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VCE Mathematical Methods ½
AOS 2 Revision [2.0]
Contour Check (Part 2) Solutions

Contour Check

Section G: [2.1-2.5] - Exam 2 Overall (113 Marks)

Question 78 (1 mark)

The maximal domain of the function $f(x) = \frac{1}{\sqrt{x^2-x-6}}$ is:

A. $x \in (0, \infty)$

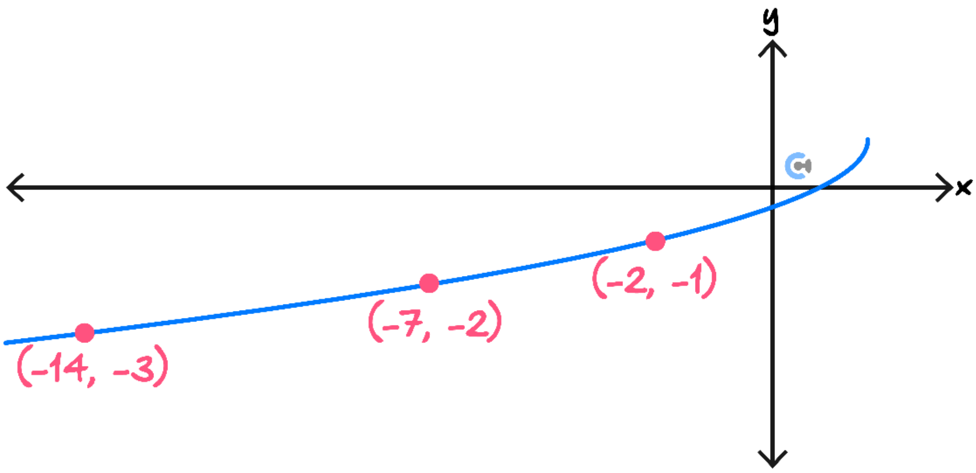
B. $x \in (-2, 3)$

C. $x \in (-\infty, 2] \cup [3, \infty)$

D. $x \in \mathbb{R} \setminus [-2, 3]$

Question 79 (1 mark)

The most likely rule for the following graph is:



A. $\sqrt{3-x}+1$

B. $-\sqrt{2-x}+1$

C. $3\sqrt{x+1}-1$

D. $-3\sqrt{2-x}+1$

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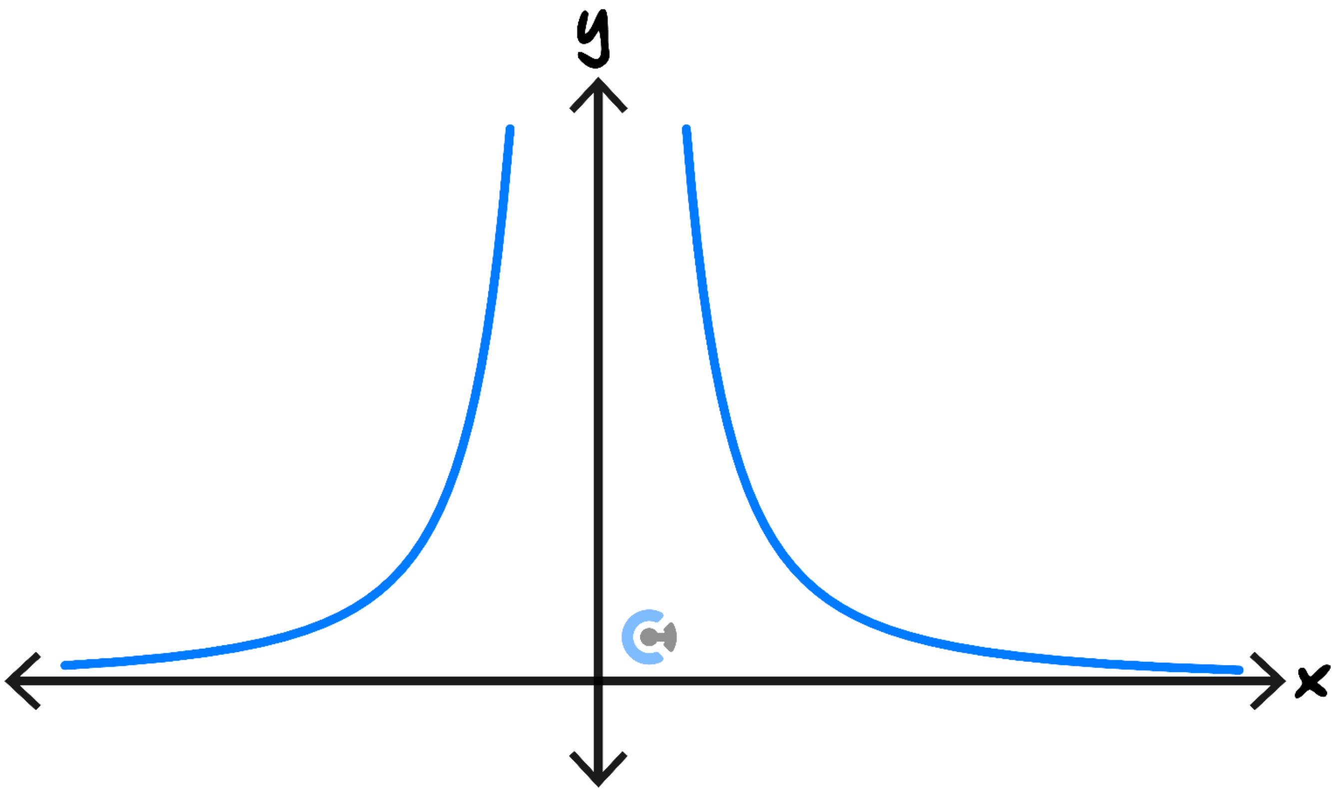
Question 80 (1 mark)

The function g with the graph shown below, is best described as:



A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

Question 81 (1 mark)

The line with the equation $4y + 3x = 25$ intersects the circle $x^2 + y^2 = 25$ exactly once at the point $P(3, 4)$. The equation for the radius of the circle that passes through P is:

A. $3y - 4x = 0$

B. $3y + 4x = 25$

C. $3y + 4x = 0$

D. $3y - 4x = 25$

Question 82 (1 mark)

Consider the function $f(x) = 12x^5 + 90x^4 + 140x^3 - 180x^2 - 480x - 200$.

The equation $f(x) = k$ will have three solutions for:

A. $k \in (-618, -24) \cup (38, 632)$

B. $k \in [-618, -24] \cup [38, 632]$

C. $k \in (-24, 38)$

D. $k \in [24, 38]$

Question 83 (1 mark)

The graph of the function f passes through the point $(2, -3)$.

If $h(x) = 3f(x - 2)$, then the graph of the function h must pass through the point:

A. $(4, -9)$

B. $(0, -9)$

C. $(4, -1)$

D. $(0, -1)$

Question 84 (1 mark)

The graph of the function $f : \mathbb{R} \to \mathbb{R}, f(x) = 3^x - 1$, is reflected in the y-axis and then translated 2 units to the left and then 3 units up.

Which one of the following is the rule of the transformed graph?

A. $y = \left(\frac{1}{3}\right)^{x+2} + 2$

B. $y = \frac{1}{3} \times 3^{x+2} + 3$

C. $y = 3^{-x} + 3$

D. $y = 3^{-x+2} + 3$

Question 85 (1 mark)

The graph of the function g is obtained from the transformed graph of the function:

$$f : [-2,6] \to \mathbb{R}, f(x) = 3x^2 + 5x - 2$$

Which undergoes a dilation of a factor 2 from the y-axis, followed by a dilation of factor $\frac{1}{4}$ from the x-axis, followed by a reflection in the x-axis, and finally followed by a translation of 6 units in the positive direction of the y-axis. The domain and range of g are respectively:

A. $[-4, 12]$ and $[-12, 4]$

B. $[-4, 12]$ and $\left[-28, \frac{337}{48}\right]$

C. $[-12, 4]$ and $\left[-\frac{239}{48}, 40\right]$

D. $[-4, 12]$ and $\left[-40, \frac{239}{48}\right]$

Question 86 (1 mark)

The image of the function $f(x) = x^4$ is $y = -40(x+2)^4$. The transformations that could have been applied are:

A. Reflection in the x-axis, then translation in the positive direction of the x-axis by 2 units, followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

B. Reflection in the x-axis, then translation in the negative direction of the x-axis by 2 units, followed by a dilation from the x-axis by a factor of 5 and a dilation by a factor 2 from the y-axis.

