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VCE Mathematical Methods ½
AOS 2 Revision [2.0]
Contour Check (Part 2)

Contour Check

Section G: [2.1-2.5] - Exam 2 Overall (113 Marks)

Question 78 (1 mark)

The maximal domain of the function $f(x) = \frac{1}{\sqrt{x^2 - x - 6}}$ is:

A. $x \in (0, \infty)$

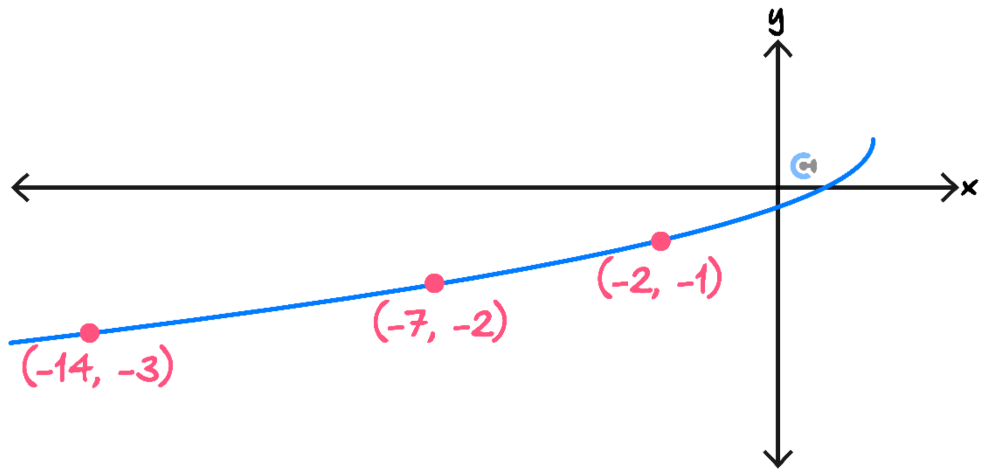
B. $x \in (-2, 3)$

C. $x \in (-\infty, 2] \cup [3, \infty)$

D. $x \in \mathbb{R} \setminus [-2, 3]$

Question 79 (1 mark)

The most likely rule for the following graph is:



A. $\sqrt{3 - x} + 1$

B. $-\sqrt{2 - x} + 1$

C. $3\sqrt{x + 1} - 1$

D. $-3\sqrt{2 - x} + 1$

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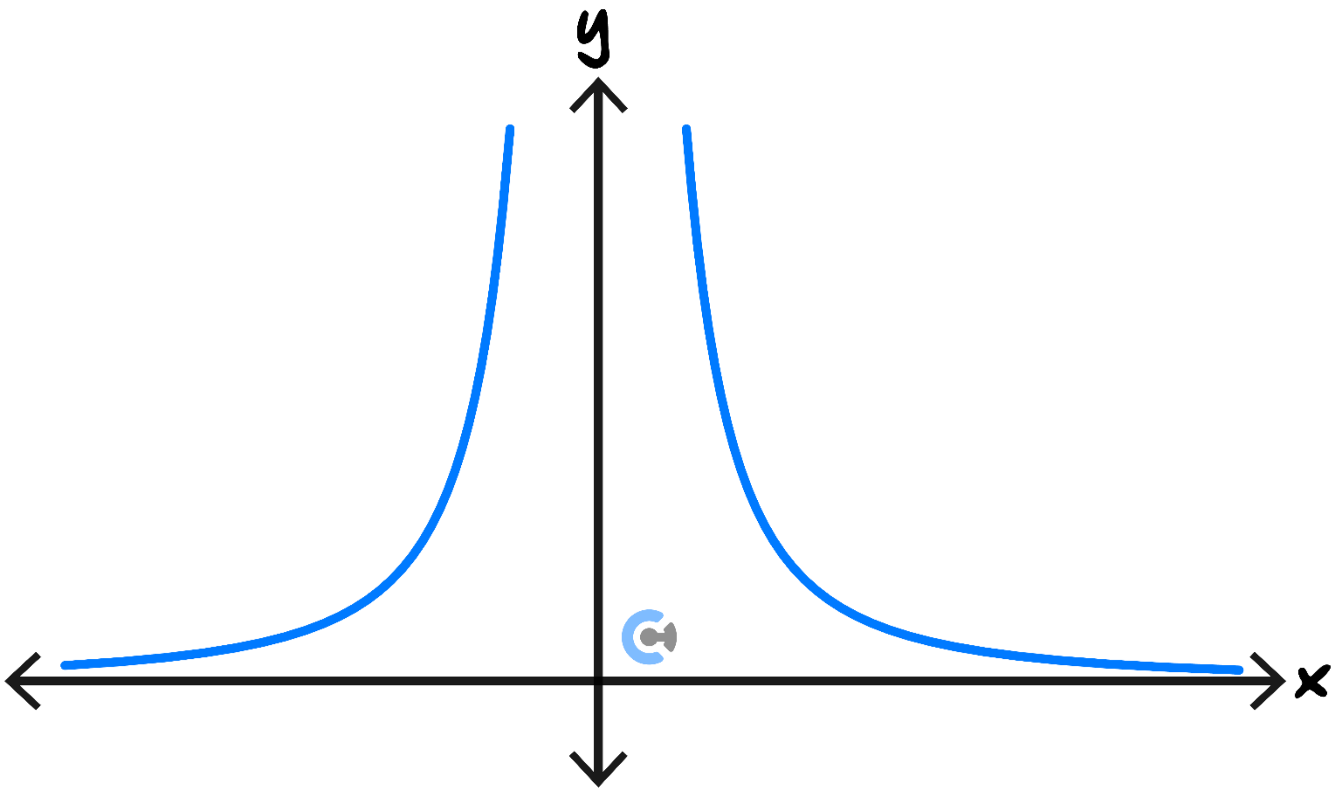
Question 80 (1 mark)

The function g with the graph shown below, is best described as:



A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

Question 81 (1 mark)

The line with the equation $4y + 3x = 25$ intersects the circle $x^2 + y^2 = 25$ exactly once at the point $P(3, 4)$. The equation for the radius of the circle that passes through P is:

A. $3y - 4x = 0$

B. $3y + 4x = 25$

C. $3y + 4x = 0$

D. $3y - 4x = 25$

Question 82 (1 mark)

Consider the function $f(x) = 12x^5 + 90x^4 + 140x^3 - 180x^2 - 480x - 200$.

