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VCE Mathematical Methods ½
Functions & Relations II [2.2]
Homework Solutions

Homework Outline:

Section A: Compulsory Questions

Sub-Section [2.2.1]: Find Domain and Range of Functions

Question 1

For the function $f(x) = \sqrt{x+3}$, find its:

a. Maximal domain.

$x \geq -3$

b. Range.

$x \geq 0$

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Question 2

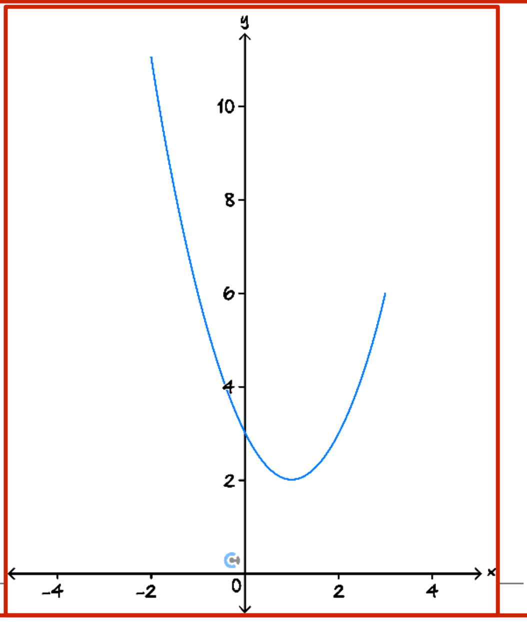
For the function $f : (-2, 3] \to \mathbb{R}, f(x) = (x-1)^2 + 2$, find its:

a. Maximal domain.

$(-2, 3]$

b. Range.

Min value is 2 and max value $f(-2) = 11$. Therefore range $[2, 11)$.



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Question 3

For the function $f : [-6, -3) \to \mathbb{R}, f(x) = \log_3(x^2 - 9)$, find its:

a. Maximal domain.

$[-6, -3)$

b. Range.

Sub in $x = -6$. $f(-6) = \log_3(36 - 9) = \log_3(27) = 3$. Therefore range is $(-\infty, 3]$

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Question 4 Tech-Active.

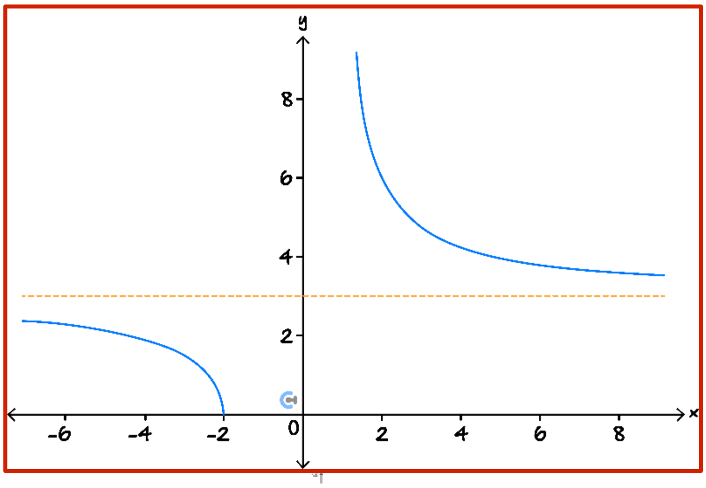
For the function $f(x) = 3\sqrt{\frac{x+2}{x-1}}$, find its:

a. Maximal domain.

We require that $\dfrac{x+2}{x-1} \geq 0$ and $x \neq 1$.
This gives dom $f = (-\infty, -2] \cup (1, \infty)$

b. Range.

The graph $y = \dfrac{x+2}{x-1}$ has an asymptote at $y = 1$. So f will have an asymptote at $y = 3$.
ran $f = [0, \infty) \setminus \{3\}$



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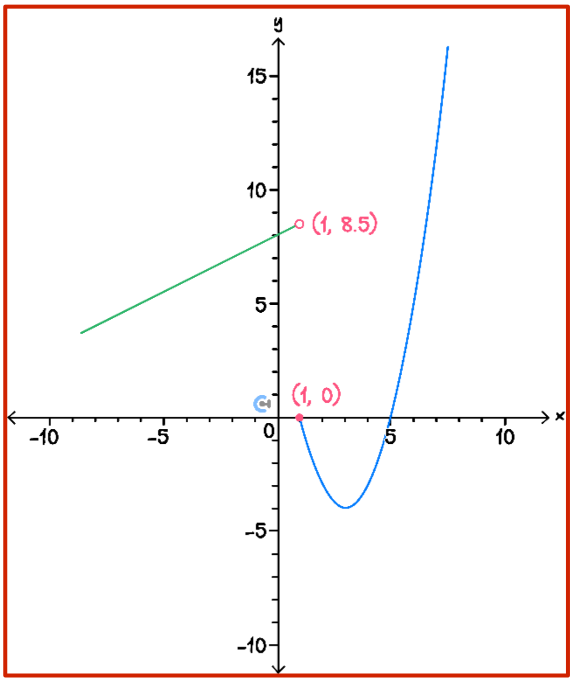
Sub-Section [2.2.2]: Sketch and Find the Domain and Range of Hybrid Functions

Question 5

Consider the hybrid function g.

$$g(x) = \begin{cases} (x-3)^2 - 4, & x \geq 1 \\ \dfrac{x}{2} + 8, & x < 1 \end{cases}$$

a. Sketch the graph $y = g(x)$.



b. Find the range of $g(x)$.

