CONTOUREDUCATION

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VCE Mathematical Methods ½
Functions & Relations I [2.1]
Workbook

Outline:

Learning Objectives:

- MM12 [2.1.1] - Sketch and find the rule of Hyperbola Functions.
- MM12 [2.1.2] - Sketch and find the rule of Truncus Functions.
- MM12 [2.1.3] - Sketch and find the rule of Root Functions.
- MM12 [2.1.4] - Sketch and find the rule of Semicircles and Circles.
- MM12 [2.1.5] - Identify the type of relations and identify whether the relation is a function.

Section A: Hyperbola

Sub-Section: Sketching Hyperbolas

Hands up if you remember what a hyperbola looks like!

Rectangular Hyperbola Definition

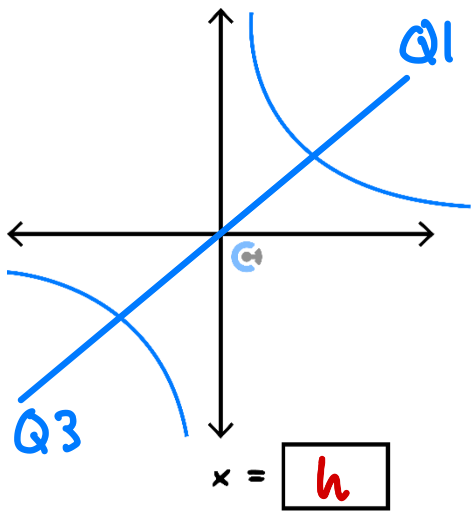
$x-h\neq 0 \Rightarrow x\neq h$

$x=h$

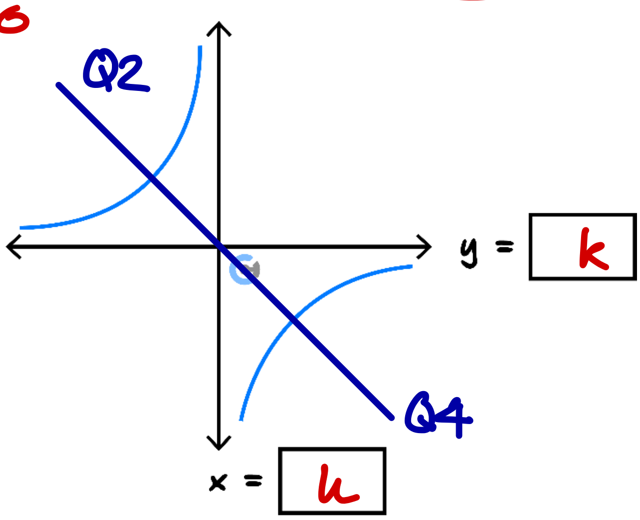
$$y=\frac{a}{\underline{x-h}}+k$$

$\neq 0$

$y=k$



where $a>0$



where $a<0$

➤ Steps

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections, and sketch the curve.

Space for Personal Notes

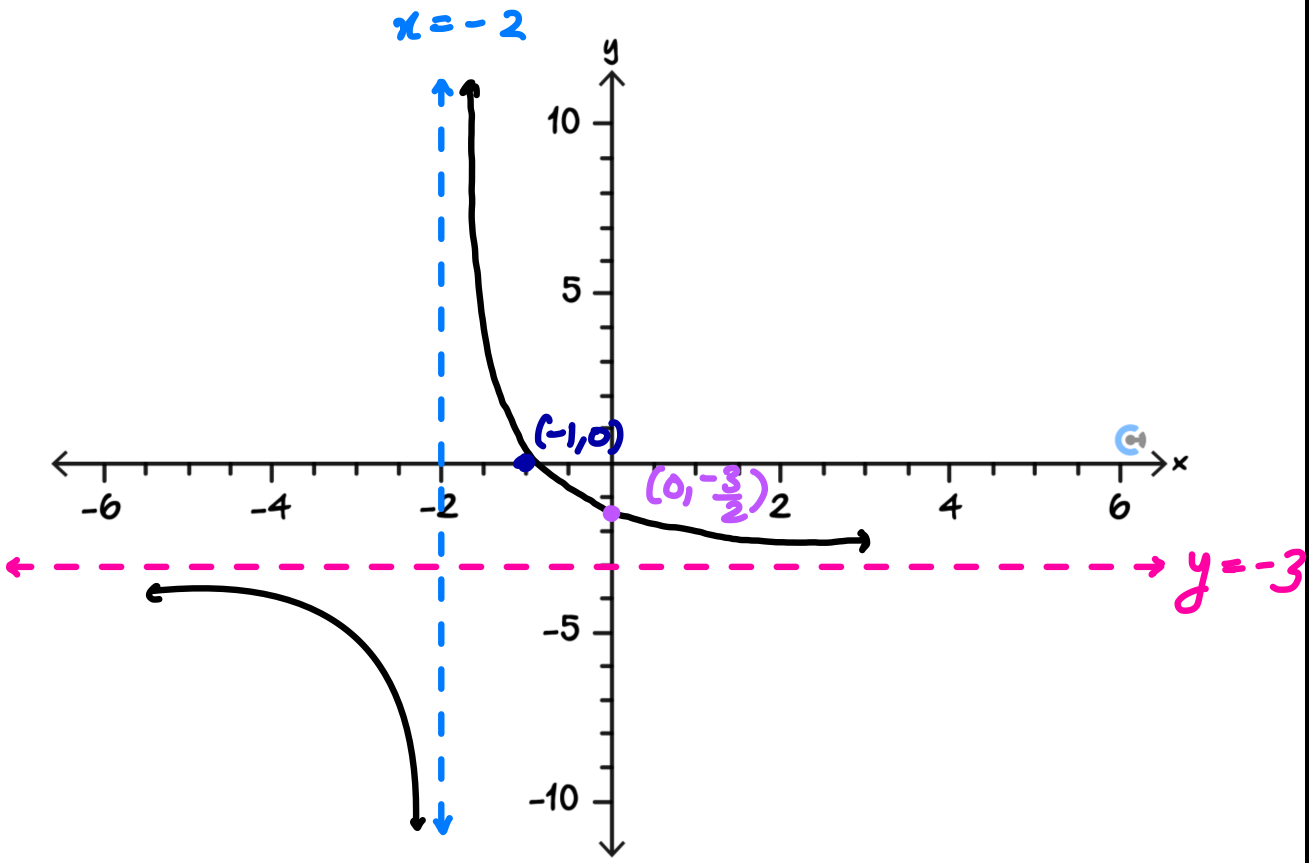
Question 1 Walkthrough.

x-asymptote : $x = -2$

y-asymptote : $y = -3$

Graph the following:

$$y = \frac{3}{x+2} - 3$$



x-int : $0 = \frac{3}{x+2} - 3$

$3 = \frac{3}{x+2}$

$\therefore x+2 = 1$

$\therefore x = -1$

y-int : $y = \frac{3}{2} - 3$

$\therefore y = \frac{-3}{2}$

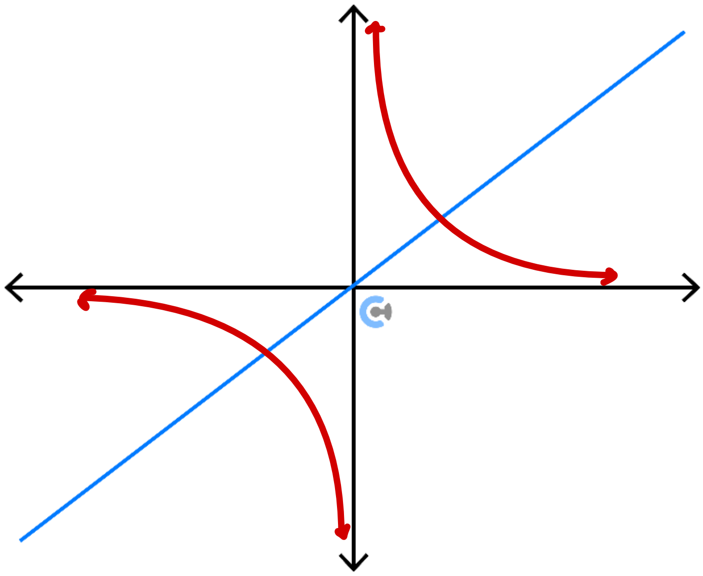
Space for Personal Notes

Why does the hyperbola look like this?

Exploration: Shape of a Hyperbola

- Consider the graph of $y = x$.

TUTORS: "As $y = x$ is the DENOMINATOR (no. of ppl you share pizza with), if $y = x$ gets bigger, $y = \frac{1}{x}$ gets smaller (you have less pizza for yourself)."



- Let's sketch $\frac{1}{x}$ on the same axes with the cues below!
- The graph of $y = x$ is the denominator of $y = \frac{1}{x}$.
- What happens to $\frac{1}{x}$ when x increases? [Increases/Decreases] $x\uparrow$ $\frac{1}{x\uparrow}$ $\downarrow$
- What happens to $\frac{1}{x}$ when x decreases? [Increases/Decreases] $x\downarrow$ $\frac{1}{x\downarrow}$ $\uparrow$
- Remembering that we cannot divide by 0, what happens to $\frac{1}{x}$ when $x = 0$?

↳ undefined. Asymptote!

Space for Personal Notes

Active Recall: Steps for sketching hyperbolas

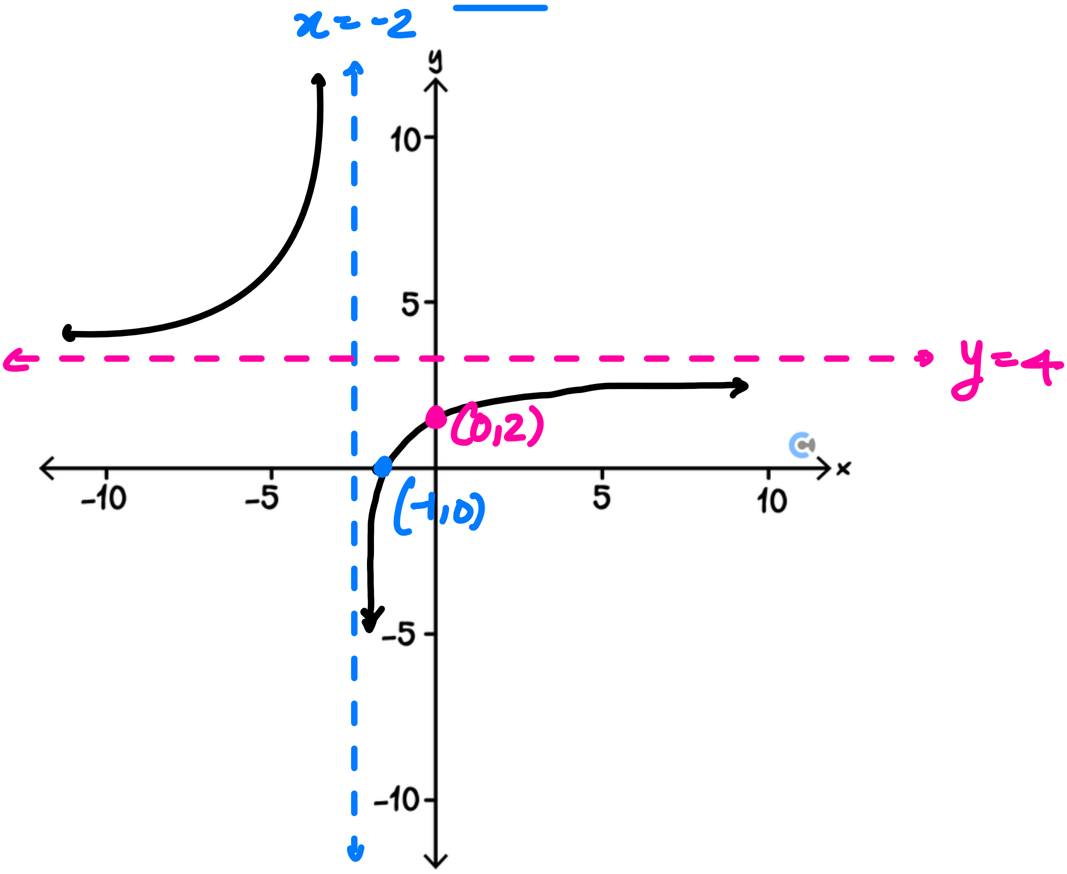
1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

Question 2

x-asymptote : $x = -2$
y-asymptote : $y = 4$

Graph the following, labelling all intercepts and asymptotes.

$$y = 4 - \frac{8}{2x+4}$$



x-int : $0 = 4 - \frac{8}{2x+4}$

$\frac{8}{2x+4} = 4$

$\therefore 2x+4 = 2$

$2x = -2$

$\therefore x = -1$

y-int : $y = 4 - \frac{8}{4}$

$= 2$

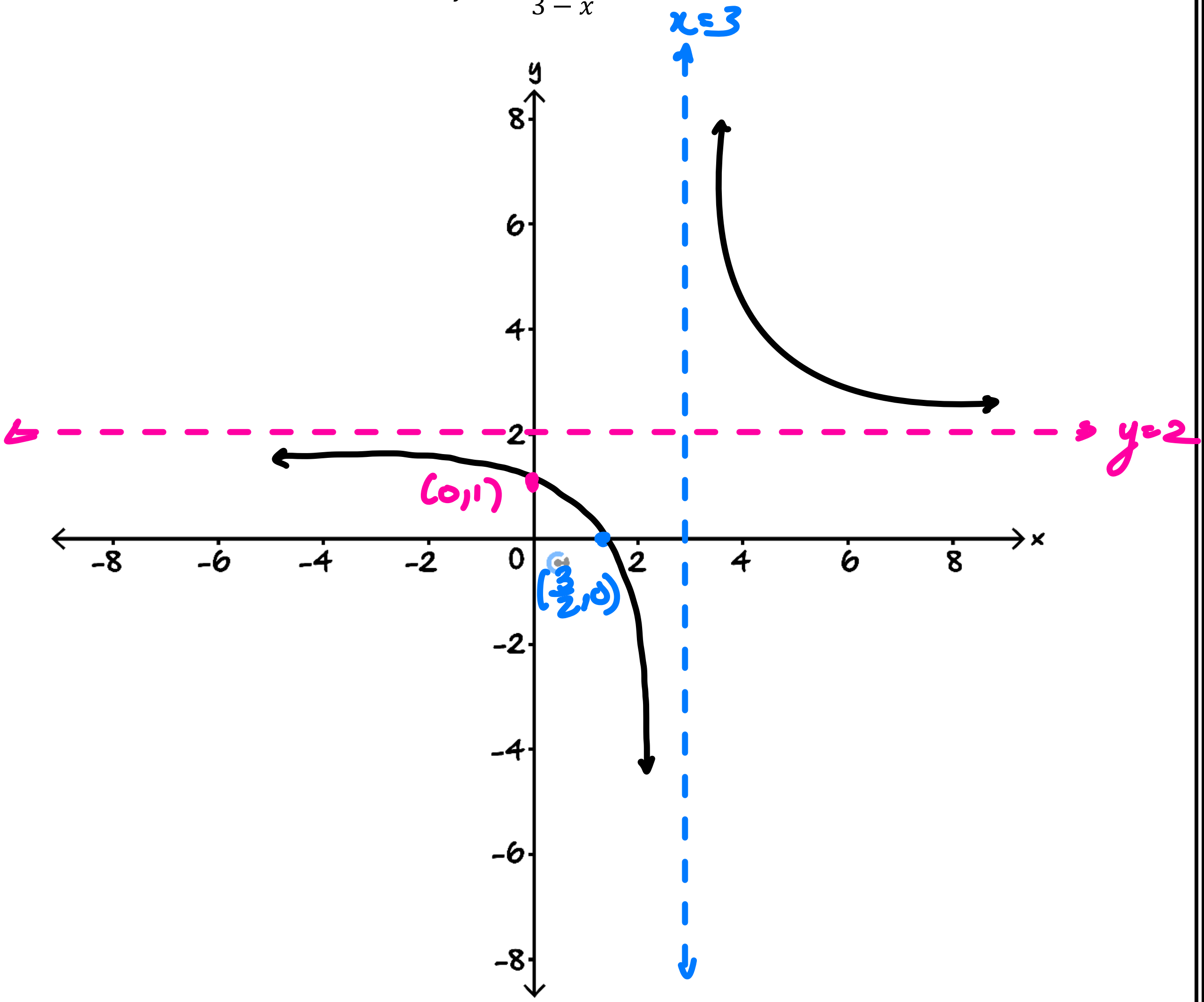
~~Question 3 Extension.~~

x-asymptote : $x = 3$

y-asymptote : $y = 2$

Graph the following, labelling all intercepts and asymptotes.

$$y = -\frac{3}{3-x} + 2$$



x-int : $0 = 2 - \frac{3}{3-x}$

$2 = \frac{3}{3-x}$

$3 - x = \frac{3}{2}$

$\therefore x = 3 - \frac{3}{2} = \frac{3}{2}$

y-int : $y = 2 - \frac{3}{3}$

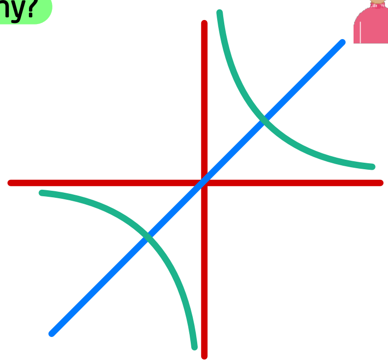
$\therefore y = 1$

Active Recall: Hyperbolas and Linears

- Hyperbolas are reciprocals of linear equations.

