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VCE Mathematical Methods ½
Functions & Relations I [2.1]
Homework Solutions

Homework Outline:

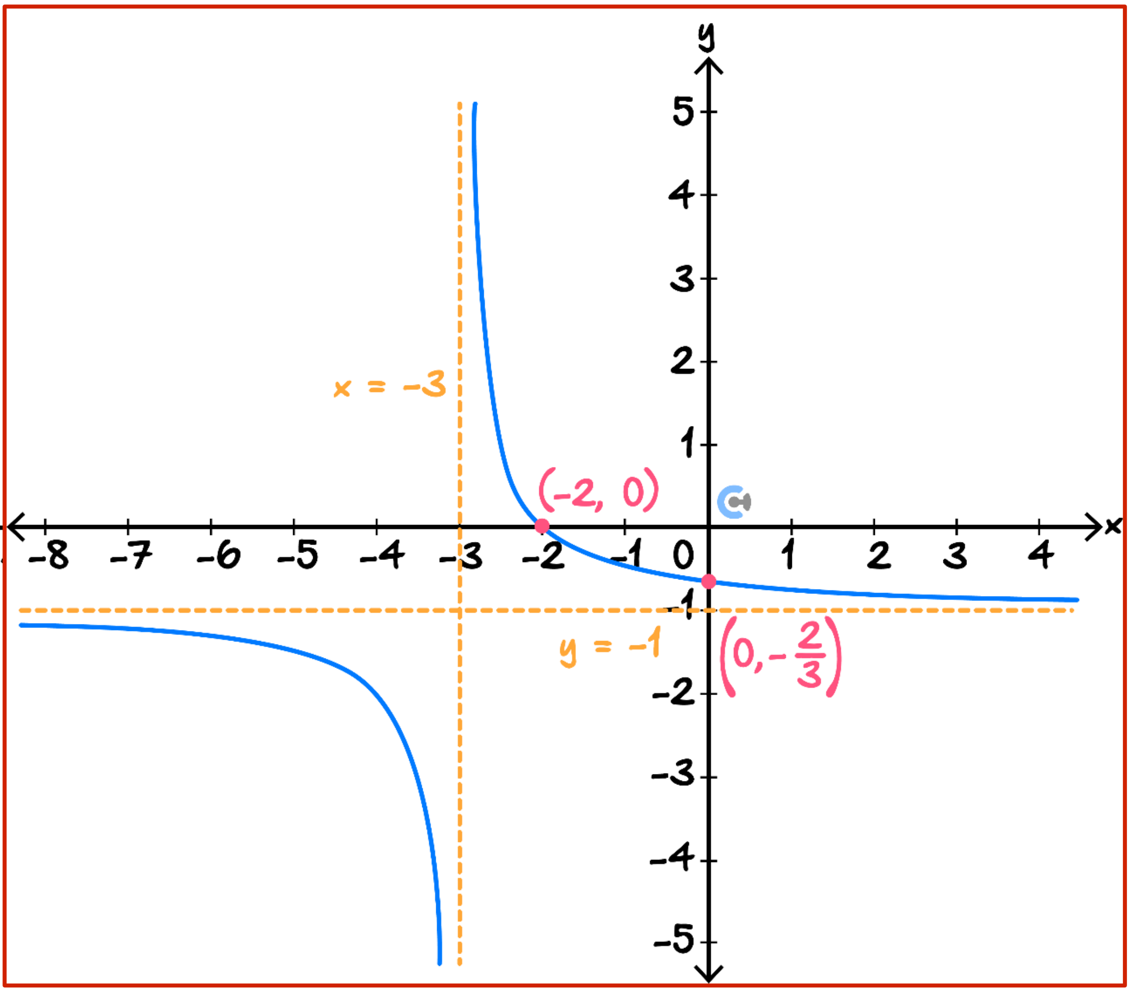
Section A: Compulsory Questions

Sub-Section [2.1.1]: Sketch and Find the Rule of Hyperbolas Functions

Question 1

Graph the following curve labelling all intercepts and asymptotes with their equations.

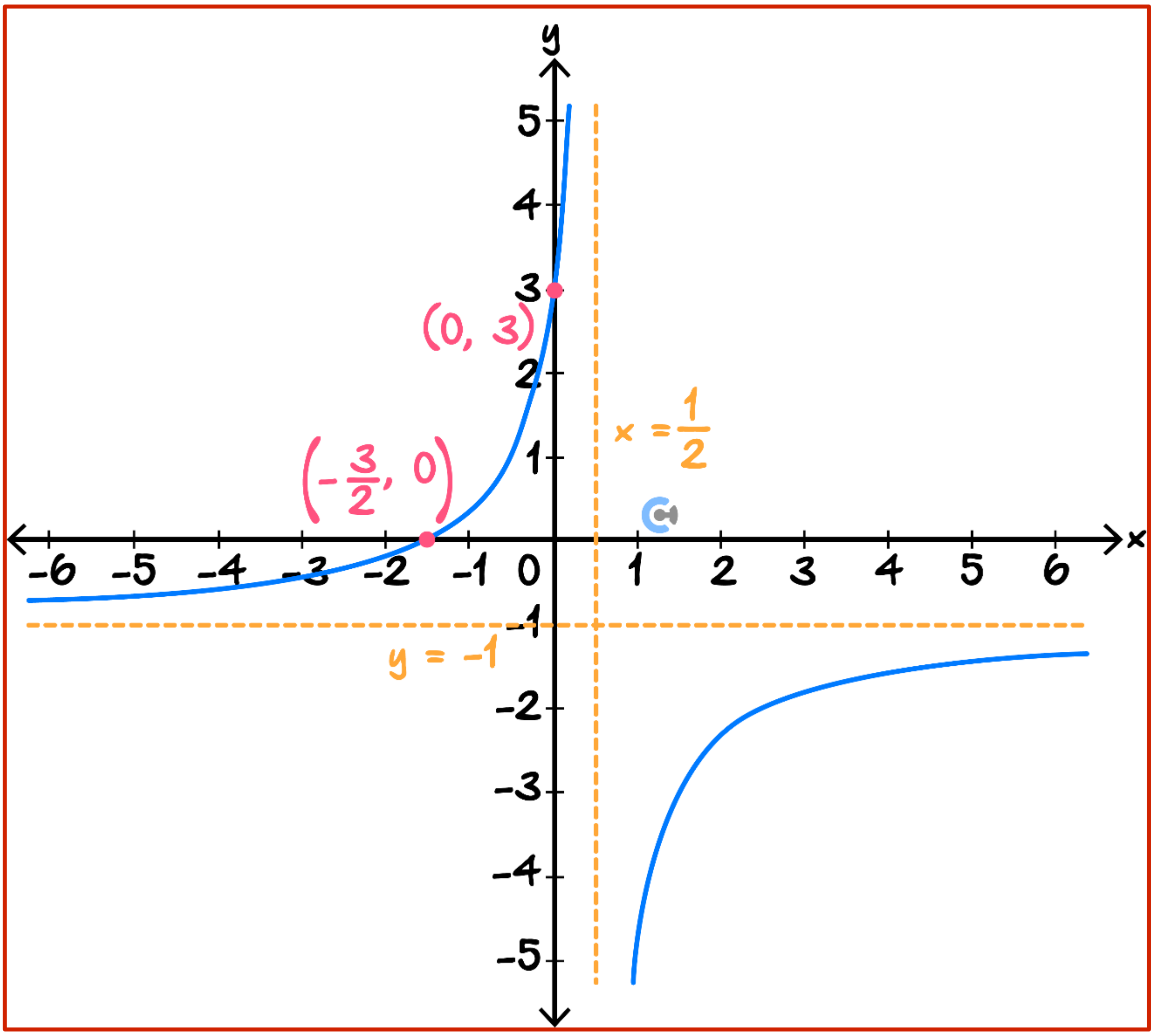
$$y = \frac{1}{x+3} - 1$$



Question 2

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = -\frac{4}{2x-1} - 1$$



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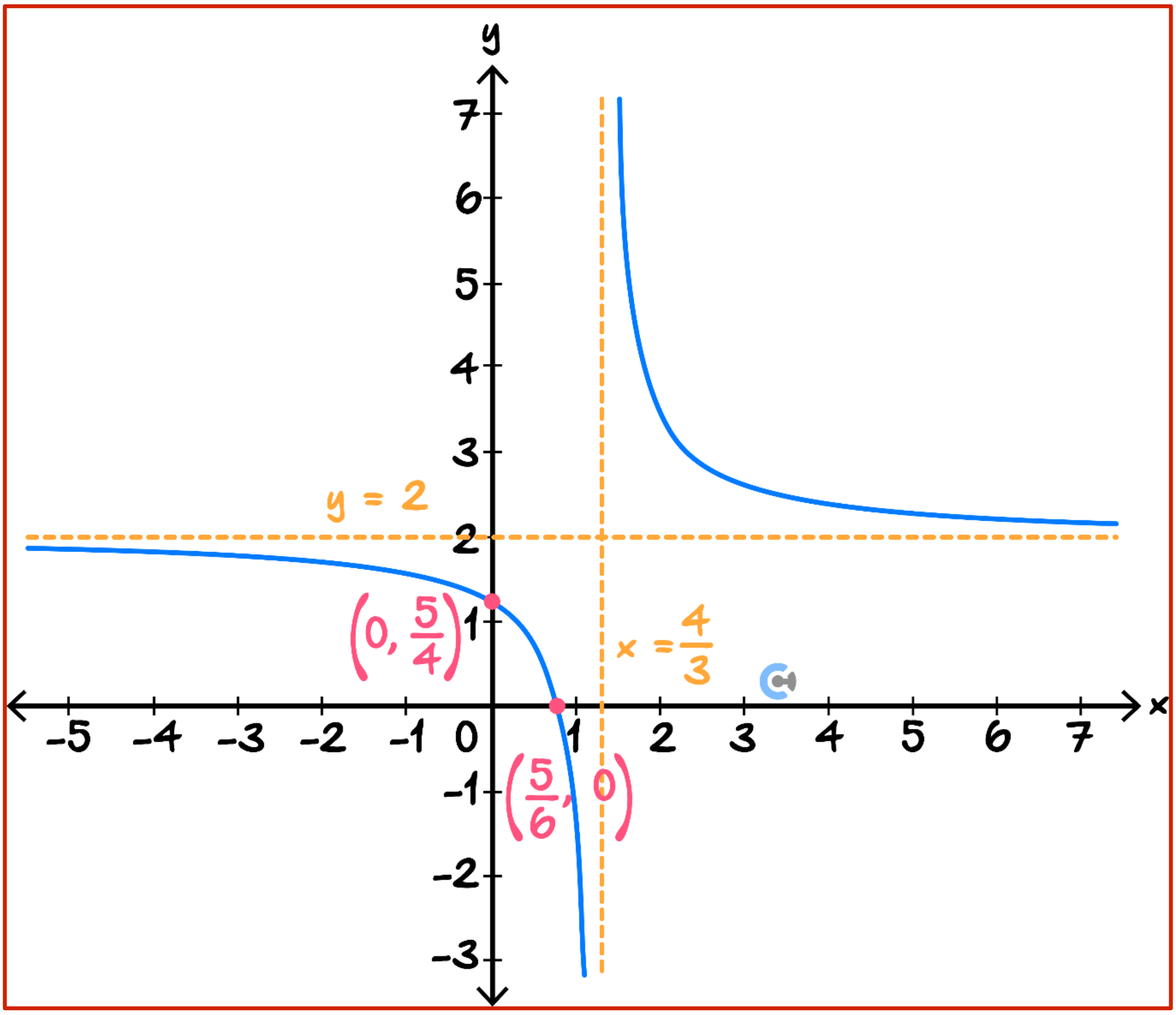
Question 3

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = 2 - \frac{3}{4 - 3x}$$



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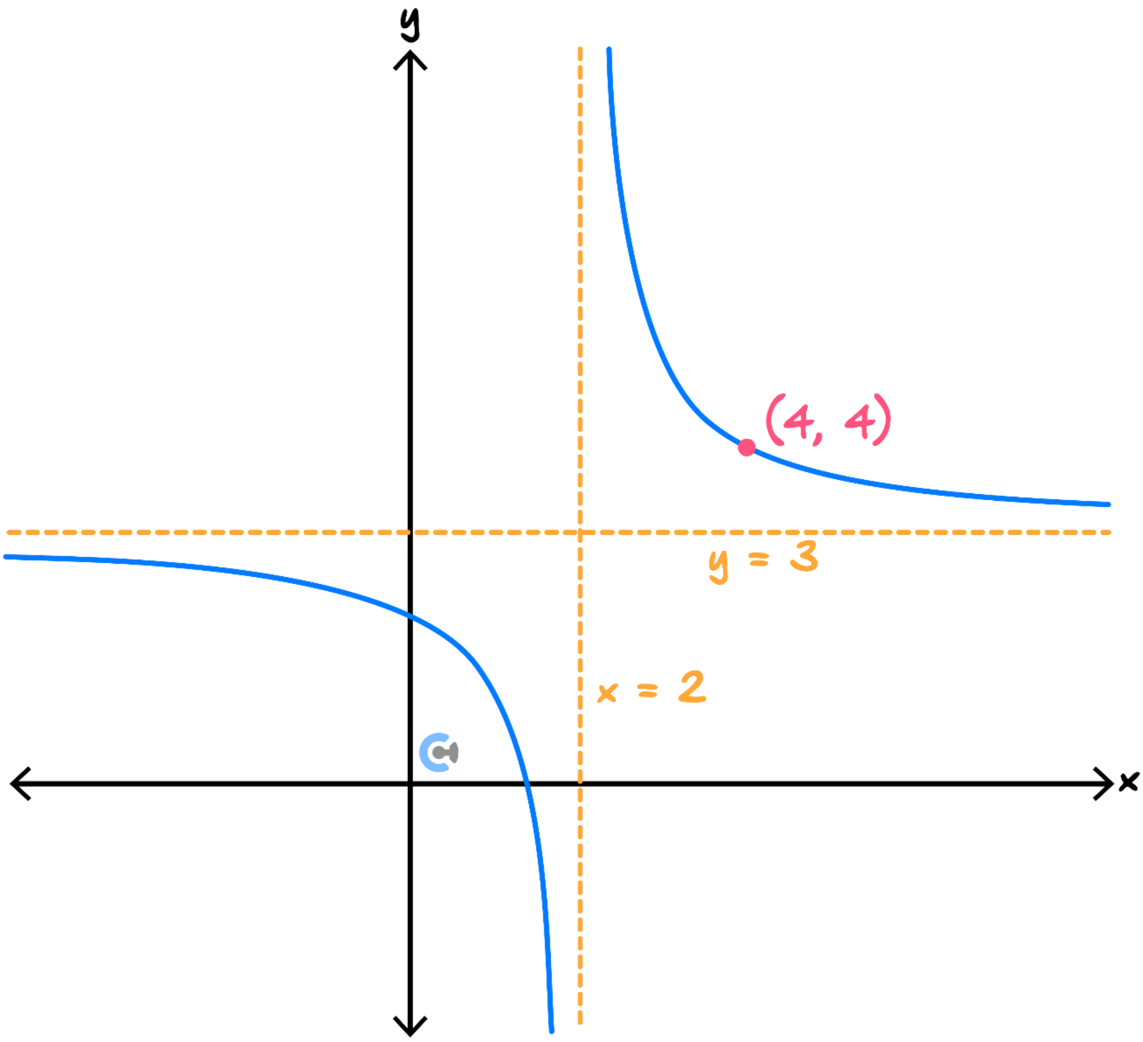
Question 4

Find the rule for the following graph, given it is of the form $y = \frac{a}{x-h} + k$.