The equation $f(x) = k$ will have three solutions for:

A. $k \in (-618, -24) \cup (38, 632)$

B. $k \in [-618, -24] \cup [38, 632]$

C. $k \in (-24, 38)$

D. $k \in [24, 38]$

Question 83 (1 mark)

The graph of the function f passes through the point $(2, -3)$.

If $h(x) = 3f(x - 2)$, then the graph of the function h must pass through the point:

A. $(4, -9)$

B. $(0, -9)$

C. $(4, -1)$

D. $(0, -1)$

Question 84 (1 mark)

The graph of the function $f : \mathbb{R} \to \mathbb{R}, f(x) = 3^x - 1$, is reflected in the y-axis and then translated 2 units to the left and then 3 units up.

Which one of the following is the rule of the transformed graph?

A. $y = \left(\frac{1}{3}\right)^{x+2} + 2$

B. $y = \frac{1}{3} \times 3^{x+2} + 3$

C. $y = 3^{-x} + 3$

D. $y = 3^{-x+2} + 3$

Question 85 (1 mark)

The graph of the function g is obtained from the transformed graph of the function:

$$f: [-2,6] \to \mathbb{R}, f(x) = 3x^2 + 5x - 2$$

Which undergoes a dilation of a factor 2 from the y-axis, followed by a dilation of factor $\frac{1}{4}$ from the x-axis, followed by a reflection in the x-axis, and finally followed by a translation of 6 units in the positive direction of the y-axis. The domain and range of g are respectively:

A. $[-4, 12]$ and $[-12, 4]$

B. $[-4, 12]$ and $\left[-28, \frac{337}{48}\right]$

C. $[-12, 4]$ and $\left[-\frac{239}{48}, 40\right]$

D. $[-4, 12]$ and $\left[-40, \frac{239}{48}\right]$

Question 86 (1 mark)

The image of the function $f(x) = x^4$ is $y = -40(x+2)^4$. The transformations that could have been applied are:

A. Reflection in the x-axis, then translation in the positive direction of the x-axis by 2 units, followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

B. Reflection in the x-axis, then translation in the negative direction of the x-axis by 2 units, followed by a dilation from the x-axis by a factor of 5 and a dilation by a factor 2 from the y-axis.

C. Reflection in the x-axis, then a dilation from the x-axis by a factor of 2, followed by a translation in the positive direction of the x-axis by 2 units, and finally a dilation from the y-axis by a factor of 2.

D. Reflection in the x-axis, then a dilation from the y-axis by a factor of $\frac{1}{2}$, followed by a translation in the negative direction of the x-axis by 2 units, and finally a dilation from the x-axis by a factor of $\frac{5}{2}$.

Space for Personal Notes

Question 87 (1 mark)

Let $h : (-1,1) \to R, h(x) = \frac{1}{x-1}$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) + h(-x) = 2h(x^2)$

C. $h(x) - h(0) = xh(x)$

D. $h(x) - h(-x) = 2xh(x^2)$

E. $\left(h(x)\right)^2 = h(x^2)$

Question 88 (1 mark)

The linear function $f : D \to R, f(x) = 5 - x$ has range $[-4, 5)$.

The domain D is:

A. $(0, 9]$

B. $(0, 1]$

C. $[5, -4)$

D. $[-9, 0)$

E. $[1, 9)$

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Question 89 (1 mark)

The function f has the property $f(x) - f(y) = (y - x)f(xy)$ for all non-zero real numbers x and y.

Which one of the following is a possible rule for the function?

A. $f(x) = x^2$

B. $f(x) = x^2 + x^4$

C. $f(x) = x \log_e(x)$

D. $f(x) = \frac{1}{x}$

E. $f(x) = \frac{1}{x^2}$

Question 90 (1 mark)

The linear function $f : D \to R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is:

A. $[-2, 6)$

B. $(-2, 2]$

C. R

D. $(-2, 6]$

E. $[-6, 2]$

Space for Personal Notes

Question 91 (1 mark)

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 92 (1 mark)

If $f : (-\infty, 1) \to R, f(x) = 2\log_e(1 - x)$ and $g : [-1, \infty) \to R, g(x) = 3\sqrt{x+1}$, then the maximal domain of the function $f + g$ is:

A. $[-1, 1)$

B. $(1, \ \infty)$

C. $(-1, 1]$

D. $(-\infty, \ -1]$

E. R

Question 93 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{x}{2}\right)$

E. $x - 2$

Question 94 (1 mark)

The range of the function $f: [-2,3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 95 (1 mark)

Let $f: R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(xy) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2\big(f(x) + f(y)\big)$

Question 96 (1 mark)

The linear function $f: D \to R, f(x) = 6 - 2x$ has range $[-4,12]$.

The domain D is:

A. $[-3, 5]$

B. $[-5, 3]$

C. R

D. $[-14, 18]$

E. $[-18, 14]$

Question 97 (1 mark)

The range of the function $f : [-2,7) \to R, f(x) = 5 - x$ is:

A. $(-2, 7]$

B. $[-2, 7)$

C. $(-2, \infty)$

D. $(-2, 7)$

E. R

Question 98 (1 mark)

The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are any non-zero real numbers.

A possible rule for the function is:

A. $f(x) = \log_e|x|$

B. $f(x) = \frac{1}{x}$

C. $f(x) = 2^x$

D. $f(x) = 2x$

E. $f(x) = \sin(2x)$

Question 99 (1 mark)

If $3f(x) = f(3x)$ for $x > 0$, then the rule for f could be:

A. $f(x) = 3x$

B. $f(x) = \sqrt{3x}$

C. $f(x) = \frac{x^3}{3}$

D. $f(x) = \log_e\left(\frac{x}{3}\right)$

E. $f(x) = x - 3$

Question 100 (1 mark)

The range of the function $f : \left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right] \to R, f(x) = 2x^3 - 3x + 4$ is:

A. $\left(4-\sqrt{2}, 4+\sqrt{2}\right)$

B. $\left(\frac{-1}{\sqrt{2}}, \sqrt{2}\right)$

C. $\left(4-\sqrt{2}, 4+\sqrt{2}\right]$

D. $\left(\frac{-2}{\sqrt{2}, \sqrt{2}}\right]$

E. $\left[4-\sqrt{2}, 4+\sqrt{2}\right]$

Question 101 (1 mark)

A function f satisfies the relation $f(x^2) = f(x) + f(x+2)$.