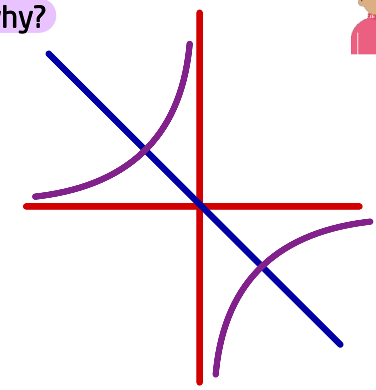
Discussion: In which quadrants, can you find positive hyperbolas and why?

$y = +x \Rightarrow$ +ve hyperbola $\Rightarrow$ Q1,3



Discussion: In which quadrants, can you find negative hyperbolas and why?

$y = -x \Rightarrow$ −ve hyperbola $\Rightarrow$ Q2,4



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Sub-Section: Finding the Rule of a Hyperbola

Let's try the other way around!

Finding the Equation of a Hyperbola from its Graph

- We generally need three facts $(h, k, and\ a)$ about the hyperbola.

$$y = \frac{a}{x-h} + k$$

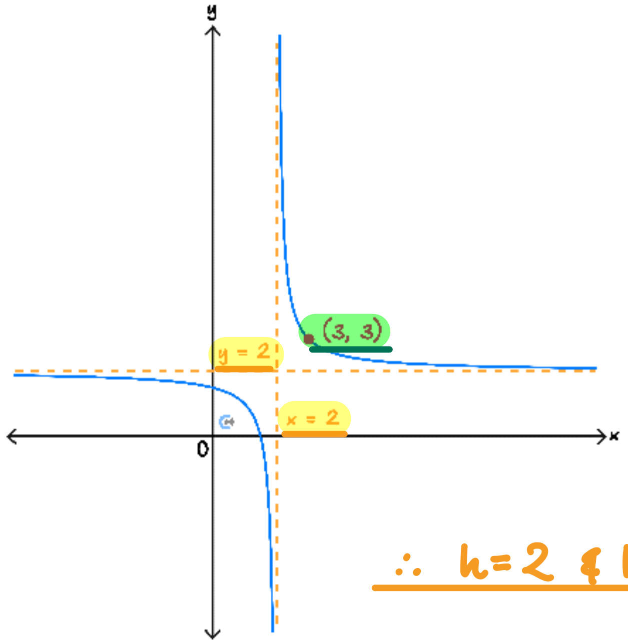
- Steps

1. Look for the asymptotes.
2. Sub in a point to find the value of a.

Space for Personal Notes

Question 4 Walkthrough.

Find the rule for the following graph, given they are in the form, $y = \frac{a}{x-h} + k$.



$\therefore\ h = 2$ & $k = 2$:

Sub(3,3):

$3 = \frac{a}{3-2} + 2$

$1 = \frac{a}{1} \Rightarrow \therefore a = 1$

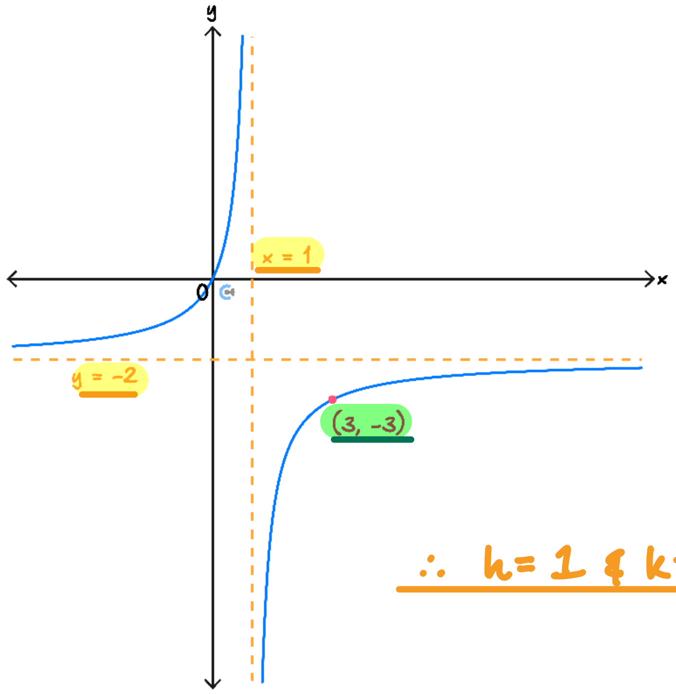
$\therefore y = \frac{1}{x-2} + 2$

Space for Personal Notes

Your turn!

Question 5

Find the rule for the following graph, given they are in the form, $y = \frac{a}{x-h} + k$.



$\therefore h = 1$ & $k = -2$:

Sub$(3, -3)$:

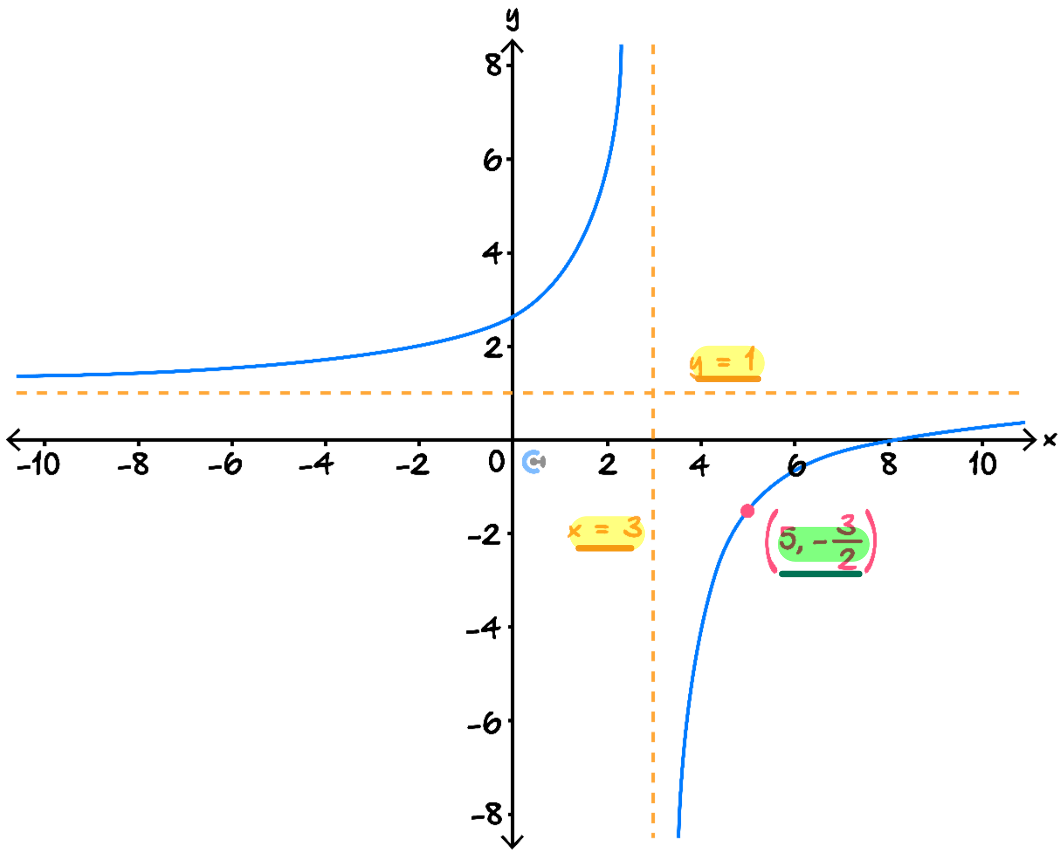
$-3 = \frac{a}{3-1} - 2$

$-1 = \frac{a}{2} \Rightarrow \therefore a = -2$

$\therefore y = \frac{-2}{x-1} - 2$

Question 6

Find the rule for the following graph, given they are in the form, $y = \frac{a}{x-h} + k$.



$\therefore\ h = 3\ \&\ k = 1\ :$

Sub$\left(5, -\frac{3}{2}\right)$:

$-\frac{3}{2} = \frac{a}{5-3} + 1$

$-\frac{5}{2} = \frac{a}{2} \Rightarrow \therefore a = -5$

$\therefore y = \frac{-5}{x-3} + 1$

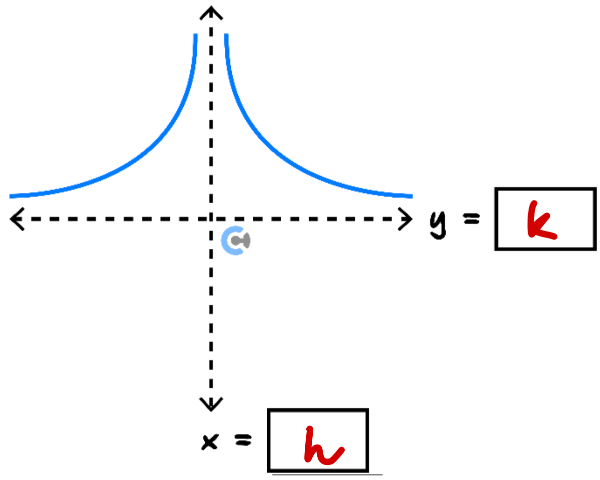
Section B: Truncus

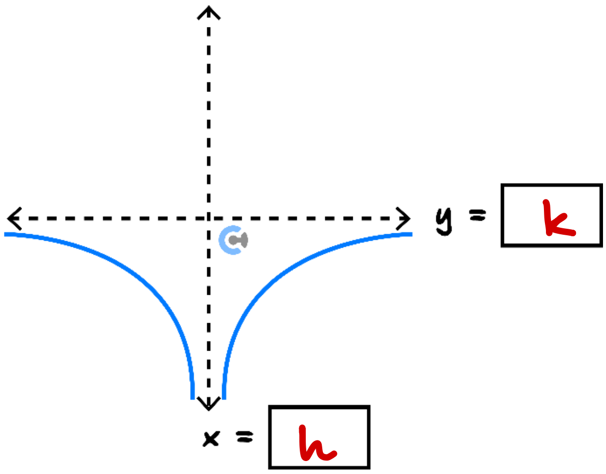
Sub-Section: Sketching Truncus

Now, truncus!

Truncus

$$y = \frac{a}{(x-h)^2} + k$$





$\text{where } a > 0$ $\text{where } a < 0$

➤ Steps

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

Space for Personal Notes

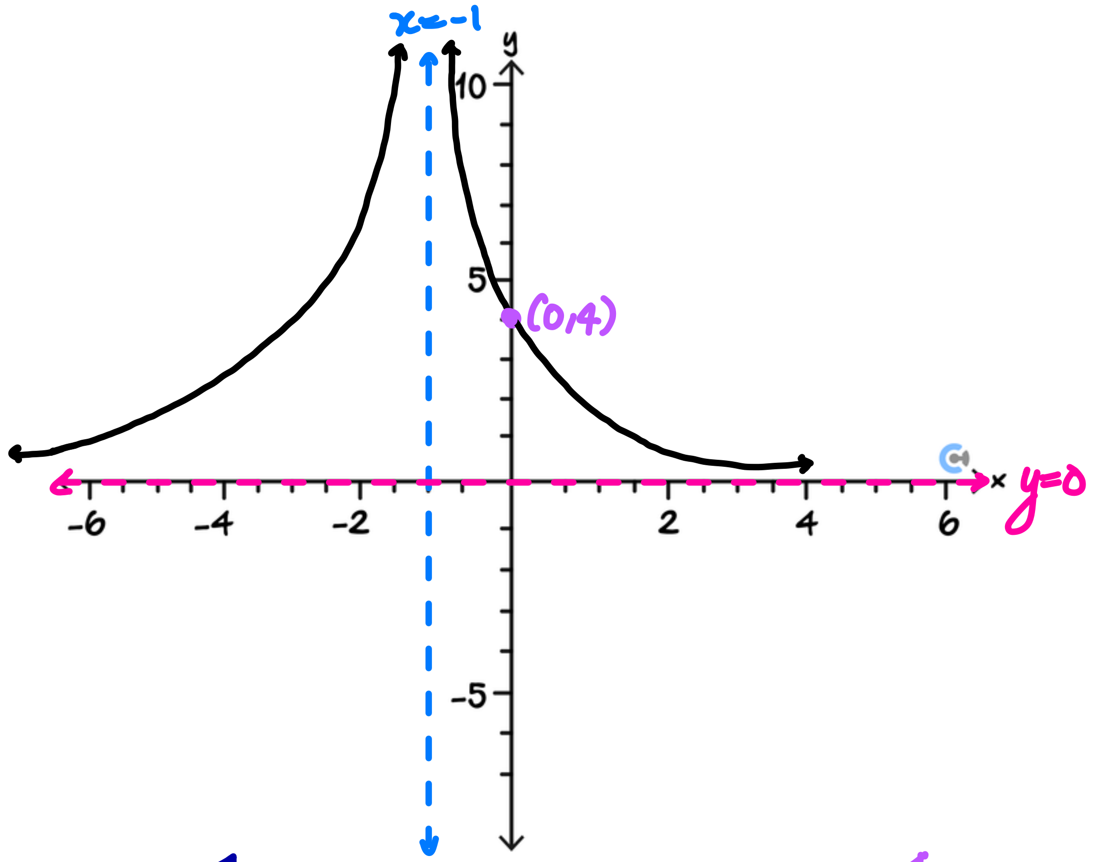
Question 7 Walkthrough.

x-asymptote : $x = -1$

y-asymptote : $y = 0$

Graph the following:

$$y = \frac{4}{(x+1)^2}$$



x-int : $0 = \frac{4}{(x+1)^2}$ ✗

y-int : $y = \frac{4}{(0+1)^2}$

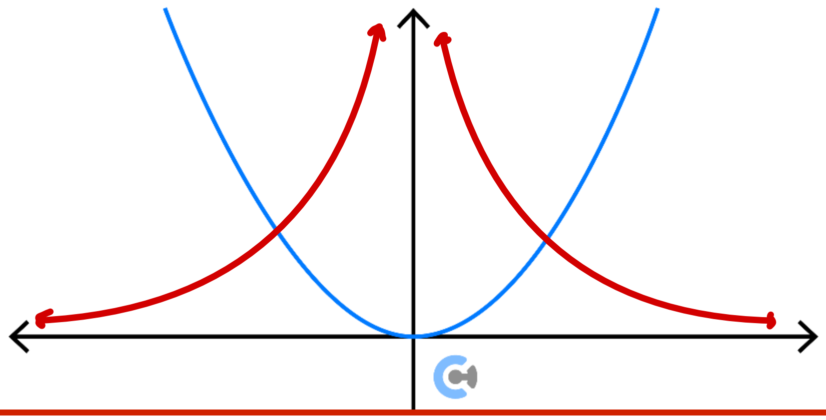
$\therefore y = 4$

Space for Personal Notes

Why is the truncus shaped like it is?

