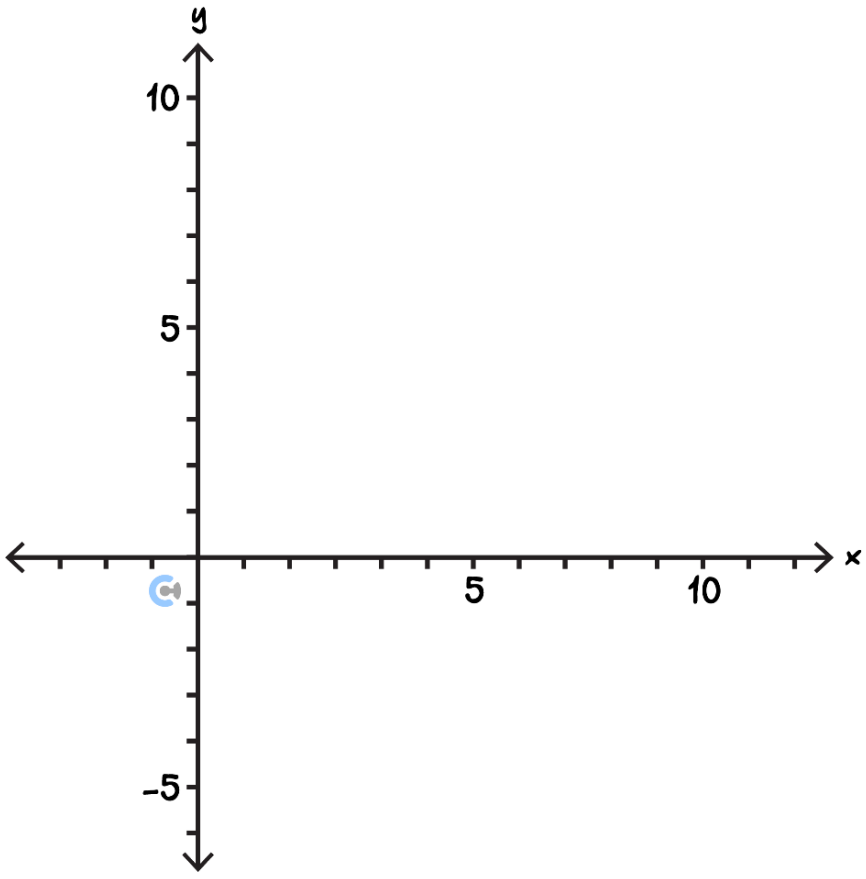
Question 20

Consider the function $f: [0, 4] \rightarrow R, f(x) = 2x + 1$.

a. Find the rule for the inverse function.

b. State the domain and range of inverse function.

c. Sketch the $f(x)$ and $f^{-1}(x)$ on the axis below.



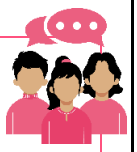
Question 21 Extension.

Consider the function $f: (-\infty, 2] \to R, f(x) = \frac{1}{2}x^2 - 2x + 4$.

a. Find the rule for the inverse function.

b. State the domain and range of inverse function.

<u>Discussion:</u> In the previous question, which line were the two inverses symmetrical to?