A possible rule for f is:

A. $f(x) = \sqrt{x+2}$

B. $f(x) = x + 2$

C. $f(x) = \log_{10}(x-1)$

D. $f(x) = \frac{1}{2}(x^2 - 1)$

E. $f(x) = \frac{1}{x-1}$

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Question 102 (1 mark)

The function f and its inverse, f^{-1}, are one-to-one for all values of x.

If $f(1) = 5$, $f(3) = 7$, and $f(8) = 10$, then $f^{-1}(7)$ and $f^{-1}(5)$ respectively are equal to:

A. 5 and 7.

B. 3 and 1.

C. 7 and 5.

D. 8 and 5.

E. 5 and 8.

Question 103 (1 mark)

The function f with rule $f(x) = 2\log_e(16 - x)$ has a maximal domain given by:

A. $x \in (16, \infty)$

B. $x \in (-\infty, 4)$

C. $x \in (4, \infty)$

D. $x \in (-4,4)$

E. $x \in (-\infty, 16)$

Question 104 (1 mark)

The range of the function with the rule $y = \sqrt{4 - x^2} + \log_e(x + 2)$ is contained within the interval:

A. $[-4, 2.8]$

B. $(-\infty, 2.8]$

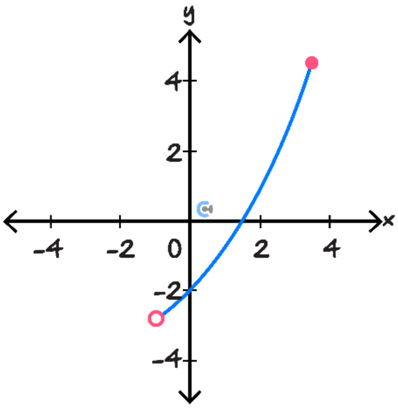
C. $(-4, 2.9)$

D. $(-\infty, 2.9)$

E. $[-4, 2.9)$

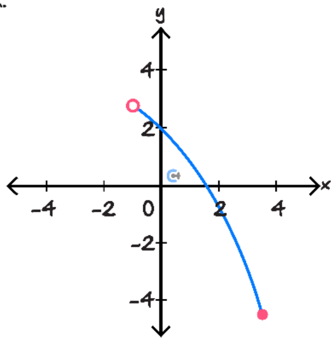
Question 105 (1 mark)

The graph of $y = f(x)$ is shown below:

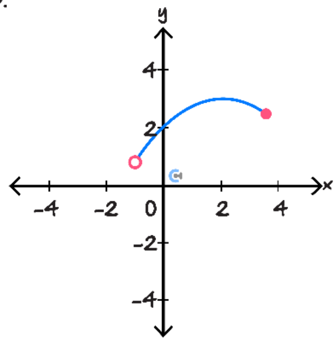


The corresponding graph of the inverse of f, $y = f^{-1}(x)$, is best represented by:

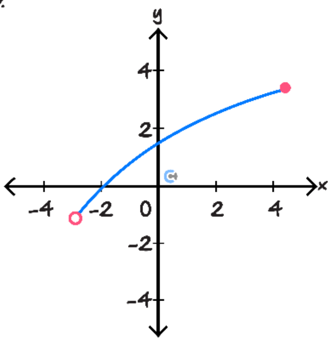
A.



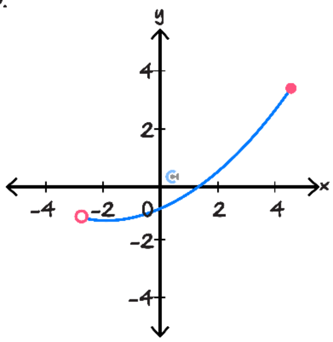
B.



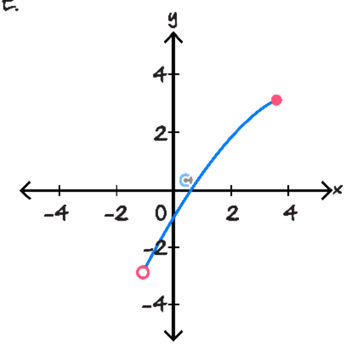
C.



D.



E.



Question 106 (1 mark)

The function $f : D \to R, f(x) = \frac{x^4}{4} - \frac{x^3}{3} - \frac{9x^2}{2} + 9x$ will have an inverse function for:

A. $D = R$

B. $D = (-3,1)$

C. $D = (1, \infty)$

D. $D = (-\infty, 0)$

E. $D = (0, \infty)$

Question 107 (1 mark)

The graph of the function $f : D \to R, f(x) = \frac{2x-3}{4+x}$, where D is the maximal domain, has asymptotes:

A. $x = -4, y = 2$

B. $x = \frac{3}{2}, y = -4$

C. $x = -4, y = \frac{3}{2}$

D. $x = \frac{3}{2}, y = 2$

E. $x = 2, y = 1$

Question 108 (1 mark)

The function $f : D \to R, f(x) = 5x^3 + 10x^2 + 1$ will have an inverse function for:

A. $D = R$

B. $D = (-2, \infty)$

C. $D = \left(-\infty, \frac{1}{2}\right]$

D. $D = (-\infty, -1]$

E. $D = [0, \infty)$

Question 109 (1 mark)

The function f has the property $f(2x) = \left(f(x)\right)^2 - 2$ for all real numbers x.

A possible rule for the function $f(x)$ is:

A. $\frac{1}{x^2+4}$

B. $\cos(x)$

C. $2\log_e(x^2 + 1)$

D. $e^x + e^{-x}$

E. x^2

Question 110 (1 mark)

Which one of the following is the inverse function of the function $f : (-\infty, 3) \to R, f(x) = \frac{2}{\sqrt{3-x}} + 1$?