Exploration: Shape of a Truncus

- Consider the graph of $y = x^2$.



TUTORS: "As $y = x^2$ is the DENOMINATOR (no. of ppl you share pizza with), if $y = x^2$ gets bigger, $y = \frac{1}{x^2}$ gets smaller (you have less pizza for yourself)."

- Let sketch $\frac{1}{x^2}$ on the same axes with the cues below!
- The graph of $y = x$ is the denominator of $y = \frac{1}{x}$.
- The graph of $y = x^2$ is the denominator of $y = \frac{1}{x^2}$.
- What happens to the $\frac{1}{x^2}$ when x^2 increases?

$\downarrow \frac{1}{x^2 \uparrow}$ $x^2 \uparrow$ $\Rightarrow$ Decrease

- What happens to the $\frac{1}{x^2}$ when x^2 decreases?

$\uparrow \frac{1}{x^2 \downarrow}$ $x^2 \downarrow$ $\Rightarrow$ Increase

- What happens to the $\frac{1}{x^2}$ when $x^2 = 0$?

undefined. Asymptote

Active Recall

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

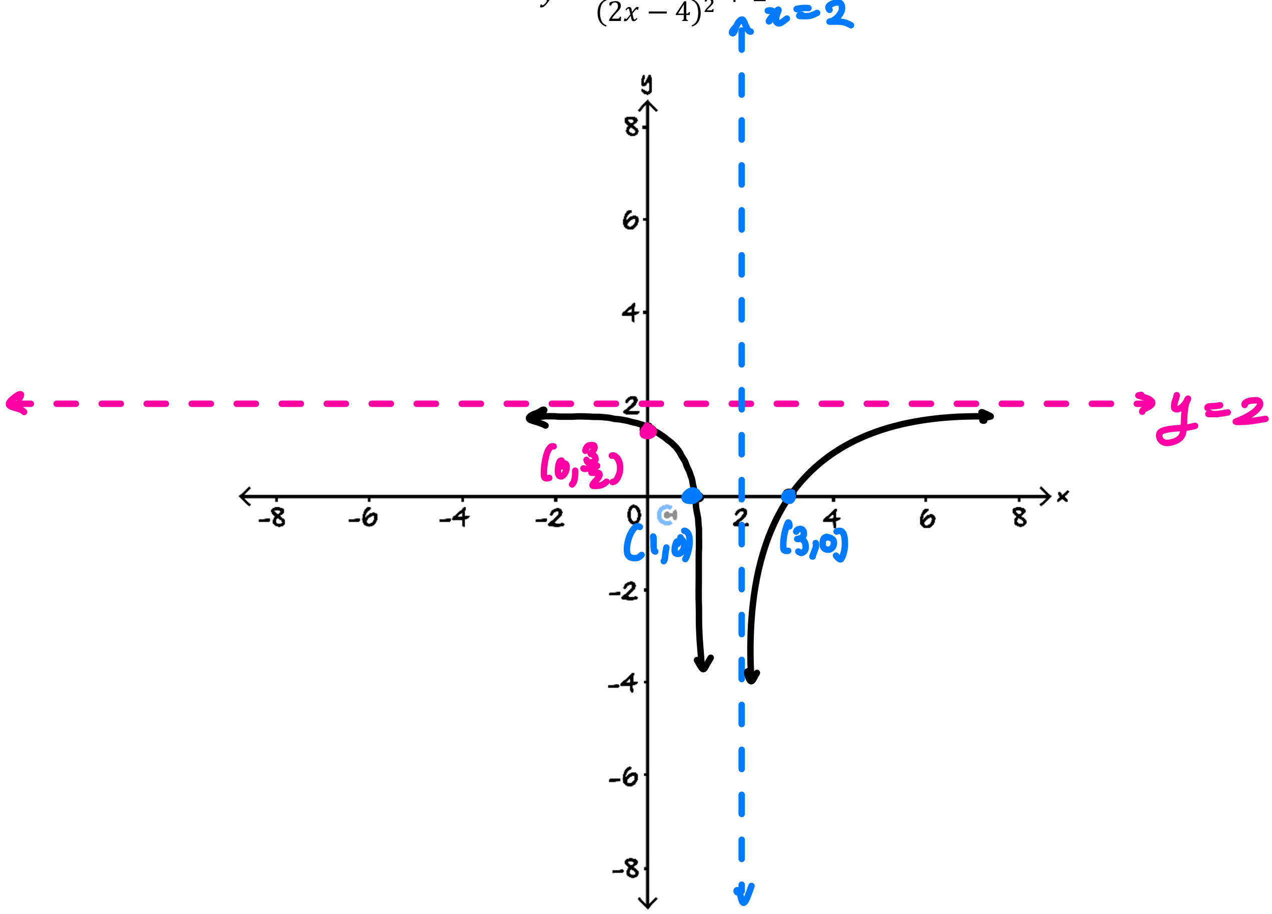
Question 8

Graph the following, labelling all intercepts and asymptotes.

x-asymptote : $x = 2$

y-asymptote : $y = 2$

$$y = \frac{-8}{(2x-4)^2} + 2$$



x-int : $0 = \frac{-8}{(2x-4)^2} + 2$

$\frac{8}{(2x-4)^2} = 2$

$(2x-4)^2 = 4$

$2x-4 = 2$ or $2x-4 = -2$

$\therefore 2x = 6$ or $2x = 2$

$x = 3$ or $x = 1$

y-int : $y = \frac{-8}{(0-4)^2} + 2$

$= \frac{-8}{16} + 2$

$= -\frac{1}{2} + 2$

$\therefore y = \frac{3}{2}$

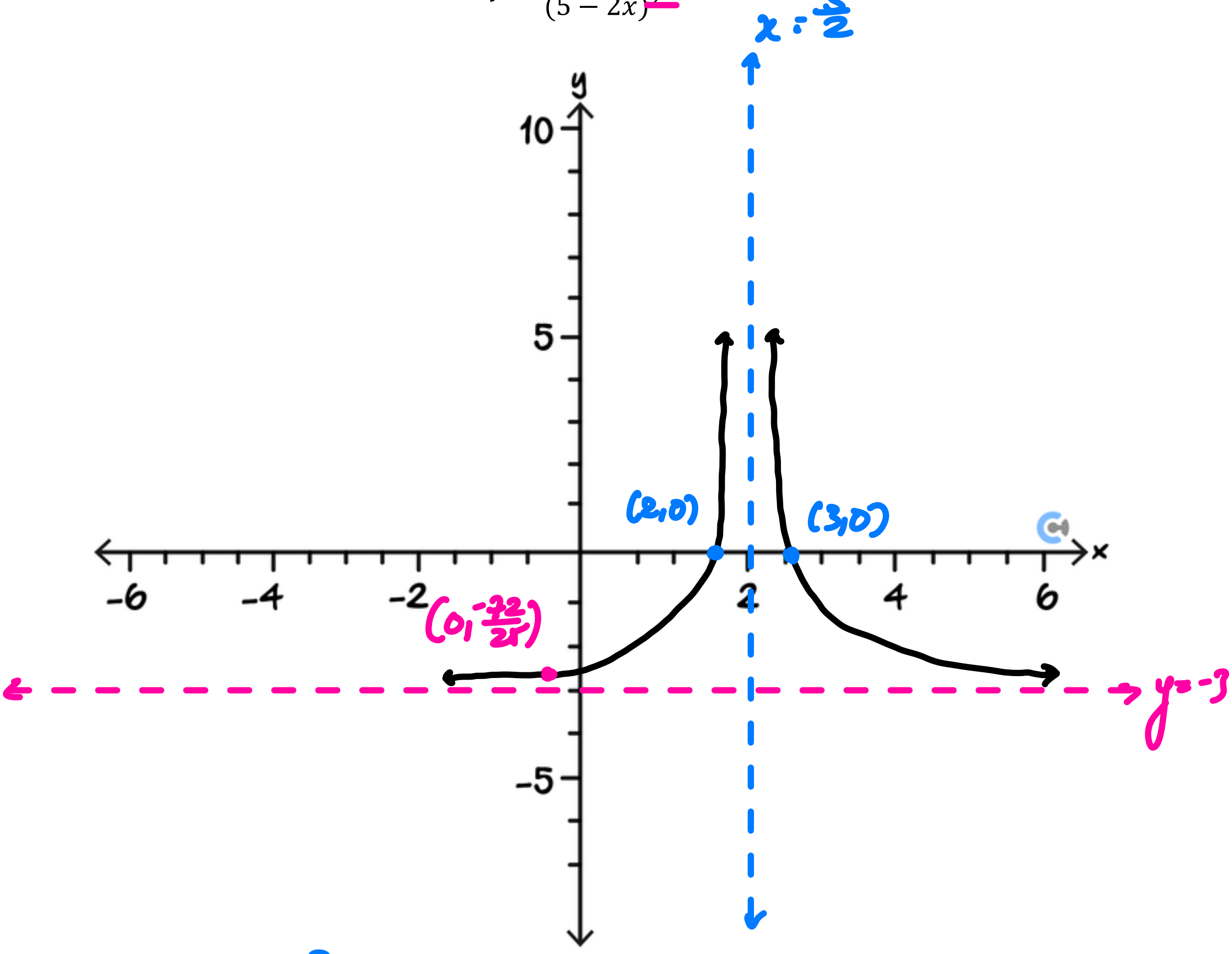
Question 9 Extension.

Graph the following, labelling all intercepts and asymptotes.

$5-2x=0 \Rightarrow x=\frac{5}{2}$

$y=-3$

$$y = \frac{3}{(5-2x)^2} - 3$$



x-int: $0 = \frac{3}{(5-2x)^2} - 3$

$3 = \frac{3}{(5-2x)^2} \Rightarrow (5-2x)^2 = 1$

$5-2x = 1 \Rightarrow 2x = 4 \therefore x = 2$

OR

$5-2x = -1 \Rightarrow 2x = 6 \therefore x = 3$

y-int: $y = \frac{3}{(5)^2} - 3$

$= \frac{3}{25} - 3 = -\frac{72}{25}$

Space for Personal Notes

<u>Active Recall:</u> Truncus and Quadratics

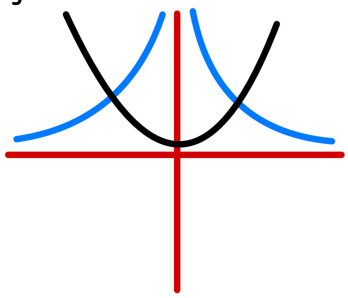
- Trunci are reciprocals of quadratic equations.

<u>Discussion:</u> In which quadrants, can you find positive trunci and why?

+ve truncus ⇒ +ve parabola

⇓

Q1,2

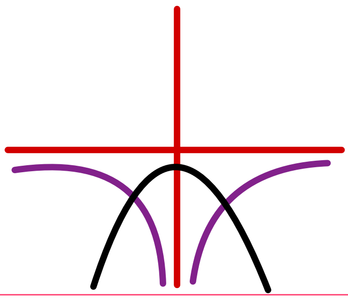


<u>Discussion:</u> In which quadrants, can you find negative trunci and why?

−ve truncus ⇒ −ve parabola

⇓

Q3,4



Space for Personal Notes

Sub-Section: Finding the Rule of a Truncus

Let's try the other way around!

Finding the Equation of a Truncus from its Graph

- We generally need three facts $(h, k, and\ a)$ about the truncus.

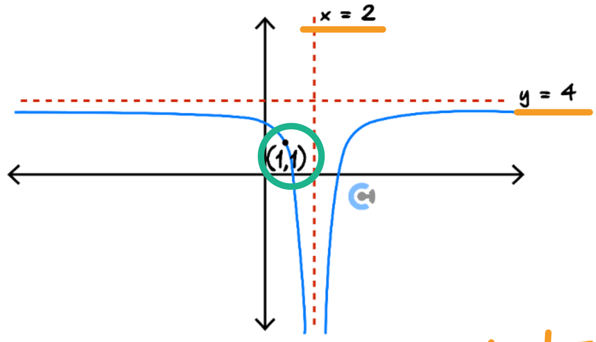
$$y = \frac{a}{(x-h)^2} + k$$

- Steps

1. Look for the asymptotes.
2. Sub in a point to solve the value of a.

Question 10 Walkthrough.

Find the rule for the following graph, given they are in the form, $y = \frac{a}{(x-h)^2} + k$.



$\therefore h = 2$ & $k = 4$:

Sub $(1, 1)$:

$1 = \frac{a}{(1-2)^2} + 4$

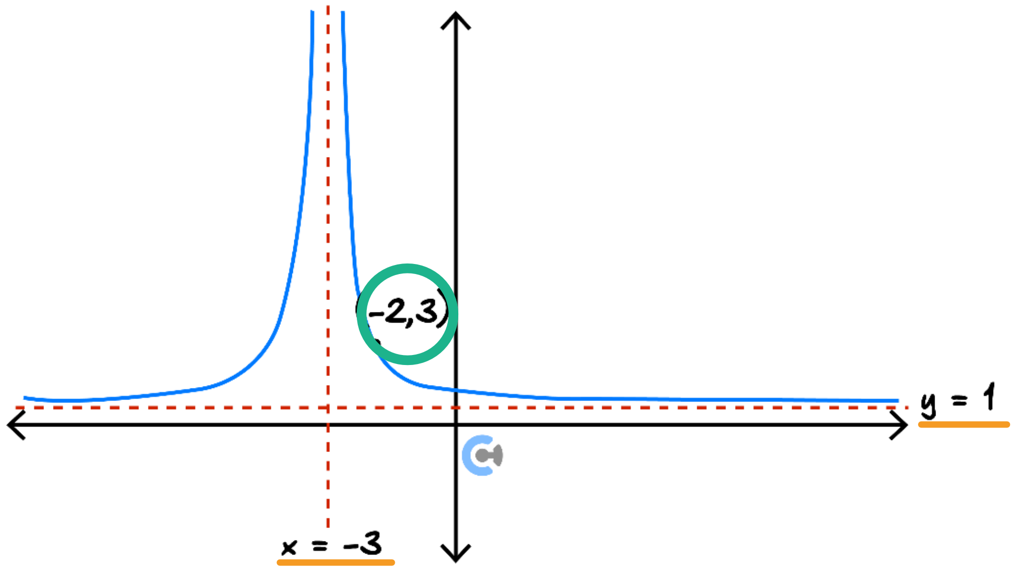
$-3 = \frac{a}{(-1)^2} \Rightarrow \therefore a = -3$

$\therefore y = \frac{-3}{(x-2)^2} + 4$

Your turn!

Question 11

Find the rule for the following graph, given they are in the form, $y = \frac{a}{(x-h)^2} + k$.