Clear that $h = 2$ and $k = 3$.
Then, $4 = \frac{a}{4-2} + 3 \implies a = 2$.

$$y = \frac{2}{x-2} + 3$$

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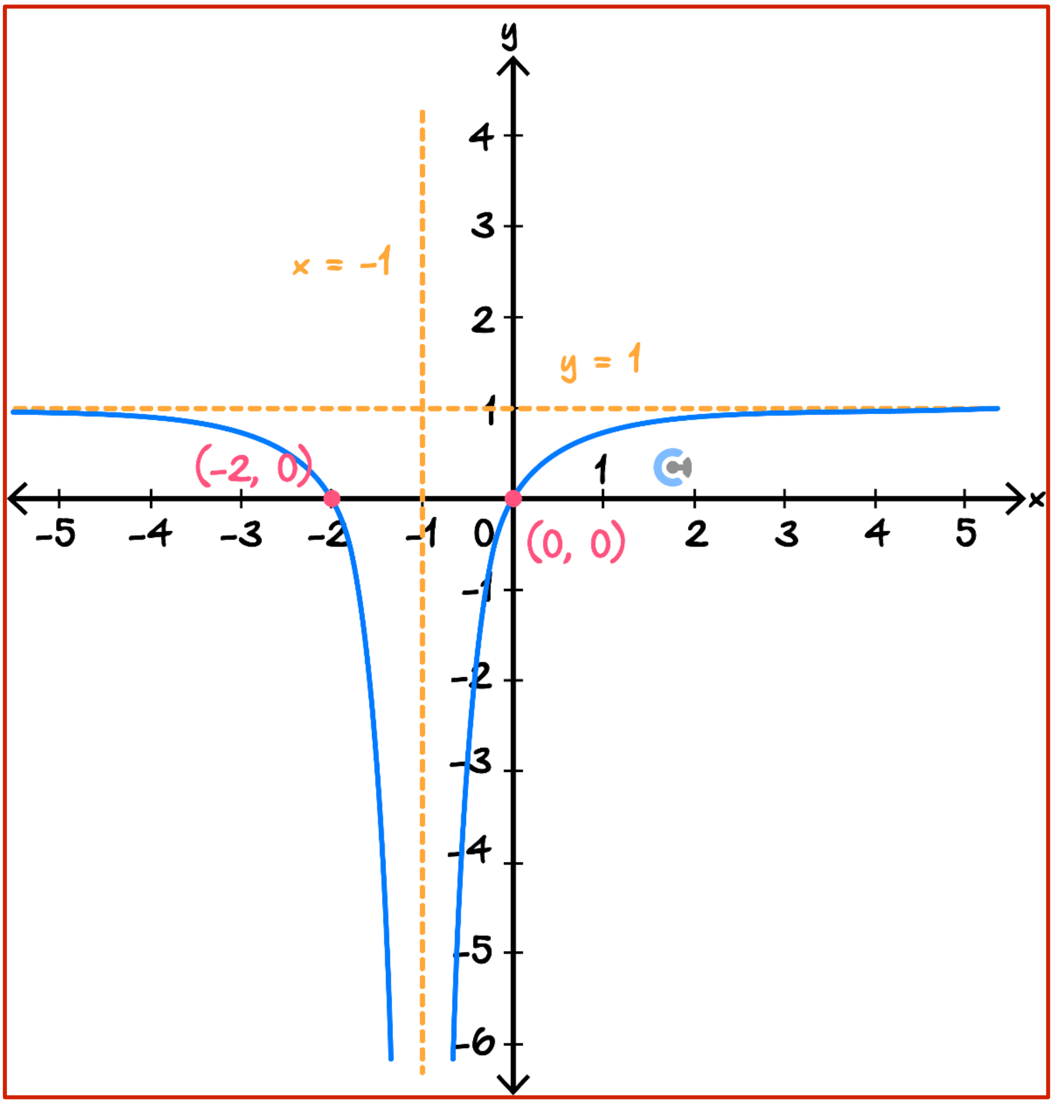
Sub-Section [2.1.2]: Sketch and Find the Rule of Truncus Functions

Question 5

Graph the following curve labelling all intercepts and asymptotes with their equations.

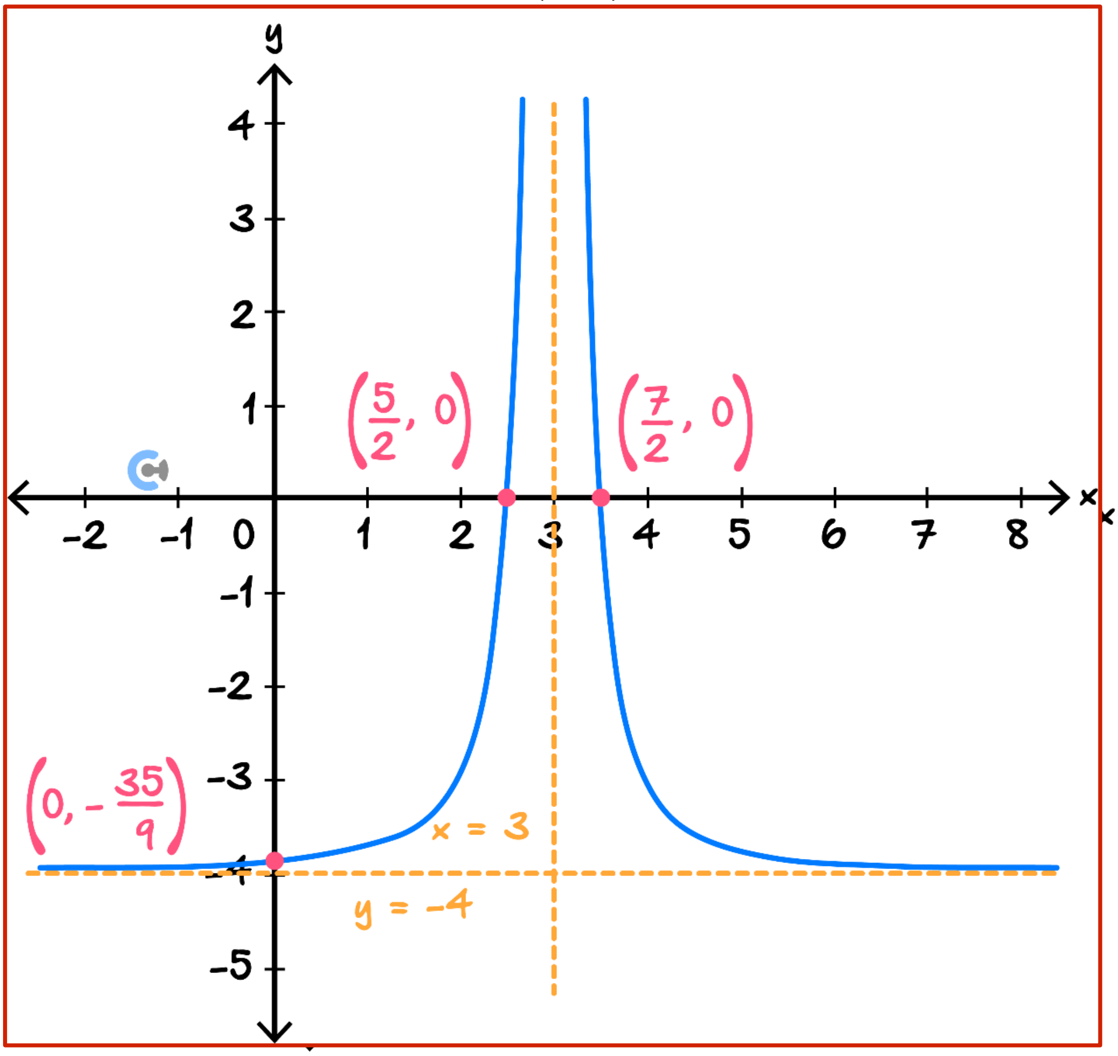
$$y = -\frac{1}{(x+1)^2} + 1$$



Question 6

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{1}{(x-3)^2} - 4$$



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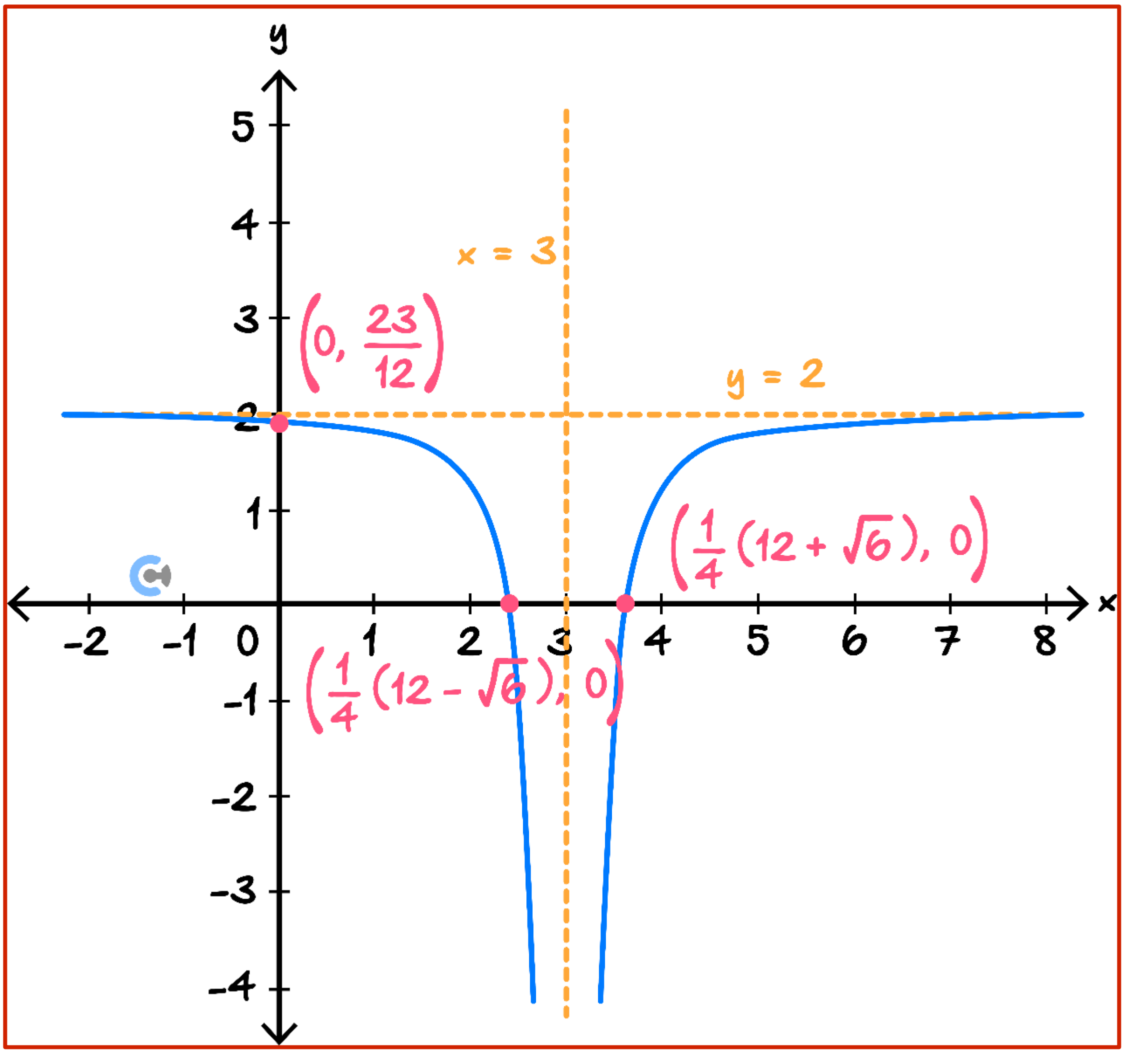
Question 7

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = 2 - \frac{3}{(6-2x)^2}$$



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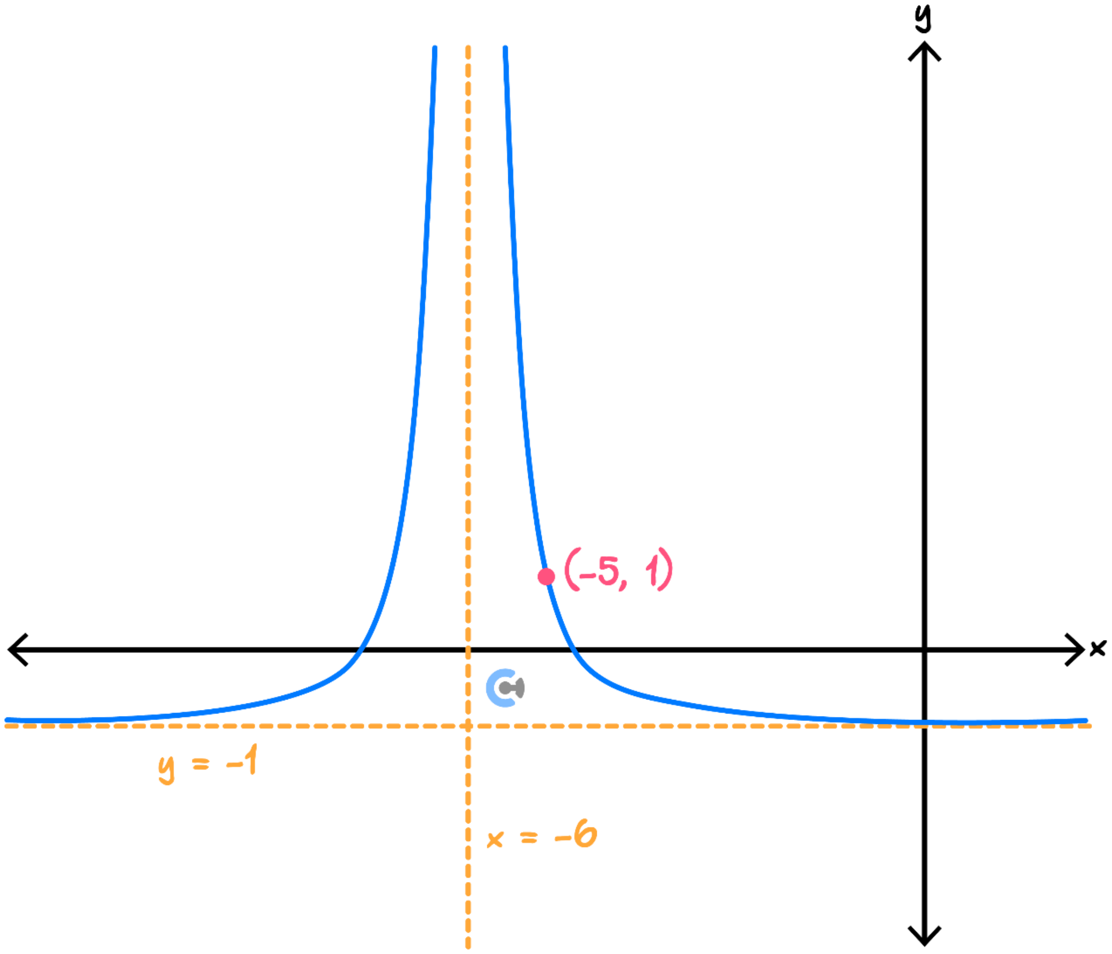
Question 8

Find the rule for the following graph, given it is of the form $y = \frac{a}{(x-h)^2} + k$.