C. Reflection in the x-axis, then a dilation from the x-axis by a factor of 2, followed by a translation in the positive direction of the x-axis by 2 units, and finally a dilation from the y-axis by a factor of 2.

D. Reflection in the x-axis, then a dilation from the y-axis by a factor of $\frac{1}{2}$, followed by a translation in the negative direction of the x-axis by 2 units, and finally a dilation from the x-axis by a factor of $\frac{5}{2}$.

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Question 87 (1 mark)

Let $h : (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $(h(x))^2 = h(x^2)$

$$h(x) = \frac{1}{x-1}$$

$$(h(x))^2 \neq h(x^2)$$

$$\left(\frac{1}{x-1}\right)^2 = \frac{1}{x^2-2x+1} \neq \frac{1}{x^2-1}$$

Question 88 (1 mark)

The linear function $f : D \to R, f(x) = 5 - x$ has range $[-4, 5)$.

The domain D is:

A. $(0, 9]$

B. $(0, 1]$

C. $[5, -4)$

D. $[-9, 0)$

E. $[1, 9)$

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Question 89 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x\log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

$$\frac{dy}{dx} = 2f(x) - f(y) = (y - x)f(xy)$$

$$f(x) = \frac{1}{x}$$

$$\text{LHS} = \frac{1}{x} - \frac{1}{y}$$

$$\text{RHS} = (y - x) \times \frac{1}{xy} = \frac{1}{x} - \frac{1}{y} = \text{LHS}$$

Question 90 (1 mark)

The linear function $f : D \to R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is:

A. $[-2, 6)$

B. $(-2, 2]$

C. R

D. $(-2, 6]$

E. $[-6, 2]$

Space for Personal Notes

Question 91 (1 mark)

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 92 (1 mark)

If $f : (-\infty, 1) \to R, f(x) = 2\log_e(1 - x)$ and $g : [-1, \infty) \to R, g(x) = 3\sqrt{x + 1}$, then the maximal domain of the function $f + g$ is:

A. $[-1, 1)$

B. $(1, \infty)$

C. $(-1, 1]$

D. $(-\infty, -1]$

E. R

Question 93 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{x}{2}\right)$

E. $x - 2$

Question 94 (1 mark)

The range of the function $f:[-2,3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9,-5]$

C. $(-5,0)$

D. $[-9,0]$

E. $[-9,-5)$

Question 95 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

$$f(x-y) = (x-y)^2 = x^2 - 2xy + y^2$$
$$f(x) - f(y) = x^2 - y^2$$
$$f(x-y) \neq f(x) - f(y)$$

A. $f(xy) = f(x)f(y)$

B. $f(xy) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x-y) = f(x) - f(y)$

E. $f(x+y) + f(x-y) = 2\big(f(x) + f(y)\big)$

Question 96 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has range $[-4,12]$.

The domain D is:

A. $[-3,5]$

B. $[-5,3]$

C. R

D. $[-14,18]$

E. $[-18,14]$

Question 97 (1 mark)

The range of the function $f : [-2,7) \to R, f(x) = 5 - x$ is:

A. $(-2, 7]$

B. $[-2, 7)$

C. $(-2, \infty)$

D. $(-2, 7)$

E. R

Question 98 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers.

A possible rule for the function is:

A. $f(x) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

If $f(x) = 2x$, then $f\left(\frac{x+y}{2}\right) = 2\left(\frac{x+y}{2}\right) = x + y$.

Moreover, $\frac{f(x)+f(y)}{2} = \frac{2x+2y}{2} = x + y$. Hence the function $f(x) = 2x$ satisfies the given functional equation.

Question 99 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 100 (1 mark)

The range of the function $f : \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4-\sqrt{2}, 4+\sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4-\sqrt{2}, 4+\sqrt{2}\right]$

D. $\left(\frac{-2}{\sqrt{2}, \sqrt{2}}\right]$

E. $\left[4-\sqrt{2}, 4+\sqrt{2}\right]$

Question 101 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x+2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x-1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

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Question 102 (1 mark)

The function f and its inverse, f^{-1}, are one-to-one for all values of x.

If $f(1) = 5, f(3) = 7$, and $f(8) = 10$, then $f^{-1}(7)$ and $f^{-1}(5)$ respectively are equal to:

A. 5 and 7.

B. 3 and 1.

C. 7 and 5.

D. 8 and 5.

E. 5 and 8.

Question 103 (1 mark)

The function f with rule $f(x) = 2\log_e(16 - x)$ has a maximal domain given by:

A. $x \in (16, \infty)$

B. $x \in (-\infty, 4)$

C. $x \in (4, \infty)$

D. $x \in (-4, 4)$

E. $x \in (-\infty, 16)$

Question 104 (1 mark)

The range of the function with the rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