$\therefore\ h = -3$ & $k = 1$:

Sub $(-2, 3)$:

$3 = \frac{a}{(-2-(-3))^2} + 1$

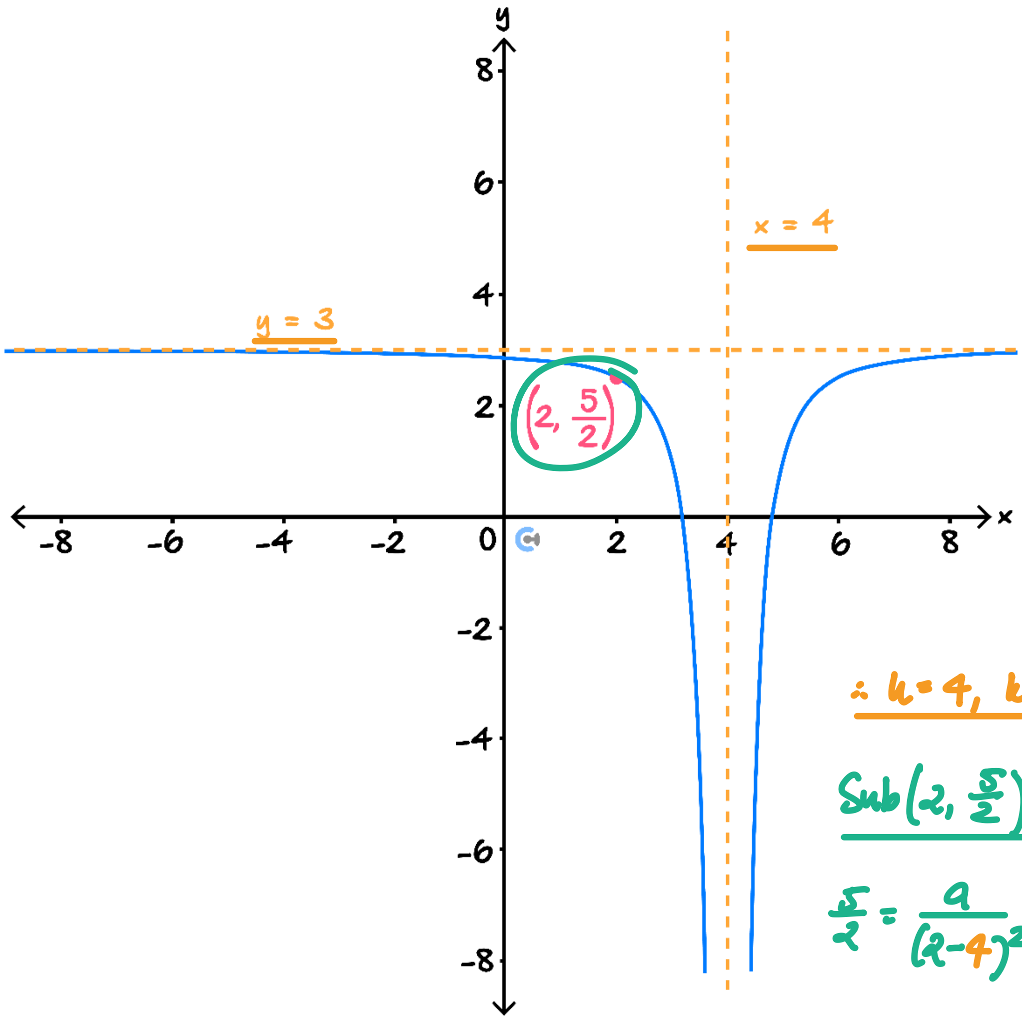
$2 = \frac{a}{(-2+3)^2} \Rightarrow \therefore a = 2$

$\therefore y = \frac{2}{(x+3)^2} + 1$

TUTORS:
1. Find h and k from asymptotes.
2. Find the value of a from the other point.

Question 12 Extension.

Find the rule for the following graph, given they are in the form, $y = \frac{a}{(x-h)^2} + k$.



$\therefore h = 4,\ k = 3$

Sub $\left(2, \frac{5}{2}\right)$:

$\frac{5}{2} = \frac{a}{(2-4)^2} + 3$

$-\frac{1}{2} = \frac{a}{4}$

$\therefore a = -2$

$\therefore y = \frac{-2}{(x-4)^2} + 3$

Section C: Root Functions

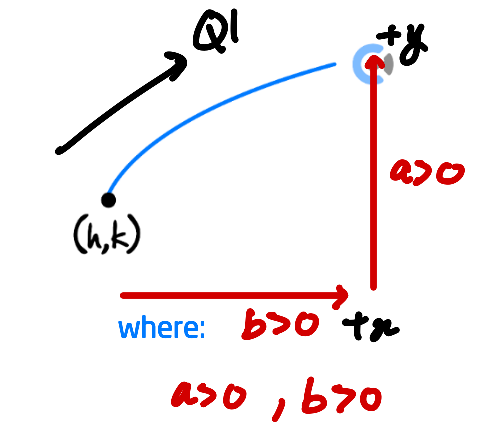
Sub-Section: Sketching Root Functions

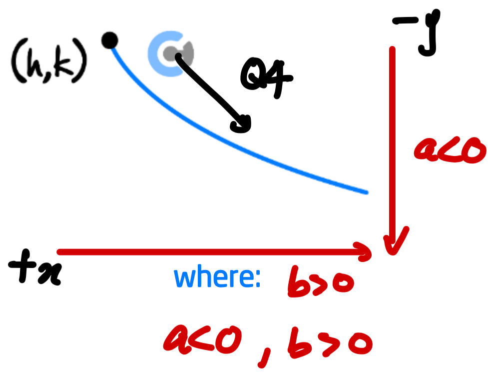
Now, root functions!

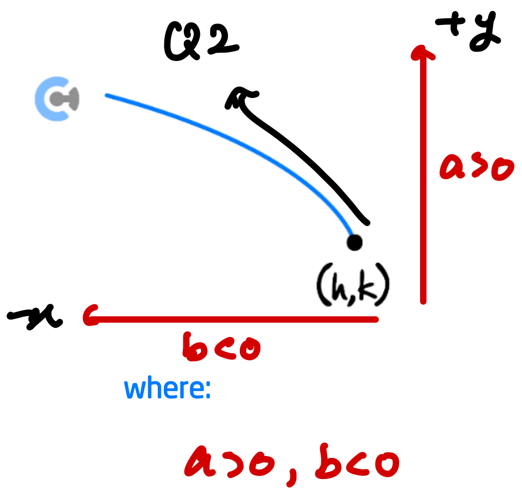
Definition

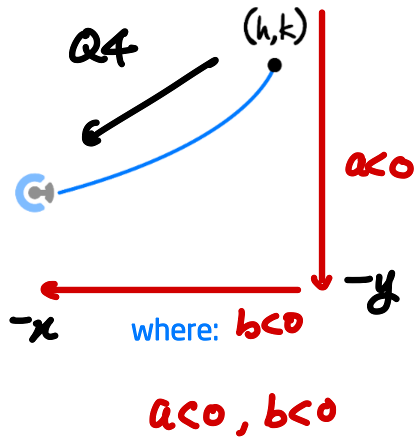
Square Root Functions

$$y = \underline{a}\sqrt{\underline{b}(x-h)} + k$$









Steps for sketching roots

1. Find the starting point (h, k).
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

Question 13 Walkthrough.

Graph the following:

$x-3=0$

$\therefore x=3 \Rightarrow \therefore y=-1$

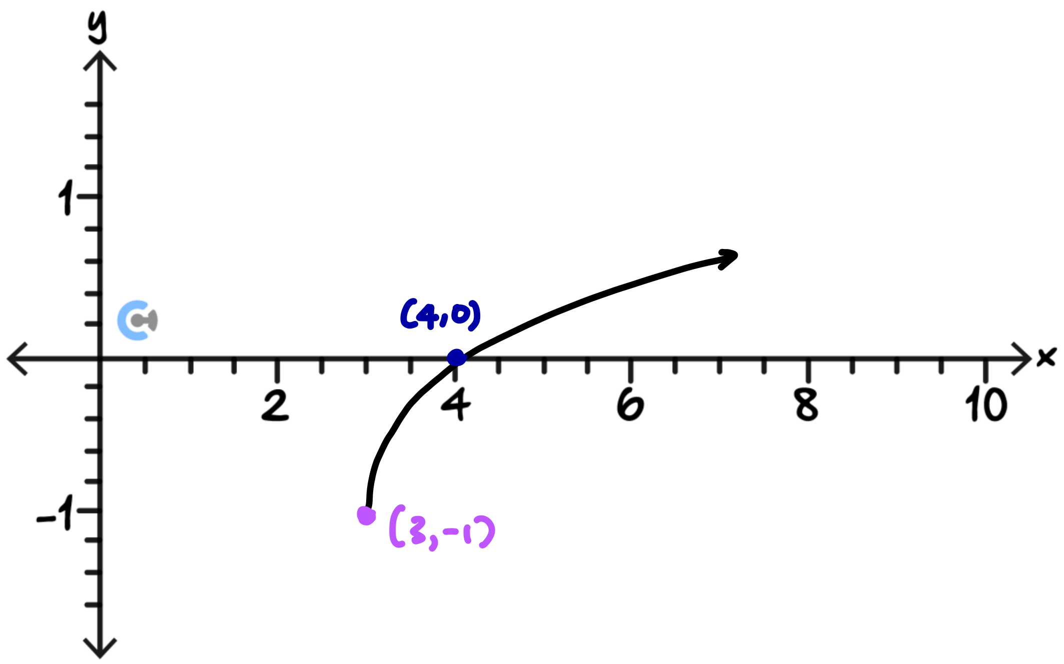
$$y = \underbrace{\sqrt{x-3}}_{=0} - 1$$

SP: $(3,-1)$

Step 1: Find the starting point of the graph and plot it on the axis.

Step 2: Find the x- and y- intercepts and plot on the axes (if they exist).

Step 3: Identify the shape of the graph by considering any reflections and sketch the curve.



x-int: $0=\sqrt{x-3}-1$

$1=\sqrt{x-3}$

$1=x-3$

$\therefore x=4$

Active Recall: Steps for sketching roots

1. Find the Starting.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the Shape of the graph by considering any reflections and sketch the curve.

Question 14

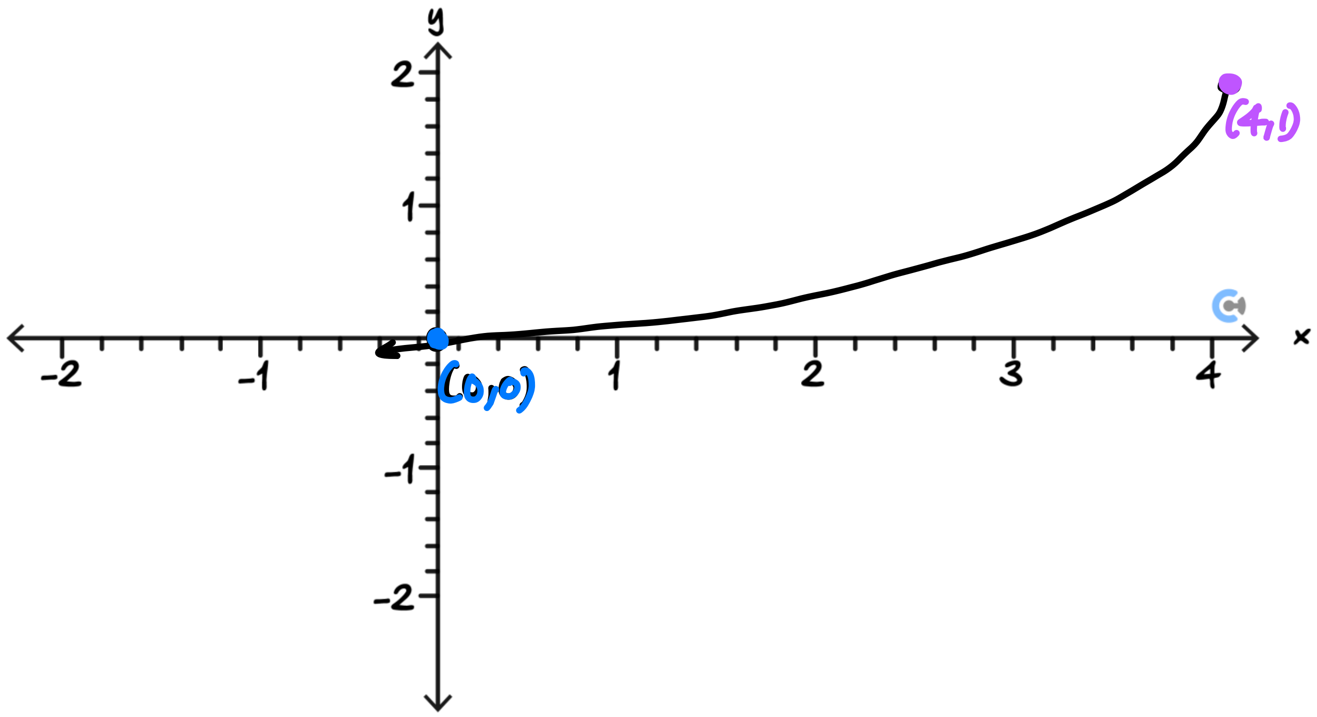
Graph the following:

$4-x=0$

$\therefore x=4 \Rightarrow \therefore y=2$

SP: $(4,2)$

$$y=-\sqrt{4-x}+2$$



x-int: $0=-\sqrt{4-x}+2$

$\sqrt{4-x}=2$

$4-x=4$

$\therefore x=0$

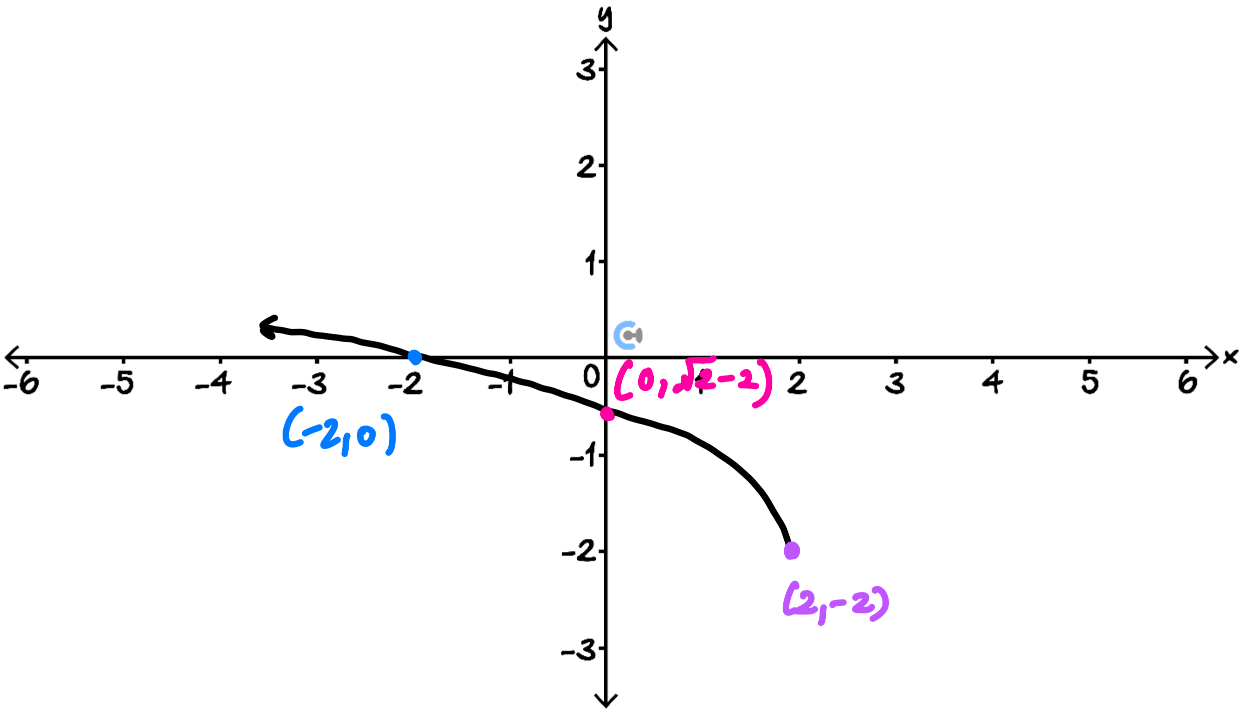
Space for Personal Notes

Question 15 Extension.