Clear that $h = -6$ and $k = -1$.
Then, $1 = \frac{a}{(-5+6)^2} - 1 \implies a = 2$.

$$y = \frac{2}{(x+6)^2} - 1$$

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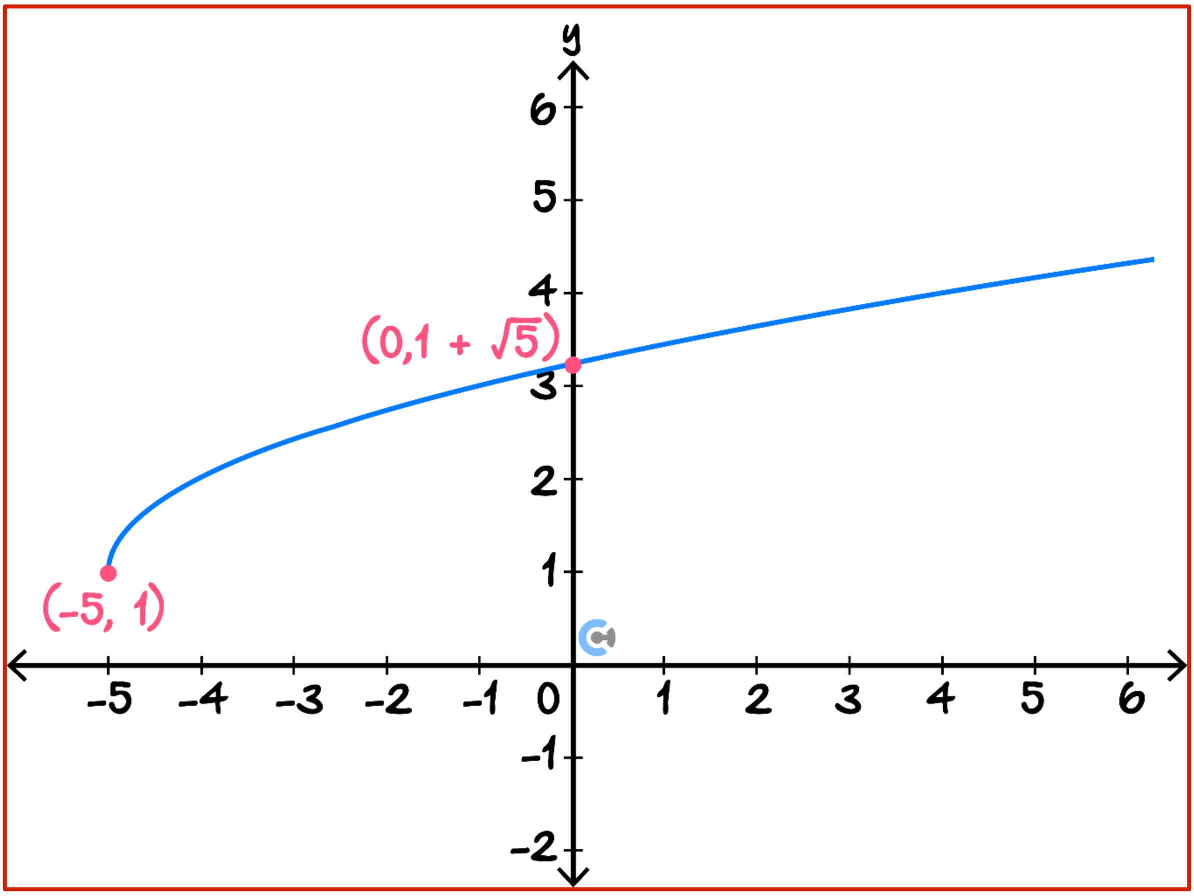
Sub-Section [2.1.3]: Sketch and Find the Rule of Root Functions

Question 9

Graph the following curve labelling all intercepts and start points.

$$y = \sqrt{x+5} + 1$$



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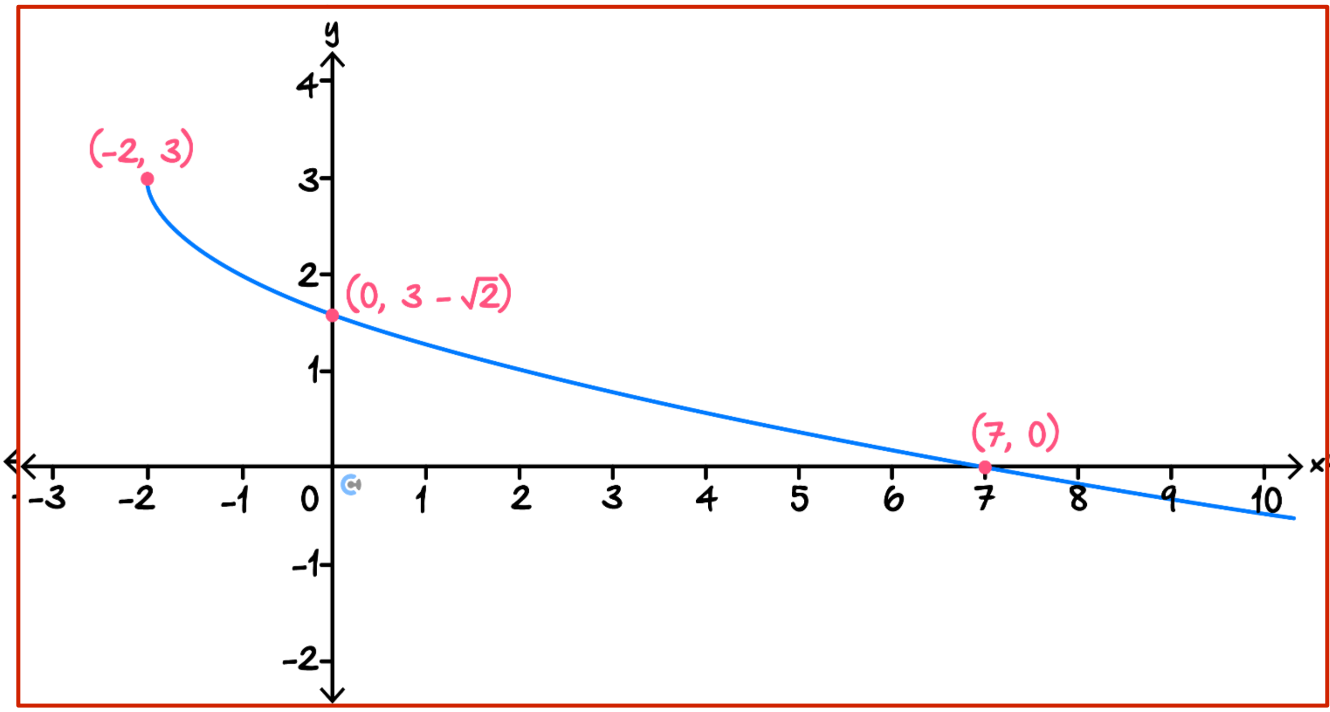
Question 10

Graph the following curve labelling all intercepts and start points.

$$y = -\sqrt{x+2} + 3$$



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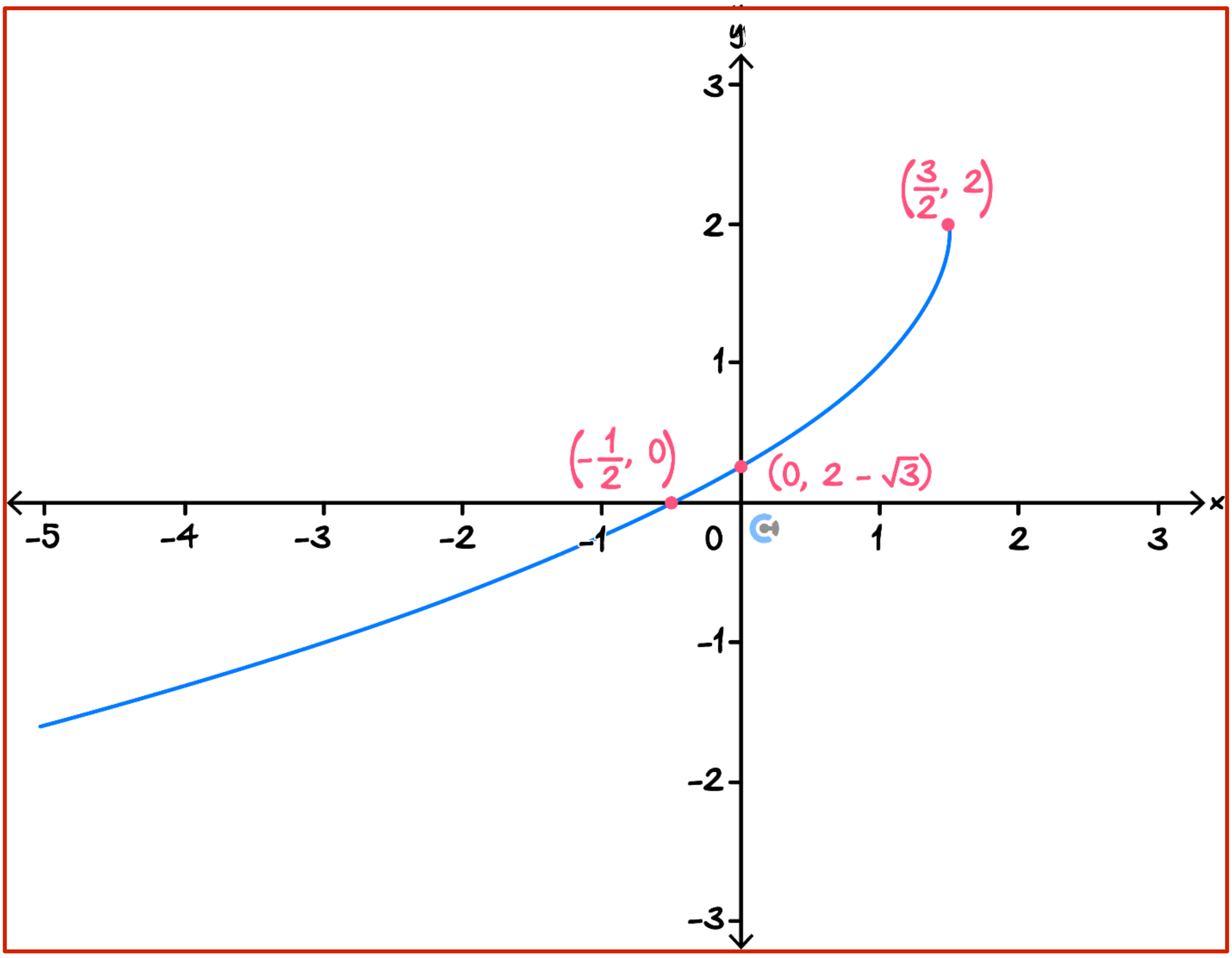
Question 11

Graph the following curve labelling all intercepts and start points.

$$y = 2 - \sqrt{3 - 2x}$$



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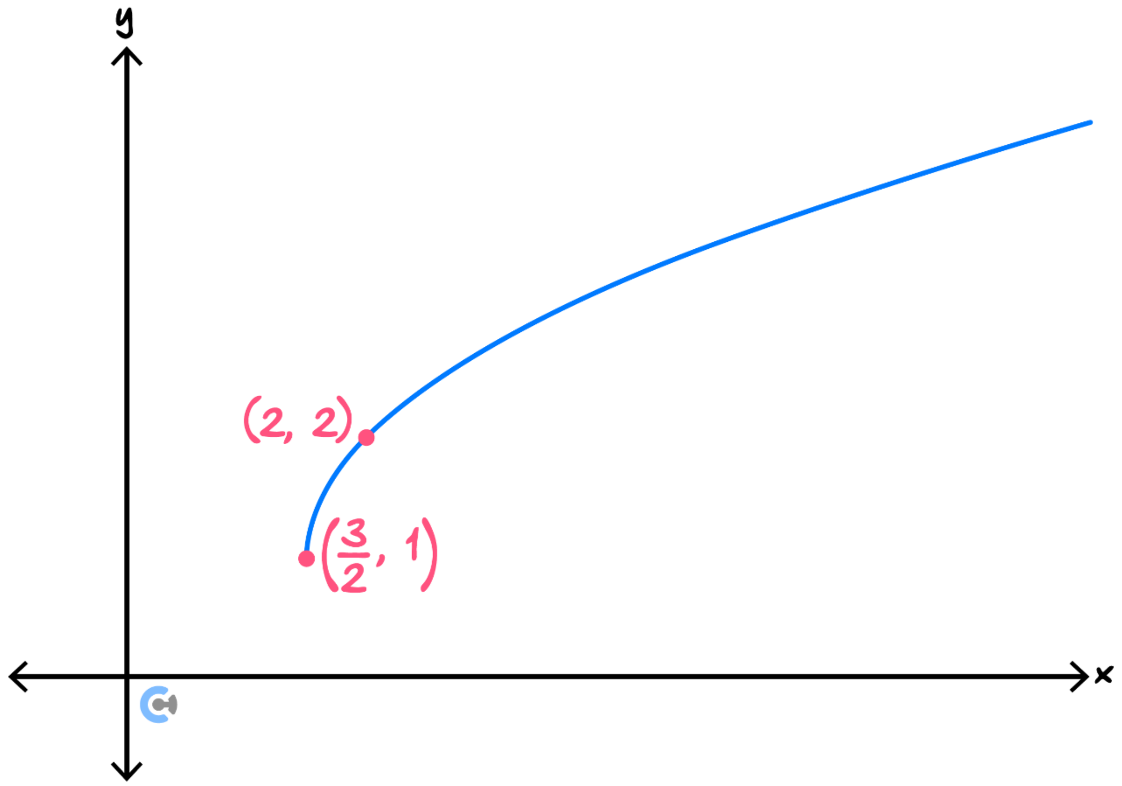
Question 12

Find the rule for the following graph, given it is of the form $y = \sqrt{a(x-h)} + k$.



From the start point $h = \frac{3}{2}$ and $k = 1$.

Then, $2 = \sqrt{a\left(2 - \frac{3}{2}\right)} + 1 \implies a = 2$.

$$y = \sqrt{2\left(x - \frac{3}{2}\right)} + 1 = \sqrt{2x - 3} + 1$$

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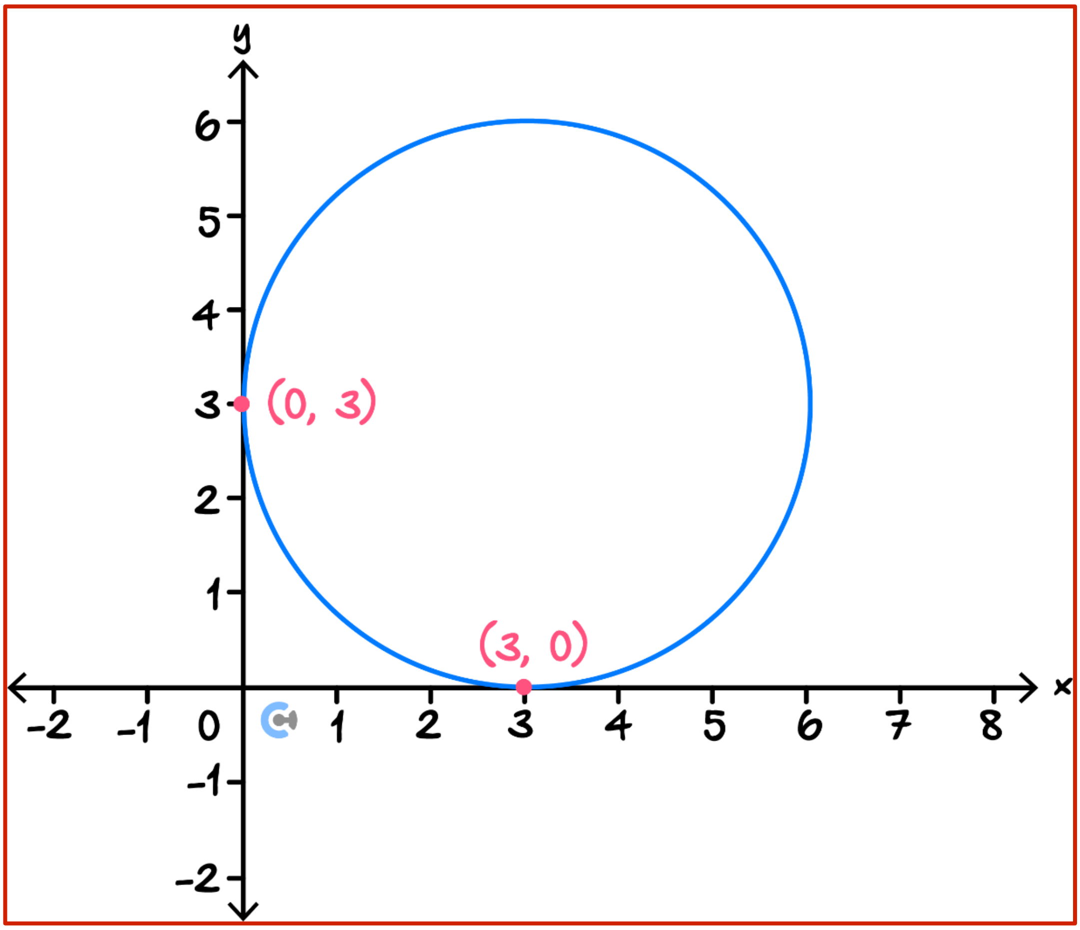
Sub-Section [2.1.4]: Sketch and Find the Rule of Semicircles and Circles