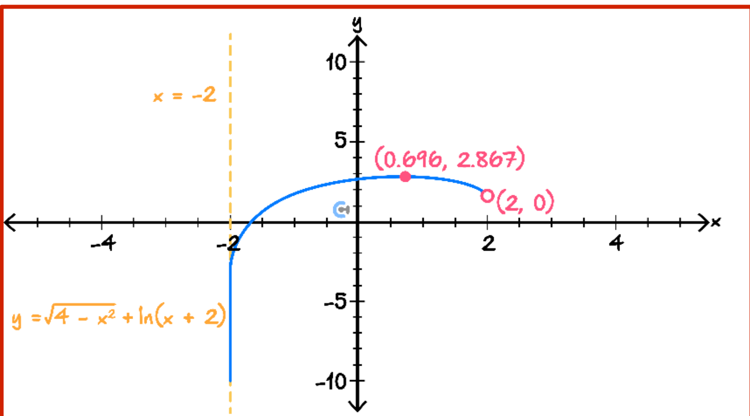
A. $[-4, 2.8]$

B. $(-\infty, 2.8]$

C. $(-4, 2.9)$

D. $(-\infty, 2.9)$

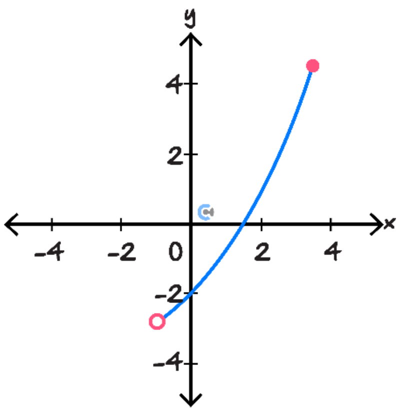
E. $[-4, 2.9)$



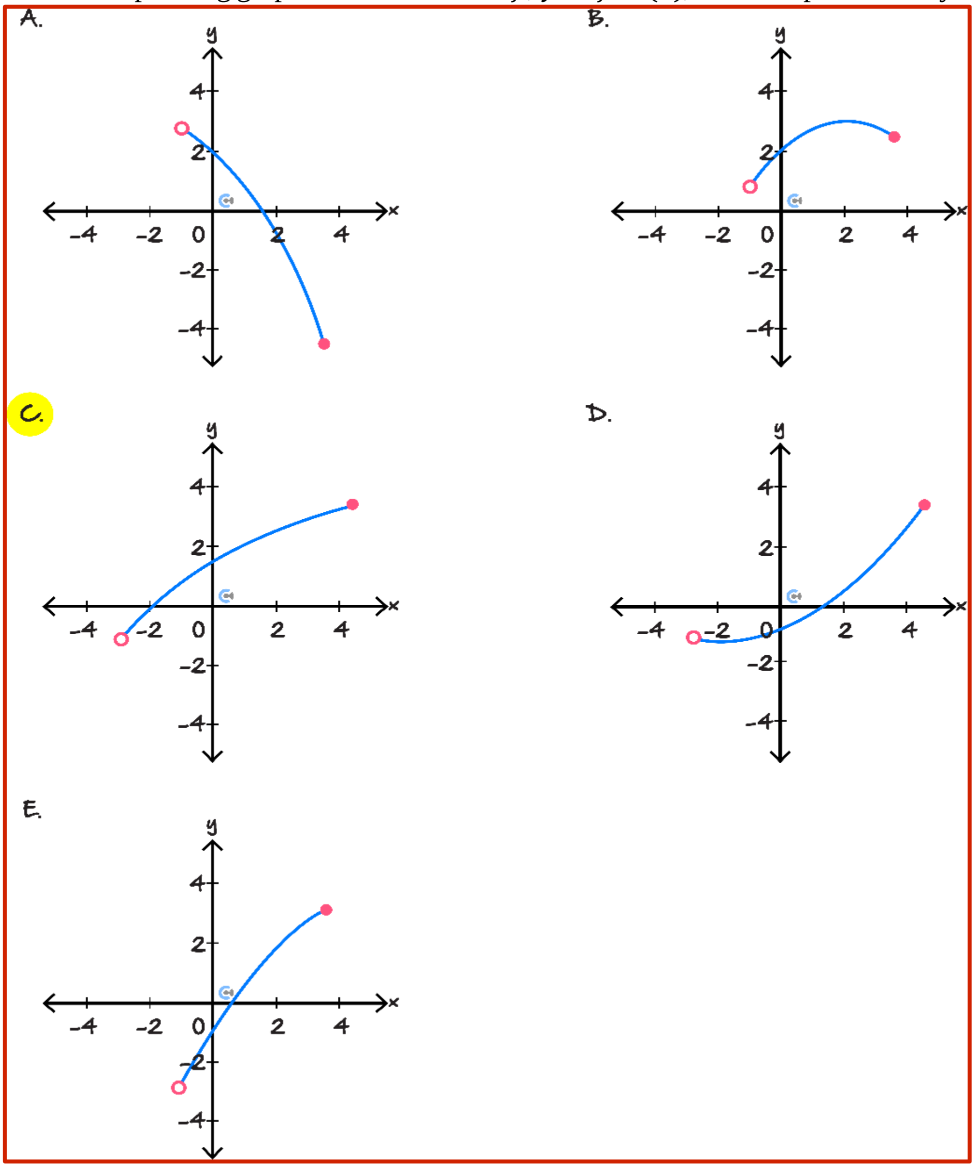
The range is $(-\infty, 2.866...]$. So it is contained within the interval $(-\infty, 2.9)$.

Question 105 (1 mark)

The graph of $y = f(x)$ is shown below:



The corresponding graph of the inverse of f, $y = f^{-1}(x)$, is best represented by:



Question 106 (1 mark)

The function $f : D \to R, f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x$ will have an inverse function for:

A. $D = R$

B. $D = (-3,1)$

C. $D = (1, \infty)$

D. $D = (-\infty, 0)$

E. $D = (0, \infty)$

Question 107 (1 mark)

The graph of the function $f : D \to R, f(x) = \frac{2x-3}{4+x}$, where D is the maximal domain, has asymptotes:

A. $x = -4, y = 2$

B. $x = \frac{3}{2}, y = -4$

C. $x = -4, y = \frac{3}{2}$

D. $x = \frac{3}{2}, y = 2$

E. $x = 2, y = 1$

A	$y = \frac{2x-3}{4+x} = 2 - \frac{11}{x+4}$, the asymptotes are $y = 2$ and $x = -4$

Question 108 (1 mark)

The function $f : D \to R, f(x) = 5x^3 + 10x^2 + 1$ will have an inverse function for:

A. $D = R$

B. $D = (-2, \infty)$

C. $D = \left(-\infty, \frac{1}{2}\right]$

D. $D = (-\infty, -1]$

E. $D = [0, \infty)$

Question 109 (1 mark)

The function f has the property $f(2x) = \left(f(x)\right)^2 - 2$ for all real numbers x.

A possible rule for the function $f(x)$ is:

A. $\frac{1}{x^2+4}$

B. $\cos(x)$

C. $2\log_e(x^2+1)$

D. $e^x + e^{-x}$

E. x^2

Question 110 (1 mark)

Which one of the following is the inverse function of the function $f : (-\infty, 3) \to R, f(x) = \frac{2}{\sqrt{3-x}} + 1$?

A. $f^{-1} : (-\infty, 3) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

B. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-3)^2} + 1$

C. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

D. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{x^2} + 3$

E. $f^{-1} : R^+ \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

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Question 111 (1 mark)

Let $f : D \to R, f(x) = \frac{3x-5}{2-x}$, where D is the maximal domain of f.

Which of the following are the equations of the asymptotes of the graph of f?

A. $x = 2$ and $y = \frac{5}{3}$.

B. $x = 2$ and $y = -3$.

C. $x = -2$ and $y = 3$.

D. $x = -3$ and $y = 2$.

E. $x = 2$ and $y = 3$.

Question 112 (1 mark)

Consider the following four functional relations:

$$f(x) = f(-x) \qquad -f(x) = f(-x) \qquad f(x) = -f(x) \qquad \left(f(x)\right)^2 = f(x^2)$$

The number of these functional relations that are satisfied by the function $f : R \to R, f(x) = x$ is:

A. 0

B. 1

C. 2

D. 3

E. 4

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Question 113 (1 mark)

Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{1-2x}$, defined over their maximal domains.

The maximal domain of the function $h = f + g$ is:

A. $\left(-2, \frac{1}{2}\right)$

B. $[-2, \infty)$

C. $(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$

D. $\left[-2, \frac{1}{2}\right]$

E. $[-2, 1]$

Question 114 (1 mark)

Let f and g be functions such that $f(-1) = 4$, $f(2) = 5$, $g(-1) = 2$, $g(2) = 7$, and $g(4) = 6$.

The value of $g\big(f(-1)\big)$ is:

A. 2

B. 4

C. 5

D. 6

E. 7

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Question 115 (1 mark)

Let $a \in (0, \infty)$ and $b \in R$.

Consider the function $h : [-a, 0) \cup (0, a] \to R, h(x) = \frac{a}{x} + b$.

The range of h is:

A. $[b - a, b + 1]$

B. $(b - a, b + 1)$

C. $(-\infty, b - 1) \cup (b + 1, \infty)$

D. $(-\infty, b - 1] \cup [b + 1, \infty)$

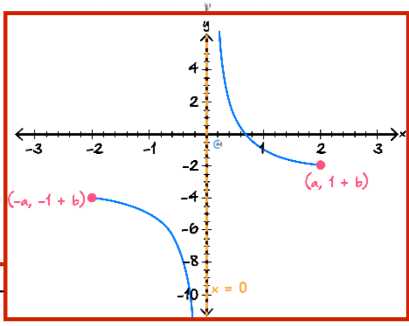
E. $[b - 1, \infty)$

$$h(x) = \frac{a}{x} + b$$

The coordinates of the endpoints are $(-a, -1+b)$ and $(a, 1+b)$.

The range is $(-\infty, -1+b] \cup [b+1, \infty)$.

An example, using the graph of $y = \frac{2}{x} - 3$ is shown below.



Question 116 (1 mark)

The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, then the graph of the function h must pass through the point:

A. $(-1, -12)$

B. $(-1, 19)$

C. $(-4, 12)$

D. $(-4, -14)$

E. $(3, 3.5)$

Space for Personal Notes

Question 117 (1 mark)

The maximal domain of the function f is $R\backslash\{1\}$.

A possible rule for f is:

A. $f(x) = \frac{x^2-5}{x-1}$

B. $f(x) = \frac{x+4}{x-5}$

C. $f(x) = \frac{x^2+x+4}{x^2+1}$

D. $f(x) = \frac{5-x^2}{1+x}$

E. $f(x) = \sqrt{x-1}$

Question 118 (1 mark)

Consider the function $f: [a,b) \to R, f(x) = \frac{1}{x}$ where a and b are positive real numbers.