Graph the following:

$2-x=0 \rightarrow x=2 \Rightarrow y=-2$

$$y = \sqrt{2-x} - 2$$



SP:

$2-x=0 \Rightarrow x=2, y=-2$

$y = \sqrt{2-0} - 2$

$= \sqrt{2} - 2$

$\approx 1.4 - 2$

≈ -0.6

$\sqrt{2} \approx 1.4$

Space for Personal Notes

x-int:

$0 = \sqrt{2-x} - 2$

$2 = \sqrt{2-x}$

$4 = 2-x$

$\therefore x = -2$

Sub-Section: Finding a Rule of a Root Function

Let's try the other way around!

Finding the Equation of a Root Function from its Graph

- We generally need three facts about the root function.

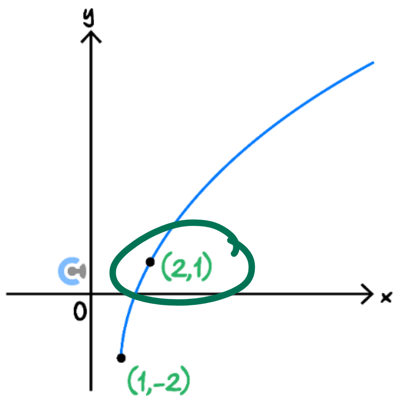
$$y = a\sqrt{\pm(x-h)} + k$$

- Steps

1. Look for the starting point (h, k).
2. Sub in a point to solve the value of a.
3. Look at Shape to determine $\pm(x-h)$

Question 16 Walkthrough.

Find the rule for the following graph, given they are in the form, $y = a\sqrt{\pm(x-h)} + k$.



SP: $(1,-2)$

$y = a\sqrt{+(x-1)} - 2$

Sub $(2,1)$:

$1 = a\sqrt{+(2-1)} - 2$

$1 = a\sqrt{1} - 2 \Rightarrow \therefore a = 3$

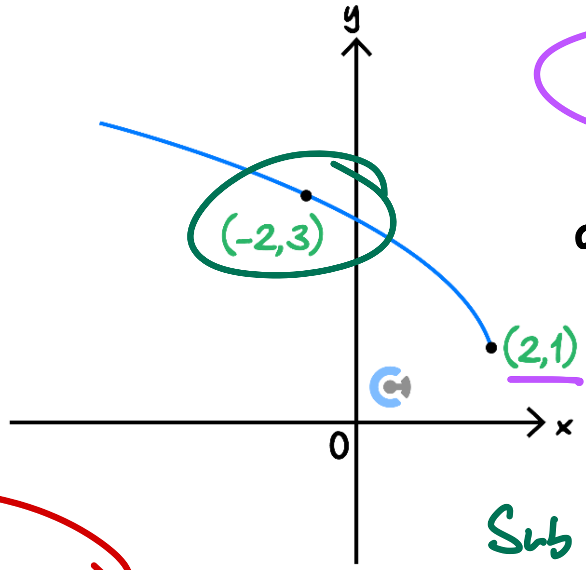
$\therefore y = 3\sqrt{x-1} - 2$

Active Recall: Steps for finding the rule for a root function

1. Look for the starting point (h, k).
2. Sub in a point to solve the value of a.

Question 17

Find the rule for the following graph, given they are in the form, $y = a\sqrt{\pm(x-h)} + k$.



$h = 2, k = 1$

$y = a\sqrt{\pm(x-2)} + 1$

Sub $(-2, 3)$:

$3 = a\sqrt{-(-2-2)} + 1$

$2 = a\sqrt{4}$

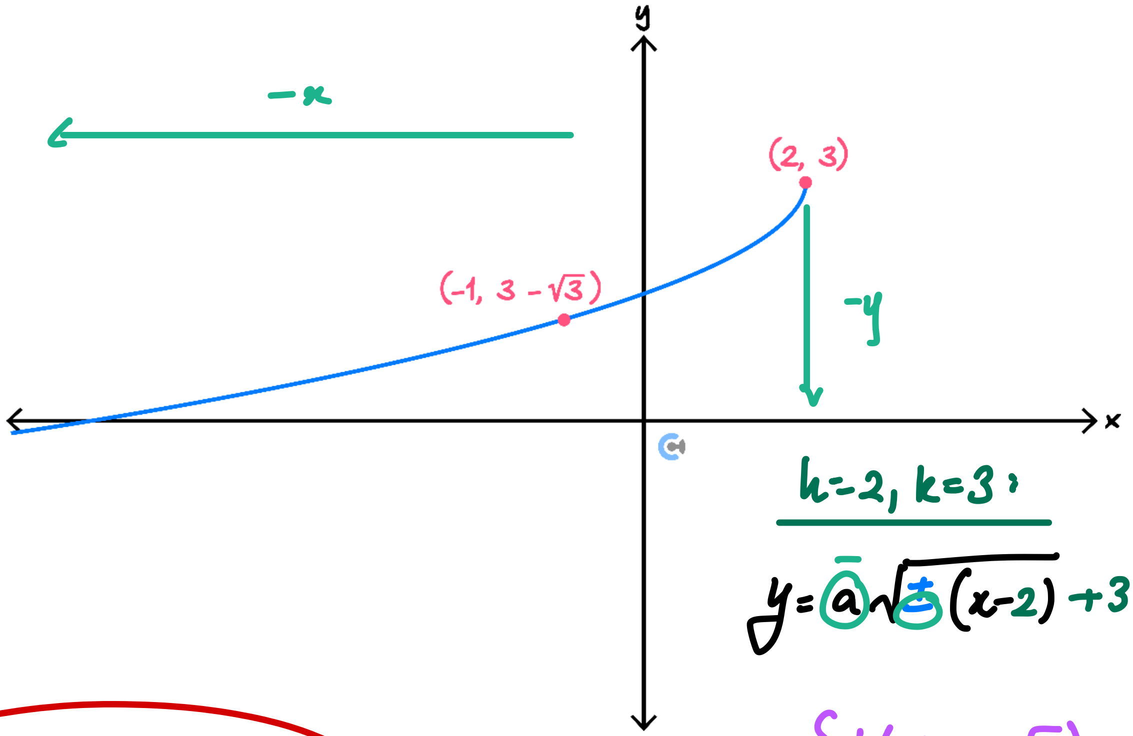
$\therefore a = 1$

$\therefore y = \sqrt{2-x} + 1$

Space for Personal Notes

Question 18 Extension.

Find the rule for the following graph, given they are in the form, $y = a\sqrt{\pm(x-h)} + k$.



$h=2, k=3$:

$y = \bar{a}\sqrt{\pm(x-2)}+3$

Sub$(-1, 3-\sqrt{3})$:

$3-\sqrt{3} = a\sqrt{-(-1-2)}+3$

$\underbrace{3-\sqrt{3} = 3+a\sqrt{3}}_{\therefore a=-1}$

$\therefore y = -\sqrt{-(x-2)}+3$

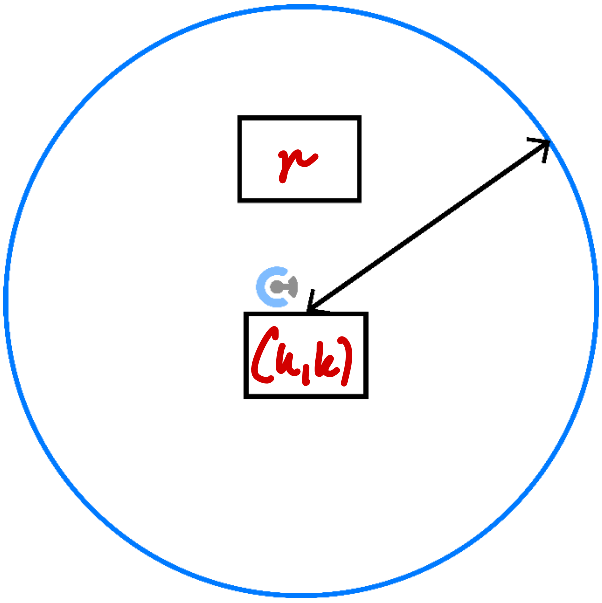
Section D: Circles and Semicircles

Sub-Section: Sketching Circles and Semi Circles

Now, circles!

Definition

Circles



$$(x-h)^2+(y-k)^2=r^2$$
$$where\ r>0$$

- Centre: (h, k)
- Radius: r
- **Steps**

1. Find the centre of the circle.
2. Find the radius of the circle.
3. Find axes intercepts (if they exist).
4. Identify the shape of the graph and sketch the curve.

Question 19 Walkthrough.

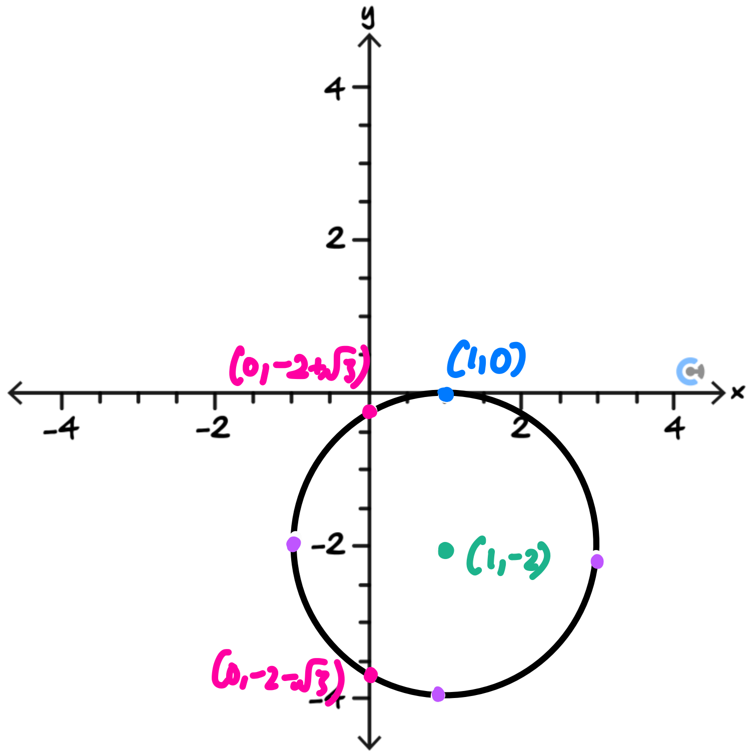
Graph the following circle:

Centre : $(1,-2)$

$r^2=4$

$r=2$

$$(x-1)^2+(y+2)^2=4$$



y-int:

$(0-1)^2+(y+2)^2=4$

$(y+2)^2=3$

$y+2=\pm\sqrt{3}$

$y=-2-\sqrt{3}$ or $-2+\sqrt{3}$

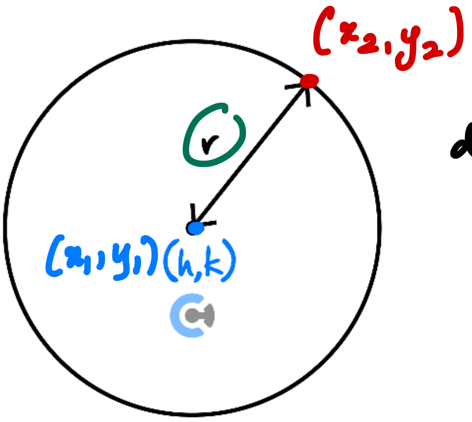
$\sqrt{3}\approx 1.7$ $\quad\approx -2-1.7 \quad\quad \approx -2+1.7$

$\approx -3.7 \quad\quad \approx -0.3$

Discussion: What do all the points on the circle have in common?

→ All radius apart from centre

Exploration: Derivation of Circle Equation



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

- The common property of all points on the circle can be written by using distance formula.

$$\sqrt{(x_2 - h)^2 + (y_2 - k)^2} = r$$

- Finally, what happens if you square both sides?

$$(x-h)^2 + (y-k)^2 = r^2$$

Active Recall: Steps for sketching a circle

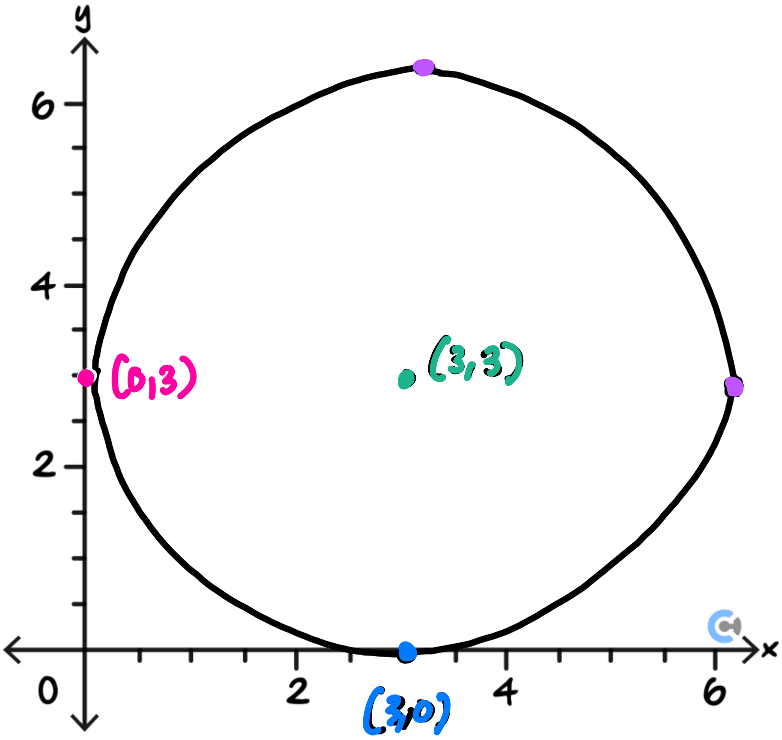
1. Find the centre of the circle.
2. Find the radius of the circle.
3. Find axes intercepts (if they exist).
4. Identify the shape of the graph and sketch the curve.

$(a+b)^2 = a^2+2ab+b^2$

Question 20

Graph the following relation and state the values of x and y over which it stretches. Include all axes intercepts.

$$x^2 - 6x + y^2 - 6y + 9 = 0$$



$(x-3)^2 - 9 + (y-3)^2 = 0$

$(x-3)^2 + (y-3)^2 = 9$

Centre : $(3,3)$

$r^2 = 9$

$r = 3$

NOTE: You will need to complete the square!

Question 21 Extension.

Graph the following relation and state the values of x and y over which it stretches. Include all axes intercepts.