Question 13

Graph the following circle, label all intercepts.

$$(x-3)^2+(y-3)^2=9$$



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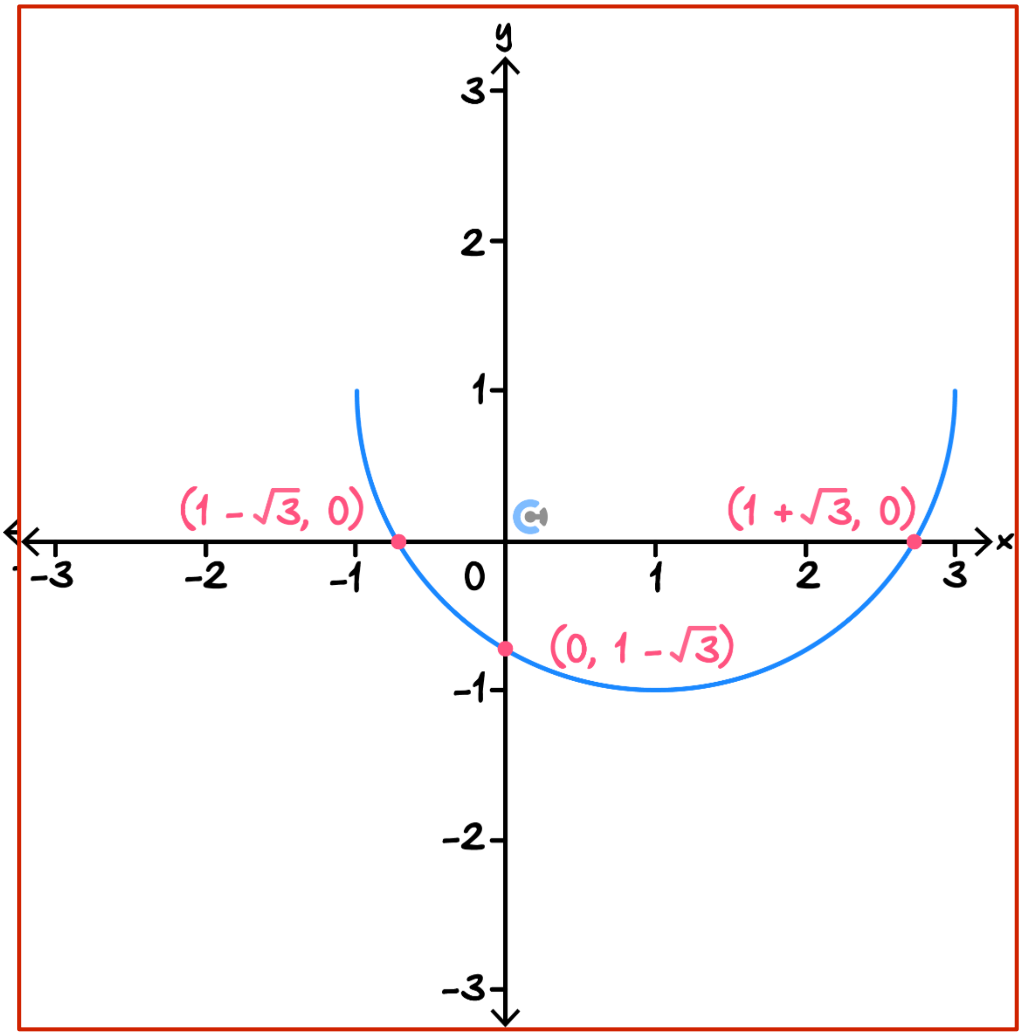
Question 14

Graph the following semi-circle, label all intercepts.

$$y = -\sqrt{4-(x-1)^2}+1$$



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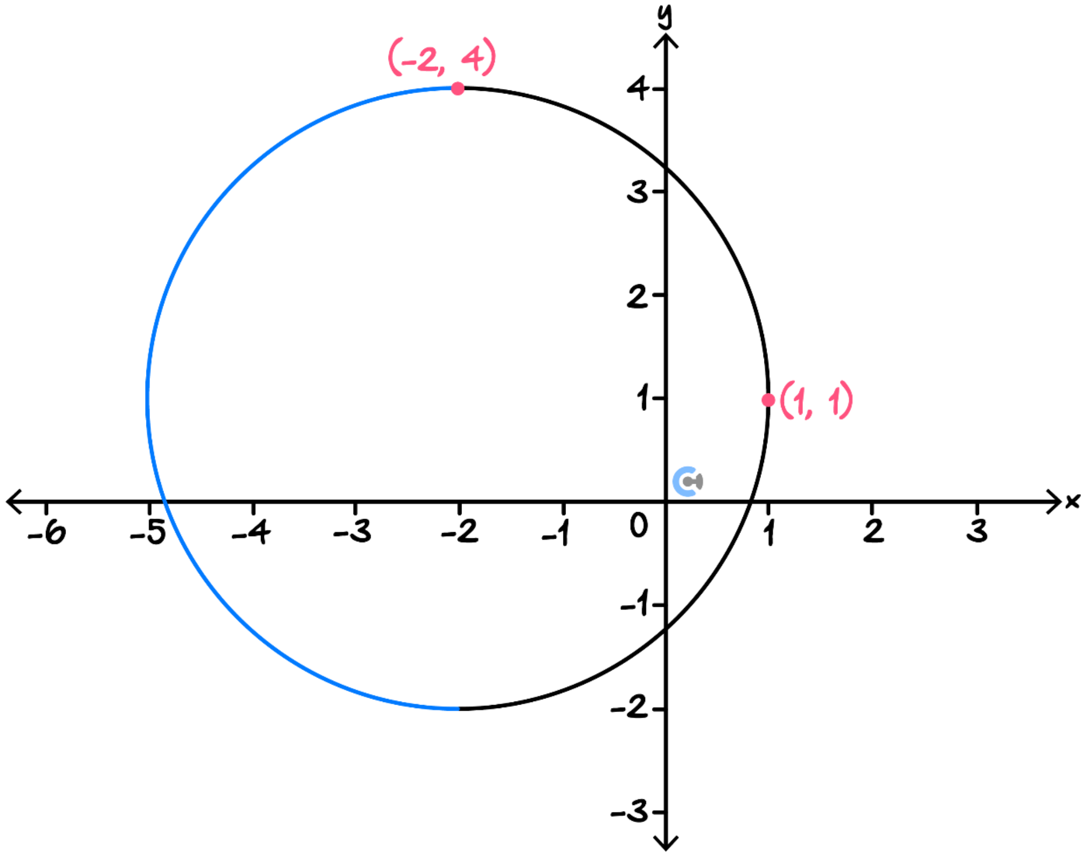
Question 15

Consider the circle with radius 3 shown on the graph below.



a. Determine the equation of the circle.

See that centre is at $(-2, 1)$ and radius is 3.
$$(x+2)^2 + (y-1)^2 = 9$$

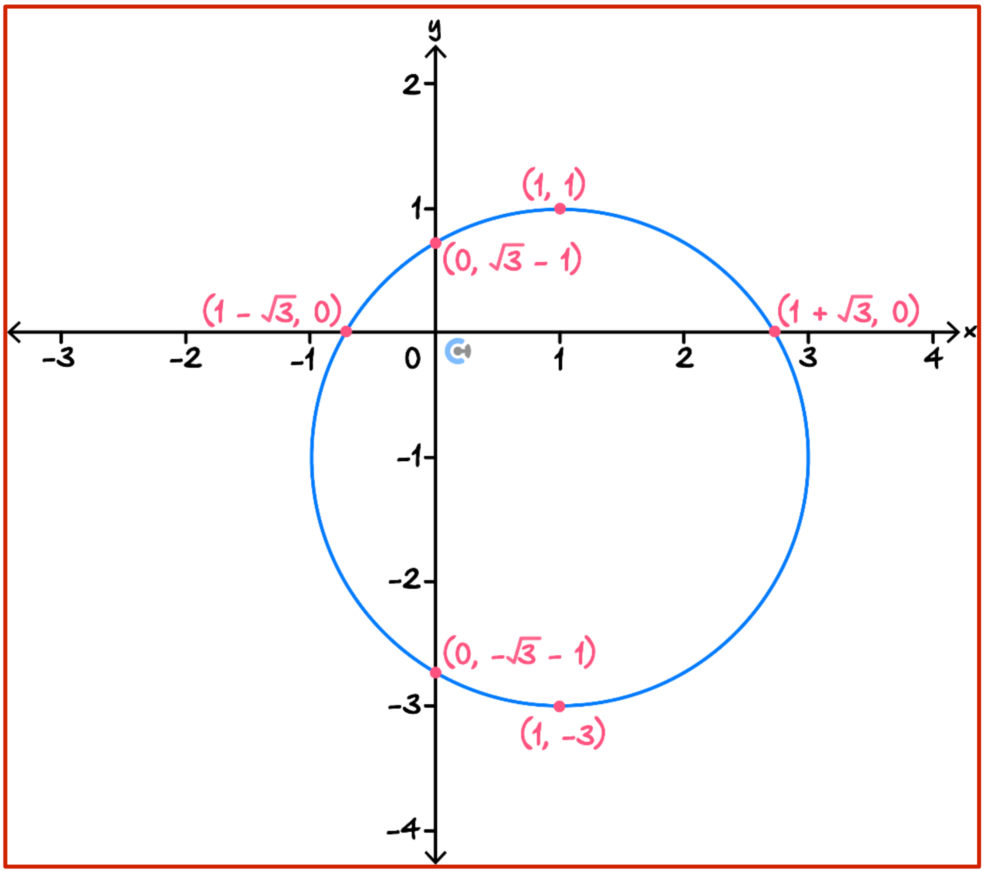
b. Hence, determine the equation of the semi-circle outlined in black.

$$x = \sqrt{9-(y-1)^2} - 2$$

Question 16

Graph the following circle, label all intercepts.

$$x^2 - 2x + y^2 + 2y - 2 = 0$$



Complete the square to get circle equation,
$(x-1)^2 + (y+1)^2 = 4$

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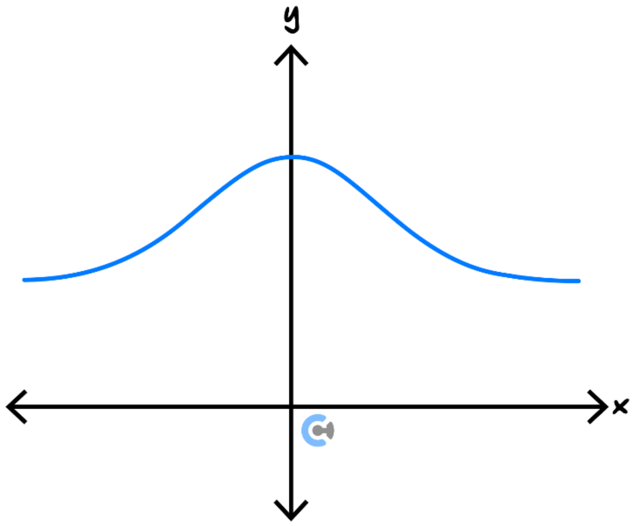
Sub-Section [2.1.5]: Identify the Type of Relations and Identify Whether the Relation is a Function

Question 17

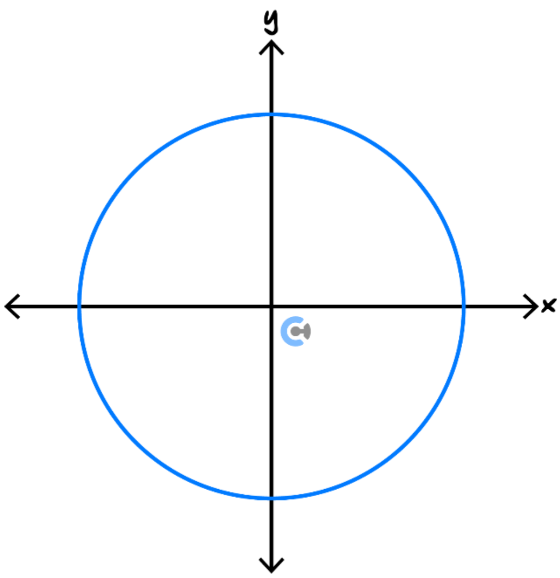
For each of the following graphs, identify the type of relation depicted and whether the relation is a function.

a.



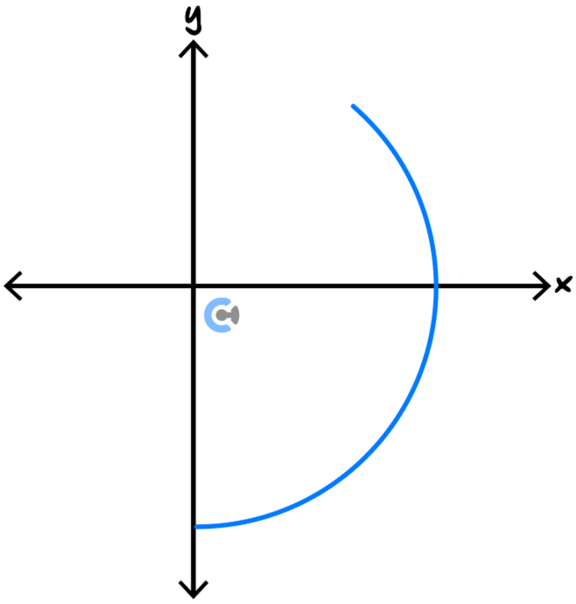
Many to one, is a function.

b.



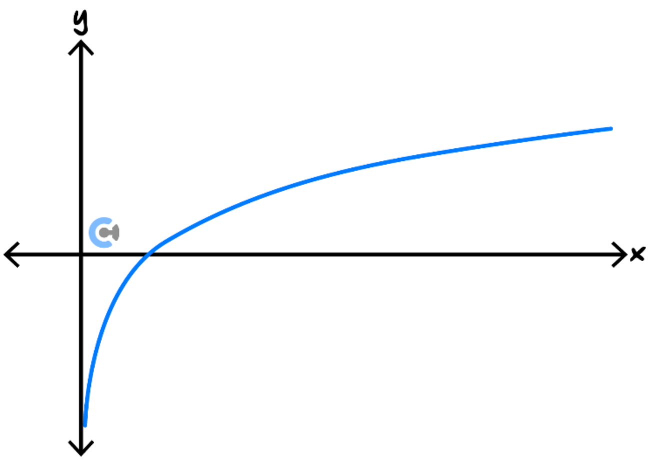
Many to many, not a function.

c.



One to many, not a function.

d.



One to one, is a function.