A. $f^{-1} : (-\infty, 3) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

B. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-3)^2} + 1$

C. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

D. $f^{-1} : (1, \infty) \to R, f^{-1}(x) = -\frac{4}{x^2} + 3$

E. $f^{-1} : R^+ \to R, f^{-1}(x) = -\frac{4}{(x-1)^2} + 3$

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Question 111 (1 mark)

Let $f : D \to R, f(x) = \frac{3x-5}{2-x}$, where D is the maximal domain of f.

Which of the following are the equations of the asymptotes of the graph of f?

A. $x = 2$ and $y = \frac{5}{3}$.

B. $x = 2$ and $y = -3$.

C. $x = -2$ and $y = 3$.

D. $x = -3$ and $y = 2$.

E. $x = 2$ and $y = 3$.

Question 112 (1 mark)

Consider the following four functional relations:

$$f(x) = f(-x) \qquad -f(x) = f(-x) \qquad f(x) = -f(x) \qquad \left(f(x)\right)^2 = f(x^2)$$

The number of these functional relations that are satisfied by the function $f : R \to R, f(x) = x$ is:

A. 0

B. 1

C. 2

D. 3

E. 4

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Question 113 (1 mark)

Consider the functions $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{1-2x}$, defined over their maximal domains.

The maximal domain of the function $h = f + g$ is:

A. $\left(-2, \frac{1}{2}\right)$

B. $[-2, \infty)$

C. $(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$

D. $\left[-2, \frac{1}{2}\right]$

E. $[-2, 1]$

Question 114 (1 mark)

Let f and g be functions such that $f(-1) = 4$, $f(2) = 5$, $g(-1) = 2$, $g(2) = 7$, and $g(4) = 6$.

The value of $g\big(f(-1)\big)$ is:

A. 2

B. 4

C. 5

D. 6

E. 7

Space for Personal Notes

Question 115 (1 mark)

Let $a \in (0, \infty)$ and $b \in R$.

Consider the function $h: [-a, 0) \cup (0, a] \to R, h(x) = \frac{a}{x} + b$.

The range of h is:

A. $[b - a, b + 1]$

B. $(b - a, b + 1)$

C. $(-\infty, b - 1) \cup (b + 1, \infty)$

D. $(-\infty, b - 1] \cup [b + 1, \infty)$

E. $[b - 1, \infty)$

Question 116 (1 mark)

The graph of the function f passes through the point $(-2, 7)$.

If $h(x) = f\left(\frac{x}{2}\right) + 5$, then the graph of the function h must pass through the point:

A. $(-1, -12)$

B. $(-1, 19)$

C. $(-4, 12)$

D. $(-4, -14)$

E. $(3, 3.5)$

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Question 117 (1 mark)

The maximal domain of the function f is $R\backslash\{1\}$.

A possible rule for f is:

A. $f(x) = \frac{x^2-5}{x-1}$

B. $f(x) = \frac{x+4}{x-5}$

C. $f(x) = \frac{x^2+x+4}{x^2+1}$

D. $f(x) = \frac{5-x^2}{1+x}$

E. $f(x) = \sqrt{x-1}$

Question 118 (1 mark)

Consider the function $f : [a,b) \to R, f(x) = \frac{1}{x}$ where a and b are positive real numbers.

The range of f is:

A. $\left[\frac{1}{a}, \frac{1}{b}\right)$

B. $\left(\frac{1}{a}, \frac{1}{b}\right]$

C. $\left[\frac{1}{b}, \frac{1}{a}\right)$

D. $\left(\frac{1}{b}, \frac{1}{a}\right]$

E. $[a,b)$

Space for Personal Notes

Question 119 (1 mark)

Which one of the following is the inverse function of $g : [3, \infty) \to R, g(x) = \sqrt{2x - 6}$?

A. $g^{-1} : [3, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

B. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = (2x - 6)^2$

C. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = \sqrt{\frac{x}{2}} + 6$

D. $g^{-1} : [0, \infty) \to R, g^{-1}(x) = \frac{x^2+6}{2}$

E. $g^{-1} : R \to R, g^{-1}(x) = \frac{x^2+6}{2}$

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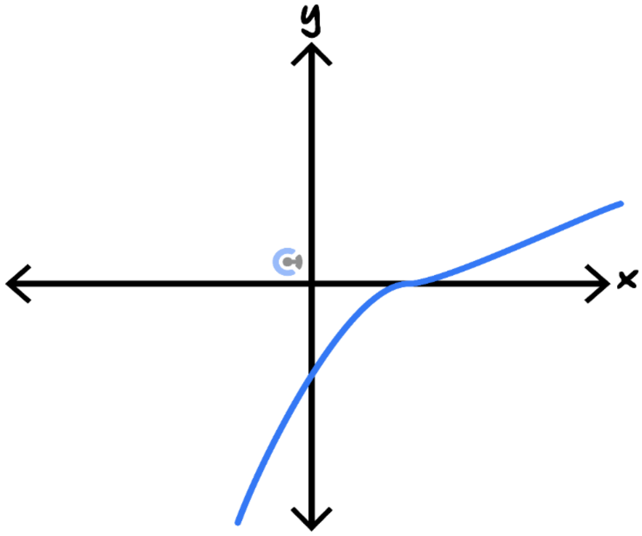
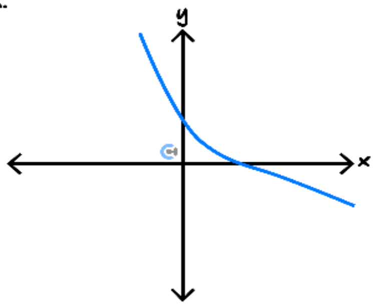
Question 120 (1 mark)

Part of the graph of the function f is shown below. The same scale has been used on both axes:

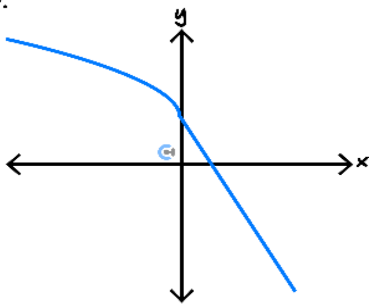


The corresponding part of the graph of the inverse function f^{-1} is best represented by:

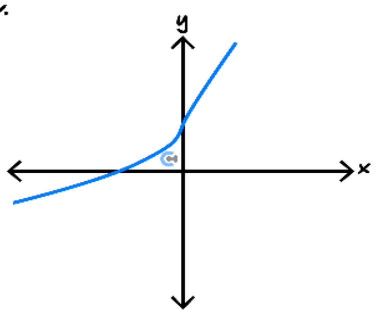
A.