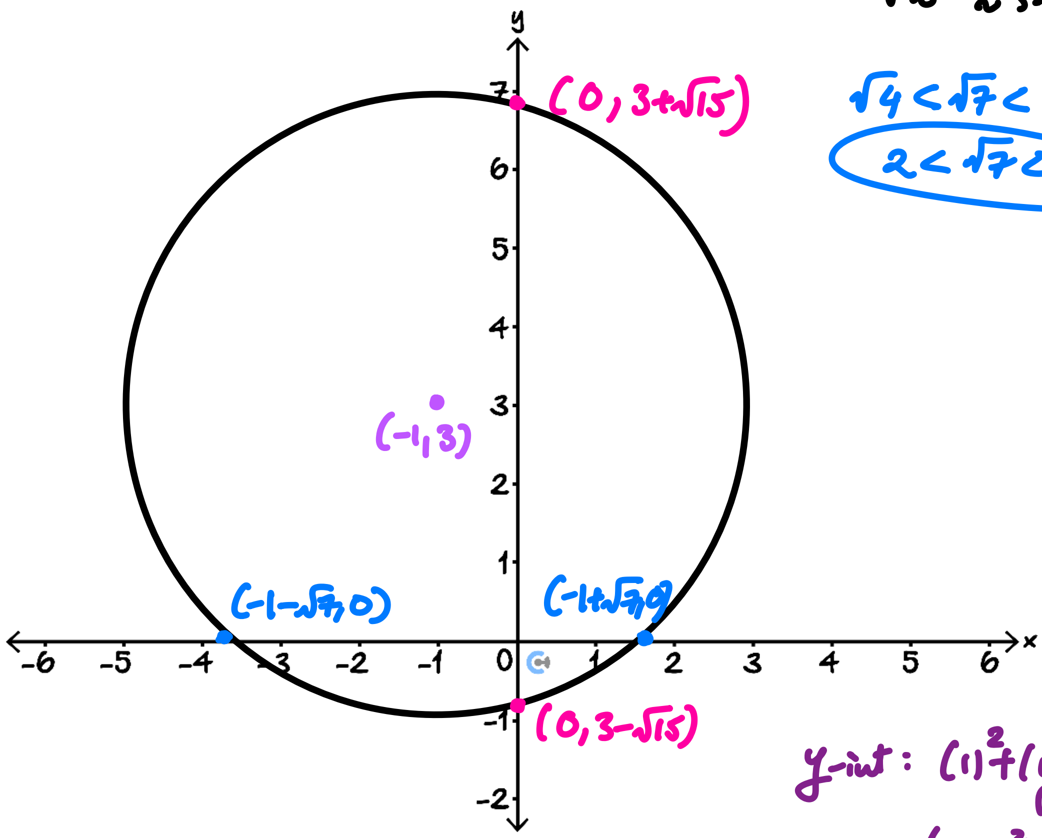
$$x^2 + 2x + y^2 - 6y - 6 = 0$$

$\sqrt{16} = 4$

$\sqrt{15} \approx 3.9$

$\sqrt{4} < \sqrt{7} < \sqrt{9}$

$2 < \sqrt{7} < 3$



$(x+1)^2 - 1 + (y-3)^2 - 9 - 6 = 0$

$(x+1)^2 + (y-3)^2 = 16$

Centre: $(-1, 3)$

radius $= 4$

y-int: $(1)^2 + (y-3)^2 = 16$

$(y-3)^2 = 15$ $\quad \sqrt{15} \approx 3.7$

$y = 3 \pm \sqrt{15}$ $\quad \approx -0.7$, ≈ 6.7

x-int: $(x+1)^2 + (-3)^2 = 16$

$(x+1)^2 = 7$

$x+1 = \pm\sqrt{7}$

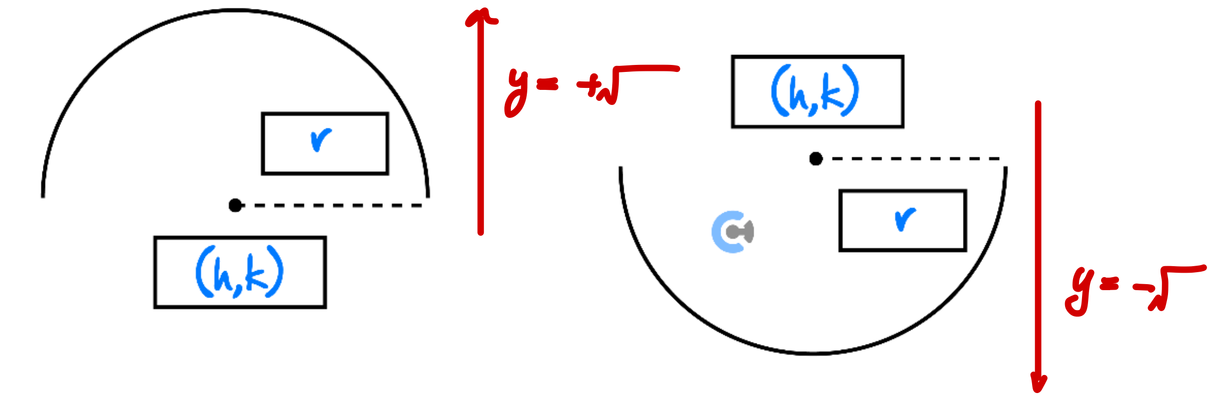
$x = -1 + \sqrt{7}$ or $-1 - \sqrt{7}$ $\quad \approx -3.6$, $\approx +1.6$

$\sqrt{7} \approx 2.6$

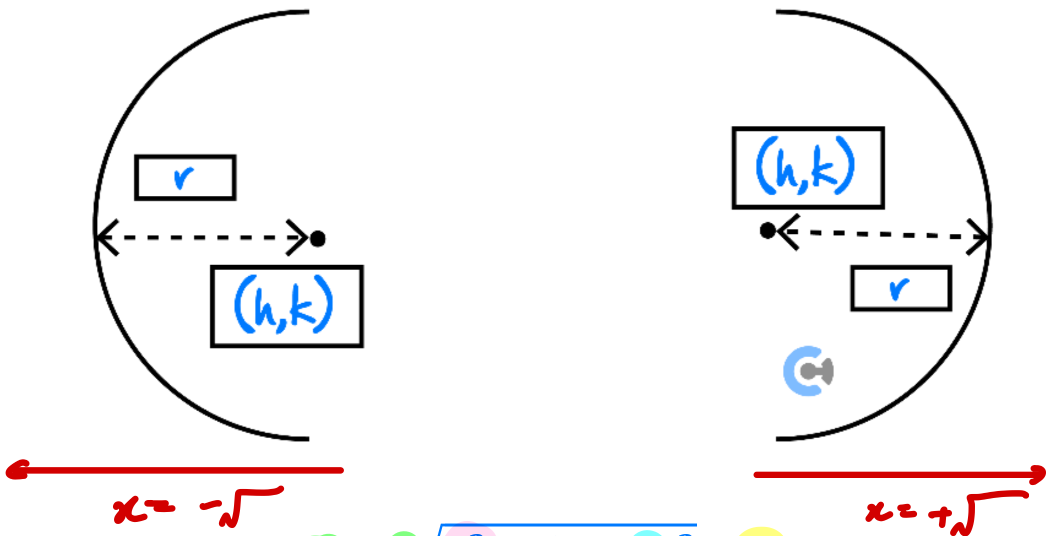
Now, semicircles!

Definition

Semicircles



$$y = \pm\sqrt{r^2 - (x - h)^2} + k$$



$$x = \pm\sqrt{r^2 - (y - k)^2} + h$$

➤ Steps

1. Find the centre of the semicircle.
2. Find the radius of the circle.
3. Find axes intercepts if they exist.
4. Identify the shape of the graph and sketch the curve.

Question 22 Walkthrough.

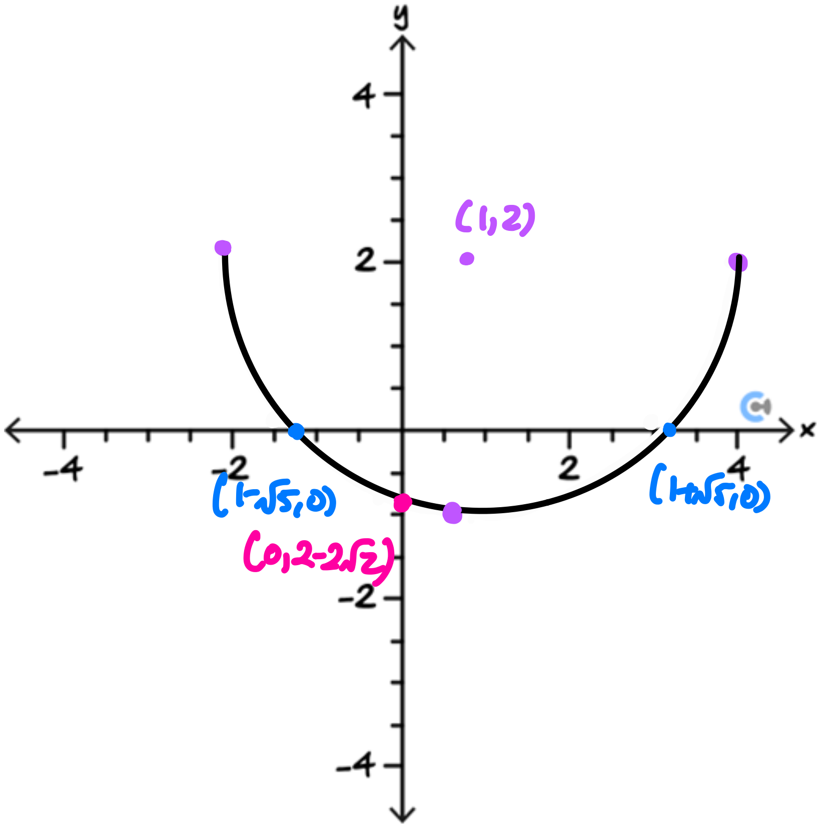
Graph the following semicircle:

$r^2 = 9$

$r = 3$

Centre : $(1, 2)$

$$y = -\sqrt{9 - (x-1)^2} + 2$$



x-int:

$0 = -\sqrt{9-(x-1)^2} + 2$

$\sqrt{9-(x-1)^2} = 2$

$9-(x-1)^2 = 4$

$(x-1)^2 = 5$

$x-1 = \pm\sqrt{5}$

$\sqrt{5} \approx 2.2$ $\quad \therefore x = 1+\sqrt{5}$ or $1-\sqrt{5}$

≈ 3.2 or ≈ -1.2

y-int:

$y = -\sqrt{9-(0-1)^2} + 2$

$= -\sqrt{8} + 2$

$= 2 - 2\sqrt{2}$ $\quad \sqrt{2} \approx 1.4$

$\approx 2 - 2.8$

≈ -0.8

Where do the semicircle equations come from?

Exploration: Derivation of Semicircle Equations

- Consider the circle equation:

$$(x-h)^2 + (y-k)^2 = r^2$$

- Try making y the subject!

$$(y-k)^2 = r^2 - (x-h)^2$$

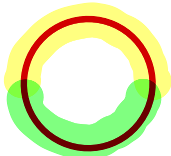
$$y-k = \pm\sqrt{r^2-(x-h)^2}$$

$$\therefore y = k \pm \sqrt{r^2-(x-h)^2}$$

- What would happen when we pick one sign over the other?

+ve sign

-ve sign



- So, by making y the subject, we get top and bottom semicircles!
- Similarly, what would happen if we make x the subject?

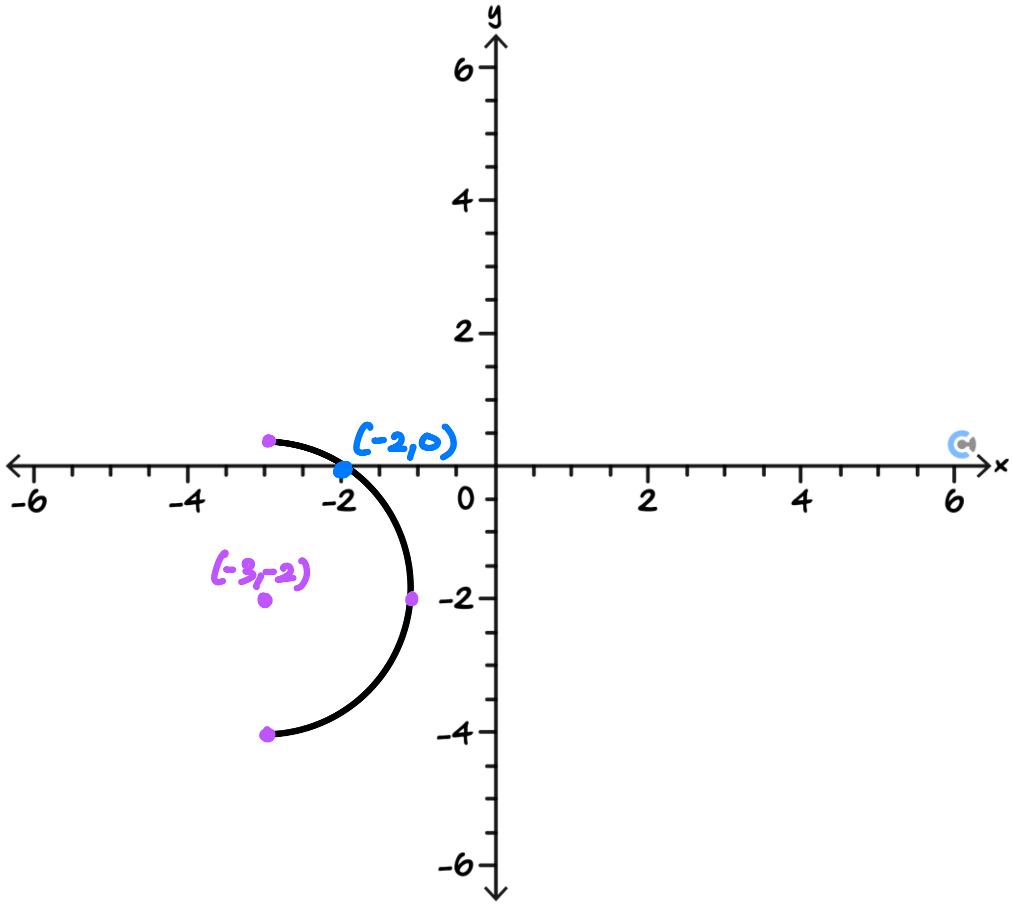
$$x = h \pm \sqrt{r^2-(y-k)^2}$$

Active Recall: Steps for sketching a semicircle

1. Find the centre of the semicircle.
2. Find the radius of the circle.
3. Find axes intercepts if they exist.
4. Identify the shape of the graph and sketch the curve.