The range of f is:

A. $\left[\frac{1}{a}, \frac{1}{b}\right)$

B. $\left(\frac{1}{a}, \frac{1}{b}\right]$

C. $\left[\frac{1}{b}, \frac{1}{a}\right)$

D. $\left(\frac{1}{b}, \frac{1}{a}\right]$

E. $[a,b)$

$f:[a, b) \rightarrow R, f(x)=\frac{1}{x}$,

$f(a)=\frac{1}{a}, f(b)=\frac{1}{b}, f(a)>f(b)$,

Range $\left(\frac{1}{b}, \frac{1}{a}\right]$

Space for Personal Notes

Question 119 (1 mark)

Which one of the following is the inverse function of $g : [3, \infty) \to R, g(x) = \sqrt{2x - 6}$?

A. $g^{-1} : [3, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

B. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = (2x - 6)^2$

C. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = \sqrt{\frac{x}{2}} + 6$

D. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

E. $g^{-1} : R \to R, g^{-1}(x) = \frac{x^2+6}{2}$

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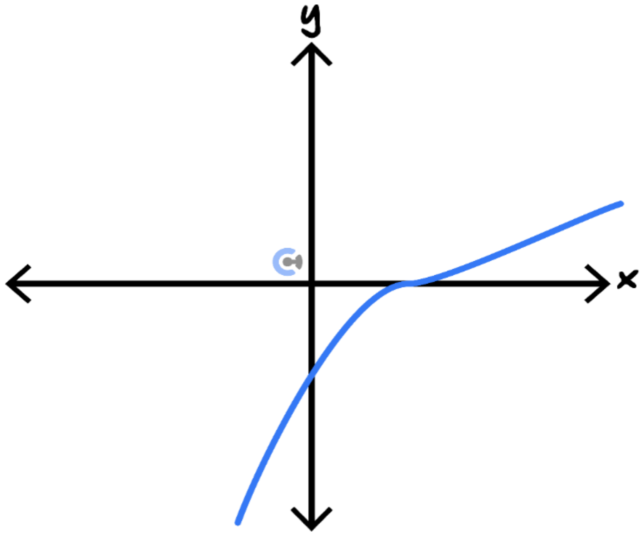
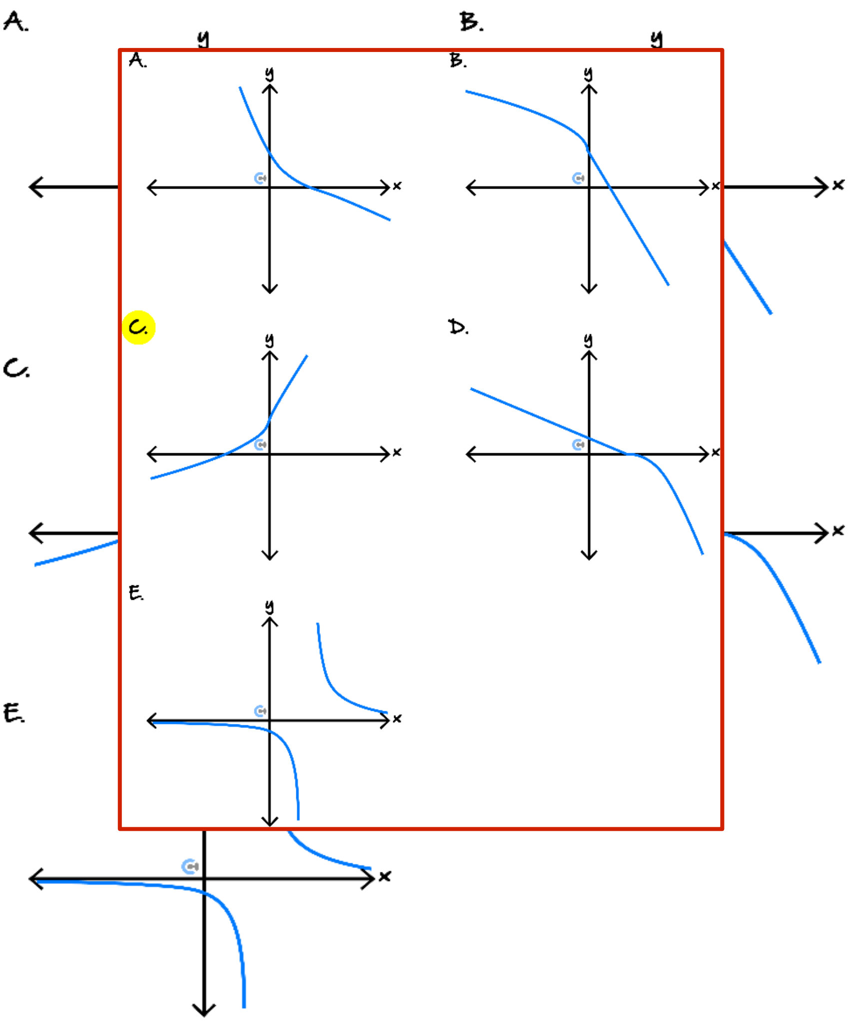
Question 120 (1 mark)

Part of the graph of the function f is shown below. The same scale has been used on both axes:



The corresponding part of the graph of the inverse function f^{-1} is best represented by:



Question 121 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

Question 122 (1 mark)

The range of the function $f : [-2, 3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 123 (1 mark)

Let $f : R \to R, f(x) = x^2$.

$$f(x-y) = (x-y)^2 = x^2 - 2xy + y^2$$
$$f(x) - f(y) = x^2 - y^2$$
$$f(x-y) \neq f(x) - f(y)$$

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(x) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2(f(x) + f(y))$

Question 124 (1 mark)

The transformation $T: \ R^2 \to R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x-1)^3 - 2(x-1) + 4$. The transformation T could be given by:

A. $T(x, y) = (x + 1, 2y + 4)$

B. $T(x, y) = \left(x + 1, \frac{1}{2}y + 4\right)$

C. $T(x, y) = (2x + 1, y + 2)$

D. $T(x, y) = \left(\frac{1}{2}x + 1, y + 2\right)$

E. $T(x, y) = (x + 1, 2y + 2)$

Question 125 (1 mark)

The transformation $T: R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x+1} - 3$ onto the graph of $y = \sqrt{x}$, has rules:

A. $T(x, y) = \left(\frac{1}{2}x - 1, -y - 3\right)$

B. $T(x, y) = \left(\frac{1}{2}x - 1, -y + 3\right)$

C. $T(x, y) = \left(\frac{1}{2}x + 1, -y - 3\right)$

D. $T(x, y) = (2x + 1, -y - 3)$

E. $T(x, y) = (2x - 1, -y + 3)$

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Question 126 (1 mark)

The point (a, b) is transformed by:

$$T(x, y) = \left(\frac{1}{2}x - \frac{1}{2}, -2y - 2\right)$$

If the image of (a, b) is $(0, 0)$, then (a, b) is:

A. $(1, 1)$

B. $(-1, 1)$

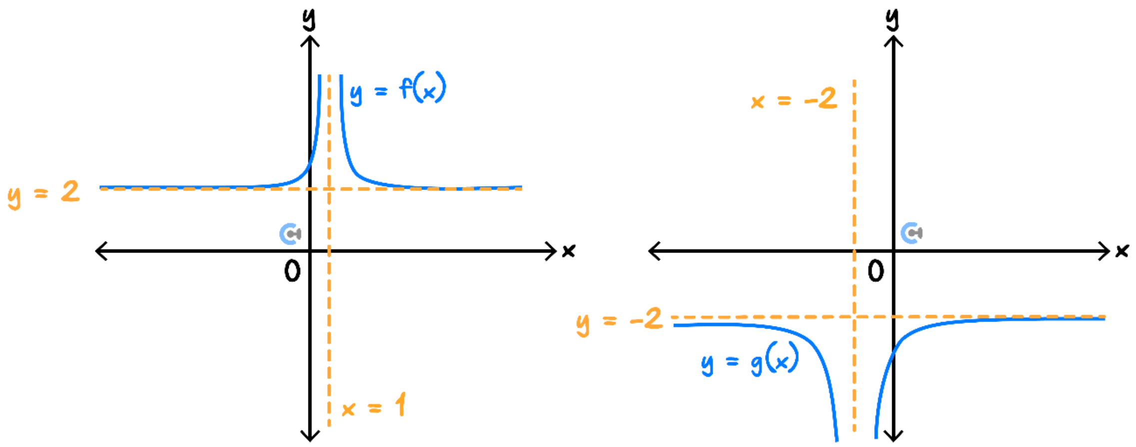
C. $(-1, 0)$

D. $(0, 1)$

E. $(1, -1)$

Question 127 (1 mark)

Consider the graphs of f and g below, which have the same scale.