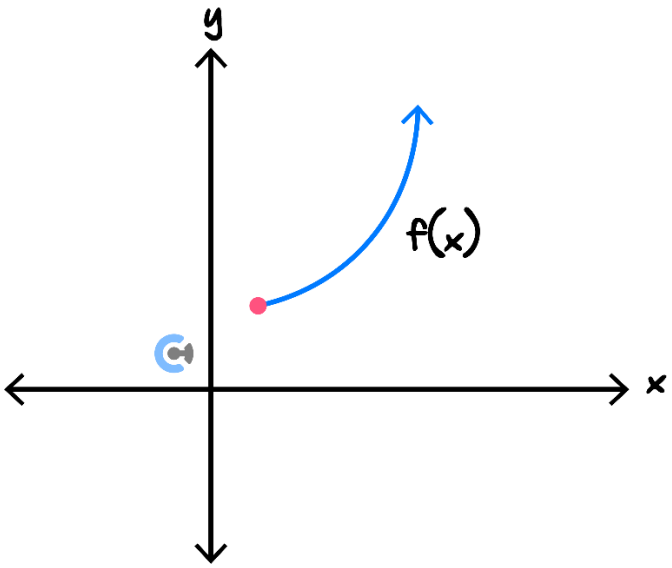


Sub-Section: Symmetry Around $y = x$

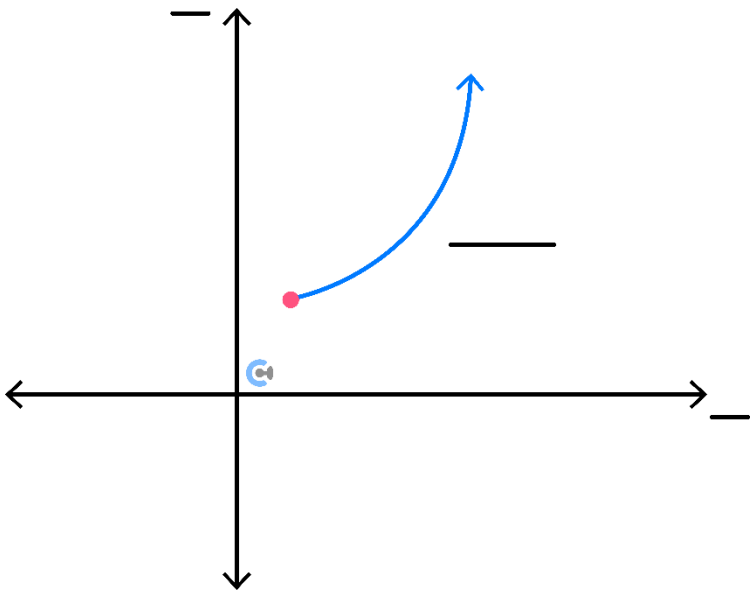
Why does this happen?

Exploration: Symmetry around $y = x$.

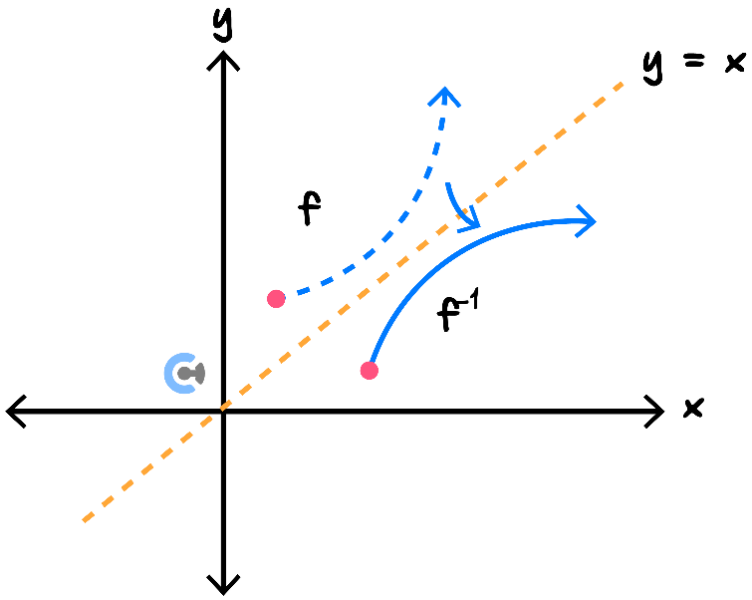
- Consider the following function:



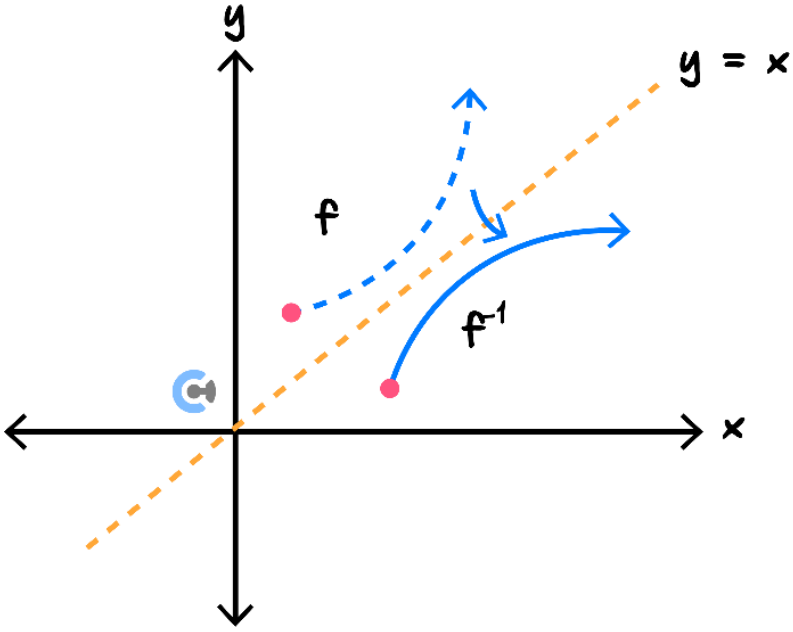
- What happens if you swap the x and y-axis on the label on our graph?