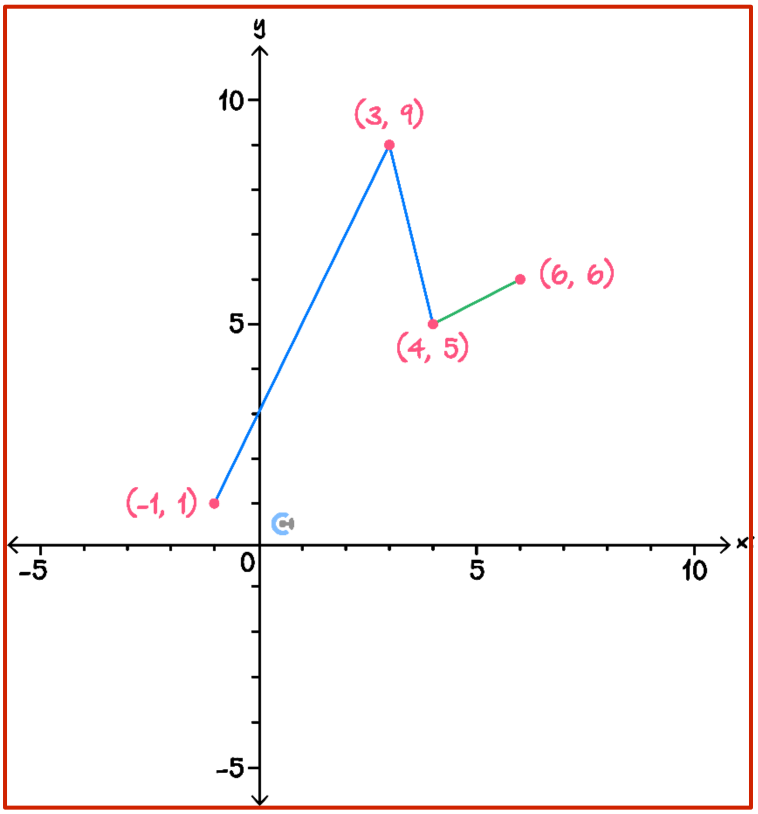
R

Question 6

Consider the hybrid function g.

$$g(x) = \begin{cases} 2x + 3, & -1 \leq x \leq 3 \\ 21 - 4x, & 3 < x < 4 \\ \frac{1}{2}x + 3, & 4 \leq x \leq 6 \end{cases}$$

a. Sketch the graph $y = g(x)$.



b. Find the range of $g(x)$.

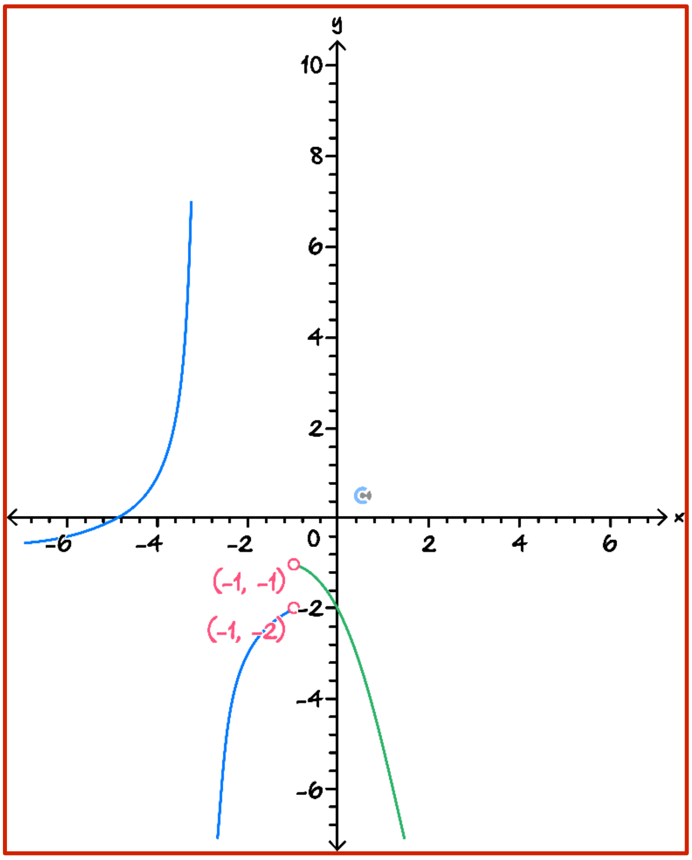
$[1, 9]$

Question 7

Consider the hybrid function g.

$$g(x) = \begin{cases} -\dfrac{2}{x+3} - 1, & x < -1 \\ -x^2 - 2x - 2 & x > -1 \end{cases}$$

a. Sketch the graph $y = g(x)$.



b. Find the range of $g(x)$.