If T transforms the graph of f onto the graph of g, then:

A. $T(x, y) = (x - 3, y - 4)$

B. $T(x, y) = (-x -$

D	Dilate the graph of *f* by a factor of 2 from the *y*-axis and reflect the image in the *x* and *y* axes.

C. $T(x, y) = (x - 3, -y)$

D. $T(x, y) = (-2x, -y)$

E. $T(x, y) = (-x, -2y)$

Question 128 (1 mark)

The graph of the function $f:\ [0,\infty)\to R$, where $f(x)=4x^{\frac{1}{3}}$, is reflected in the x-axis and then translated five units to the right and six units vertically down.

Which one of the following is the rule of the transformed graph?

A. $y=4(x-5)^{\frac{1}{3}}+6$

B. $y=-4(x+5)^{\frac{1}{3}}-6$

C. $y=-4(x+5)^{\frac{1}{3}}+6$

D. $y=-4(x-5)^{\frac{1}{3}}-6$

E. $y=4(x-5)^{\frac{1}{3}}+1$

Question 129 (1 mark)

The point $A(3,2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g where $g(x)=\frac{1}{2}f(x-1)$. The same transformation maps the point A to the point P.

The coordinates of the point P are:

A. $(2,1)$

B. $(2,4)$

C. $(4,1)$

D. $(4,2)$

E. $(4,4)$

A (3, 2), $g(x)=\frac{1}{2}f(x-1)$,

Dilate by a factor of a $\frac{1}{2}$ from the x-axis: $(3,\ 1)$

Translate 1 unit to the right: $(4,\ 1)$

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Question 130 (1 mark)

The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x-5}$ by a reflection in the x-axis followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f?

A. $f(x) = \sqrt{5-4x}$

B. $f(x) = -\sqrt{x-5}$

C. $f(x) = \sqrt{x+5}$

D. $f(x) = -\sqrt{4x-5}$

E. $f(x) = -\sqrt{4x-10}$

Question 131

The function f is defined as $f:[a, a+2] \to \mathbb{R}, f(x) = x^2 - 4x - 8$.

a. Find the turning point of $f(x)$.

$f(x) = (x-2)^2 - 12$.
Thus turning point is $(2, 12)$.

b. Find the values of a such that:

i. The range of $f(x)$ is $[-8, 4]$.

First consider $f(a) = -8 \implies a = 0, 4$.
$f(0+2) = -12 \neq 4$ so reject $a = 0$.
$f(4+2) = 4$, therefore accept $a = 4$.
Now consider $f(a) = 4 \implies a = -2, 6$.
$f(-2+2) = -8$ therefore accept $a = -2$.
$f(6+2) = 24 \neq -8$, therefore reject $a = 6$.
Conclude that $a = -2$ or $a = 4$.

ii. The inverse function f^{-1} exists.

We need f to be a one to one function.
Consider when the endpoints are on the turning point.
$a = 0$ or $a = 2$.
Therefore f^{-1} exists for $x \in (-\infty, 0] \cup [2, \infty)$

iii. $\sqrt{f(x)}$ does not exist.

Does not exist when $f(x) < 0$ this occurs when $2 - 2\sqrt{3} < x < 2 + 2\sqrt{3}$.
Hence for $a \in (2 - 2\sqrt{3}, 2\sqrt{3})$

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Question 132

The line with the equation $y = mx$ intersects the circle with the centre $(0, 4)$ and radius 2 exactly once at the point $P(x, y)$.

(**Note:** A line which intersects a circle exactly once is called a line that is tangent to the circle.)

a. Find the equation of the circle.

$$(x-4)^2 + y^2 = 4$$

b. Show that the x-coordinate of the point P satisfies the equation:

$$(1 + m^2)x^2 - 8x + 12 = 0$$

Sub in $y = mx$ into the circle equation.

$$\begin{aligned} (x-4)^2 + (mx)^2 &= 4 \\ x^2 - 8x + 16 + m^2x^2 &= 4 \\ (1+m^2)x^2 - 8x + 12 &= 0 \end{aligned}$$

c. Use the discriminant to find the possible values of m.

We want $(1 + m^2)x^2 - 8x + 12 = 0$ to have only one solution. Therefore

$$\Delta = 64 - 48(1 + m^2) = 0 \implies m = \pm\frac{1}{\sqrt{3}}$$

d. Hence, find the two possible sets of coordinates for P.

Sub in $y = \pm\frac{1}{\sqrt{3}}x$ into the equation of the circle to find the points of intersection.
When $m = \frac{1}{\sqrt{3}}$ intersection at $(3, \sqrt{3})$.
When $m = -\frac{1}{\sqrt{3}}$ intersection at $(3, -\sqrt{3})$.

e. Find the distance of P from the origin.

$$d = \sqrt{9+3} = 2\sqrt{3}$$

f. Find the acute angle that the two lines tangent to the circle make at the origin.

The line $y = \frac{1}{\sqrt{3}}x$ makes an angle of $30°$ with the positive x-axis. By symmetry the angle made by the two tangents at the origin is $60°$.

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Question 133

Consider the function $f : (-3,1) \to \mathbb{R}, f(x) = (x+3)(x+2)(3x-3)$.

a. State the range of f, correct to 3 decimal places.

$\text{ran}(f) = (-18.194, 2.638)$

b. The following sequence of transformations, T, map the graph of f onto the graph of g.

- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 2 units up and 1 unit left, followed by,
- A reflection in the x-axis.

i. State the rule of g.

$g(x) = -\left(f\left(2(x+1)\right) + 2\right) = -62 - 174x - 120x^2 - 24x^3$

ii. State the domain of g.

Apply the transformation $x \to \frac{1}{2}x - 1$ onto the interval $(-3, 1)$ to get the domain of g.
Thus, the domain of g is $\left(-\frac{5}{2}, -\frac{1}{2}\right)$.

iii. State the range of g correct to 3 decimal places.

Apply the transformation $y \to -(y+2)$ onto the interval $(-18.194, 2.638)$ to get the domain of g.
Thus, the range of g is $(-4.638, 16.194)$

iv. Find the image of the point $(1, 0)$ under T.

$$\left(-\frac{1}{2}, -2\right)$$

Question 134 (7 marks)

The equation of a circle is given by $(x-6)^2 + (y-3)^2 = 9$, and a line with the equation $y = nx, n > 0$ is tangent to this circle at the point $R(p, q)$.

a. Write down the centre and radius of the circle. (1 mark)

Compare $(x-6)^2 + (y-3)^2 = 9$ *with* $(x-h)^2 + (y-k)^2 + r^2$,
where $(h, k)$is centre and r is radius.
$(h, k) = (6, 3)$
$r^2 = 9 \Rightarrow r = \pm\sqrt{9} = \pm 3$ (radius cannot be negative)
$\Rightarrow r = 3$
Thus, centre of circle is $(6, 3)$ and radius is 3.

b. Show that the x-coordinate of the point R satisfies the equation: (2 marks)

$$(1+n^2)x^2 - 6(2+n)x + 36 = 0$$

The line with equation $y = nx$ meets the circle with equation $(x-6)^2 + (y-3)^2 = 9$.
Therefore, x satisfies the equation of circle: $(x-6)^2 + (nx-3)^2 = 9$
Expanding and rearranging: $x^2 - 12x + 36 + n^2x^2 - 6nx = 0$
and therefore, $(1+n^2)x^2 - 6(2+n)x + 36 = 0$

c. Use the discriminant to determine the exact value of n. (1 mark)

The line is a tangent to a circle, there is only one point of contact and hence only one solution to the equation obtained in **part b**. Therefore the discriminant equal 0.