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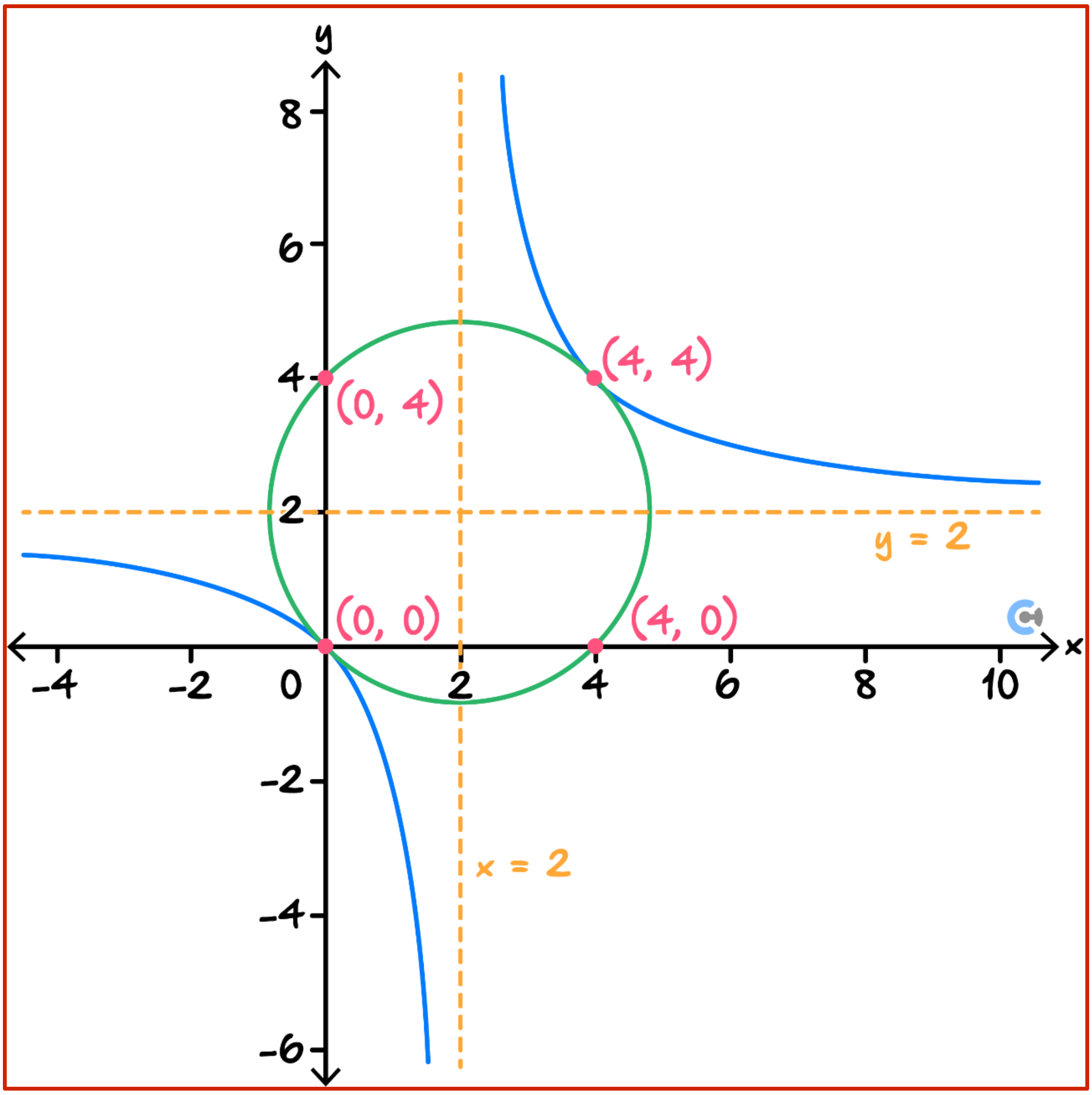
Sub-Section: The 'Final Boss'

Question 18

Consider the hyperbola $y = \frac{4}{x-2} + 2$.

a. Sketch the graph of $y = \frac{4}{x-2} + 2$ on the axes below. Label all axes intercepts and asymptotes.



A circle with centre $(2, 2)$ is such that it hits each branch of the hyperbola exactly once.

b. Use the fact that the shortest distance between both branches of the hyperbola lies on the line $y = x$ in order to find the equation of the circle.

The line $y = x$ intersects the hyperbola at $(0, 0)$ and $(4, 4)$ so the distance between branches is $\sqrt{4^2 + 4^2} = 4\sqrt{2}$. Note that the centre of the circle $(2, 2)$ is the midpoint of $(0, 0)$ and $(4, 4)$. Therefore, the circle has radius $r = 2\sqrt{2} \implies r^2 = 8$. The circle has equation $(x - 2)^2 + (y - 2)^2 = 8$.

c. Sketch the circle from **part b.** on the same axes as **part a.** Label all axes intercepts and intersections with the hyperbola with coordinates.

Solution is in graph of **part a.**

d. Determine the function that describes the lower half of the circle from **part b**.

$$y = -\sqrt{8 - (x - 2)^2} + 2$$

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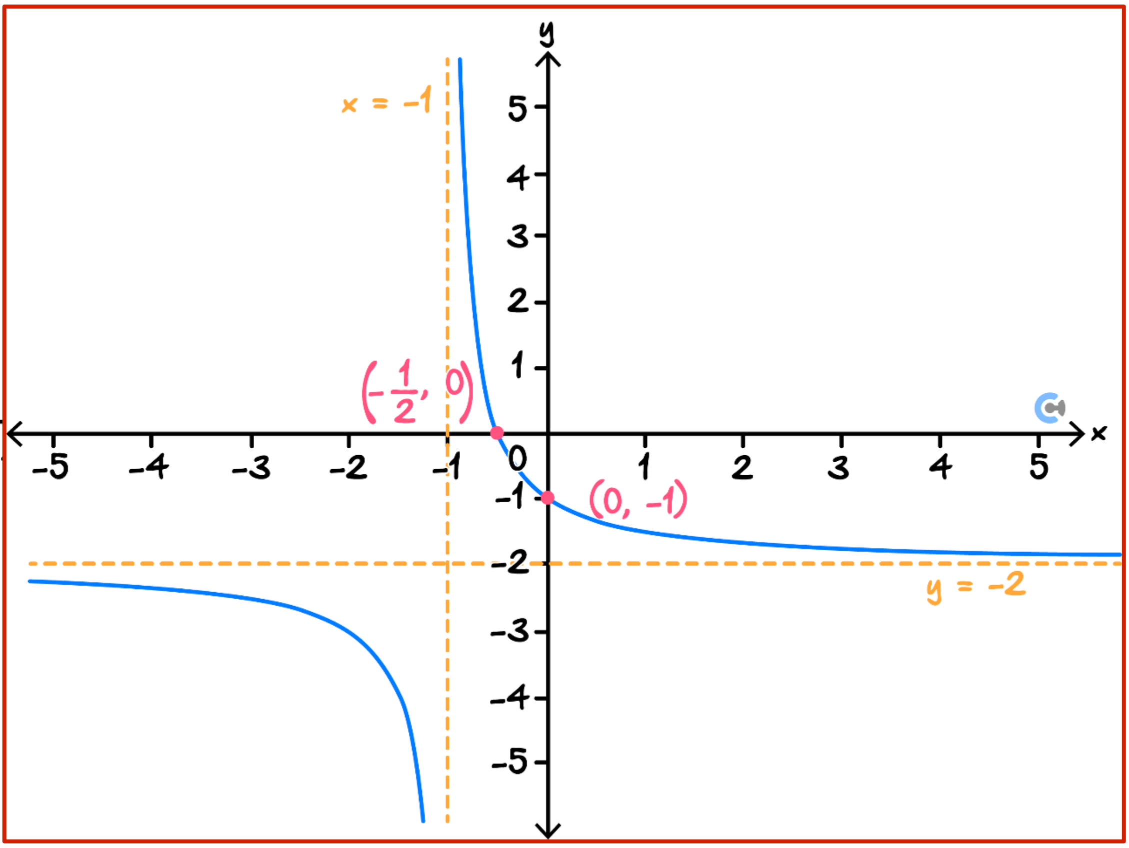
Section B: Supplementary Questions

Sub-Section [2.1.1]: Sketch and Find the Rule of Hyperbolas Functions

Question 19

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{1}{x+1} - 2$$



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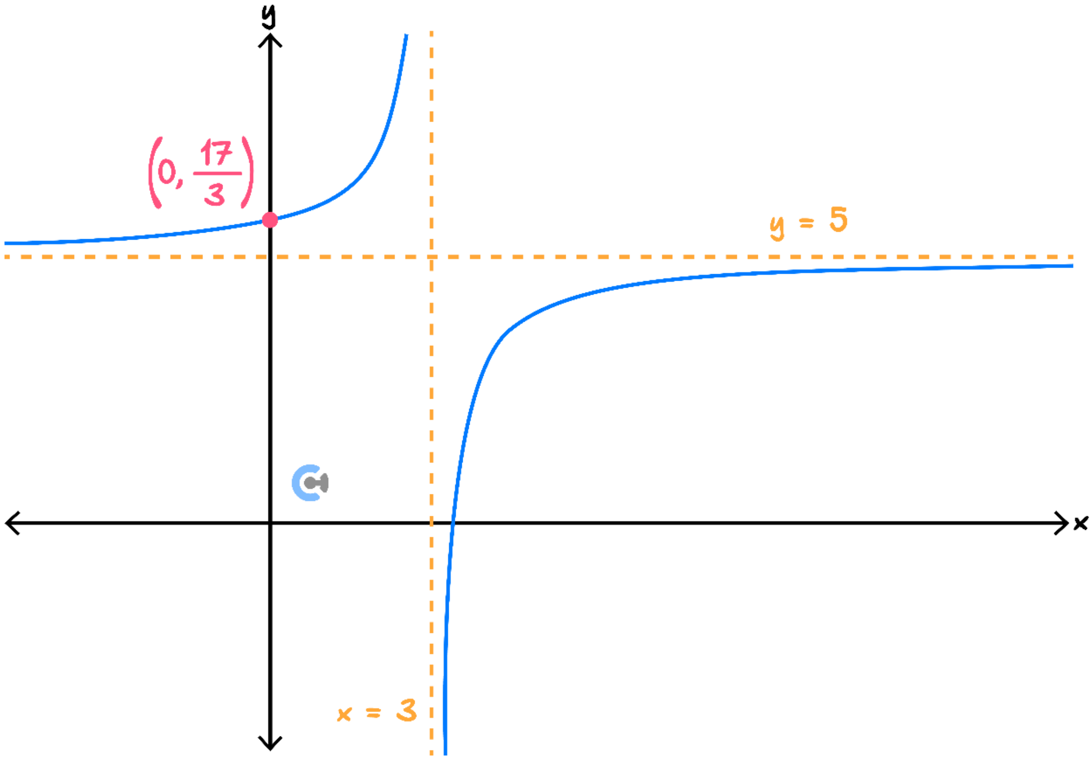
Question 20

Find the rule for the following graph, given it is of the form $y = \frac{a}{x-h} + k$.



Clear that $h = 3$ and $k = 5$.

Then, $\frac{17}{3} = \frac{a}{-3} + 5 \implies a = -2$

$$y = \frac{-2}{x-3} + 5$$

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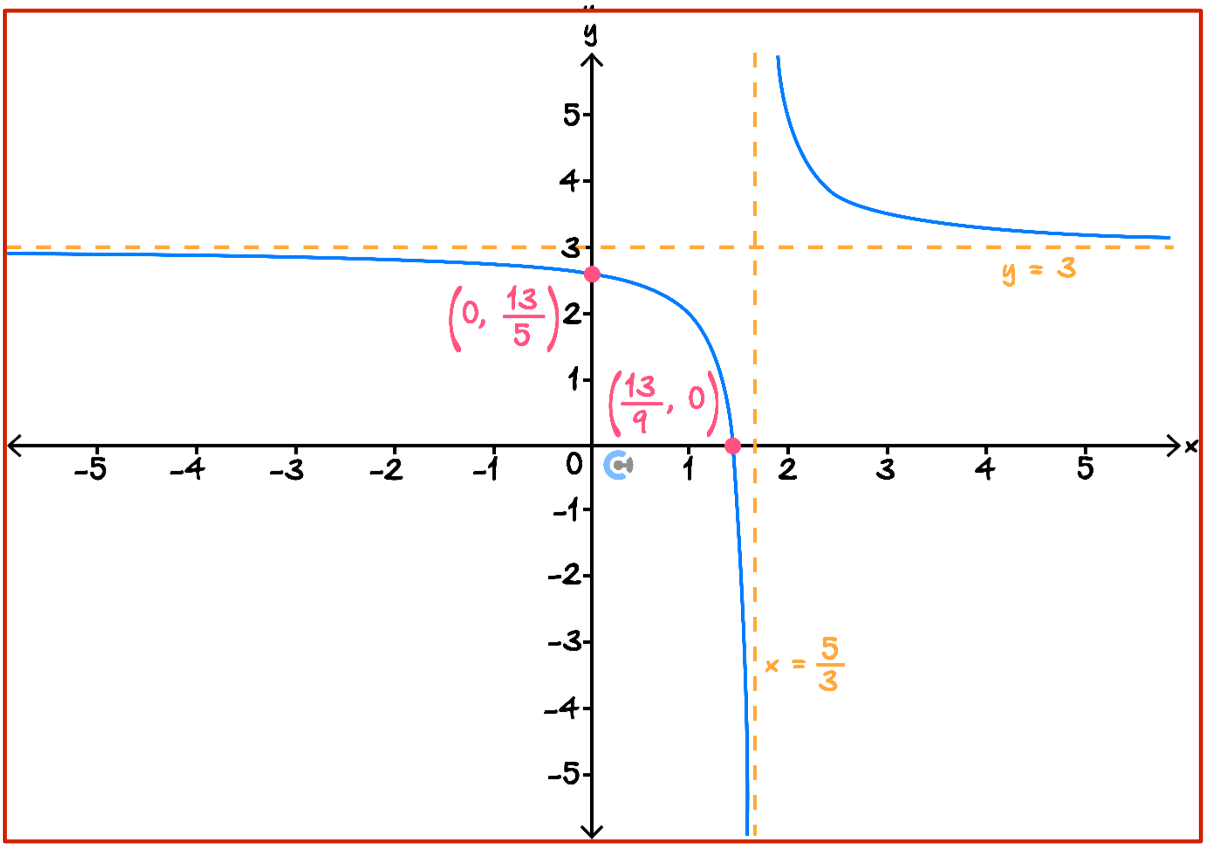
Question 21

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = 3 - \frac{2}{5 - 3x}$$



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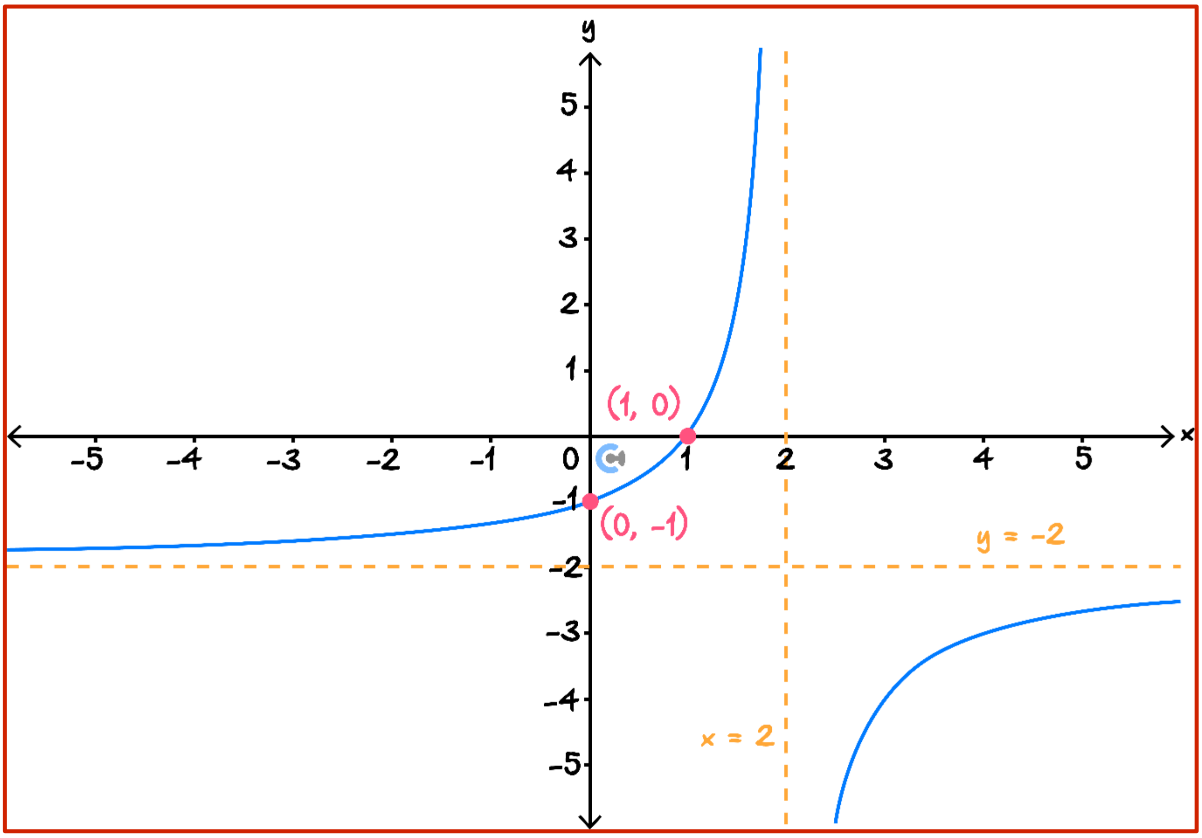
Question 22