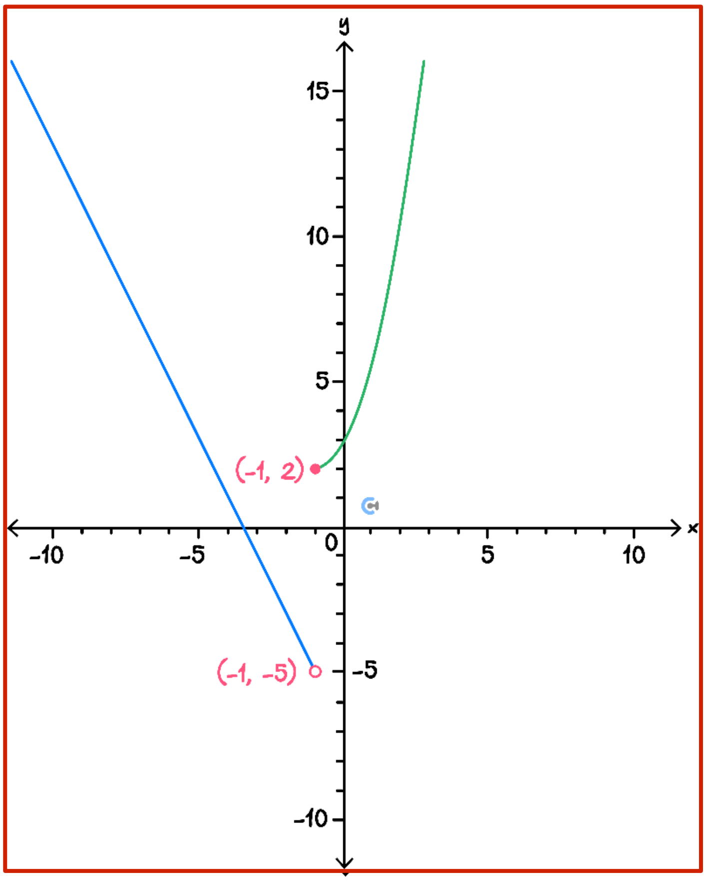
$R\backslash\{-1\}$

Question 8 Tech-Active.

Consider the hybrid function g.

$$g(x) = \begin{cases} (x+1)^2 + 2, & x \geq -1 \\ -7 - 2x, & x < -1 \end{cases}$$

a. Sketch the graph $y = g(x)$.



b. Find the range of $g(x)$.

$(-5, \infty)$

Sub-Section [2.2.3]: Find the Rule, Domain, Range, and Intersections between Inverse Functions

Question 9

Consider the function $f(x) = 4x - 1$.

a. Find the rule for the inverse function f^{-1}.

$$f^{-1}(x) = \frac{x+1}{4}$$

b. State the domain and range of f^{-1}.

dom $f^{-1} = \mathbb{R}$ and ran $f^{-1} = \mathbb{R}$

c. Find the coordinates for any points of intersection between f and f^{-1}.

Solve $4x - 1 = x \implies 3x = 1 \implies x = \frac{1}{3}$.

Intersection at $\left(\frac{1}{3}, \frac{1}{3}\right)$

Question 10

Consider the function $f : [-2, 4] \to \mathbb{R}, f(x) = -3x + 1$.

a. Find the rule for the inverse function f^{-1}.

$$f^{-1}(x) = \frac{1-x}{3}$$

b. State the domain and range of f^{-1}.

$$\text{dom } f^{-1} = \text{ran } f = [-11, 7]$$
$$\text{ran } f^{-1} = \text{dom } f = [-2, 4]$$

c. Find the coordinates for any points of intersection between f and f^{-1}.

Solve $-3x + 1 = x \implies 4x = 1 \implies x = \frac{1}{4}$.

Itersection at $\left(\frac{1}{4}, \frac{1}{4}\right)$

Question 11

Consider the function $f(x) = -2\sqrt{4-x} + 5$.

a. Find the rule for the inverse function f^{-1}.

Let $y = f^{-1}(x)$.

$$x = -2\sqrt{4-y} + 5$$
$$-\frac{1}{2}(x-5) = \sqrt{4-y}$$
$$4 - y = \frac{1}{4}(x-5)^2$$
$$y = -\frac{1}{4}(x-5)^2 + 4$$

so $f^{-1}(x) = -\frac{1}{4}(x-5)^2 + 4$

b. State the domain and range of f^{-1}.