Discriminant, $\Delta = 0$

$$\left(-6(2+n)\right)^2 - 4(1+n^2)36 = 0$$
$$(12+6n)^2 - 144 - 144n^2 = 0$$
$$144 + 36n^2 + 144n - 144 - 144n^2 = 0$$
$$144n - 108n^2 = 0$$
$$36n(4-3n) = 0$$
$$n = 0 \quad or \quad 4 - 3n = 0 \Rightarrow n = \frac{4}{3}$$

Since, $n > 0 \Rightarrow$Exact value of n is $\frac{4}{3}$.

d. Find the coordinate of R. (2 marks)

Sub $n = \frac{4}{3}$ and $n^2 = \frac{16}{9}$ into $(1+n^2)x^2 - 6(2+n)x + 36 = 0$.

$$\left(1+\frac{16}{9}\right)x^2 - 6\left(2+\frac{4}{3}\right)x + 36 = 0$$

$$\frac{25}{9}x^2 - 20x + 36 = 0 \rightarrow 25x^2 - 180x + 324 = 0 \rightarrow (5x-18)^2 = 0$$

Thus, $x = \frac{18}{5}$

Now, to find y-coordinate sub $n = \frac{4}{3}$ and $x = \frac{18}{5}$ into $y = nx$.

$y = \left(\frac{4}{3}\right)\left(\frac{18}{5}\right) = \frac{24}{5}$ →Therefore the coordinate of R is $\left(\frac{18}{5}, \frac{24}{5}\right)$.

e. Calculate the distance of R from the origin. (1 mark)

Distance of R from the origin $= \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{24}{5}\right)^2} = \frac{1}{5}\sqrt{900} = \frac{30}{5} = 6$

Question 135 (8 marks)

Consider the curve with equation $y = \sqrt{x-c} - d$.

a. Show that if the curve meets the line with equation $y = 3x$ at the point $(u, 3u)$, then u satisfies the equation: (1 mark)

If $(u, 3u)$ lies on $y = 3x$ and on the curve $y = \sqrt{x-c} - d$,

then $3u = \sqrt{u-c} - d$

$$3u + d = \sqrt{u-c}$$
$$(3u+d)^2 = u - c$$
$$9u^2 + 6ud + d^2 = u - c$$
$$9u^2 + (6d-1)u + d^2 + c = 0$$

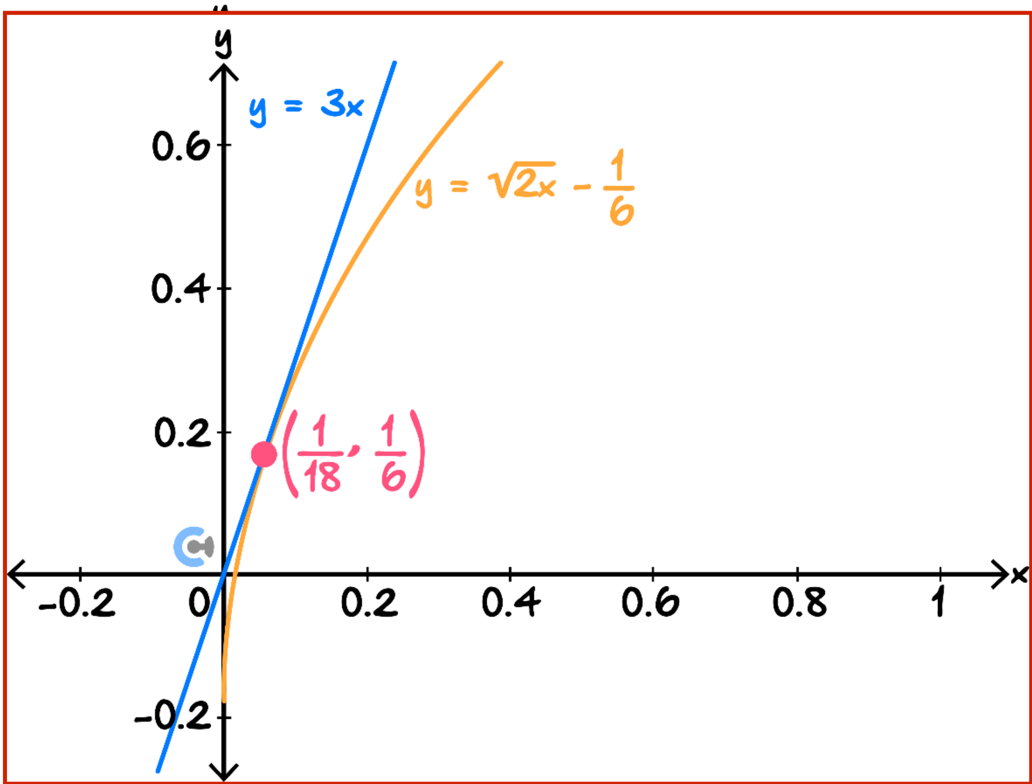
b.

i. If the line with equation $y = 3x$ is a tangent to the curve, show that $d = \frac{1-36c}{12}$. (2 marks)

The line meets the curve at one point if the discriminant of the quadratic in u is zero.

$$\Delta = 0$$
$$(6d-1)^2 - 4(9)(d^2+c) = 0$$
$$36d^2 - 12d + 1 + 36d^2 - 36c = 0$$
$$12d = 1 - 36c$$
$$d = \frac{1-36c}{12}$$

ii. Sketch the graph of $y = \sqrt{2x} - \frac{1}{6}$ and find the coordinates of the point on the graph at which the line with equation $y = 3x$ is a tangent. (2 marks)



Solving equation $3x = \sqrt{2x} - \frac{1}{6}$ *will give the required coordinates.*

$$3x + \frac{1}{6} = \sqrt{2x}$$
$$\left(3x + \frac{1}{6}\right)^2 = 2x$$
$$9x^2 + \frac{1}{36} + x = 2x$$
$$324x^2 + 1 - 36x = 0$$
$$(18x-1)^2 = 0$$

1

c. Find the values of k for which the line with equation $y = 3x - k$:

i. Meets the curve with equation $y = \sqrt{2x} - \frac{1}{6}$ twice. (1 mark)

From the above, we know that the line with the equation $y = 3x$ is a tangent to the curve with equation $y = \sqrt{2x} - \frac{1}{6}$.

Hence, if $0 < k < \frac{1}{6}$, the line will cross the curve twice.

ii. Meets the curve with equation $y = \sqrt{2x} - \frac{1}{6}$ once. (1 mark)

If $k = 0$ or $k > \frac{1}{6}$, the line will cross the curve once.

iii. Does not meet the curve with equation $y = \sqrt{2x} - \frac{1}{6}$. (1 mark)

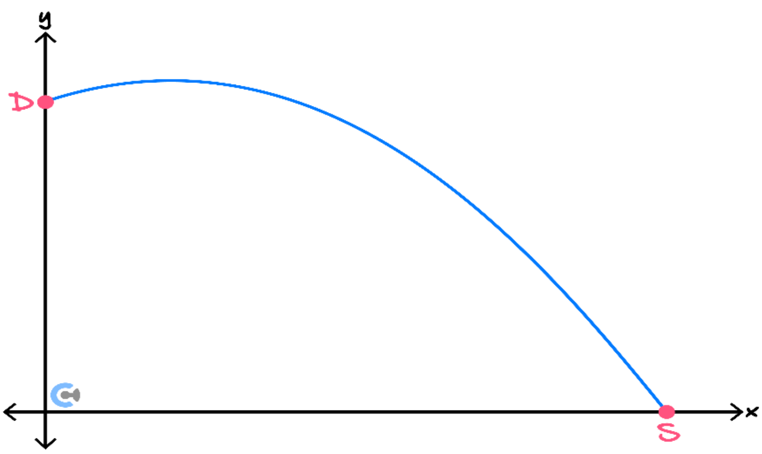
It will not meet the curve if $k < 0$.

Question 136 (11 marks)

Sophia and Daniel are testing how far Daniel can throw a frisbee from a raised hill. Daniel stands at a point D, while Sophia stands at the point S. The frisbee's path follows the curve:

$$h(x) = a(x - b)^2 + c$$

where h is the height of the frisbee above the ground, and x is the horizontal distance from the hill.



a. Sophia is positioned 25 metres from the hill, at the point $(25, 0)$. If the frisbee begins to descend when it reaches a height of 16 metres at a horizontal distance of 5 metres, what is the equation for $h(x)$? (2 marks)

It reaches a height of 16 metres at a horizontal distance of 5 metres means maximum height is at coordinate $(5,16)$ i.e., vertex is $(5,16)$.

$$h(x) = a(x-5)^2 + 16 \quad (1)$$

Now, Sophia is positioned 25 metres from the hill means point S is $(25,0)$.