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{2-2x}{x-2}$$



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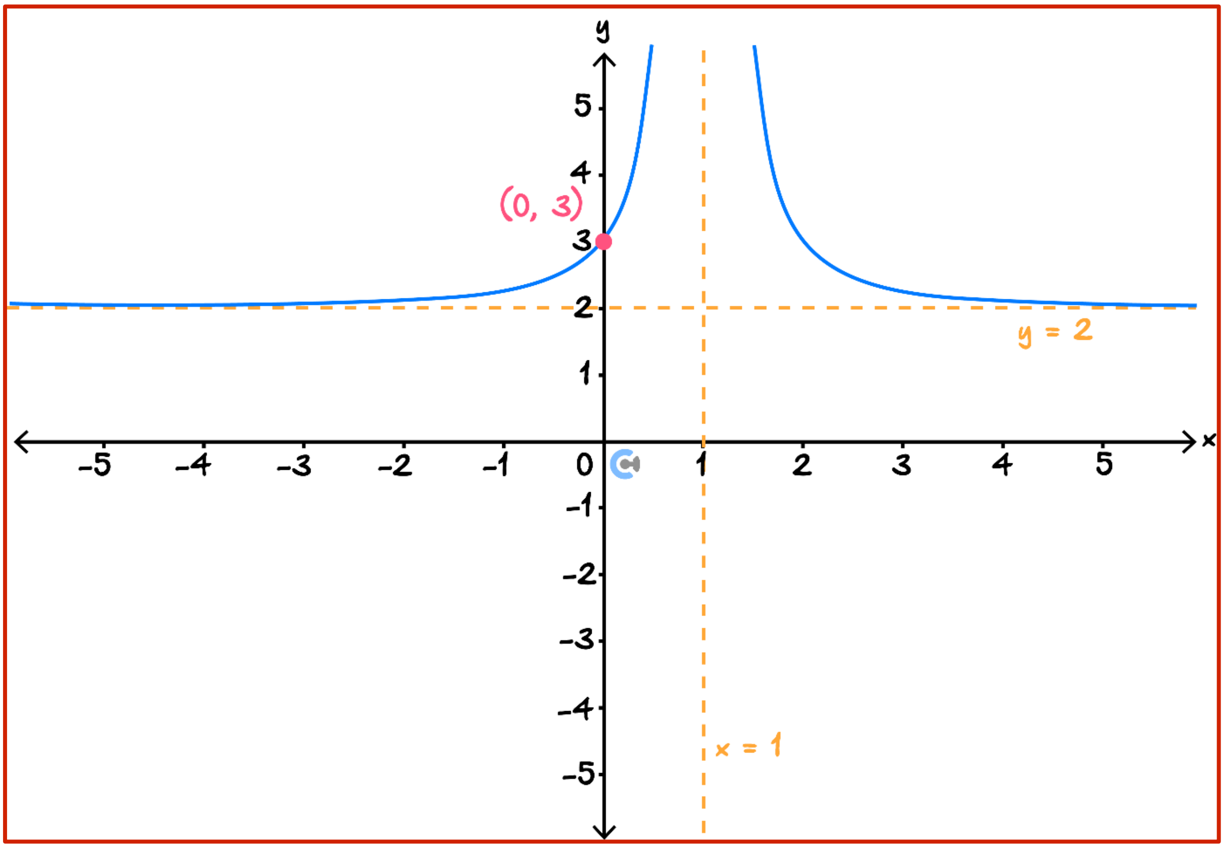
Sub-Section [2.1.2]: Sketch and Find the Rule of Truncus Functions

Question 23

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{1}{(x-1)^2} + 2$$



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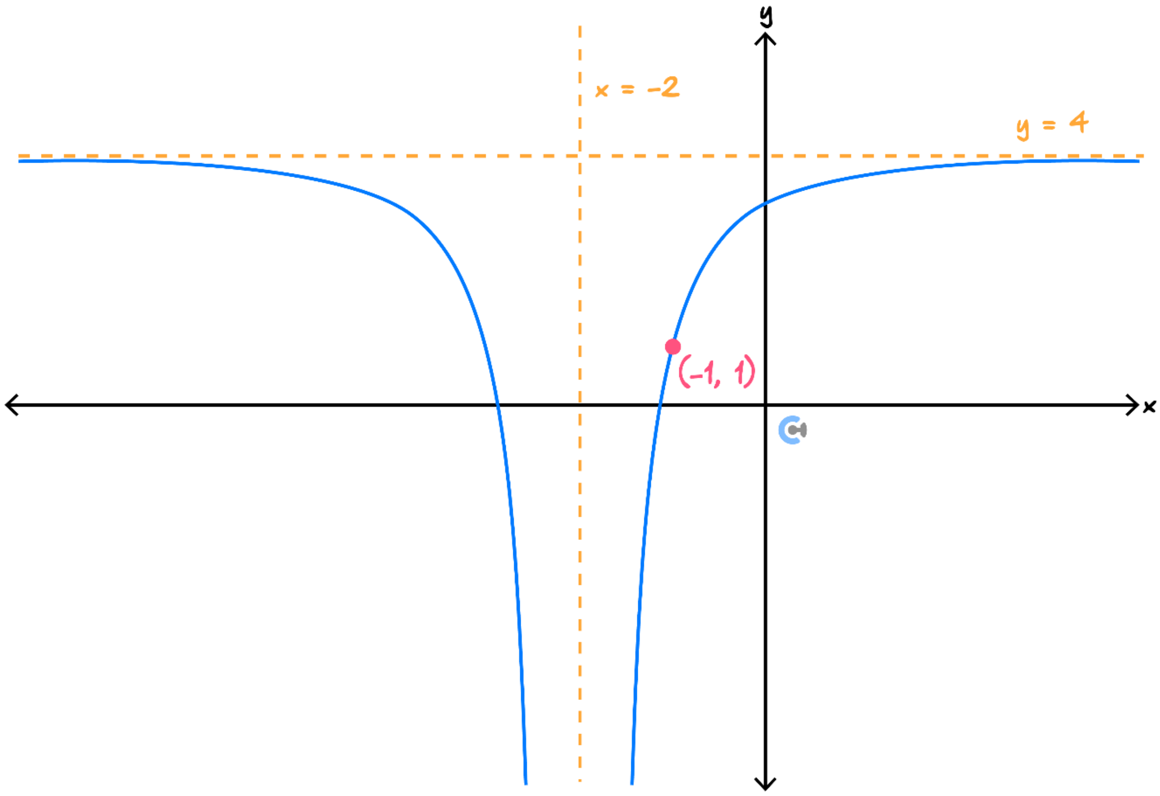
Question 24

Find the rule for the following graph, given it is of the form $y = \frac{a}{(x-h)^2} + k$.



Clear that $h = -2$ and $k = 4$.

Then, $1 = \frac{a}{(-1+2)^2} + 4 \Longrightarrow a = -3$.

$$y = \frac{-3}{(x+2)^2} + 4$$

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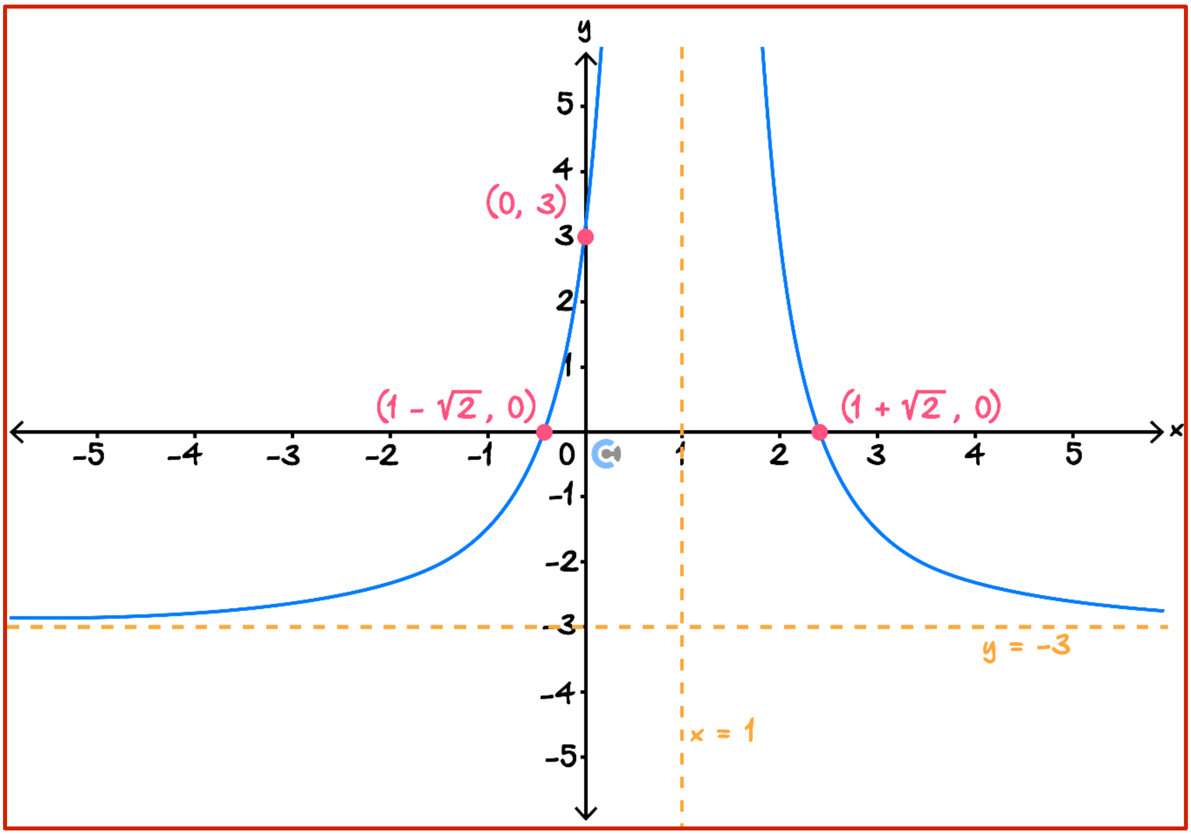
Question 25

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{6}{(1-x)^2} - 3$$



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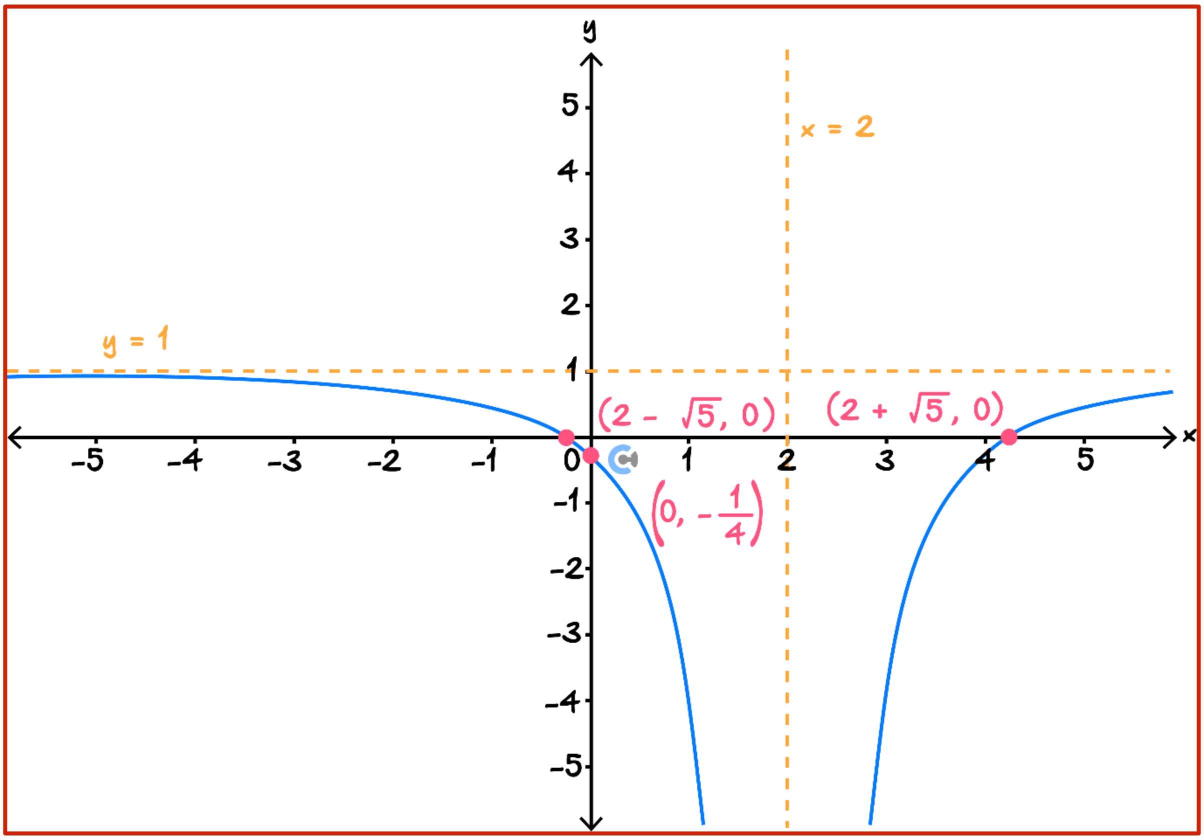
Question 26

Graph the following curve labelling all intercepts and asymptotes with their equations.

$$y = \frac{x^2 - 4x - 1}{(x-2)^2}$$



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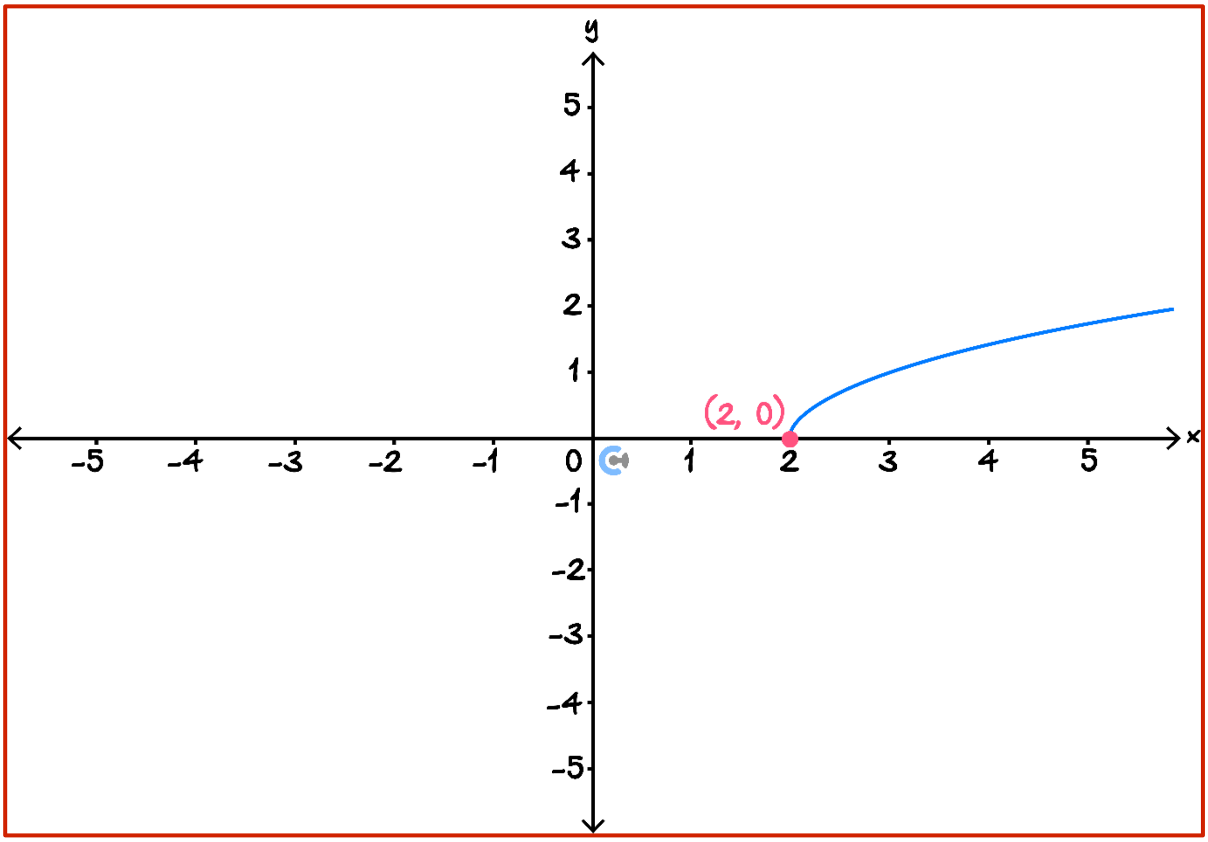
Sub-Section [2.1.3]: Sketch and Find the Rule of Root Functions