$\text{dom } f^{-1} = \text{ran } f = (-\infty, 5]$

$\text{ran } f^{-1} = \text{dom } f = (-\infty, 4]$

c. Find the coordinates for any points of intersection between f and f^{-1}.

Solve $x = -2\sqrt{4-x} + 5$. This yields

$$\frac{1}{4}(x-5)^2 = 4 - x$$
$$x^2 - 10x + 25 = 16 - 4x$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$
$$x = 3$$

Therefore intersection at $(3, 3)$

Question 12 Tech-Active.

Consider the function $f(x) = \frac{1}{x-5} + 2$.

a. Find the rule for the inverse function f^{-1}.

Define function on cas then solve $f(y) = x$ for y.

$f^{-1}(x) = \frac{1}{x-2} + 5$

b. State the domain and range of f^{-1}.

$\text{dom } f^{-1} = \text{ran } f = \mathbb{R} \setminus \{2\}$

$\text{ran } f^{-1} = \text{dom } f = \mathbb{R} \setminus \{5\}$

c. Find the coordinates for any points of intersection between f and f^{-1}.

Solve $f(x) = x$ or $f(x) = f^{-1}(x)$ on CAS. This yields $x = \frac{1}{2}(7 - \sqrt{13}), \frac{1}{2}(\sqrt{13} + 7)$

Therefore intersections at

$$\left(\frac{1}{2}\left(7 - \sqrt{13}\right), \frac{1}{2}\left(7 - \sqrt{13}\right)\right) \text{ and } \left(\frac{1}{2}\left(\sqrt{13} + 7\right), \frac{1}{2}\left(\sqrt{13} + 7\right)\right)$$

Sub-Section: Final Boss

Question 13

Consider the function $f: [a, \infty) \to \mathbb{R}, f(x) = x^2 - 6x + 10$.

a. Write $f(x)$ in turning point by completing the square.

$$f(x) = (x-3)^2 + 1$$

b. Hence, state the smallest value of a such that, the inverse function f^{-1} exists.

$$a = 3$$

c. Use functional notation to define f^{-1}.

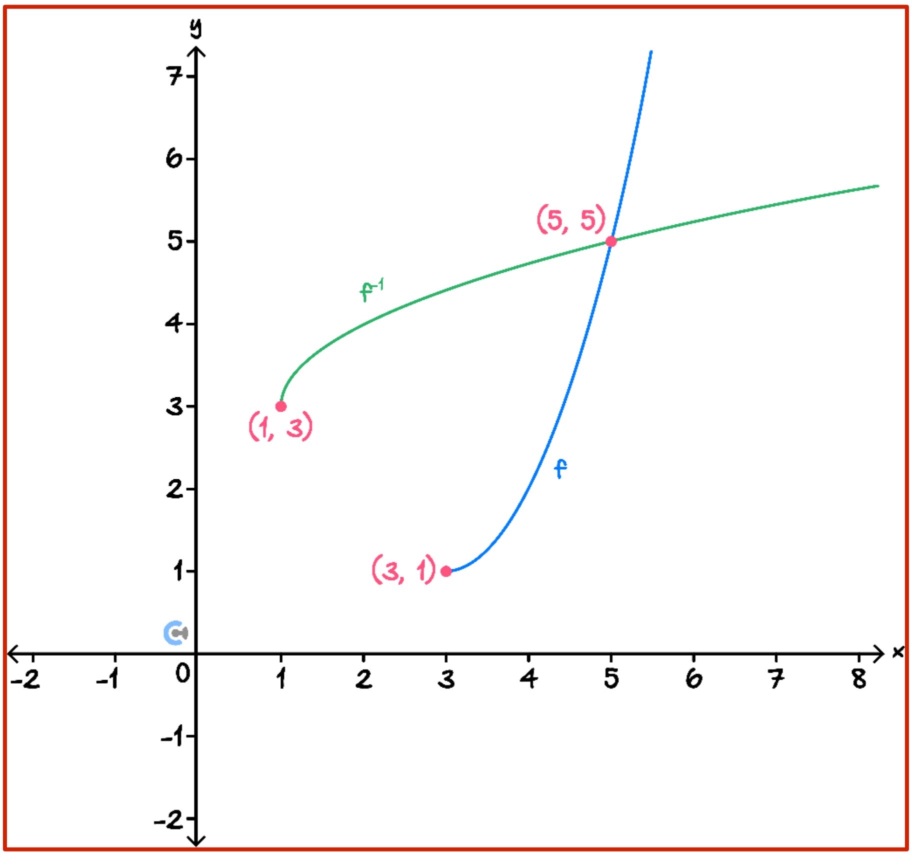
Let $y = f^{-1}(x)$. Then

$$x = (y-3)^2 + 1 \implies y - 3 = \pm\sqrt{x-1} \implies y = \pm\sqrt{x-1} + 3$$

note that dom f^{-1} = ran $f = [1, \infty)$ and ran f^{-1} = dom $f = [3, \infty)$. This implies we take the positive root. Therefore

$$f^{-1}: [1, \infty) \to \mathbb{R},\ f^{-1}(x) = \sqrt{x-1} + 3$$

d. Sketch the graphs of f and f^{-1} on the axes below. Label endpoints and any points of intersection with coordinates.