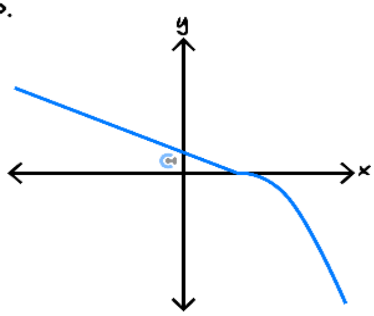
B.



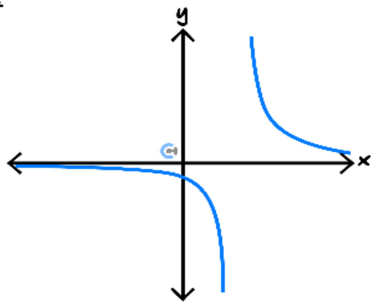
C.



D.



E.



Question 121 (1 mark)

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then the rule for f could be:

A. $\frac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $\log_e\left(\frac{|x|}{2}\right)$

E. $x - 2$

Question 122 (1 mark)

The range of the function $f : [-2, 3) \to R, f(x) = x^2 - 2x - 8$ is:

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 123 (1 mark)

Let $f : R \to R, f(x) = x^2$.

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(x) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

E. $f(x + y) + f(x - y) = 2(f(x) + f(y))$

Question 124 (1 mark)

The transformation $T:\ R^2 \to R^2$ maps the graph of $y = x^3 - x$ onto the graph of $y = 2(x-1)^3 - 2(x-1) + 4$. The transformation T could be given by:

A. $T(x, y) = (x + 1, 2y + 4)$

B. $T(x, y) = \left(x + 1, \frac{1}{2}y + 4\right)$

C. $T(x, y) = (2x + 1, y + 2)$

D. $T(x, y) = \left(\frac{1}{2}x + 1, y + 2\right)$

E. $T(x, y) = (x + 1, 2y + 2)$

Question 125 (1 mark)

The transformation $T:\ R^2 \to R^2$, which maps the graph of $y = -\sqrt{2x + 1} - 3$ onto the graph of $y = \sqrt{x}$, has rules:

A. $T(x, y) = \left(\frac{1}{2}x - 1, -y - 3\right)$

B. $T(x, y) = \left(\frac{1}{2}x - 1, -y + 3\right)$

C. $T(x, y) = \left(\frac{1}{2}x + 1, -y - 3\right)$

D. $T(x, y) = (2x + 1, -y - 3)$

E. $T(x, y) = (2x - 1, -y + 3)$

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Question 126 (1 mark)

The point (a, b) is transformed by:

$$T(x, y) = \left(\frac{1}{2}x - \frac{1}{2}, -2y - 2\right)$$

If the image of (a, b) is $(0, 0)$, then (a, b) is:

A. $(1, 1)$

B. $(-1, 1)$

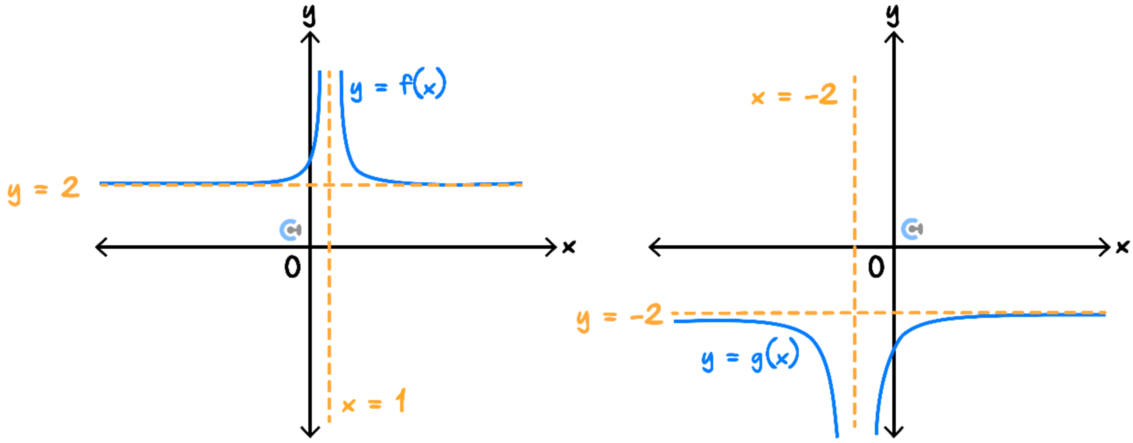
C. $(-1, 0)$

D. $(0, 1)$

E. $(1, -1)$

Question 127 (1 mark)

Consider the graphs of f and g below, which have the same scale.



If T transforms the graph of f onto the graph of g, then:

A. $T(x, y) = (x - 3, y - 4)$

B. $T(x, y) = (-x - 3, y - 4)$

C. $T(x, y) = (x - 3, -y)$

D. $T(x, y) = (-2x, -y)$

E. $T(x, y) = (-x, -2y)$

Question 128 (1 mark)

The graph of the function $f : [0, \infty) \to R$, where $f(x) = 4x^{\frac{1}{3}}$, is reflected in the x-axis and then translated five units to the right and six units vertically down.

Which one of the following is the rule of the transformed graph?

A. $y = 4(x-5)^{\frac{1}{3}} + 6$

B. $y = -4(x+5)^{\frac{1}{3}} - 6$

C. $y = -4(x+5)^{\frac{1}{3}} + 6$

D. $y = -4(x-5)^{\frac{1}{3}} - 6$

E. $y = 4(x-5)^{\frac{1}{3}} + 1$

Question 129 (1 mark)

The point $A(3, 2)$ lies on the graph of the function f. A transformation maps the graph of f to the graph of g where $g(x) = \frac{1}{2}f(x-1)$. The same transformation maps the point A to the point P.