Question 27

Graph the following curve labelling all intercepts and start points.

$$y = \sqrt{x-2}$$



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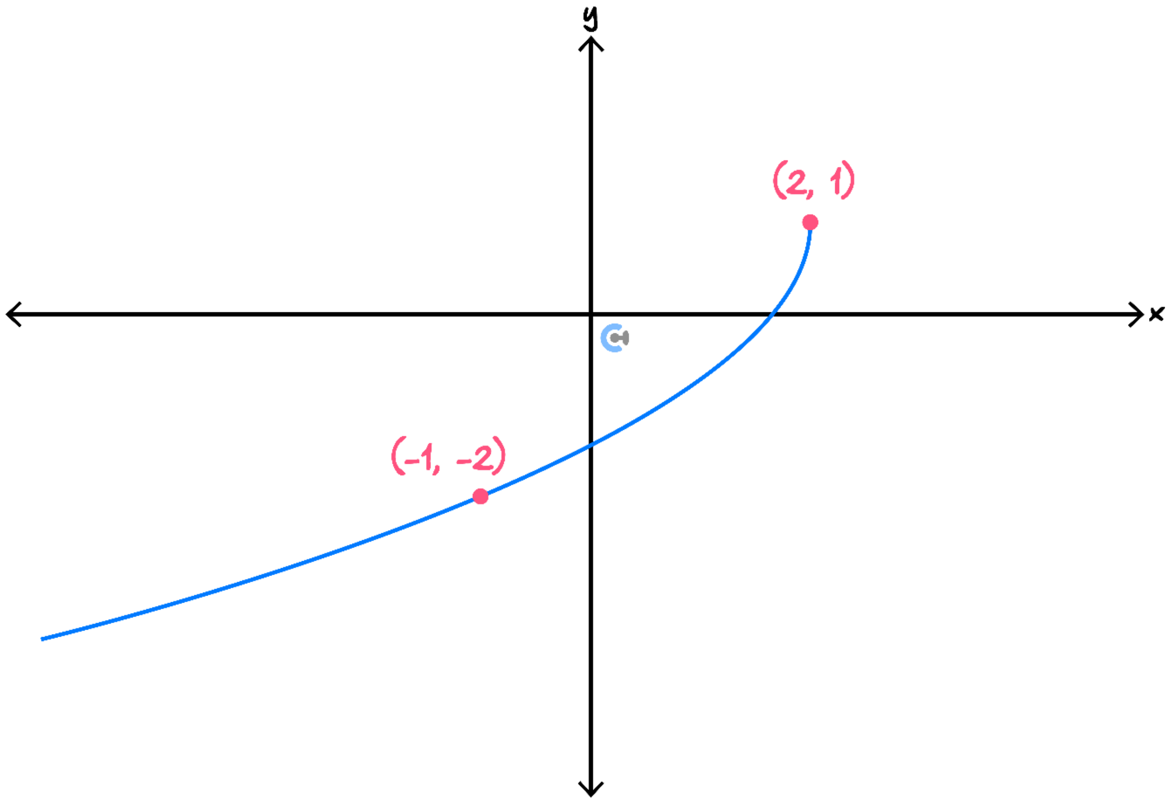
Question 28

Find the rule for the following graph, given it is of the form $y = a\sqrt{h - x} + k$.



From the start point $h = 2$ and $k = +1$.
Then, $-2 = a\sqrt{2 - (-1)} + 1 \implies a = -\sqrt{3}$.
$y = -\sqrt{3}\sqrt{2 - x} + 1$

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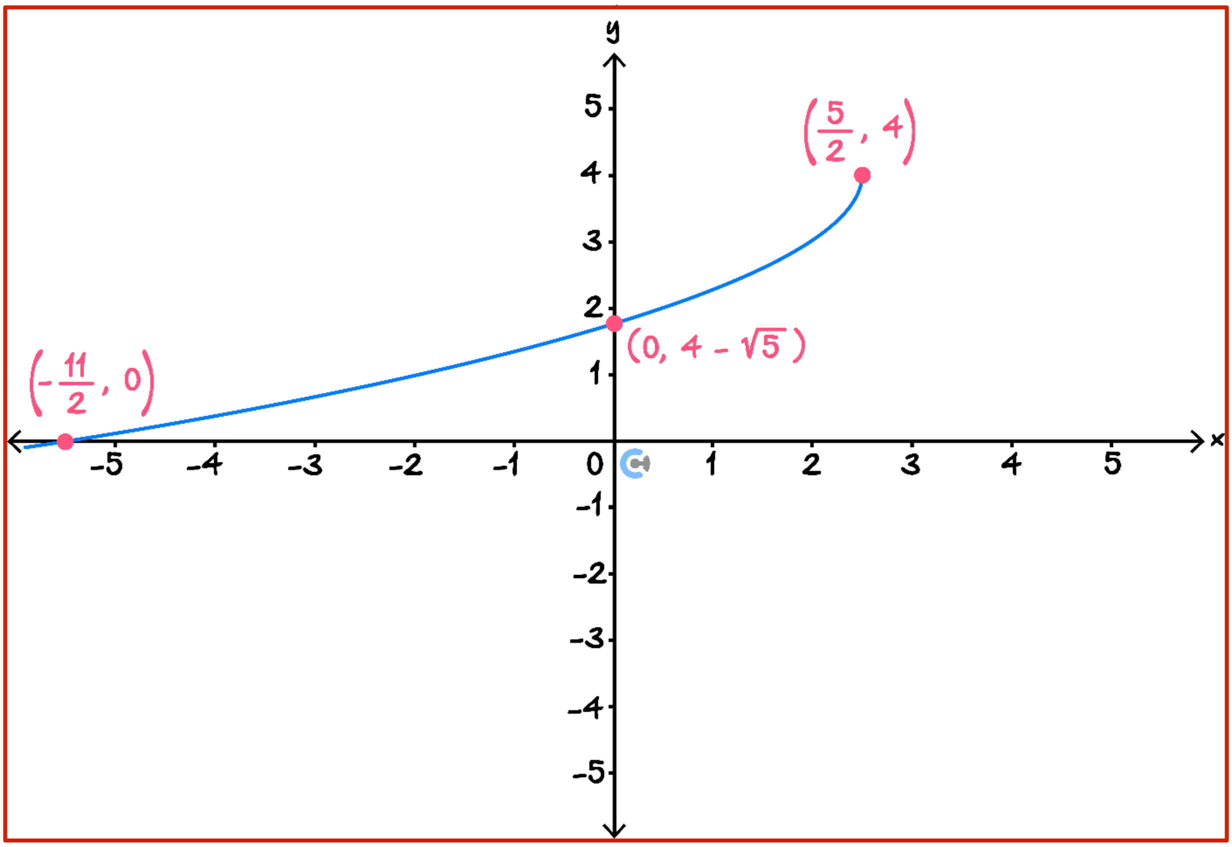
Question 29

Graph the following curve labelling all intercepts and start points.

$$y = 4 - \sqrt{5 - 2x}$$



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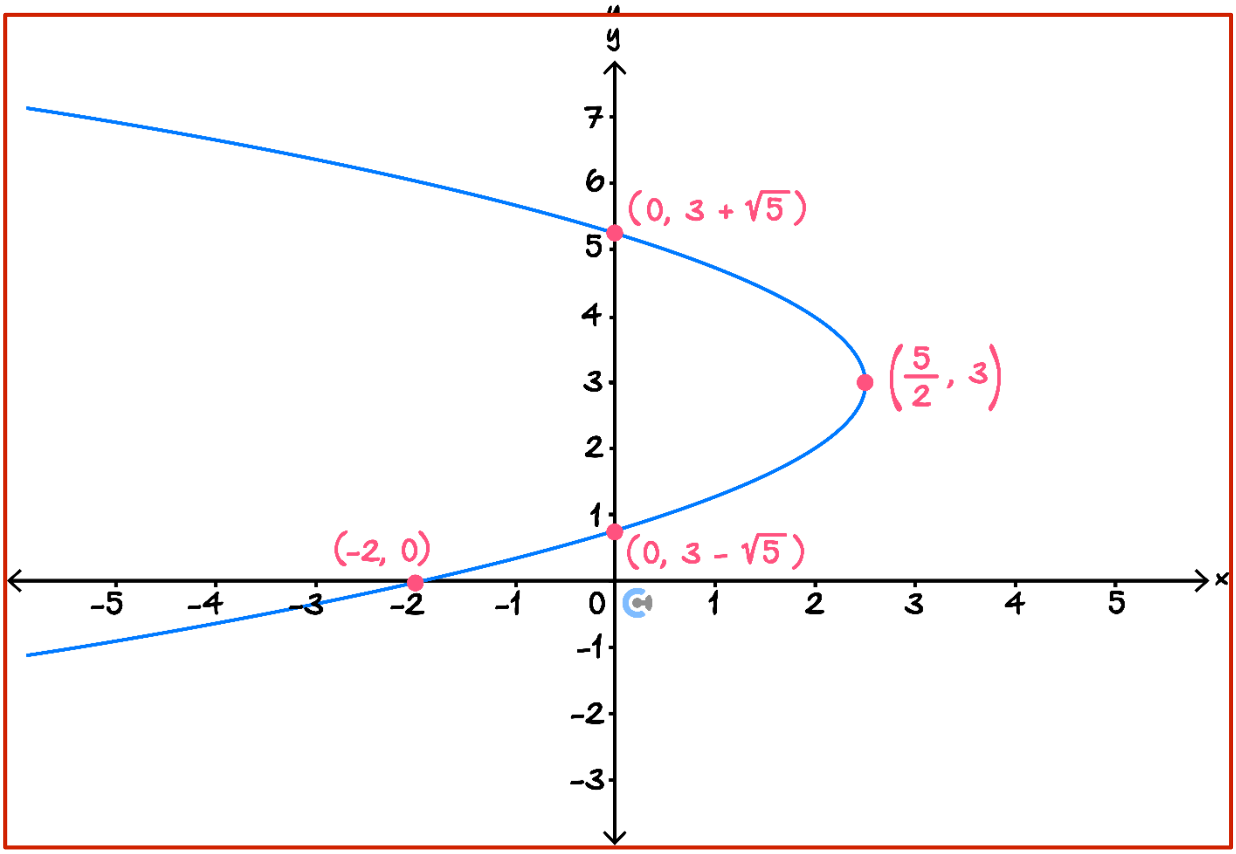
Question 30

Graph the following curve labelling all intercepts and turning points.

$$(y-3)^2 = 5 - 2x$$



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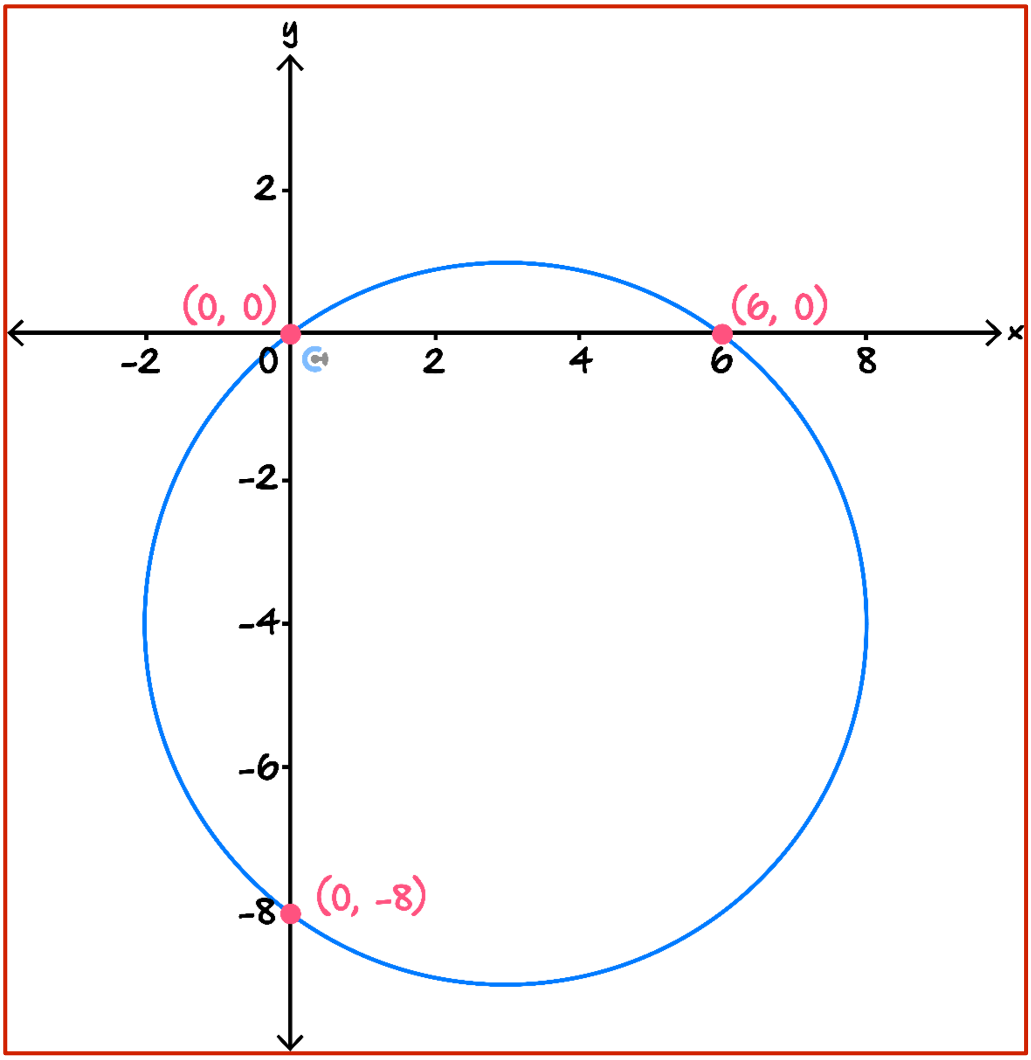
Sub-Section [2.1.4]: Sketch and Find the Rule of Semicircles and Circles

Question 31

Graph the following circle, label all intercepts.