Intersection occurs when $x^2-6x+10=x \implies x^2-7x+10=0 \implies (x-5)(x-2)=0$. So $x=2$ or $x=5$. But note that $x=2$ is not in the domain of f. Therefore only point of intersection is $(5,5)$.

e. Let g be a one-to-one function with the same rule as f but a different domain. g is defined as:

$$g:(k,\infty)\to\mathbb{R}, g(x)=x^2-6x+10.$$

Find the smallest value of k such that, g and g^{-1} do not intersect each other.

By looking at the graph from the previous part we see that $k=5$.

Section B: Supplementary Questions

Sub-Section [2.2.1]: Find Domain and Range of Functions

Question 14

Find the domain of the following functions:

a. $y = \sqrt{5 - 2x}$.

$$5 - 2x \geq 0$$
$$5 \geq 2x$$
$$x \leq \frac{5}{2}$$

b. $y = -\frac{3}{x^2+4x-12}$.

$$x^2 + 4x - 12 \neq 0$$
$$(x+6)(x-2) \neq 0$$
$$x \neq -6,2$$

c. $y = 2\log_e(x+1)$.

$$x + 1 > 0$$
$$x > -1$$

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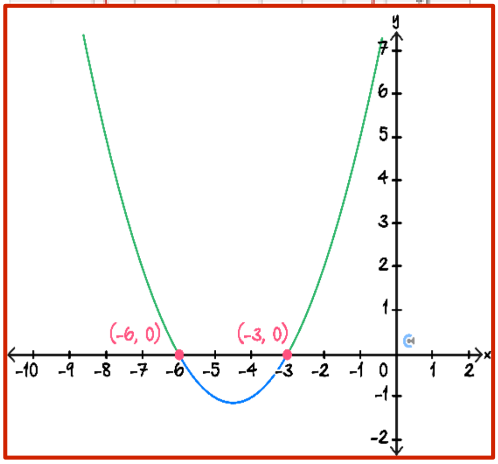
Question 15

Find the maximal domain of the following functions:

a. $y = \frac{(\sqrt{x^2+9x+18}\ )}{2}$.

$x^2 + 9x + 18 \geq 0$

$(x+6)(x+3) \geq 0$

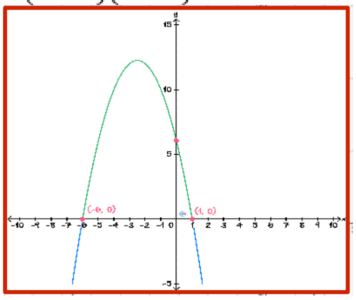


$x \in (-\infty, -6] \cup [-3, \infty)$

b. $y = \frac{3}{\sqrt{6-5x-x^2}} - 4$

$6 - 5x - x^2 > 0$

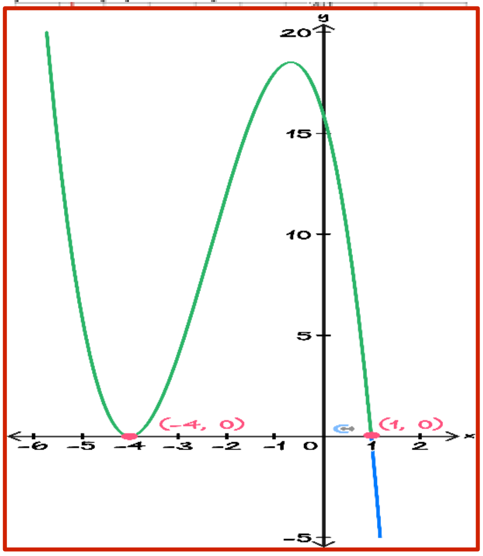
$-(x+6)(x-1) > 0$



$x \in (-6,1)$

c. $y = \log_5\left((1-x)(x+4)^2\right) + 1$.

$(1-x)(x+4)^2 > 0$



$x \in (-\infty, 4) \cup (4,1)$

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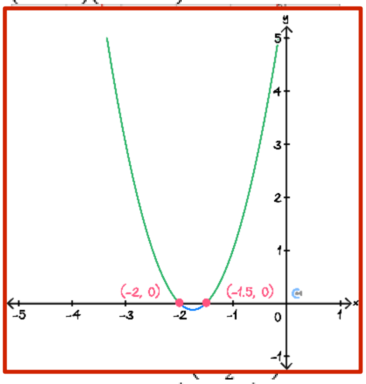
Question 16

Express $f(x)$ in full function mapping notation:

a. $f(x) = \log_2(2x^2 + 7x + 6)$.