Question 23

Graph the following: $x = \sqrt{-y^2 - 4y + 1} - 3$



$x = \sqrt{-(y^2+4y)+1} - 3$

$x = \sqrt{-((y+2)^2-4)+1} - 3$

$\therefore x = \sqrt{-(y+2)^2+5} - 3$

Centre: $(-3,-2)$

$r^2 = 5$

$r = \sqrt{5}$

x-int:

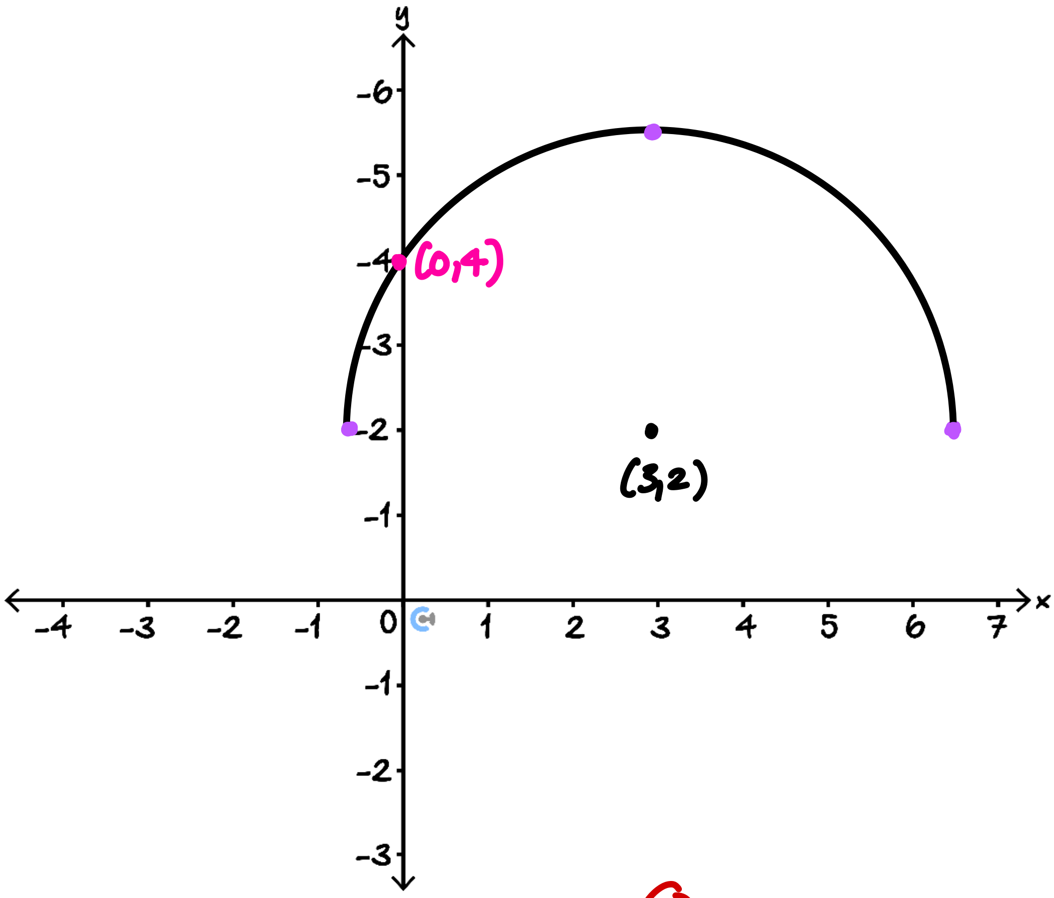
$x = \sqrt{1} - 3$

$\therefore x = -2$

NOTE: You need to complete the square for the function that is inside the root!

Question 24 Extension.

Graph the following: $y = \sqrt{-x^2 + 6x + 4} + 2$



① $y = \sqrt{-x^2+6x+4} + 2$

$= \sqrt{-(x^2-6x)+4} + 2$

$= \sqrt{-(x-3)^2+9+4} + 2$

$y = \sqrt{-(x-3)^2+13} + 2$

$h = 3$

$k = 2$

$r^2 = 13$

$r = \sqrt{13}$

② Centre : $(3,2)$

③ y-int:

$y = \sqrt{-(-3)^2+13} + 2$

$= \sqrt{4} + 2$

$y = 4$

Sub-Section: Finding a Rule for Circles and Semicircles

Again, another way!

Finding the Equation of a Root Function from its Graph

- We need generally three facts about the circles/semicircles.

$$(x-h)^2+(y-k)^2=r^2$$

$$y=\pm\sqrt{r^2-(x-h)^2}+k$$

$$x=\pm\sqrt{r^2-(y-k)^2}+h$$

- Steps

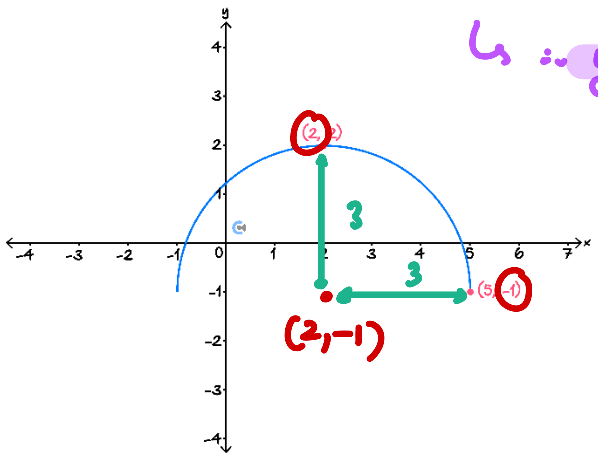
1. Identify the center, (h,k).
2. Identify the radius, r.

Question 25 Walkthrough.

Find the rule for the following semicircle.

$y=+\sqrt{3^2-(x-2)^2}-1$

$\therefore y=\sqrt{9-(x-2)^2}-1$

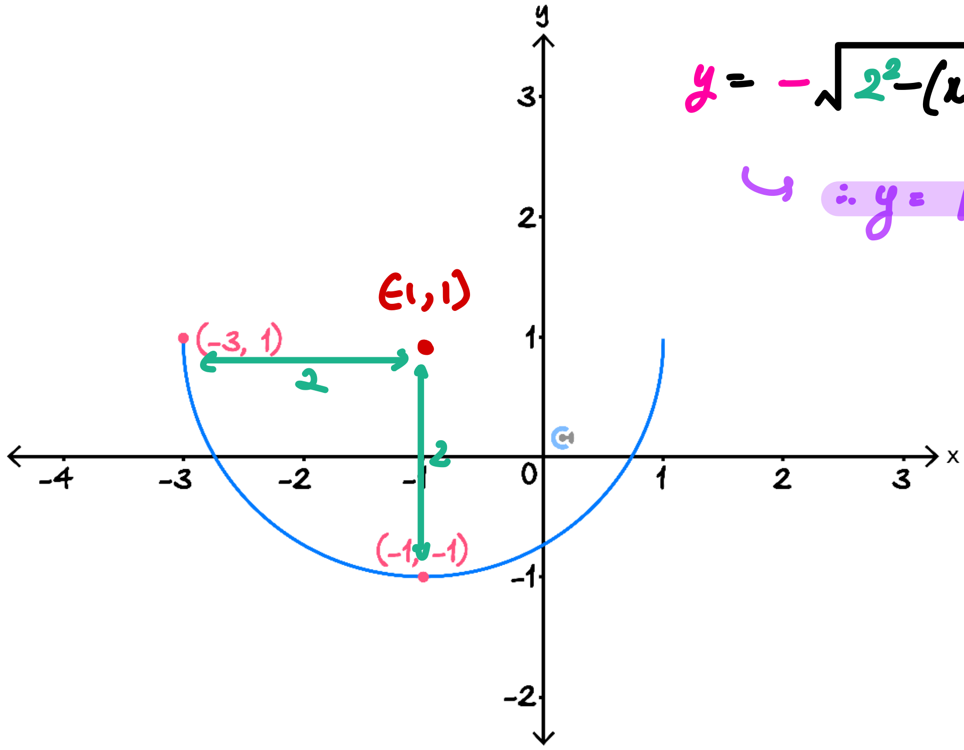


<u>Active Recall:</u> **Steps for finding the rule of circles and semicircles**

1. Identify the centre, (h, k).
2. Identify the radius, r.

Question 26

Find the rule for the following semicircle.

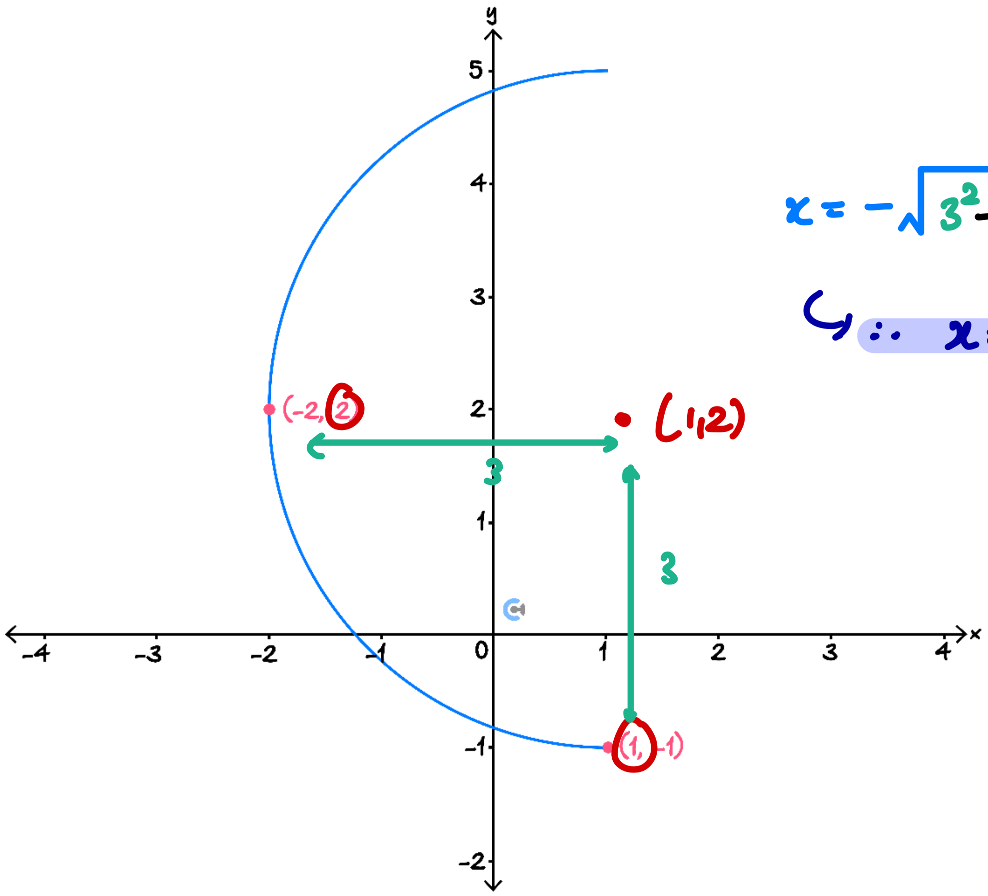


$y = -\sqrt{2^2-(x-(-1))}+1$

$\therefore y = 1-\sqrt{4-(x+1)^2}$

Question 27 Extension.

Find the rule for the following semicircle.



$x = -\sqrt{3^2 - (y-2)^2} + 1$

$\therefore \ x = 1 - \sqrt{9 - (y-2)^2}$

Section E: Functions and Relations

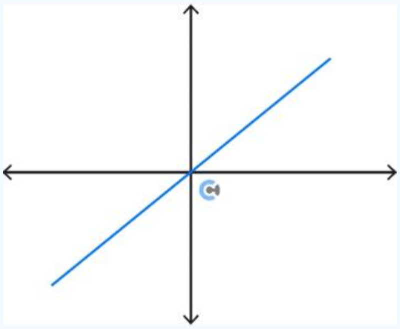
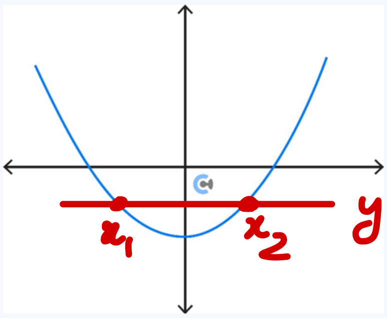
Sub-Section: Relations

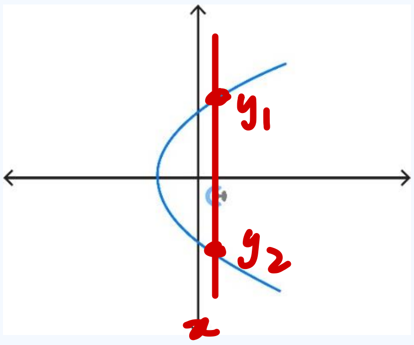
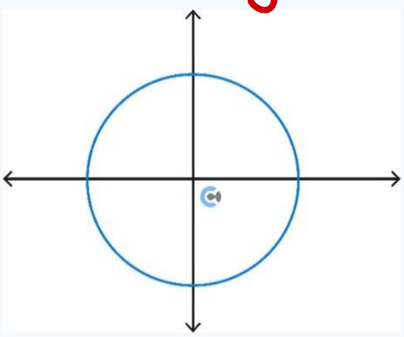
Let's take a look at all types of relations!

Definition

Types of Relations → relationship b/w x & y

- There are four types of relations:

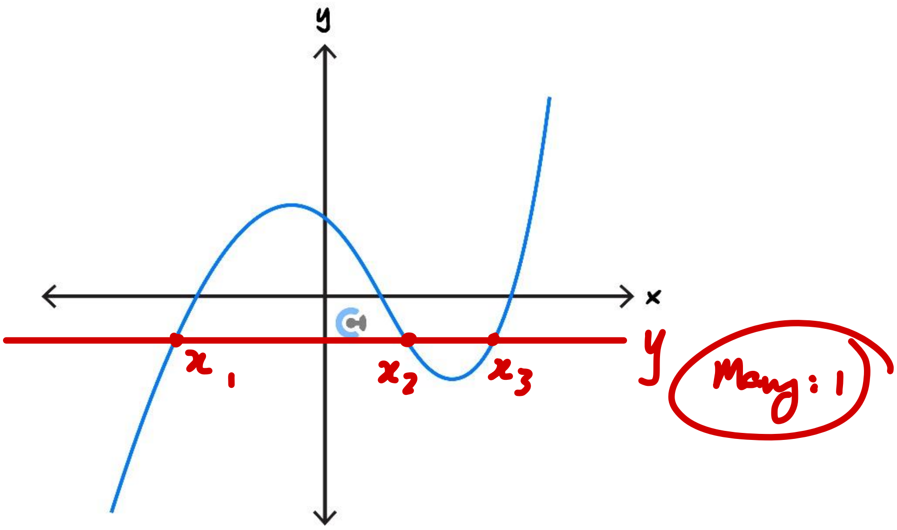
One to One (x y)	Many to One (x y)
	
One x to One y.	Many x's to One y.

One to Many (x y)	Many to Many (x y)
	
One x to Many y's.	Many x's to Many y's.

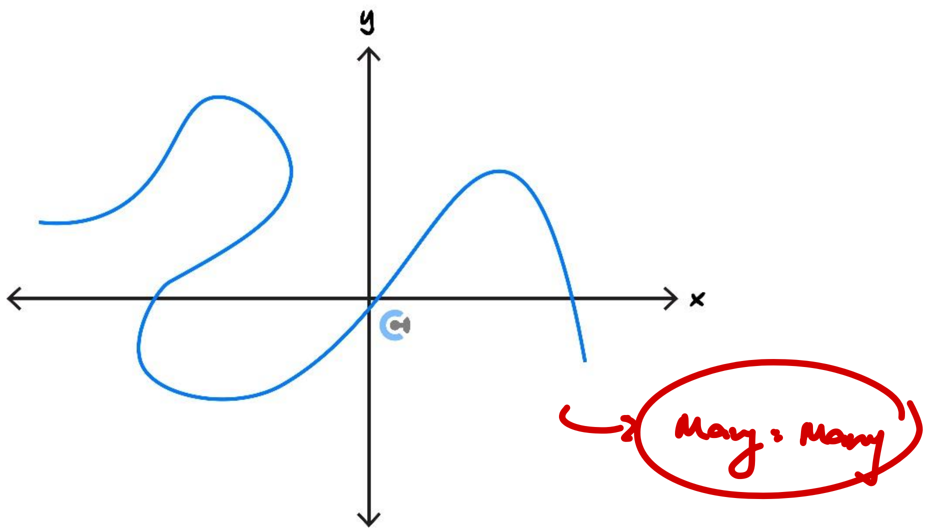
Question 28

State the type of relation for each of the following graphs.

a.



b.



Sub-Section: Functions

What is a function?

Functions

$$y = f(x)$$

- Functions are relations which make one y-value at any given x-value.

Discussion: What types of relations are functions?

$x \to \boxed{f} \to$ 1 output

1 y for 1 x

Many$:1$ & $1:1$

Misconception

Misconception: "An equation between x and y can either be a function or a relation. In other words, functions are not relations."

Truth: Functions are in fact a subset of relations. All functions are relations.

BUT all relations are NOT necessarily functions.