The coordinates of the point P are:

A. $(2, 1)$

B. $(2, 4)$

C. $(4, 1)$

D. $(4, 2)$

E. $(4, 4)$

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Question 130 (1 mark)

The graph of a function f is obtained from the graph of the function g with rule $g(x) = \sqrt{2x-5}$ by a reflection in the x-axis followed by a dilation from the y-axis by a factor of $\frac{1}{2}$.

Which one of the following is the rule for the function f?

A. $f(x) = \sqrt{5-4x}$

B. $f(x) = -\sqrt{x-5}$

C. $f(x) = \sqrt{x+5}$

D. $f(x) = -\sqrt{4x-5}$

E. $f(x) = -\sqrt{4x-10}$

Question 131

The function f is defined as $f : [a, a+2] \to \mathbb{R}, f(x) = x^2 - 4x - 8$.

a. Find the turning point of $f(x)$.

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b. Find the values of a such that:

i. The range of $f(x)$ is $[-8, 4]$.

ii. The inverse function f^{-1} exists.

iii. $\sqrt{f(x)}$ does not exist.

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Question 132

The line with the equation $y = mx$ intersects the circle with the centre $(0, 4)$ and radius 2 exactly once at the point $P(x, y)$.

(**Note:** A line which intersects a circle exactly once is called a line that is tangent to the circle.)

a. Find the equation of the circle.

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b. Show that the x-coordinate of the point P satisfies the equation:

$$(1 + m^2)x^2 - 8x + 12 = 0$$

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c. Use the discriminant to find the possible values of m.

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d. Hence, find the two possible sets of coordinates for P.

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e. Find the distance of P from the origin.

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f. Find the acute angle that the two lines tangent to the circle make at the origin.

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Question 133

Consider the function $f : (-3,1) \to \mathbb{R}, f(x) = (x+3)(x+2)(3x-3)$.

a. State the range of f, correct to 3 decimal places.

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b. The following sequence of transformations, T, map the graph of f onto the graph of g.

- A dilation by a factor of $\frac{1}{2}$ from the y-axis, followed by,
- A translation of 2 units up and 1 unit left, followed by,
- A reflection in the x-axis.

i. State the rule of g.

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ii. State the domain of g.

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iii. State the range of g correct to 3 decimal places.

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iv. Find the image of the point $(1, 0)$ under T.

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Question 134 (7 marks)

The equation of a circle is given by $(x-6)^2 + (y-3)^2 = 9$, and a line with the equation $y = nx, n > 0$ is tangent to this circle at the point $R(p, q)$.

a. Write down the centre and radius of the circle. (1 mark)

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b. Show that the x-coordinate of the point R satisfies the equation: (2 marks)

$$(1+n^2)x^2 - 6(2+n)x + 36 = 0$$

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c. Use the discriminant to determine the exact value of n. (1 mark)

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d. Find the coordinate of R. (2 marks)

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e. Calculate the distance of R from the origin. (1 mark)

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Question 135 (8 marks)

Consider the curve with equation $y = \sqrt{x - c} - d$.

a. Show that if the curve meets the line with equation $y = 3x$ at the point $(u, 3u)$, then u satisfies the equation: (1 mark)

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b.

i. If the line with equation $y = 3x$ is a tangent to the curve, show that $d = \frac{1-36c}{12}$. (2 marks)

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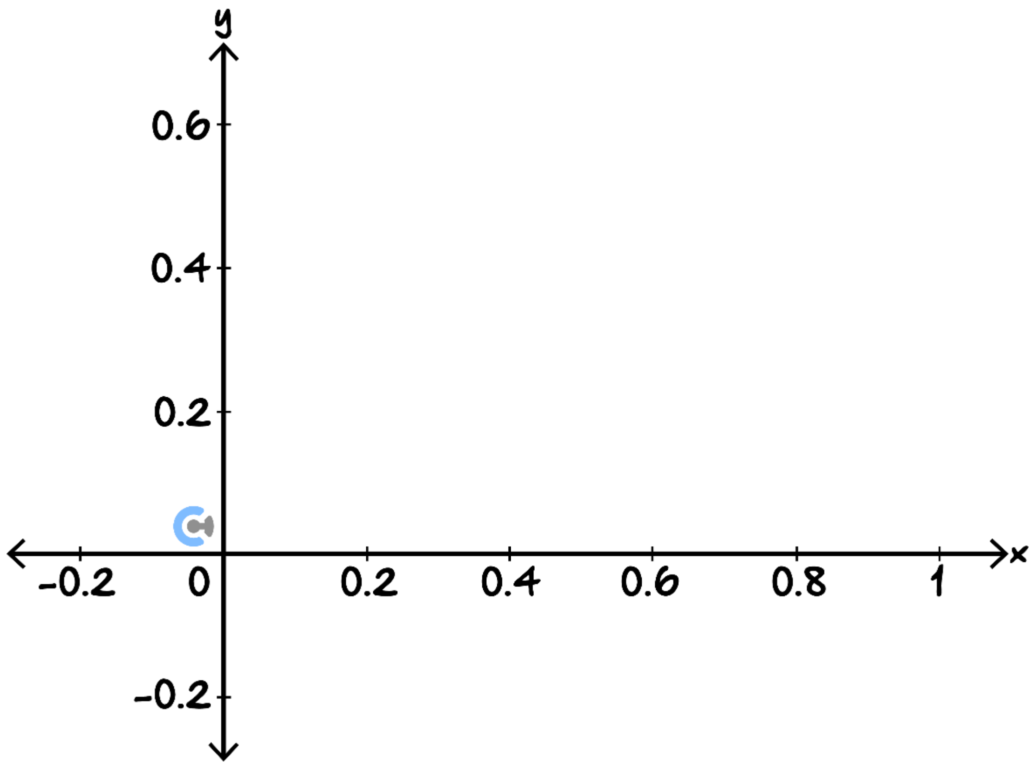
ii. Sketch the graph of $y = \sqrt{2x} - \frac{1}{6}$ and find the coordinates of the point on the graph at which the line with equation $y = 3x$ is a tangent. (2 marks)



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c. Find the values of k for which the line with equation $y = 3x - k$:

i. Meets the curve with equation $y = \sqrt{2x} - \frac{1}{6}$ twice. (1 mark)