$2x^2 + 7x + 6 > 0$

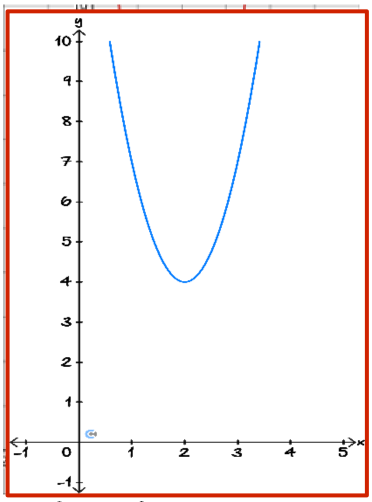
$(x+2)(2x+3) > 0$



$f: (-\infty, -2) \cup \left(-\frac{3}{2}, \infty\right) \to \mathbb{R}, f(x) = \log_2(2x^2 + 7x + 6)$

b. $f(x) = (3x^2 - 12x + 16)^{\frac{3}{2}}$.

$3x^2 - 12x + 16 \geq 0$

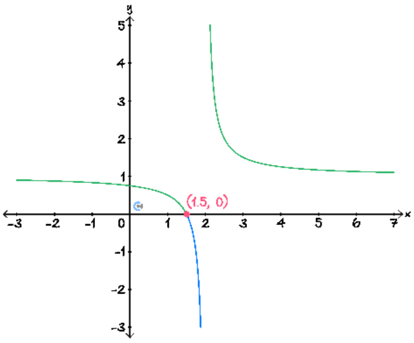


$x \in (-\infty, \infty)$

$f: \mathbb{R} \to \mathbb{R}, f(x) = (3x^2 - 12x + 16)^{\frac{3}{2}}$

c. $f(x) = 3\sqrt{-\frac{1}{4-2x} + 1}$.

$$-\left(\frac{1}{4 - 2x}\right) + 1 \geq 0$$



$$x \in \left(-\infty, \frac{3}{2}\right] \cup (2, \infty)$$

$$f: \left(-\infty, \frac{3}{2}\right] \cup (2, \infty) \to \mathbb{R}, f(x) = 3\sqrt{-\frac{1}{4 - 2x} + 1}$$

Question 17

Find the maximal domain of the function $f(x) = x^2 + 4x + 12$ such that, the range of $f(x)$ is $[8,17)$.

$f(x) = 8 \Rightarrow x = -2$
$f(x) = 17 \Rightarrow x = -5,1$
$x \in (-5,1)$

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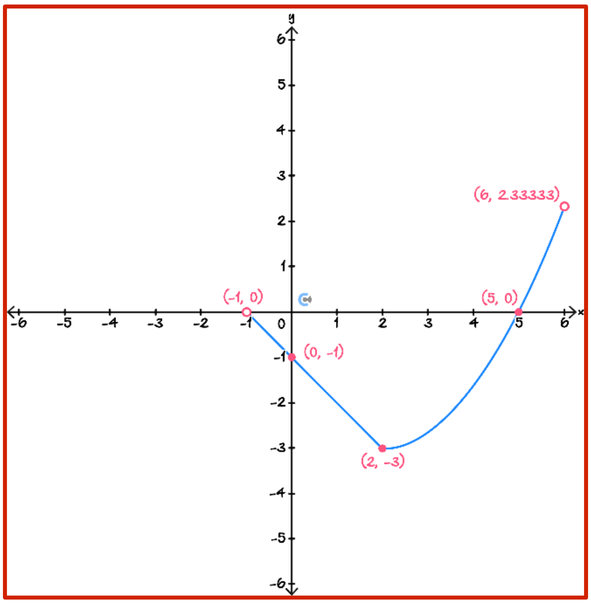
Sub-Section [2.2.2]: Sketch and Find the Domain and Range of Hybrid Functions

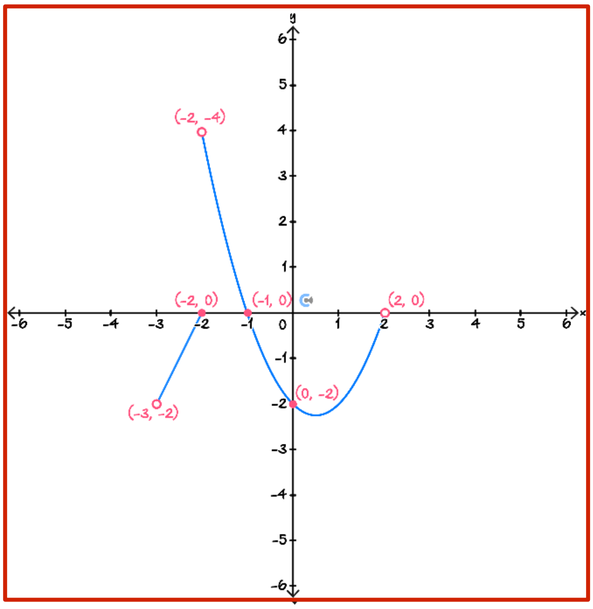
Question 18

Sketch the following graphs. Label all intercepts and endpoints.