- Wait...do we want the x-axis to be the vertical one? [Yes/No]
- How should we reflect the graph so that the x and y-axis becomes horizontal and vertical again?



Symmetry of Inverse Functions



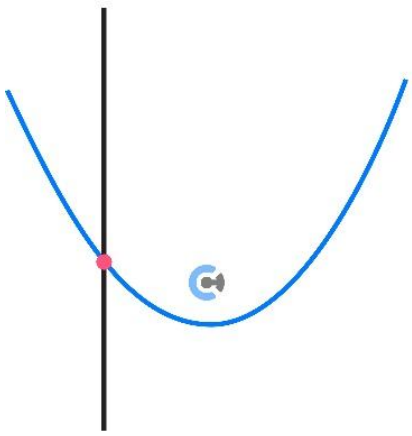
- Inverse functions are always symmetrical around $y = x$.

Sub-Section: Validity of Inverse Function

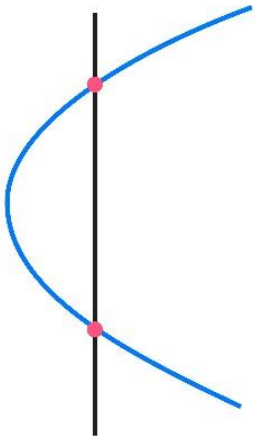
Does an inverse function always exist?

Discussion: If you find an inverse, can you guarantee that it is always a function?
Hence, is it always an inverse function?

REMINDER: Functions



Passes : Function

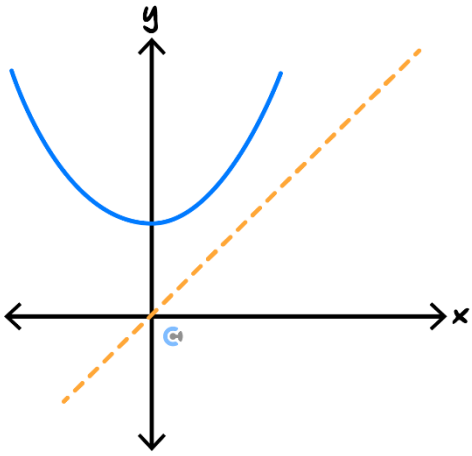


Fails : Not function

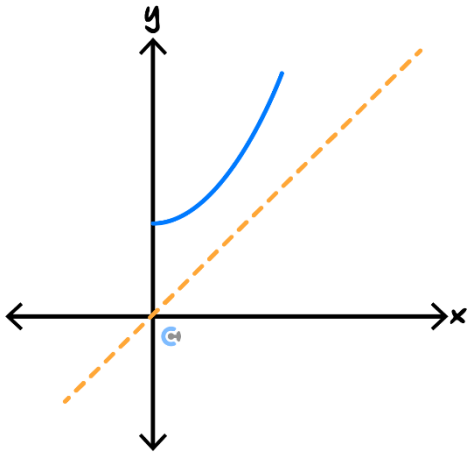
- Functions pass a vertical line test.

Exploration: Validity of inverse functions.

- Consider the many to one and one to one functions.



Many to one



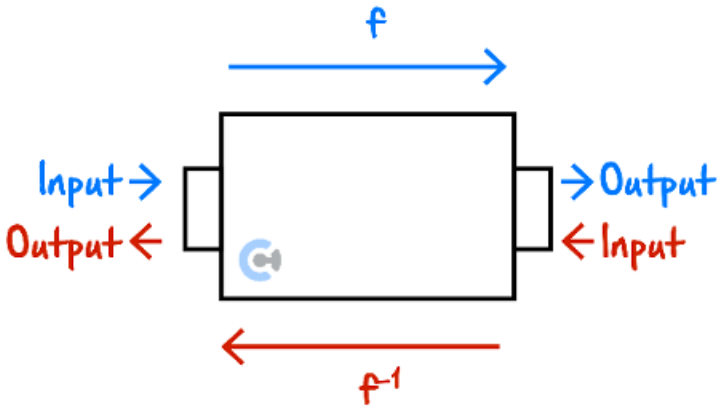
One to one

- Reflect them around $y = x$ and sketch the inverse! *(Label Above)*
- Which inverse is a function? (Passes through a vertical line test?)