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ii. Meets the curve with equation $y = \sqrt{2x} - \frac{1}{6}$ once. (1 mark)

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iii. Does not meet the curve with equation $y = \sqrt{2x} - \frac{1}{6}$. (1 mark)

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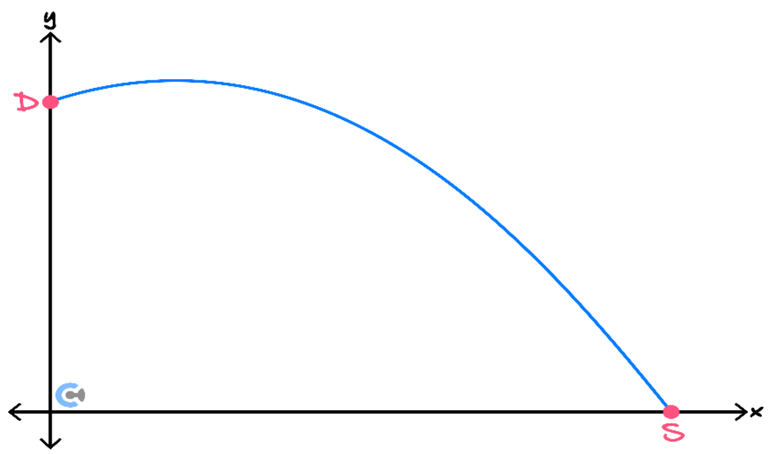
Question 136 (11 marks)

Sophia and Daniel are testing how far Daniel can throw a frisbee from a raised hill. Daniel stands at a point D, while Sophia stands at the point S. The frisbee's path follows the curve:

$$h(x) = a(x - b)^2 + c$$

where h is the height of the frisbee above the ground, and x is the horizontal distance from the hill.



a. Sophia is positioned 25 metres from the hill, at the point $(25, 0)$. If the frisbee begins to descend when it reaches a height of 16 metres at a horizontal distance of 5 metres, what is the equation for $h(x)$? (2 marks)

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b. What is the height of the hill that Daniel is standing on? (1 mark)

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c. What is the angle of depression from Daniel to Sophia? Give your answer in degrees, correct to two decimal places. (2 marks)

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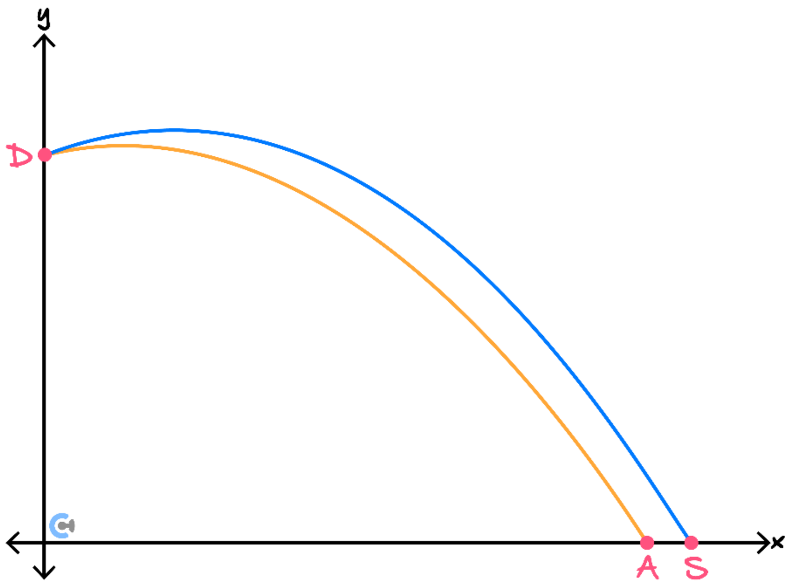
d. After some time, Daniel's throw weakens, and his frisbee now follows a different curve $g(x)$, where $h(x)$ undergoes these transformations:



- A dilation by a factor of $\frac{3}{5}$ from the y-axis.
- A dilation by a factor of $\frac{1}{3}$ from the x-axis.
- A translation of 10 metres in the positive y-axis.

i. Find the equation for $g(x)$. (3 marks)

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ii. Determine the coordinate of the point A. By how many metres has the horizontal distance of the frisbee's landing point? Give your answers correct to two decimal places. (2 marks)

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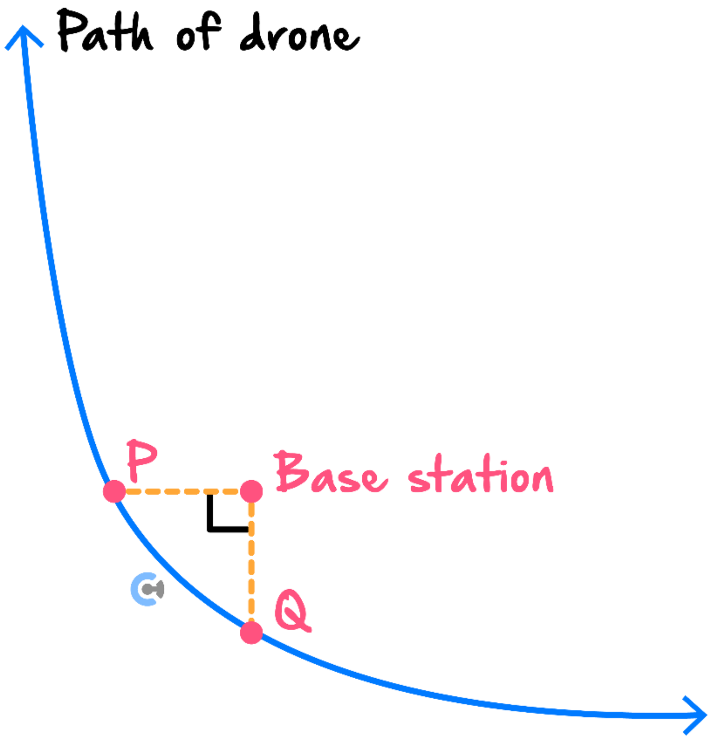
iii. At what height does the frisbee start descending in the second experiment? (1 mark)

Question 137 (12 marks)