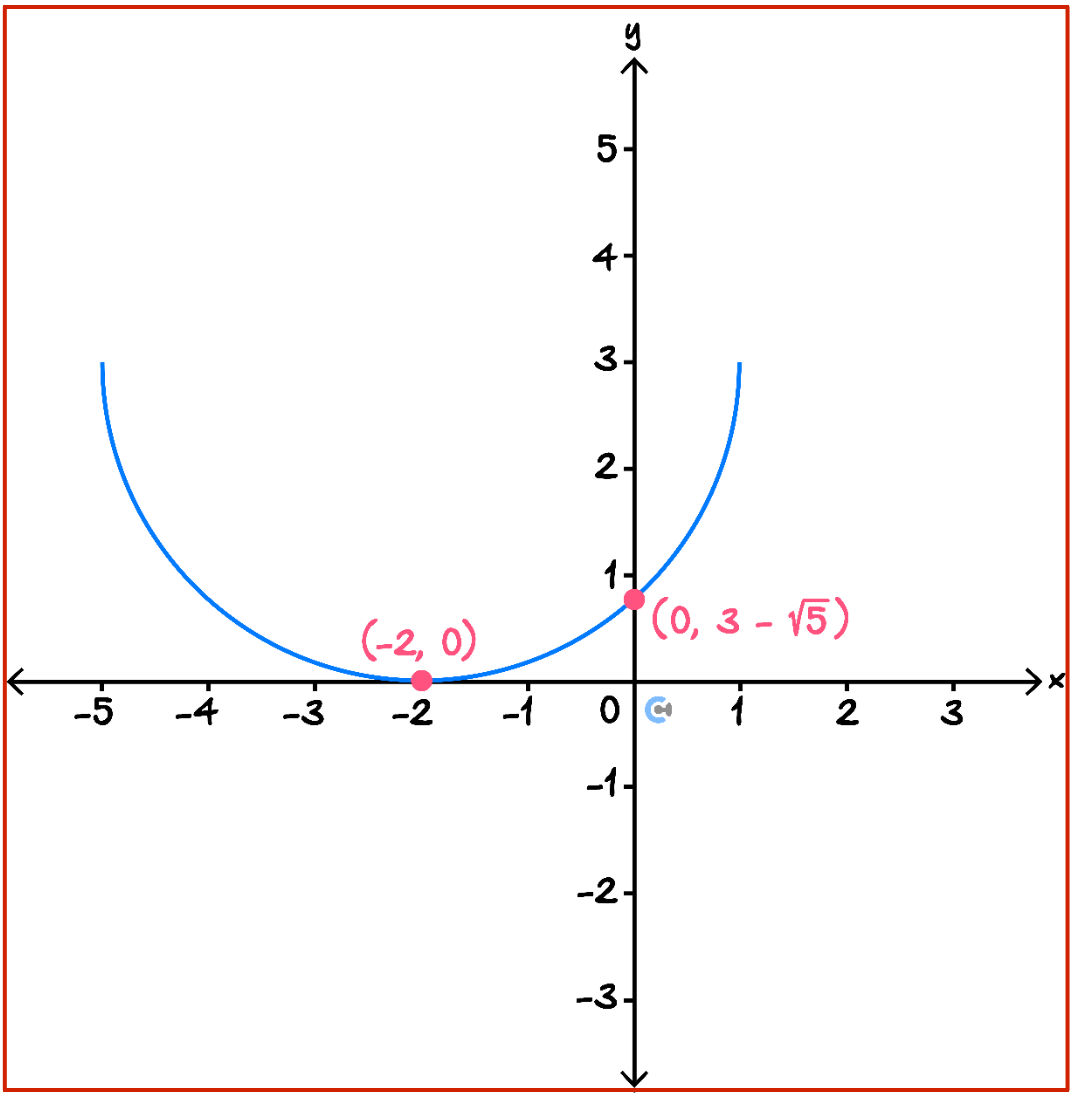
$$(x-3)^2+(y+4)^2=25$$



Question 32

Graph the following semi-circle, label all intercepts.

$$y = 3 - \sqrt{9 - (x+2)^2}$$



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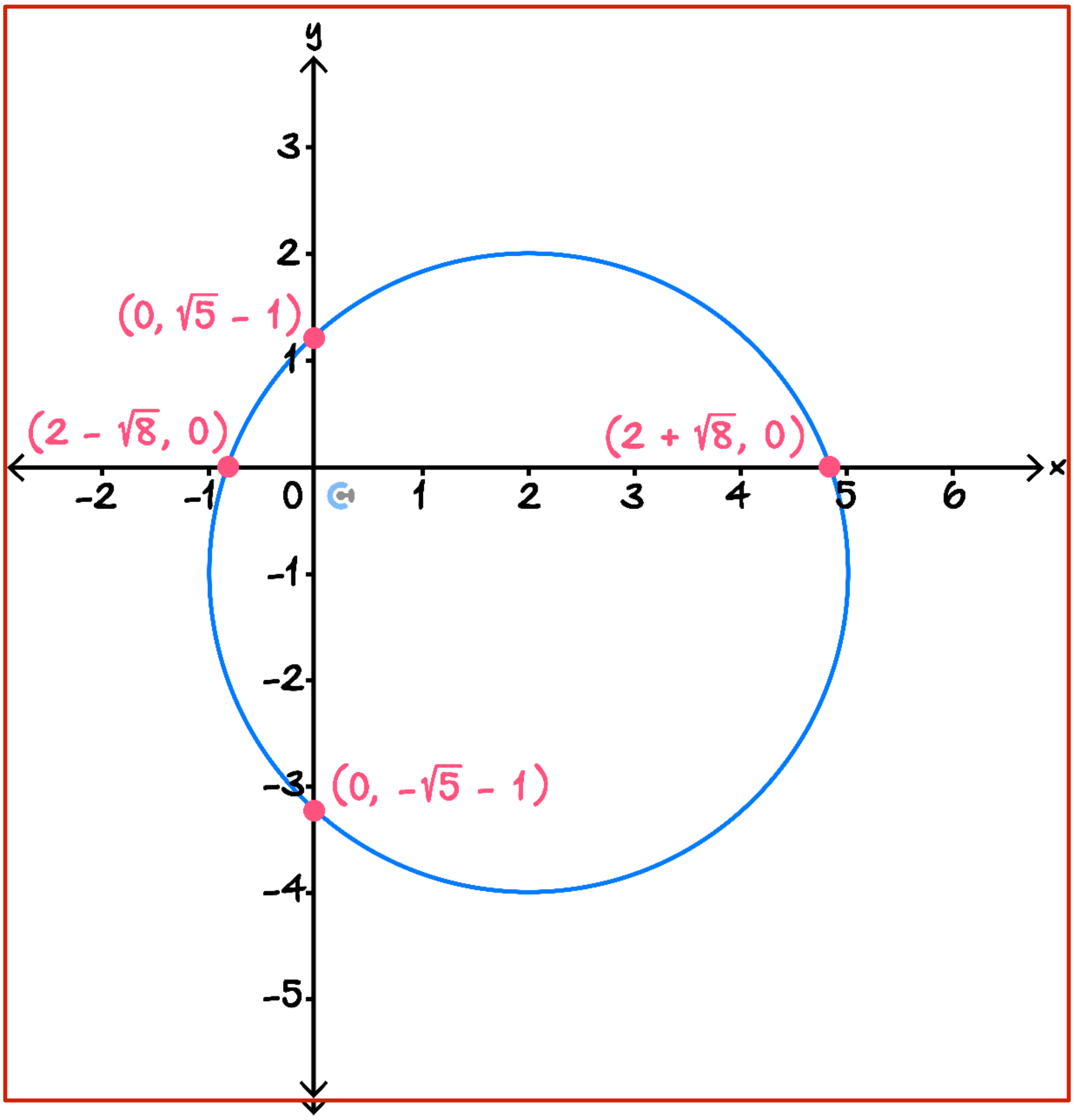
Question 33

Graph the following circle, label all intercepts.

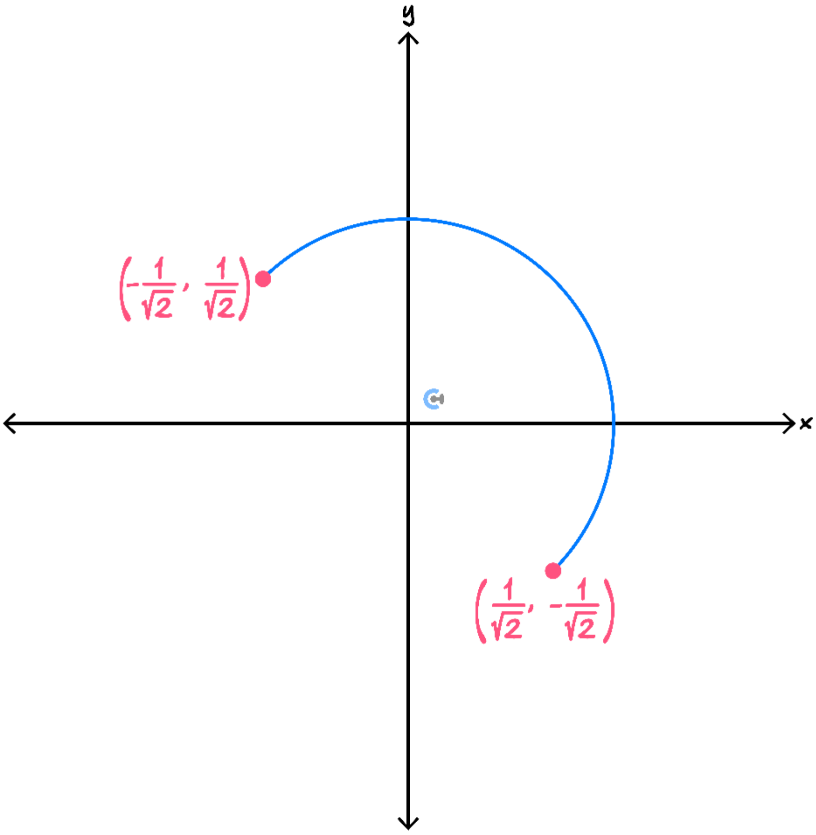
$$x^2 + y^2 + 2y - 4x = 4$$



Complete the square to get circle equation:
$(x-2)^2 + (y+1)^2 = 9.$

Question 34

Determine the equation of the semi circle with radius 1 shown on the graph below.



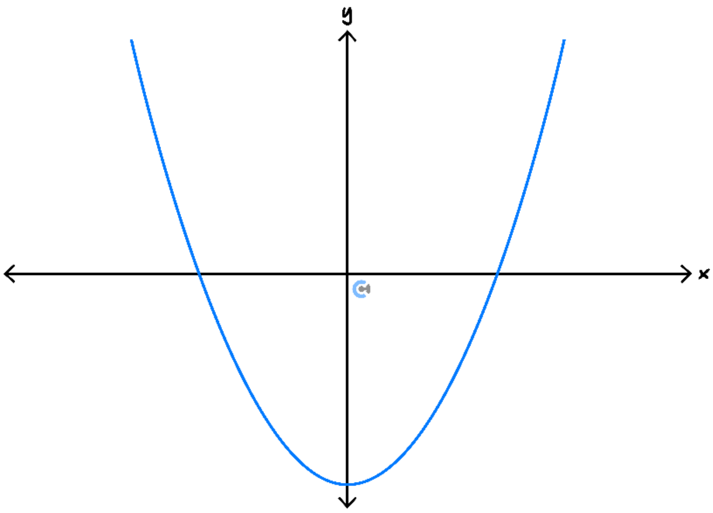
The line segment joining the two end points has a length of 2 units, which is the diameter of the circle.
Thus the center of our circle is the midpoint of our two end-points, the origin.
Thus our semi circle can be described with the following equation and restriction, $x^2+y^2=1$ and $x+y \geq 0$.
Now we can rearrange $x^2+y^2=1$ to be of the form, $(x+y)^2 = 1+2xy$.
Then to implement our desired restriction we simply square root both sides taking the positive square root, resulting in $x+y=\sqrt{1+2xy}$.
This forces $x+y>0$ giving us our desired graph.

Sub-Section [2.1.5]: Identify the Type of Relations and Identify Whether the Relation is a Function

Question 35

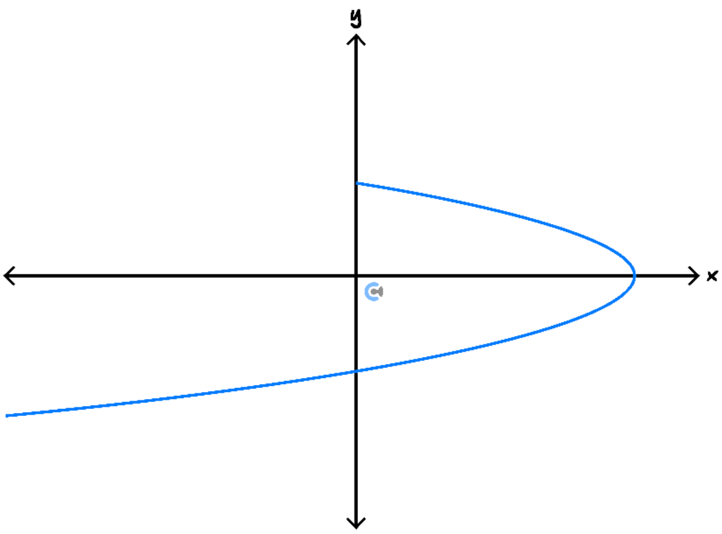
For each of the following graphs, identify the type of relation depicted and whether the relation is a function.

a.



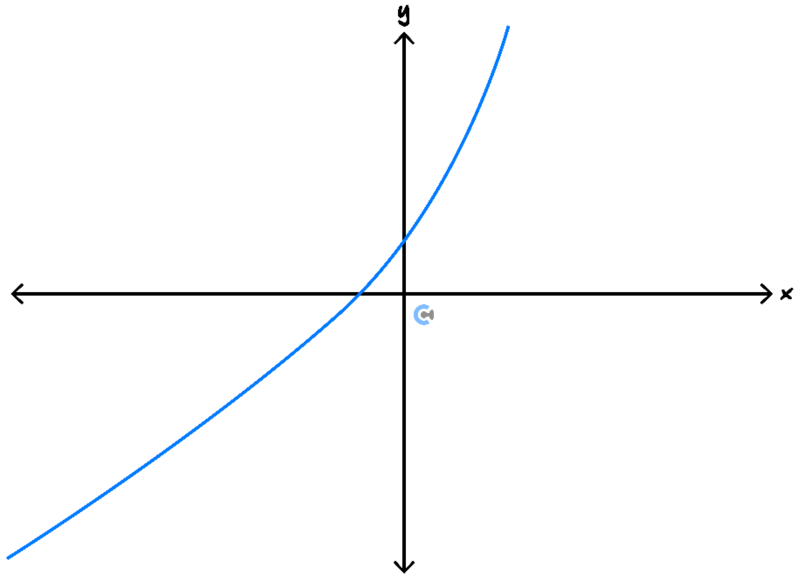
Many to one, is a function.

b.



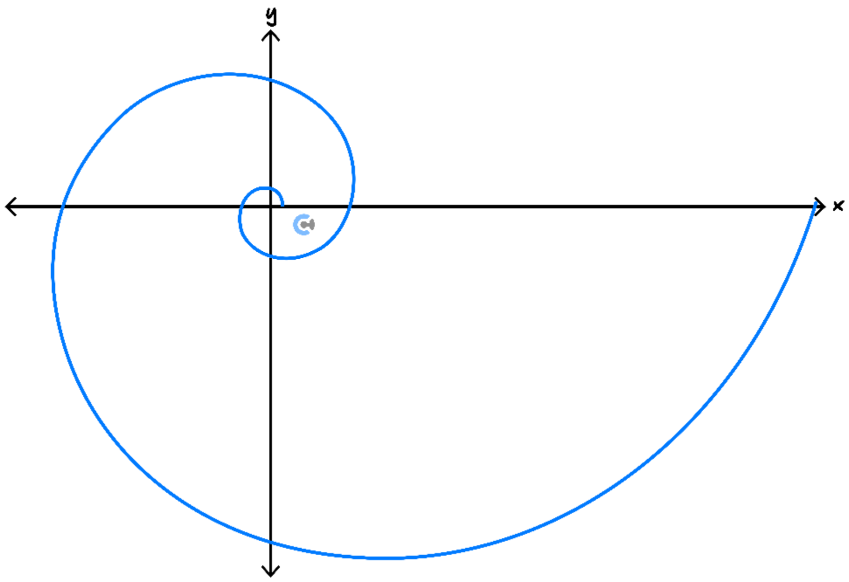
One to many, not a function.

c.



One to one, is a function.

d.



Many to many, not a function.

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