[neither] / [left] / [right] / [both]

- For an inverse **function** to exist, what must the original function be? [many to one] / [one to one]

Validity of Inverse Functions



- Requirement for Inverse Function:

f *needs to be* _____

Question 22 Walkthrough.

Consider the function $f: (-\infty, a] \to \mathbb{R}, f(x) = (x-2)^2 + 3$.

a. Find the largest possible value of a such that the inverse function f^{-1} exists.

b. Find the domain and range of the inverse function. (2 marks)

c. Find the rule for the inverse function. (2 marks)

TIP: Always try sketching the function to find the domain such that an inverse function can exist!

NOTE: You will need to complete the square when finding the inverse of quadratic functions!

Space for Personal Notes

Your turn!

Question 23

Consider the function $g: [b, \infty) \to \mathbb{R}, g(x) = (x + 2)^2 + 1$.

a. Find the smallest possible value of b such that the inverse function g^{-1} exists.

b. Find the domain and range of the inverse function. (2 marks)

c. Find the rule for the inverse function. (2 marks)

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Question 24 Extension.

Consider the function $g: (-\infty, b] \to \mathbb{R}, g(x) = -x^2 + 4x - 3$.

a. Find the largest possible value of b such that the inverse function g^{-1} exists.

b. Find the domain and range of the inverse function. (2 marks)

_____

_____

_____

_____

c. Find the rule for the inverse function. (2 marks)

_____

_____

_____

_____

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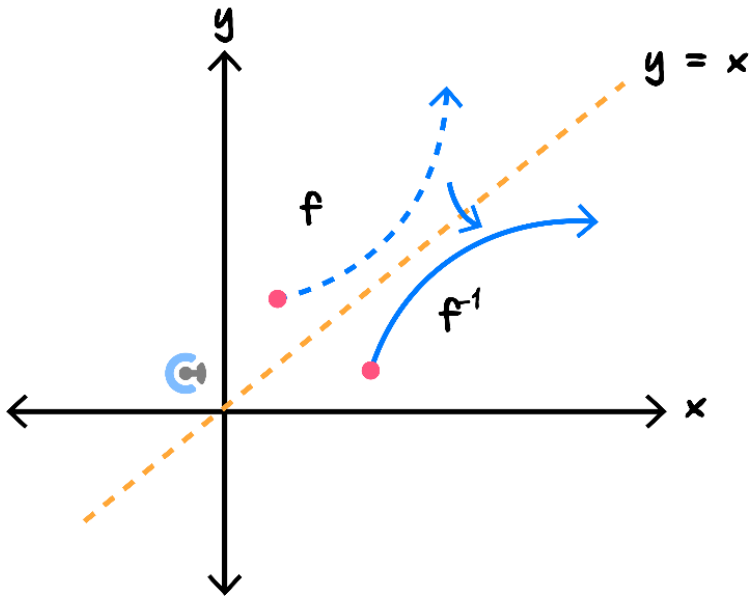
Sub-Section: Intersection Between Inverses

Where do inverses meet?

Active Recall: Symmetry around $y = x$.

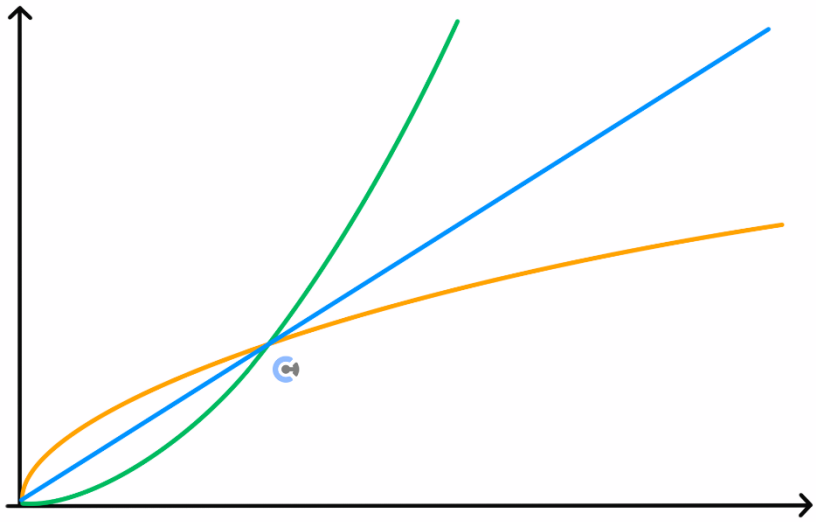


- Inverse functions are always symmetrical around $y = x$.