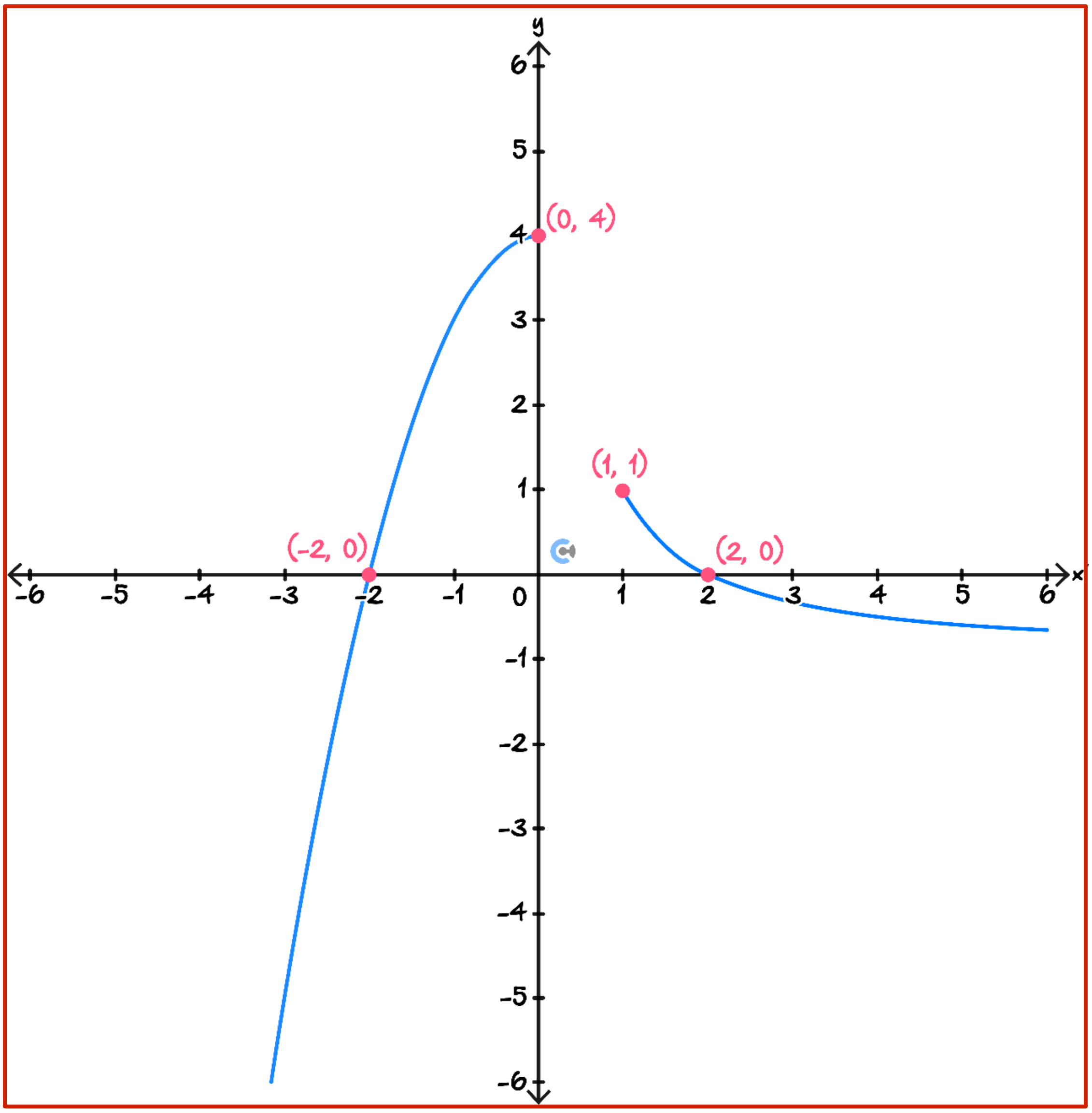
a. $f(x) = \begin{cases} \frac{1}{3}(x-2)^2 - 3, & 2 \le x < 6 \\ -x - 1, & -1 < x < 2 \end{cases}$



b. $f(x) = \begin{cases} 2x + 4, & -3 < x \le -2 \\ x^2 - x - 2, & -2 < x < 2 \end{cases}$



c. $f(x) = \begin{cases} 4 - x^2, & x \leq 0 \\ \frac{2}{x} - 1, & x \geq 1 \end{cases}$