Sub $x = 25$ and $h(x) = y = 0$ into (1)

$$0 = a(25-5)^2 + 16$$

$$a = -\frac{16}{400} = -\frac{1}{25}$$

Thus, $h(x) = -\frac{1}{25}(x-5)^2 + 16$

b. What is the height of the hill that Daniel is standing on? (1 mark)

The point where Daniel is standing is on the y-axis means x-coordinate is zero.

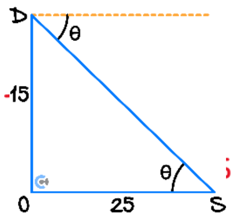
Sub $x = 0$ into $h(x) = -\frac{1}{25}(x-5)^2 + 16$

$$h(x) = -\frac{1}{25}(0-5)^2 + 16 = 15 \text{ metres}$$

c. What is the angle of depression from Daniel to Sophia? Give your answer in degrees, correct to two decimal places. (2 marks)

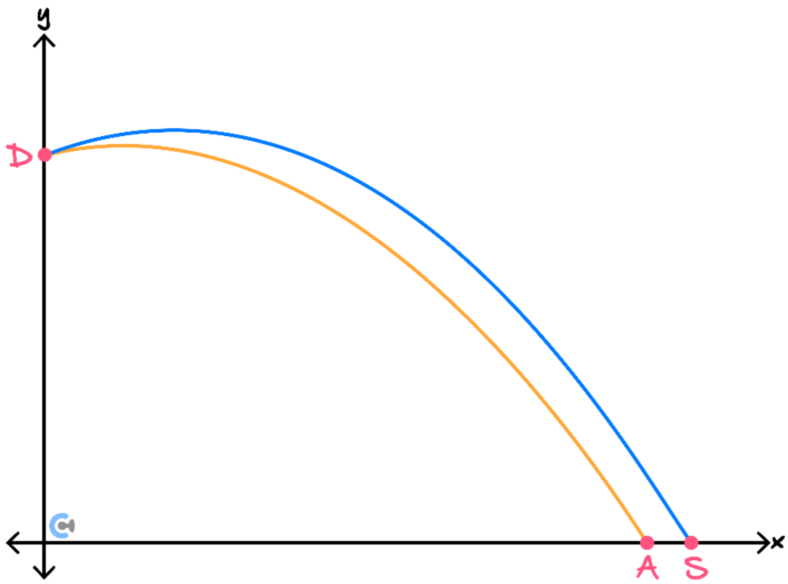
Angle of depression $=$Angle of elevation $= \theta$

$$\tan\theta = \frac{DO}{OS} = \frac{15}{25} = \frac{3}{5}$$



$$\theta = \tan^{-1}\left(\frac{3}{5}\right) \approx 30.96°$$

d. After some time, Daniel's throw weakens, and his frisbee now follows a different curve $g(x)$, where $h(x)$ undergoes these transformations:



- A dilation by a factor of $\frac{3}{5}$ from the y-axis.
- A dilation by a factor of $\frac{1}{3}$ from the x-axis.
- A translation of 10 metres in the positive y-axis.

i. Find the equation for $g(x)$. (3 marks)

$$x' = \frac{3}{5}x \Rightarrow x = \frac{5}{3}x' \quad (1)$$

$$y' = \frac{1}{3}y + 10 \Rightarrow y = 3(y' - 10) \quad (2) \quad \{y = h(x)\}$$

Sub (1) and (2) *into* $h(x) = -\frac{1}{25}(x-5)^2 + 16$

$$3(y' - 10) = -\frac{1}{25}\left(\frac{5}{3}x' - 5\right)^2 + 16$$

$$y' = -\frac{1}{75}\left(\frac{5}{3}x' - 5\right)^2 + \frac{46}{3}$$

$$\Rightarrow g(x) = -\frac{1}{75}\left(\frac{5}{3}x - 5\right)^2 + \frac{46}{3}$$

ii. Determine the coordinate of the point A. By how many metres has the horizontal distance of the frisbee's landing point? Give your answers correct to two decimal places. (2 marks)

Point A lies on x-axis sub $y = g(x) = 0$ into $g(x) = -\frac{1}{75}\left(\frac{5}{3}x - 5\right)^2 + \frac{46}{3}$.

$$0 = -\frac{1}{75}\left(\frac{5}{3}x - 5\right)^2 + \frac{46}{3}$$

$$x \approx 23.35$$

Coordinate of $A = (23.35, 0)$

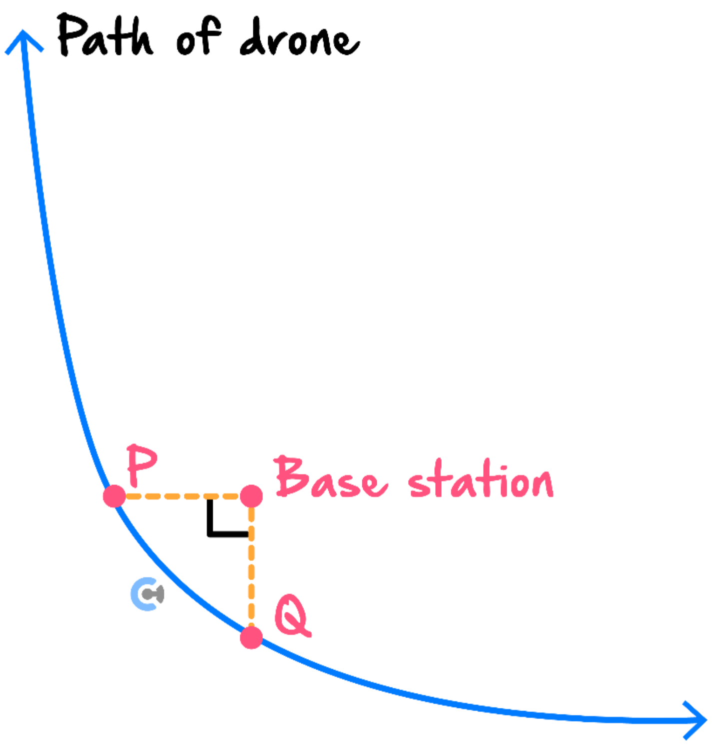
Distance between A and $S = 25 - 23.35 = 1.65$ metres

iii. At what height does the frisbee start descending in the second experiment? (1 mark)

$\frac{46}{3}$ metres

Question 137 (12 marks)

A drone is navigating a valley using GPS signals. The path is modelled by a hyperbola, and at two points, P and Q, the drone is at a fixed distance of 400 m from the base station. The angle between P and Q with the base station is 90°.



a. Taking 800 m as 1 unit and placing the base station at the origin, determine the coordinates of points P and Q. (2 marks)

$$P = (\_\_\_\_, 0) \quad \text{and} \quad Q = (0, \_\_\_\_)$$

$$P = \left(-\frac{400}{800}, 0\right) \text{ and } Q = \left(0, -\frac{400}{800}\right)$$
$$P = (-0.5, 0) \text{ and } Q = (0, -0.5)$$