Discussion: Where could a function and its inverse meet?

Exploration: Intersections between a function and its inverse.

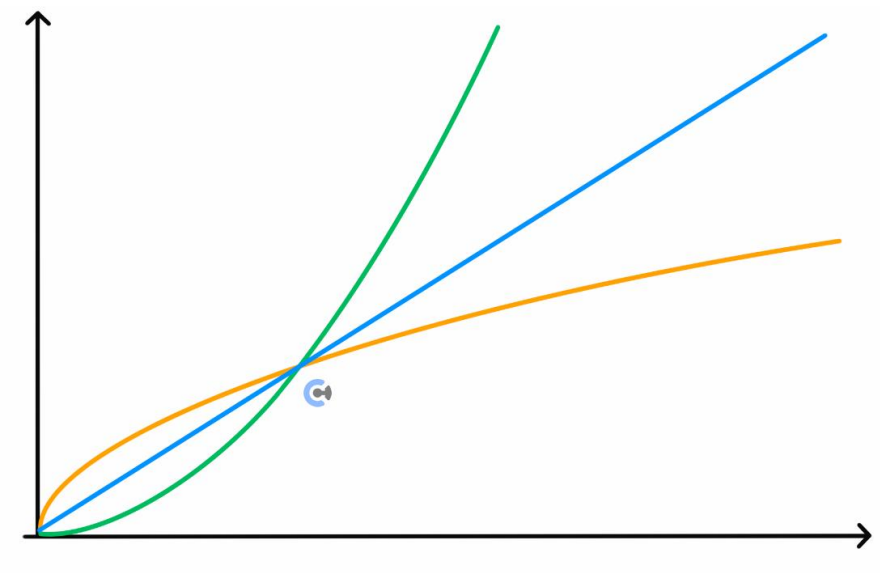
- Consider a function and its inverse below.



- Note the symmetry around $y = x$ for inverses!
- Circle the point where the two functions intersect.
- Where does this point also lie?

Discussion: Hence, instead of solving $f(x) = f^{-1}(x)$, what can we solve instead of finding the point where a function and its inverse intersect?

Intersection Between a Function and Its Inverse



$$f(x) = x \quad \text{OR} \quad f^{-1}(x) = x$$

Question 25

Find the intersection between $f: [0, \infty) \to R, f(x) = x^3$ and its inverse, without finding the inverse.

NOTE: We can always equate the function to x instead of the inverse function itself!

ALSO NOTE: This only works for an increasing function, however in VCAA, this is always the case. Something to note for SACS is that there could be intersections that are NOT on $y = x$.

Contour Checklist

☐ Learning Objective: [2.2.1] – Find Domain and Range of Functions

Key Takeaways

Interval Notation:

- Parentheses (non-inclusive):

$$x \in (a, b) \Rightarrow a < x < b$$

- Square brackets [inclusive]:

$$x \in [a, b] \Rightarrow a \leq x \leq b$$

Maximal Domain:

- Inside of a $\log$ must be _____.
- Inside of a root must be _____.
- Denominator _____.

Learning Objective: [2.2.2] – Sketch and Find the Domain and Range of Hybrid Functions

Key Takeaways

Piecewise (Hybrid) Functions:

- Series of functions.

$$h(x) = \begin{cases} f(x), & Domain_1 \\ g(x), & Domain_2 \end{cases}$$

- When we have an x intercept for one graph, sum graph intersects the other graph.
- $Domain_1$ and $Domain_2$ represent the x values for which the two functions are _____.
- The two domains do not have to join!

Learning Objective: [2.2.3] – Find the Rule, Domain, Range, and Intersections Between Inverse Functions

Key Takeaways

- f needs to be _____ for f^{-1} to exist.
- Domain of the inverse function equals to _____ **and** vice versa.
- Symmetrical around _____.
- For intersections of inverses, we can equate the function to _____.

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