A drone is navigating a valley using GPS signals. The path is modelled by a hyperbola, and at two points, P and Q, the drone is at a fixed distance of $400\ m$ from the base station. The angle between P and Q with the base station is $90°$.



a. Taking 800 m as 1 unit and placing the base station at the origin, determine the coordinates of points P and Q. (2 marks)

$$P = (\_\_\_\_,0) \quad \text{and} \quad Q = (0,\_\_\_\_)$$

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b. The flight path follows the function $g(x) = \frac{b}{x+2} - 2$. Given that a third point $R\left(-\frac{5}{4}, 2\right)$ lies on the path, show that $b = 3$. (2 marks)

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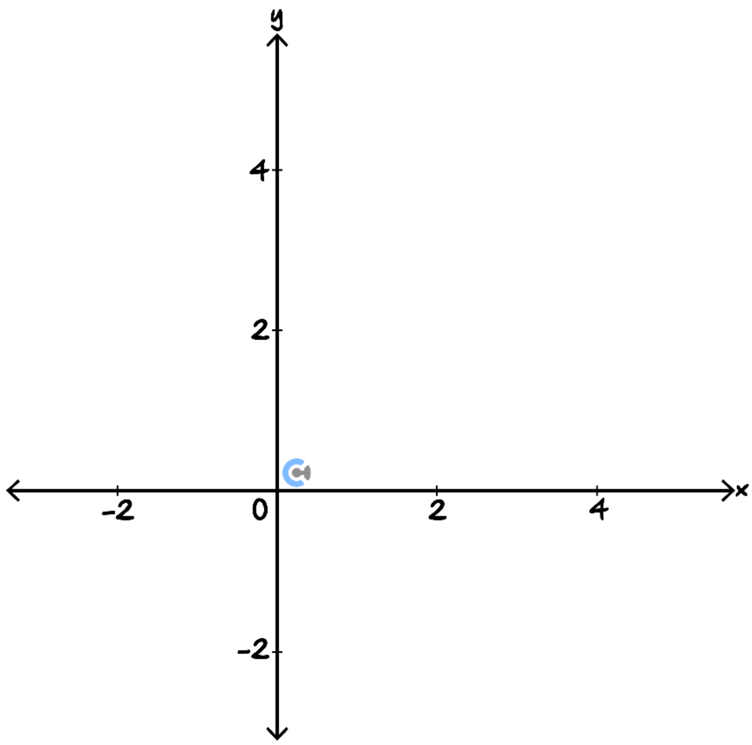
c. Draw a diagram representing the hyperbolic path, including labelled axes, asymptotes, and a proper scale. (3 marks)



d. Using the hyperbola equation and the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Derive an equation for the drone's distance from the base station at any point. (2 marks)

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e. Use a calculator to determine the minimum distance between the drone and the base station, rounding to the nearest metre. (3 marks)

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Question 138 (11 marks)

Consider the function $f: \mathbb{R} \to \mathbb{R}, f(x) = -(x+2)^2$.

a. Write down:

i. The domain of f. (1 mark)

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ii. The range of f. (1 mark)

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iii. The maximum value of f. (1 mark)

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iv. The value of $f(-1)$. (1 mark)

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b. The inverse function f^{-1} does not exist. Explain why. (1 mark)

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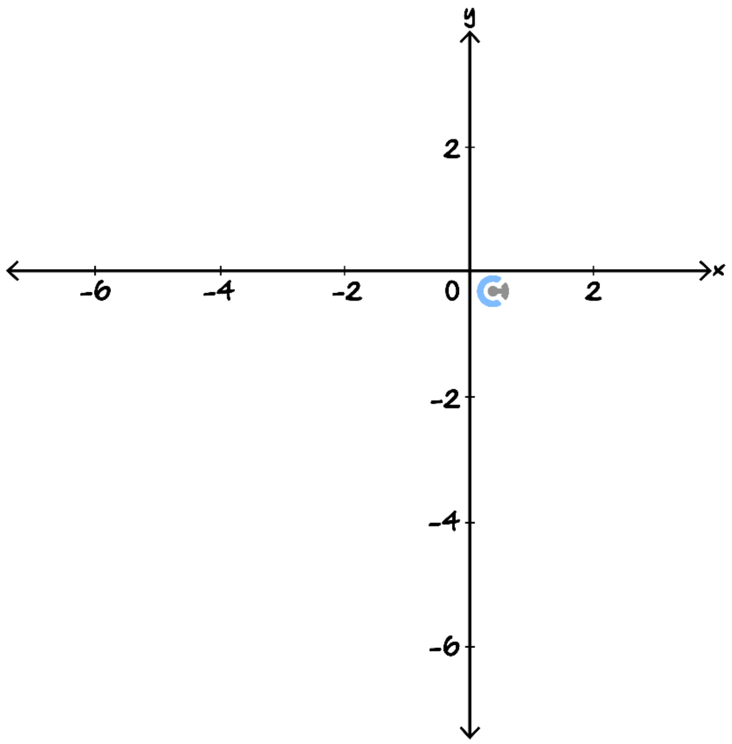
Consider the function $g : [-2, \infty) \to \mathbb{R}$, where $g(x) = -(x+2)^2$.

c. On the set of axes below, sketch the graph of $y = g(x)$. Clearly label any endpoints and/or intercepts. (2 marks)



d. On the same set of axes, sketch the graph of the inverse function of $g(x)$, that is, sketch the graph of $y = g^{-1}(x)$. Clearly label any endpoints and/or intercepts. (2 marks)

e. State the rule for $g^{-1}(x)$. (2 marks)

Question 139 (11 marks)

A cubic function $h(x)$ has the rule $h(x) = (x-4)(x+6)(x-7)$.

a. Solve the equation $h(x-5) = 0$. (2 marks)

b. Solve the equation $h(x+4) = 0$. (2 marks)

c. It is known that the equation $h(x) + n = 0$ has a solution $x = -2$. Find the value of n and solve the equation $h(x) + n = 0$. (3 marks)

d. The equation $h(x-q)=0$ has a solution $x=0$. Find the possible values of q. (2 marks)

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e. Find the values of q such that $h(x-q)=0$ has only one positive solution. (2 marks)

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