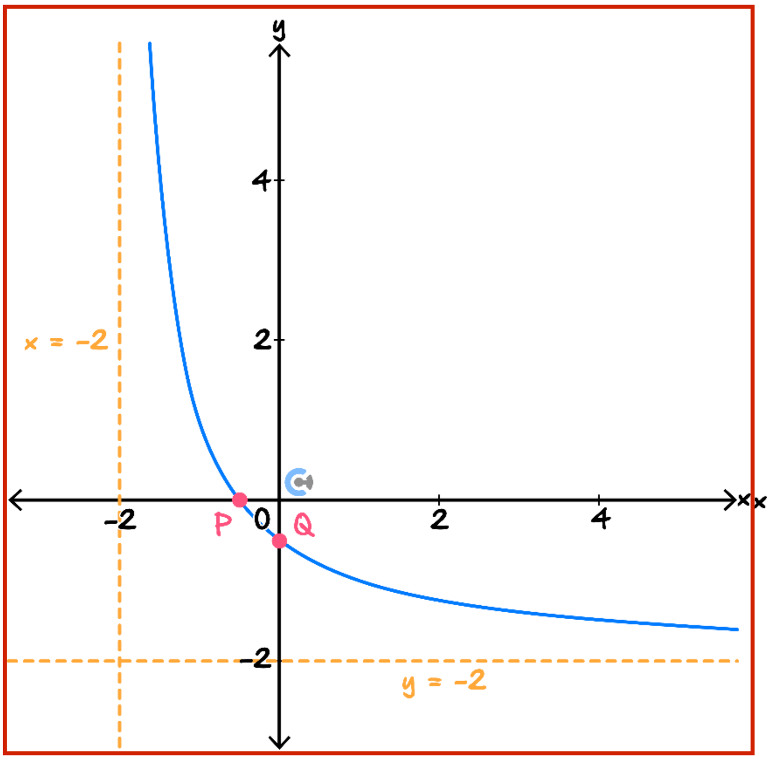
b. The flight path follows the function $g(x) = \frac{b}{x+2} - 2$. Given that a third point $R\left(-\frac{5}{4}, 2\right)$ lies on the path, show that $b = 3$. (2 marks)

Point R lies on the path

i.e., $g(x)$, so sub $x = -\frac{5}{4}$ and $g(x) = 2$

$$2 = \frac{b}{-\frac{5}{4} + 2} - 2$$
$$2 + 2 = \frac{b}{\frac{3}{4}}$$
$$4 = \frac{4b}{3}$$
$$b = 3$$

c. Draw a diagram representing mptotes, and a proper scale. (3 marks)



d. Using the hyperbola equation and the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Derive an equation for the drone's distance from the base station at any point. (2 marks)

Sub $x_1 = 0, y_1 = 0, x_2 = x$ and $y_2 = \frac{3}{x+2} - 2$ into $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{x^2 + \left(\frac{3}{x+2} - 2\right)^2}$$

e. Use a calculator to determine the minimum distance between the drone and the base station, rounding to the nearest metre. (3 marks)

We want the function $\sqrt{x^2 + \left(\frac{3}{x+2} - 2\right)^2}$ to be as small as possible

Sketch the graph and find the minimum at $(-0.26795, 0.37894)$

Therefore, $0.37894 \times 800 = 303$ metres

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Question 138 (11 marks)

Consider the function $f : \mathbb{R} \to \mathbb{R}, f(x) = -(x+2)^2$.

a. Write down:

i. The domain of f. (1 mark)

Domain of f is $\mathbb{R}$.

ii. The range of f. (1 mark)

Range of f is $[0, -\infty)$

iii. The maximum value of f. (1 mark)

Maximum is zero.

iv. The value of $f(-1)$. (1 mark)

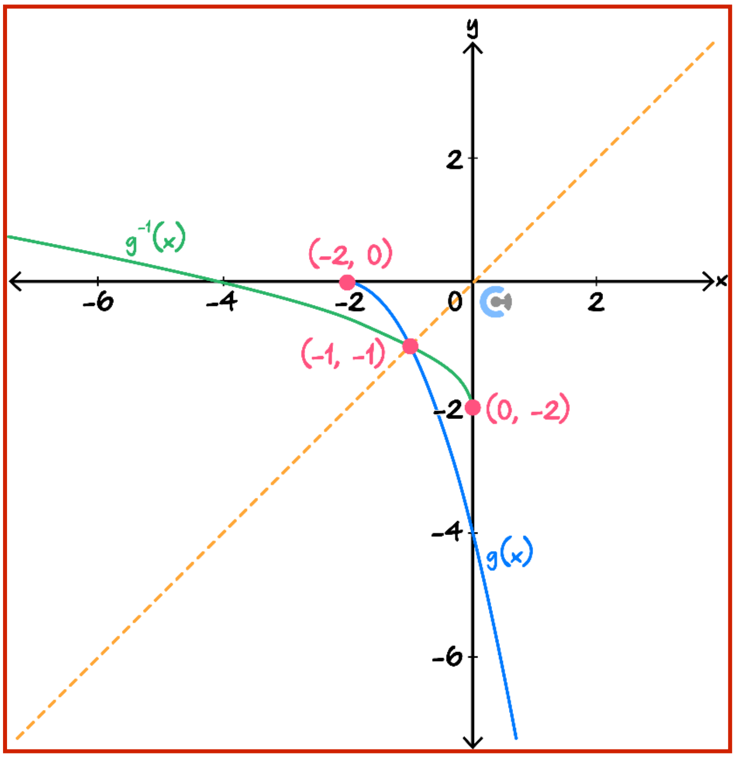
Sub $x = -1$ into $\mathrm{f}(x) = -(x+2)^2$
$f(-1) = -(-1+2)^2 = -1$

b. The inverse function f^{-1} does not exist. Explain why. (1 mark)

Because f is not $1:1$ function.

Consider the function $g: [-2, \infty) \to \mathbb{R}$, where $g(x) = -(x+2)^2$.

c. On the set of axes below, sketch the graph of $y = g(x)$. Clearly label any endpoints and/or intercepts. (2 marks)



d. On the same set of axes, sketch the graph of the inverse function of $g(x)$, that is, sketch the graph of $y = g^{-1}(x)$. Clearly label any endpoints and/or intercepts. (2 marks)

Solution Pending

e. State the rule for $g^{-1}(x)$. (2 marks)

$g(x) = y = -(x+2)^2$

Swap x and y

$x = -(y+2)^2$

$-x = (y+2)^2$

$\sqrt{-x} = y + 2$

$y = \sqrt{-x} - 2$

$g^{-1}(x) = \sqrt{-x} - 2$

Question 139 (11 marks)

A cubic function $h(x)$ has the rule $h(x) = (x - 4)(x + 6)(x - 7)$.

a. Solve the equation $h(x - 5) = 0$. (2 marks)

Sub $x = x - 5$ into $h(x) = (x - 4)(x + 6)(x - 7)$.

$h(x - 5) = (x - 5 - 4)(x - 5 + 6)(x - 5 - 7)$

$= (x - 9)(x + 1)(x - 12)$

$h(x - 5) = 0$

$(x - 9)(x + 1)(x - 12) = 0$

$x = 9, -1, 12$

b. Solve the equation $h(x + 4) = 0$. (2 marks)

Sub $x = x + 4$ into $h(x) = (x - 4)(x + 6)(x - 7)$.

$h(x + 4) = (x + 4 - 4)(x + 4 + 6)(x + 4 - 7)$

$= (x - 0)(x + 10)(x - 3)$

$h(x + 4) = 0$

$x(x + 10)(x - 3) = 0$

$x = 0, -10, 3$

c. It is known that the equation $h(x) + n = 0$ has a solution $x = -2$. Find the value of n and solve the equation $h(x) + n = 0$. (3 marks)

Since $x = -2$ *is* a solution of $h(x) + n = 0$, sub $x = -2$ into $h(x) + n = 0$.

$h(-2) + n = 0$

$(-2 - 4)(-2 + 6)(-2 - 7) + n = 0$

$216 + n = 0$

$n = -216$

Sub $n = 216$ *into* $h(x) + n = 0$ and solve

$(x - 4)(x + 6)(x - 7) - 216 = 0$

$$x = -2, \frac{7 + \sqrt{145}}{2}, \frac{7 - \sqrt{145}}{2}$$

d. The equation $h(x-q)=0$ has a solution $x=0$. Find the possible values of q. (2 marks)

Since $x=0$ is a solution of $h(x-q)=0$, sub $x=0$ into $h(x-q)=0$.

$h(0-q)=0$

$(-q-4)(-q+6)(-q-7)=0$

$-(q+4)(q-6)(q+7)=0$

$\Rightarrow q=-4,6,-7$

e. Find the values of q such that $h(x-q)=0$ has only one positive solution. (2 marks)

Solutions of $h(x-q)=0$ are $q+7, q-6, q+4$.

$\therefore$ at $-7<x\le -4$, $h(x-q)=0$ has only one positive solution.

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