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Question 19

Find the range of the following piecewise functions.

a. $f(x) = \begin{cases} x - 2, & -4 \le x < 1 \\ 2x - 2, & 1 \le x \le 3 \end{cases}$

Range of $x - 2,\ -4 \le x < 1$ is $y \in [-6, -1)$
Range of $2x - 2,\ 1 \le x \le 3$ is $y \in [0,4]$
Range of $f(x)$ is $y \in [-6,-1) \cup [0,4]$

b. $f(x) = \begin{cases} x^2 - 4x + 6, & 0 < x < 5 \\ \frac{1}{2}x + 6, & -6 < x < 0 \end{cases}$

Range of $x^2 - 4x + 6,\ 0 < x < 5$ is $y \in [2,11)$
Range of $\frac{1}{2}x + 6,\ -6 < x < 0$ is $y \in [3,6]$
Range of $f(x)$ is $y = [2,11)$

c. $f(x) = \begin{cases} x+4, & -3 < x < -1 \\ x^2, & -1 \le x \le 2 \\ x-4, & 2 < x < 8 \end{cases}$

Range of $x+4,\ -3 < x < -1$ is $y \in (1,3)$
Range of $x^2,\ -1 \le x \le 2$ is $y \in [0,4]$
Range of $x-4,\ 2 < x < 8$ is $y \in (-2,4)$
Range of $f(x)$ is $y \in (-2,4]$

Question 20

Find the maximal domain of the function $f(x) = \begin{cases} \sqrt{8-2x} \\ \log_e(-x^2+5x+6) \end{cases}$.

For $\sqrt{8-2x}$ to be defined, $8-2x \ge 0$. Therefore $x \in (-\infty, 4]$
For $\log_e(-x^2+5x+6)$ to be defined, $-x^2+5x+6 > 0$. Therefore $x \in (-1,6)$
Maximal domain is $(-\infty, 6)$

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Sub-Section [2.2.3]: Find the Rule, Domain, Range, and Intersections Between Inverse Functions

Question 21

The function $f(x)$ is defined as $f: [-5,1) \to \mathbb{R}, f(x) = \frac{2}{3-x} + 6$.

a. Find the equation of $f^{-1}(x)$.

$f^{-1}(x) = -\frac{2}{x-6} + 3$

b. Determine the domain of $f^{-1}(x)$.

Domain of $f^{-1}(x)$ is Range of $f(x)$

$x \in \left[\frac{25}{4}, 7\right)$

c. State the range of $f^{-1}(x)$.

Range of $f^{-1}(x)$ is Domain of $f(x)$

$y \in [-5,1)$

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Question 22

Consider the function $g: (-\infty, 0] \to \mathbb{R}, g(x) = 2x^2 - 12x + 16$.

a. Find the equation of the inverse function.

$y = \pm\sqrt{\frac{1}{2}(x+2)} + 3$

Range of $g^{-1}(x)$ must be Domain of $g(x)$. Therefore we take the negative root

$g^{-1}(x) = -\sqrt{\frac{1}{2}(x+2)} + 3$

b. Find the domain of the inverse function.

Domain of $g^{-1}(x)$ is Range of $g(x)$

$x \in [16, \infty)$

c. State the range of the inverse function.

Range of $g^{-1}(x)$ is Domain of $g(x)$

$y \in (-\infty, 0]$

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Question 23

Consider the function $f(x) = 1 - \sqrt{7 - x}$.

a. Define the inverse function of $f(x)$, using full function mapping notation.

$y = -x^2 + 2x + 6$

Domain of $f^{-1}(x)$ is Range of $f(x)$. Therefore $x \in (-\infty, 1]$

$f: (-\infty, 1] \to \mathbb{R}, f(x) = -x^2 + 2x + 6$

b. Find the point of intersection between $f(x)$ and $f^{-1}(x)$.

Intersection between $f(x)$ and $f^{-1}(x)$ is the intersection between $f^{-1}(x)$ and $y = x$

$$-x^2 + 2x + 6 = x$$

$$x = -2, 3$$

$x \neq 3$ since $x \in (-\infty, 1]$

Intersection at $(-2, -2)$

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Question 24

Find the values of k such that, the graph $f: [0, \infty) \to \mathbb{R}, f(x) = x^2 + k$ and $f^{-1}(x)$ have 2 solutions.

Maximum k value occurs when $y = x^2 + k$ has 1 solution with $y = x$

$x^2 + k = x$

$x^2 - x + k = 0$

$\Delta = (-1)^2 - 4(1)(k)$

$\Delta = 1 - 4k$

$\Delta = 0$

$k = \frac{1}{4}$

Minimum k value occurs when endpoint of $f(x)$ occurs on $y = x$

Endpoint of $f(x)$ is $(0, k)$

$k = 0$

$k \in \left[0, \frac{1}{4}\right)$

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