{All Functions} ⊆ {All Relations}

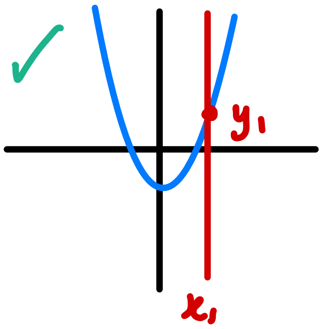
Space for Personal Notes

Question 29

State whether the following relations are also a function.

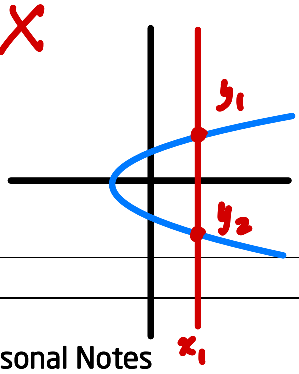
a. $y = x^2$

Function ✓



b. $y^2 = x$

Function ✗



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Question 30

For the following tables of inputs and outputs, identify which are (i) valid relations, and (ii) valid functions.

a.

x	y
-1	6
2	6
6	-1
-1	2

Relation ✓

Function ✗

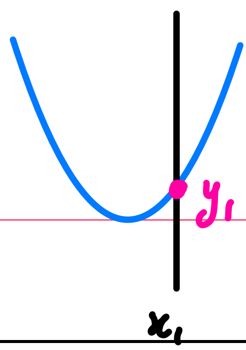
b.

x	y
-1	2
-1	1
1	1
1	-2

Relation ✓

Function ✗

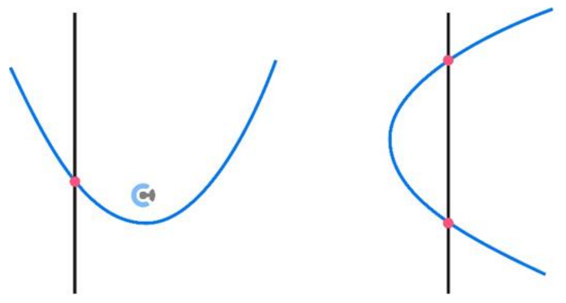
<u>Discussion:</u> What is the maximum number of times a function can hit any vertical line?



Max = 1

Vertical Line Test

- **Definition**: Tells apart between functions and non-function relations.

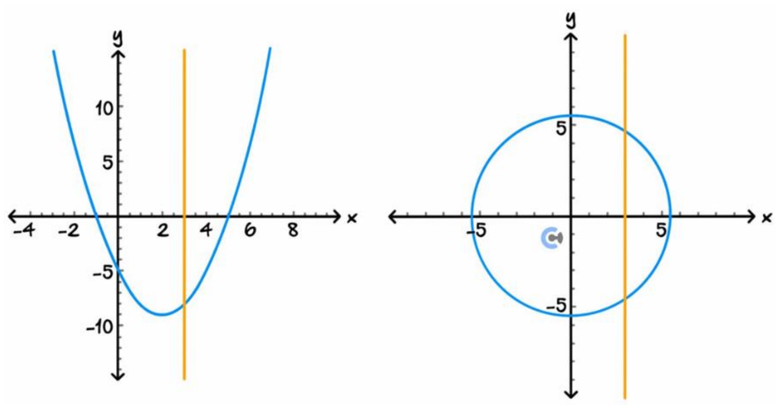


Passes : Function   Fails : Not function

Every function only intersects a vertical line once.

Question 31

Which of the following graph(s) describes a function? Which of the following graph(s) show a relation?

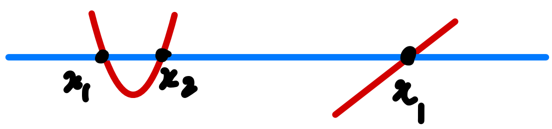


Relation ✓   Relation ✓

Function ✓   Function ✗

2x   1x

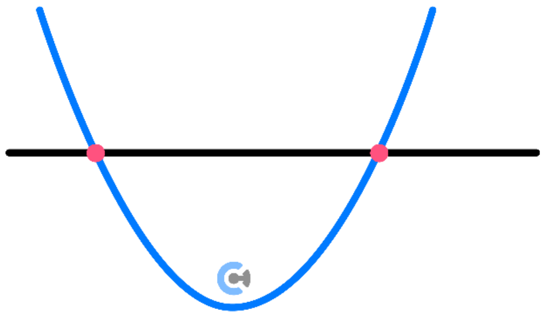
Discussion: How many times would a many to one and one to one function hit a horizontal line?



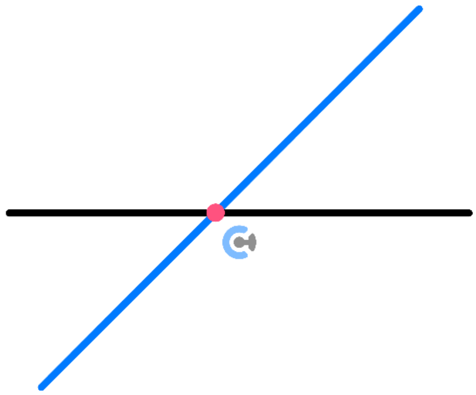
Definition

Horizontal Line Test

- **Definition**: Tells apart between many to one and one to one functions. (And relations.)



Fails: Many to one



Passes: One to one

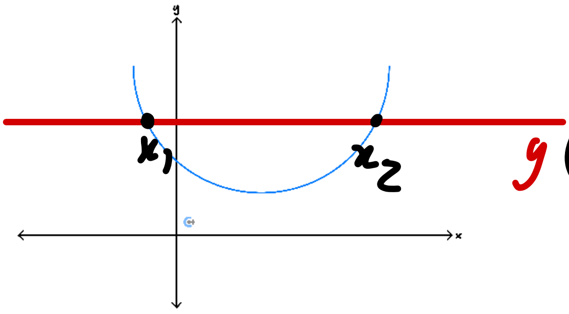
NOTE: One to one function hits **any** horizontal line drawn maximum once.

Space for Personal Notes

Question 32

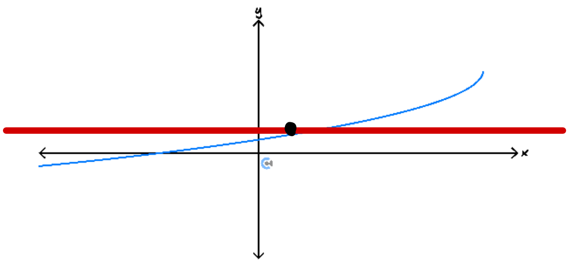
Which of the following graph(s) are one to one, and which are many to one?

a.



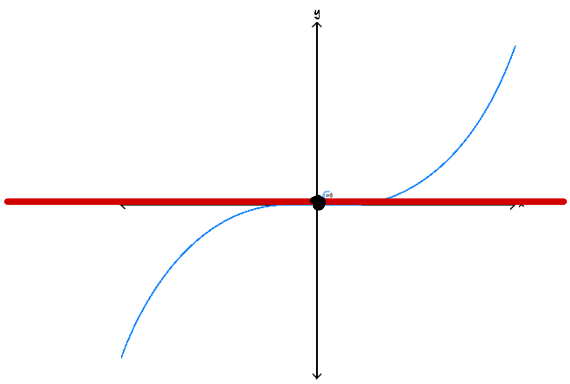
Many : 1

b.



1:1

c.



1:1

Contour Check

Learning Objective: [2.1.1] – Sketch and find the rule of hyperbola Functions

Key Takeaways

- Rectangular Hyperbola

$$y = \frac{a}{x-h} + k$$

- Steps for sketching:

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

- Finding the Equation of a Hyperbola from its Graph

  - We need generally three facts about the hyperbola.

$$y = \frac{a}{x-h} + k$$

- Steps

1. Look for the asymptotes.
2. Sub in a point to find the value of a.

Learning Objective: [2.1.2] – Sketch and find the rule of Truncus Functions

Key Takeaways

- Truncus

$$y = \frac{a}{(x-h)^2} + k$$

- Steps for sketching:

1. Find the horizontal and vertical asymptotes and plot them on the axis.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

- Finding the Equation of a Truncus from its Graph

  - We need generally three facts about the Truncus.

$$y = \frac{a}{(x-h)^2} + k$$

- Steps

1. Look for the asymptote.
2. Sub in a point to find the value of a.

Learning Objective: [2.1.3] - Sketch and find the rule of Root Functions

Key Takeaways

- Square Root Functions

$$y = a\sqrt{b(x-h)} + k$$

- Steps for sketching

1. Find the starting point.
2. Find the x- and y- intercepts and plot on the axes (if they exist).
3. Identify the shape of the graph by considering any reflections and sketch the curve.

- Finding the Equation of a Root Function from its Graph
  - We need generally three facts about the root function.

$$y = a\sqrt{\pm(x-h)} + k$$

- Steps

1. Look for the starting point (h, k).
2. Sub in a point to solve the value of a.

Learning Objective: [2.1.4] - Sketch and find the rule of Semicircles and Circles

Key Takeaways

- **Circles**

$$(x-h)^2+(y-k)^2=r^2$$
$$\text{where } r>0$$

- Centre: (h,k)
- Radius: r
- **Steps**

1. Find the centre of the circle.
2. Find the radius of the circle.
3. Find axes intercepts (if they exist).
4. Identify the shape of the graph and sketch the curve.

- **Semicircles**

$$y=\pm\sqrt{r^2-(x-h)^2}+k$$

$$x=\pm\sqrt{r^2-(y-k)^2}+h$$

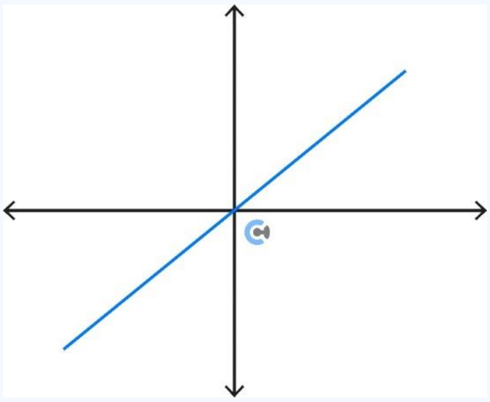
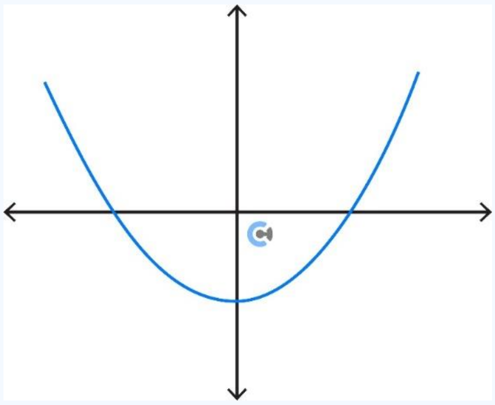
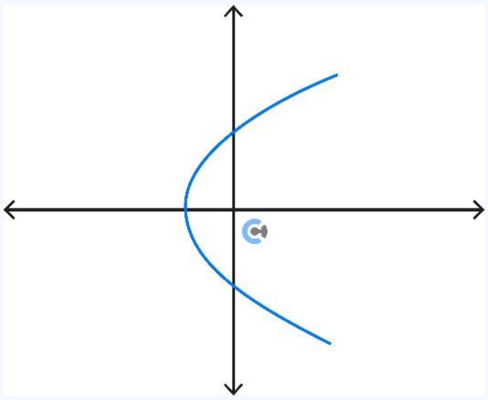
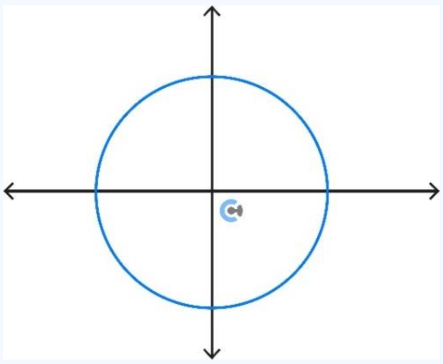
- Steps for finding the rule of circles and semicircles

1. Identify the centre, (h,k).
2. Identify the radius, r.

Learning Objective: [2.1.5] - Identify the type of relations and identify whether the relation is a function

Key Takeaways

- **Types of Relations**
  - There are four types of relations:

One to One	Many to One
	
One to Many	**Many to Many**
	

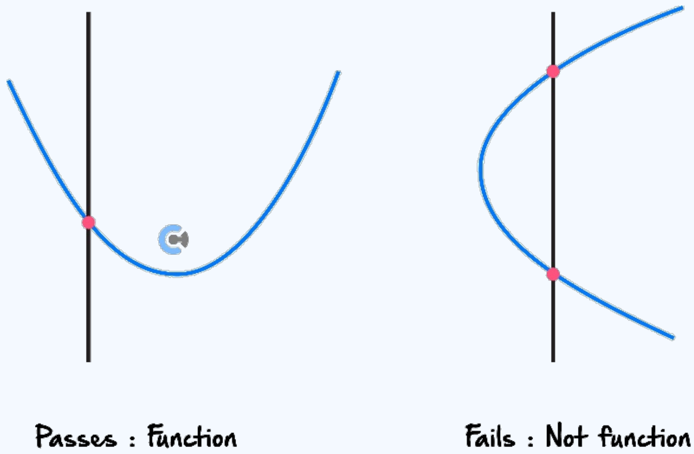
Functions

$$y = f(x)$$

- Functions are relations which make one y-value at any given x-value.

Vertical Line Test

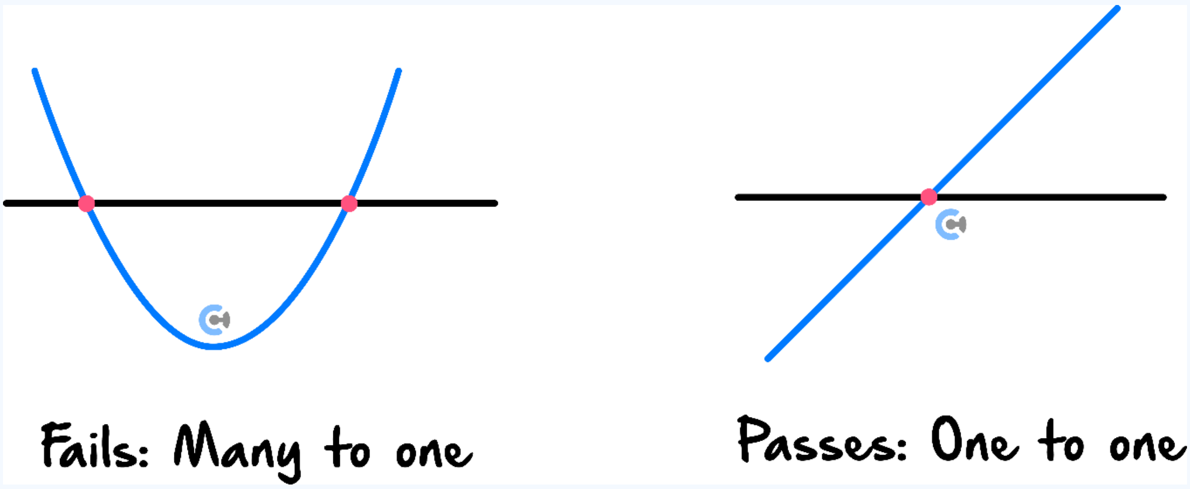
- **Definition**: Tells apart between functions and non-function relations.



Every function only intersects a vertical line once.

Horizontal Line Test

- **Definition**: Tells apart between many to one and one to one functions. (And relations.)



- One to one function hits **any** horizontal line drawn at most Once.

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- **Price?** Completely free!
- **One Active Booking Per Subject:** Must attend your current consultation before scheduling the next. :)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

Booking Link

bit.ly/contour-methods-consult-2025